Dynamics of aging transition of an array of gloablly coupled oscillators using the mean field theory

Vishal Subedi

BIT Mesra, Mesra, Ranchi-835215

Dr. V.K. Chandrasekar

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Abstract

Due to the loss of activity(deterioration) of certain number of Stuart Landau oscillators in the coupled system, the ensemble may shift towards aging transition. Aging refers to the phenomenon of the oscillators going from active state to inactive state. We propose a method to study the dynamic robustness of the system by examining the aging transition of the system of globally coupled oscillators, valid for any population size by using the mean field theory and by introducing a feedback parameter for the mean field term. The values of the parameter specifying the distance from Hopf Bifurcation are drawn from a random distribution. The discussion in this paper focuses on using mean field approach to study the dynamical robustness of the system by analytically examining the AD state of the centroid of the ensemble of oscillators. In this work, we investigate the various AD regions analytically with the help of two parameter curves. The results are then verified numerically for N oscillators.

Keywords or phrases: HB , aging, Stuart Landau - Oscillators, Mean Field Theory, Globally Coupled Oscillators

Abbreviations

Abbreviations

SL	Stuart Landau
HB	Hopf Bifurcation

1 INTRODUCTION

1.1 Background/Rationale

Vishal Subedi

A complex system is a system composed of many components which may interact with each other. Examples of complex systems are Earth's global climate, organisms, the human brain, infrastructure such as power grid, transportation or communication systems, social and economic organizations (like cities), an ecosystem, a living cell, and ultimately the entire universe. These complex systems transit between various states when coupled with each other like synchronization, amplitude death, oscillation death, chimera and cluster state. One of the most familiar non-linear complex oscillators are SL Oscillators. SL oscillators represent the behavior of a general nonlinear system near HB .

1.2 Statement of the Problems

There have been much recent developments in aging transitions by considering some proportion of oscillators in inactive state [1, 2, 3]. Most of these models have a proportion of inactive oscillators with a parameter $a > 0$ specifying the active state and $b < 0$ specifying inactive state. All of the above discussions consider SL equations with slight variations. Oscillators are coupled globally as well as locally in the case of studying aging transitions. Most of the above-mentioned models consider the probability value specifying the proportion of inactive oscillators in the ensemble, to be used for studying the aging transition. One can always use a distribution to randomly draw the values of the parameter a_j's and then use the mean of the distribution and the variance to study the population of globally coupled oscillators using mean field approach. Aging transitions with time-delays have also been studied using two coupled oscillators and further studying the AD state [4]. The work on aging transition and dynamical robustness by G. Tanaka et al considers aging for heterogenous coupling where not all of the oscillators are coupled with everyone [1].

1.3 Objectives of the Research

1.3.1 Overall objective

The level of global oscillation can be measured by the order parameter $|Z|$, denoted by r in our case. As the value of a_j continuously decreases, the system further deteriorates due to the oscillators going to inactive state as a result of amplitude death. This can be characterized by finding the average $a = \frac{1}{N}\sum_{j=1}^{N} a_j$. Using a random distribution to draw values of a_j and by using the mean field approach to study the dynamics of aging transition of array of N oscillators, this case study proposes an analytical behaviour of AD states of oscillators near the HB using the centroid of array of oscillators and a shape parameter, which signifies the dispersion of individual oscillators from their centroid. The new mean field feedback term, denoted by η, also helps in further analysis of the AD region. As the value of a decreases, the oscillations still exist but the amplitude reduces up to a critical value $a = a_c$, after which there is an AD state of the ensemble. Our basic aim to determine this cutoff value for the different values of K, σ and a and then compare the results obtained from the mean field approach with the numerical simulation of the equations for different distributions.

1.4 Scope

2 LITERATURE REVIEW

2.1 Information

We are studying a collection of oscillators using the mean field approach similar to the approach and model used by S. DE Monte et al [5]. Array of coupled limit cycles have been addressed showing that, the array of coupled oscillators as a whole can display, both in the globally coupled [6-13] and spatially extended case [14-16], extremely complex behaviours.

Dynamic robustness of oscillators can also be enhanced in case of coupled non-linear oscillators. Rigorous work has been done by Y. Liu et al by varying the parameter signifying the distance from HB (with proportion of inactive oscillators as p) and introducing a new term α that controls the degree of diffusion, thus enhancing the dynamical robustness of the system [2]. A similar work by G. Tanaka *et al* focuses on heterogenous coupling of N oscillators with an adjacency coefficient which shows the strength of connection between each connected pair of oscillators [1]. Mean field approach is used to study the resultant amplitude of N globally coupled oscillators. A similar work by G.M. Pritula *et al* focuses on the use of mean field approach to evaluate the amplitude of the centroid of the globally coupled oscillators [17].

The appearance or the disappearance of a periodic orbit through a local change in the stability properties of a steady point is known as the HB. It has been shown that random perturbation in the parameter representing distance from HB slightly increases the robustness of the system when the random perturbations follow uniform distribution [3]. Aging can also be influenced by time-varying transition of active and inactive oscillators and can have complex effects on the dynamical robustness of the system [4]. Discussions on aging and clustering of globally coupled oscillators have concluded a hornlike region in the (K, p) phase diagram which is called the 'desynchronization horn', which validates the phenomenon of active oscillators desynchronizing to form clusters [18].

2.2 Summary

The recent developments have focused much on the approach of self feedback to control the diffusion parameter. Oscillators are segregated into active and inactive ones, with p proportion of oscillators going to inactive state. The parameter a_j signifies the state of the (isolated)jth oscillator, whether active(>0) when the oscillator is a stable limit cycle with amplitude $\sqrt{\alpha_j}$ or inactive(<0), as the oscillator settles down to a fixed point. Without loss of generality, one cas always draw a sample of N a_j's from a given random distribution with mean a and variance σ^2, which can be used in the mean field theory. In our discussions, a mean feedback term(η), similar to the self feedback term α, also needs to be introduced, owing to the fact that the mean field approach is being used to deduce the results analytically.

3 METHODOLOGY

3.1 Concepts

SL model oscillators were first used for studying the effects and transition of turbulence by L.D. Landau [19]. More of its applications include chemical systems [20], electronic circuits [21-23], and biological systems [24-26]. A system of coupled oscillators exhibits much more complex behavior than known. Examples include the interaction of arrays of Josephson Junctions [27-29], Belousov-Zhabotinskii reactions in coupled Brusselator models [30-31] and modeling the behavior of social networks [32]. For these systems, the onset of synchronization presents features similar to a thermodynamic phase transition, with the centroid playing the role of an order parameter [33, 34]

Aging, in coupled SL oscillators, is referred to the diffusion of the amplitudes of the oscillations after a large interval of time. Consider a population of N non-identical globally coupled SL oscillators with a linear mean field coupling:

$$\frac{\mathrm{d}z_j}{\mathrm{d}t} = (a_j + i\omega - |z_j|^2)z_j + \frac{K}{N}\sum_{k=1}^{N}(z_k - \alpha z_j) + \eta\sum_{k=1}^{N} z_k \quad (1)$$

where z_j represents the position of the j^{th} oscillator in the complex plane. Every oscillator has the same intrinsic frequency ω ; a_j is the parameter specifying the distance of the j^{th} oscillator from HB , drawn from a distribution with mean a and variance σ^2 .

Let $< z_j >$ denote the mean field $\frac{1}{N}\sum_{j=1}^{N} z_j$.

This method is based on a moment expansion and a closure assumption, in analogy with series expansions of classical statistical mechanics [35-37].

As we can see Z is the centroid of the N oscillators, writing the positions of oscillators in terms of their distance from the centroid as $z_j = Z + \epsilon_j$

Since, $Z =< z_j >$, therefore, $< \epsilon_j >= 0$ which means the average distance of all the oscillators around their centroid is 0, considering the distance follows cartesian sign rule.

By definition, $< a_j >= a$

We need to also account for the dispersion of individual oscillators around their mean field, hence introducing the shape parameter, denoted by W, as follows:

$$W =< a_j\epsilon_j >$$

3.2 Methods

By using $z_j = Z + \epsilon_j$, the time derivative of the centroid can be written as:

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = (a + i\omega - K(1-\alpha) + \eta - |Z|^2)Z + W \quad (2)$$

which has the same functional form as that of individual uncoupled elements except the term . Now, the time derivative of the shape parameter W can be written as:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}{<}a_j\epsilon_j{>}}{\mathrm{d}t} - < a_j > \frac{\mathrm{d}Z}{\mathrm{d}t} \quad (3)$$

Simplifying and neglecting higher orders of $O(<\epsilon_j^2>)$ and $O<(a_j - a_0)^2\epsilon_j>$, we get:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \sigma^2 Z + (a - \alpha K - 2|Z|^2 + i\omega)W - Z^2W^* \quad (4)$$

Hence, combining eq. (2) with eq (4), we get the compete complex differential equations of the order parameters viz. Z and W as:

$$\begin{aligned}\frac{\mathrm{d}Z}{\mathrm{d}t} &= (a + i\omega - K(1-\alpha) + \eta - |Z|^2)Z + W \\ \frac{\mathrm{d}W}{\mathrm{d}t} &= \sigma^2 Z + (a - \alpha K - 2|Z|^2 + i\omega)W - Z^2W^*\end{aligned} \quad (5)$$

Equations (5) constitutes a 4-dimensional system of equations when broken down to real and imaginary parts which is very complicated to solve. Hence, we would stick to the mean field approach and use the polar substitution for further simplification. The plane of oscillators has no intrinsic natural frequency as in [11], because of the fact that is a constant.

Setting $Z = re^{i\phi}$ and $W = we^{i\theta}$, we reduce the eq. (5) to their respective magnitudes, yielding:

$$\dot{r} = (a - r^2 + K(1 - \alpha) + \eta)r + wcos(\phi - \theta)$$

$$\dot{w} = \sigma^2 rcos(\phi - \theta) + (a - \alpha K - 2r^2)w - r^2 wcos[2(\phi - \theta)]$$

$$\dot{\phi} = \omega - (\tfrac{w}{r})sin(\phi - \theta) \quad (6)$$

$$\dot{\theta} = \omega - r^2 sin[2(\phi - \theta)] + \tfrac{\sigma^2 r}{w} sin(\phi - \theta)$$

4 RESULTS AND DISCUSSION

4.1 Numerical Simulations

It is very easy for a reader to observe that the value ,Φ - Θ denotes the phase difference between the order parameters Z and W. Now, the obvious question arises, "What should be the values of parameters for which the aging transition occurs?". Henceforth, we further need to find out for which value of the parameters K , σ , ω , a , α , η, does the values of magnitude of mean field and dispersion go to death state, i.e. become inactive. Here, aging refers to the death of an oscillator after a transient time interval.

Solving (6) for an exact solution is out of the scope of this paper. Hence, the results are approximated using numerical methods for t = 6000 sec. All the numerical simulations were carried out in python with the help of the module odeint from the famous scipy module. The values of parameters for these approximations are $K = 10$, $\sigma = 0.5$, $\omega = 4$, $a = -0.45$, $\alpha = 1$, $\eta = 0$. Our main concern, for this paper, is studying the oscillation amplitude of the centroid, as stated previously, using the mean field theory. The results approximated via numerical methods, when plotted vs time for r and w, show amplitude death for the above values -Fig.1 and Fig. 2 respectivey. Fig. 3 shows that the phase difference between the mean field and shape parameter also tend towards 0 as the amplitude and frequency death occurs. From the figures 1, 2, and 3, one can easily conclude that as the value of the phase of mean field goes closer and closer to the phase of dispersion around the centroid, the array of coupled oscillators start exhibiting aging transition. This can also be understood in a different sense. Saying that the array of coupled oscillators start showing aging transition is sufficient to argue on the fact that a certain proportion of oscillators are now in an inactive state. But as previously stated, we wont dig deep into that part of active and inactive oscillators as it has been already discussed in [11] for hetrogenous coupled oscillators. But we still don't have an exact cutoff value after which one can guarantee the occurrence of this aging transition.

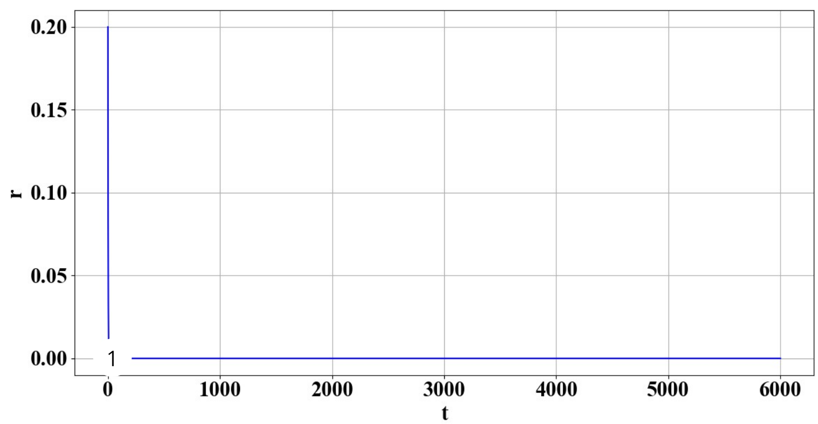

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Fig 1 r vst t. The numerical solution of r from (6) shows amplitude death afrom t = 16 sec. This amplitude, denoted by r, is the amplitude of the array of globally coupled oscillators. Point 1 shows the point from which the AD state initialises.

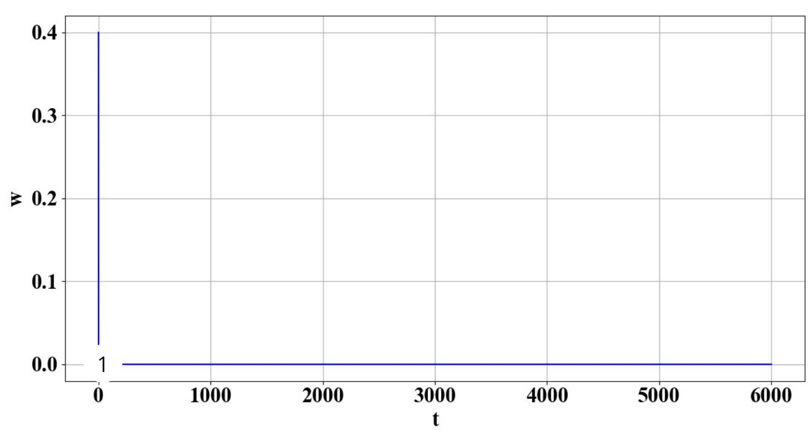

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Fig 2 w vs t. The numerical solution of w from (6) shows aging from t = 10 sec. This value w is the magnitude of dispersion for the array of globally coupled SL oscillators.

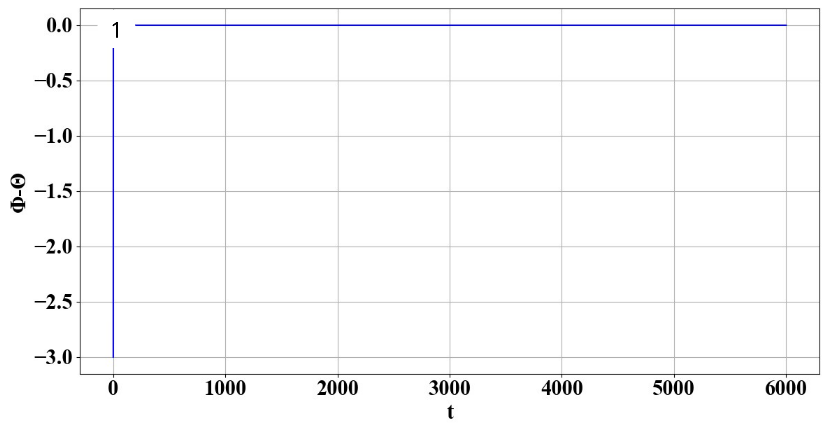

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Fig 3 Φ-Θ vs t. The numerical solution of Φ-Θ calculated from (6) shows that initially the value was -3.0. But as the time instant is increased, the value gets moe positive and goes to 0 asymptotically, which marks the onset of aging in the ensemble of oscillators.

4.2 $(K\ ,\sigma)$: Two parameter plot analysis

The AD state marks the onset of aging transition in the globally coupled array of SL oscillators. This basically means that a particular proportion of oscillators have gone to inactive state. As already discussed, after a particular value of **a**, the array, as a whole, goes to AD state. Coupling strength, between 2 oscillators, determines how well they are connected with each other. Similary, the coupling strength of whole population signifies how good is the interaction between individual oscillators. The standard deviation, σ, denotes the deviation of a_j's from their mean, for N oscillators. An obvious intution says, if the individual oscillators are very far away from their centroid, the array of oscillators may show aging transition(depending on the value of the mean **a** of the distribution). Moreover, if the coupling is very low, the interaction between the oscillators will be reduced, thus reducing the overall coupling of the array of the oscillators. Fig. 4 shows the two parameter plot fo K vs σ for α = 1 and keeping all the other paramters same. It can be easily seen from the two parameter plot that as the mean value of the distribution of a_j's is increased, the AD region vanishes smoothly. This result matches exactly with the argument stated above. As in [11], there is a critical probability p_c

where the aging transition occurs. In this paper, as we are concerned on the parameters of the distribution, we would try and search for a particular a_c after which there is no aging transition. Over here, it can be seen that as the mean value is increased from -0.5 to 0, the AD region reduces and vanishes for a = 0. In Fig. 4, 5, 6, 7, the white curve shows the aging transition from NAD to AD state and vice-versa. But we cannot possibly argue that this(a = 0) is the exact point after which the NAD region kicks in beacause we have ruled out the values between -0.005 and 0. To be honest, it is practically not possible to search precisely each and every point for various values of α_j. So, a quick question arises, "What could possibly be done to solve this dilemma ?".

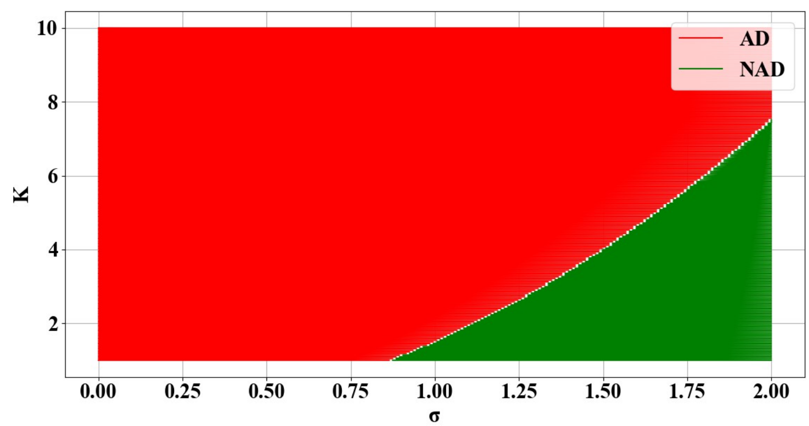

Fig 4 Order parameter K as a function of σ for a = -0.5. It consists 70% of the AD region. For higher values of coupling strength, the region is predominantly AD.

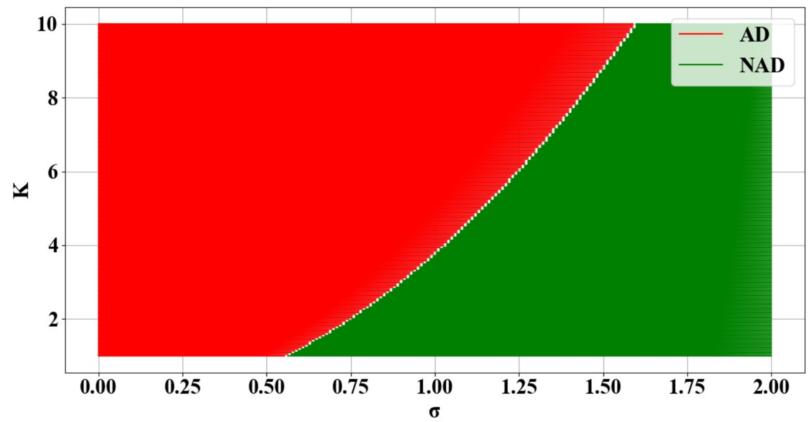

Fig 5 K vs σ for a = -0.25. Shows significant reduction in the AD state for lower as well as higher coupling strength.

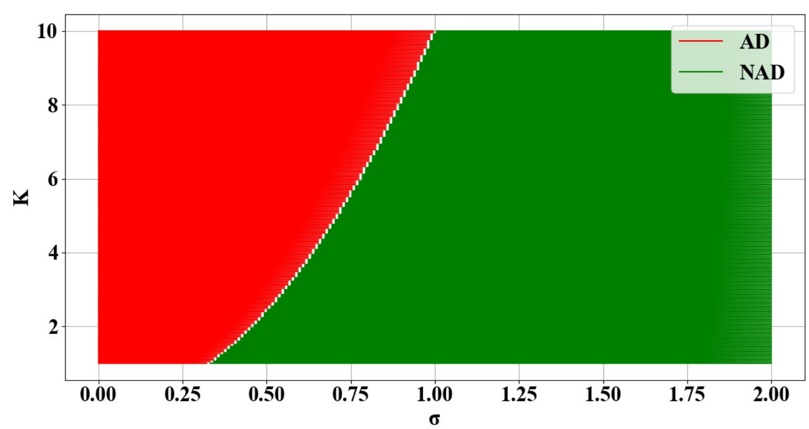

Fig 6 K vs σ for a = -0.01. Shows further reduction in the AD state for lower as well as higher coupling strength. Array of oscillators with lower coupling strength and higher deviation are in active state.

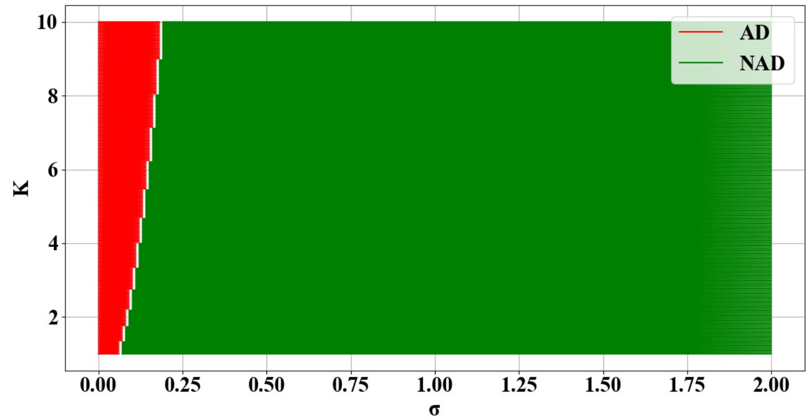

Fig 7 K vs σ for a = -0.005. The AD has nearly been reduced to 0, meaning there is not much deterioration of individual oscillators in the array.

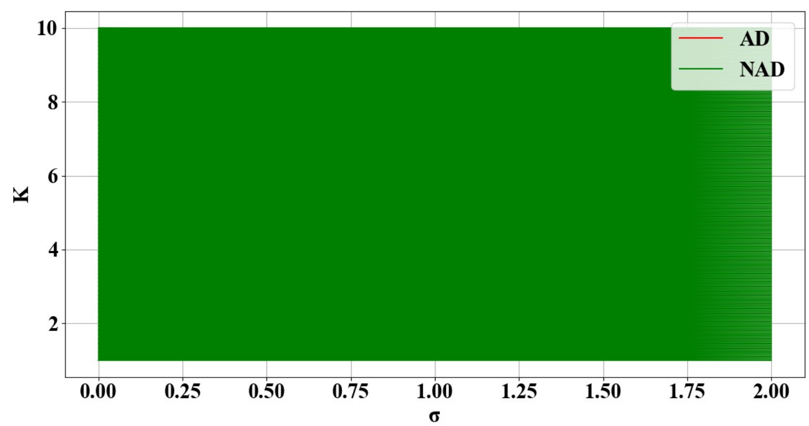

Fig 8 K vs σ for a = 0; shows that there is no AD state as the mean of a_j's is now reduced to 0.

4.2.1 Variation in α

From the previous section, it was pretty clear that the aging transition is affected by the change in the value of **a**, as well as the values of K and σ. But we couldn't come up with a precise value of **a** for which the array of globally coupled SL oscillators go to an inactive state. The self feedback term, first introduced in [11] plays a crucial role in aging transition. As mentioned earlier, it controls the degree of diffusion between the oscillators. When $\alpha = 1$, the case is symmetrical coupling and when $\alpha = 0$, the coupling is said to be direct. Changing the feedback term should cause some changes in the dynamics of aging transition for different values of **a.** A closer look at the $\dot{r}$ eq^n from (6) shows that the value of (1 - α) is multiplied with the coupling strength K. It is pellucid from this fact that as the value of α is decreased, (1 - α) is increased, thus increasing the coupling strength. One may think that as 0 < α < 1, 1 - α also has the same range. Owing to this fact, when this factor is multiplied with K, it should make the coupling strength lower than 1 per se. In this discussion, we are more interested to see the change in α. May it be less than 1, the fact that 1 - α increases monotonically is sufficient to show the aging transition due to deterioration of individual oscillators in the array. One can easily infer from Fig. 8 that as the value of α is decreased, the AD region of the array of oscillators reduces to a great extent.

The dispersion also plays a vital role in the aging transition, as mentioned earlier. Taking a closer look at the $\dot{w}$ eq^n from (6), it can be seen that, contrary to the previous case, the value of α is multiplied with the coupling strength and the result is subtracted from the whole. So, now the reader can easily deduce that as the value of α is decresed, the value of αK is also decresed. But as the value is subtracted from the whole, the net value increases. But this increment is very low as compared to the previous case. Hence, this increment is supressed and that is the vey reason it takes a very large valueof 1-α (closer to 1) to reduce the AD region to 0.

As the value of the self feedback term α is reducesd, the dynamics of aging transition also change w.r.t to the coupling strength and deviation. Fig. 8-11 show the transition of AD state as α is reduced. One can quickly infer that the decrement in the area of AD region is proportional to the change in α from its previous value.

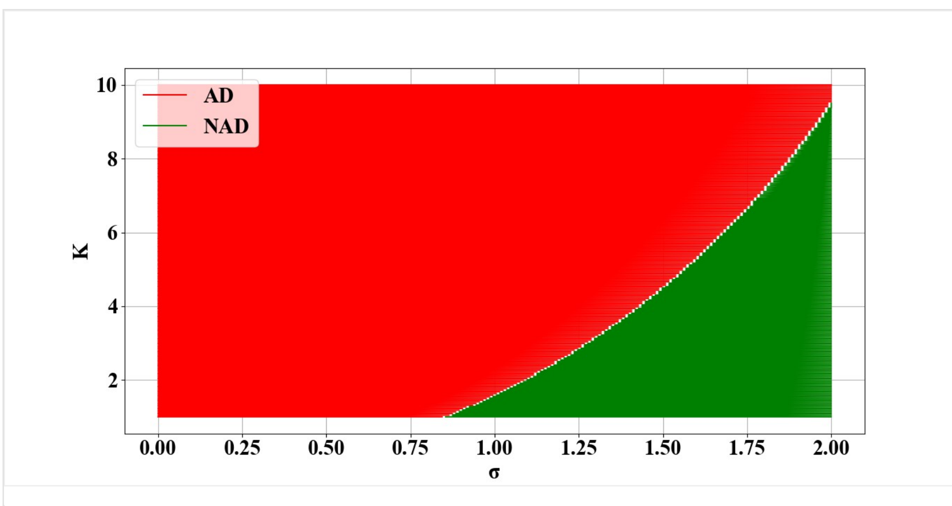

Fig 9 K vs σ for a = -0.5 and α = 0.99. The curve depicts a slightly higher amount of NAD region as compared to Fig. 1, which is due to the reduction of the value of α by 0.01 units.

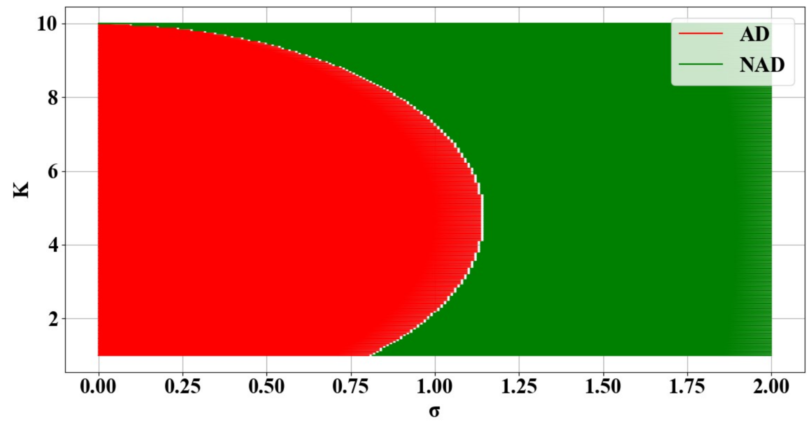

Fig 10 K vs σ for a = -0.5 and α = 0.95. The curve shows a higher amount of NAD region as compared to Fig. 9, which is due to the reduction of the value of α by 0.03 units.

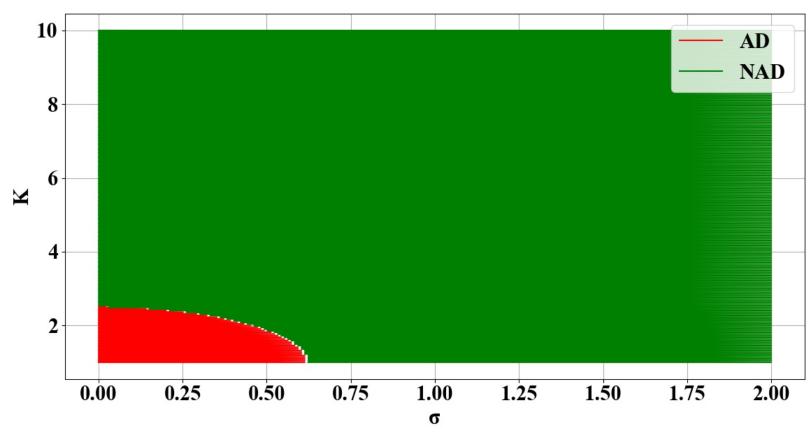

Fig 11 K vs σ for a = -0.5 and α = 0.8. The NAD region is reduced to a very less value as the vale of α is now decreased to 0.8.

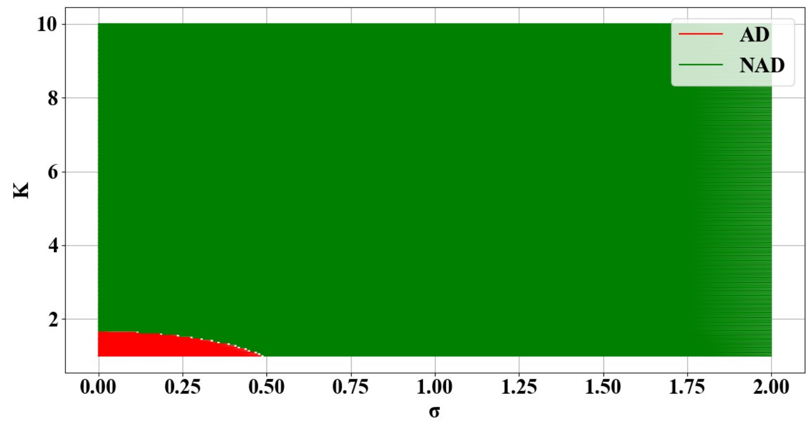

Fig 12 K vs σ for a = -0.5 and α = 0.70. The NAD region is now negligible w.r.t the AD region due to further reduction of α. We can possibly argue that around α = 0.6, the AD region vanishes.

4.2.2 Variation in η

Contrary to the value of α, η denotes the mean field feedback term. It bascially controls the contribution of the centroid of the array of oscillators in the whole equation. Keeping the value of α constant, the value of η can also be varied in order to have a more closer look at the aging dynamics of the array of oscillators. According to the figures 9-12, as the value of η is increased from 0.1 to 0.495, the AD region starts to get reduced and they nearly die out at η = 0.495. For all the variations in η, tha value of the mean is lept constant at -0.5. The dynamics aging transition, with the aame variation in η, also change if the value of the mean is changed.

From the RHS of (6), it can be clearly seen that the value of η is added to the whole equation. So, one can easily infer from this condition that when the value of η is increasesd, so will the rate of change of r. Hence, the value of the amplitude of the centroid of the array undergoes a large change when eta is increasd, thus increasing the likelihood of the aging tranition of the arrray of coupled oscillators.

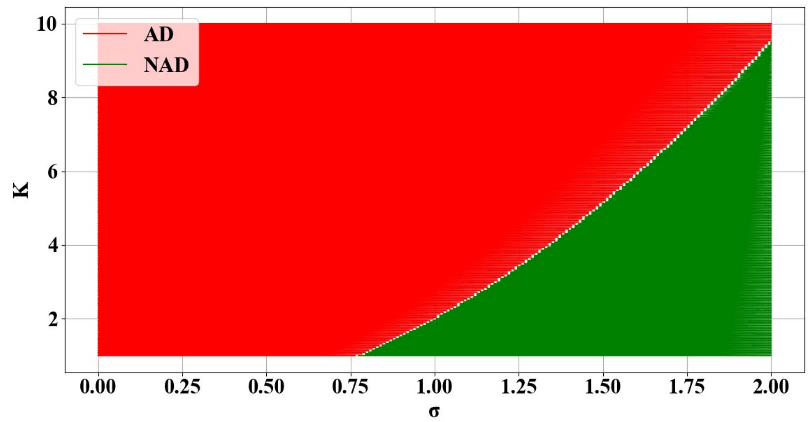

Fig 13 K vs σ for a = -0.5 and η = 0.1.

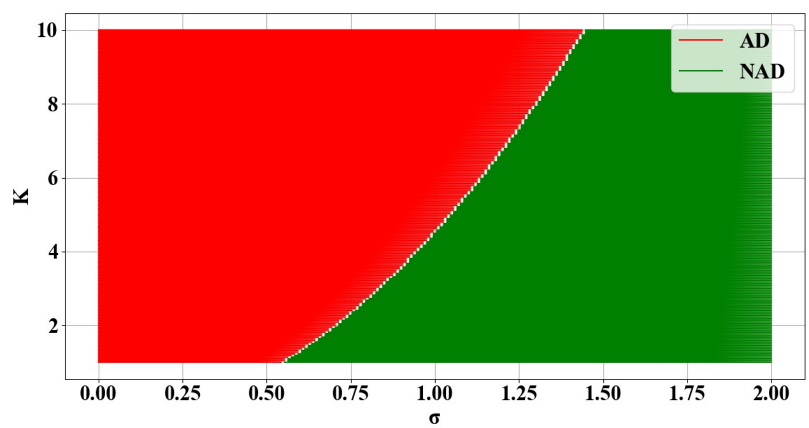

Fig 14 K vs σ for a = -0.5 and η = 0.3.

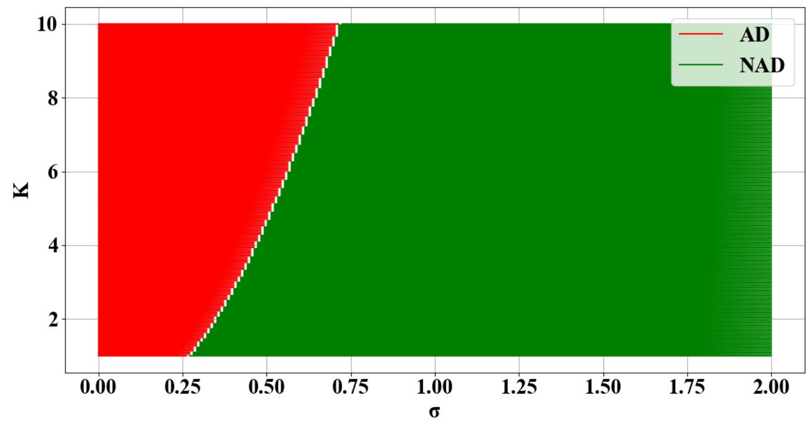

Fig 15 K vs σ for a = -0.5 and η = 0.45.

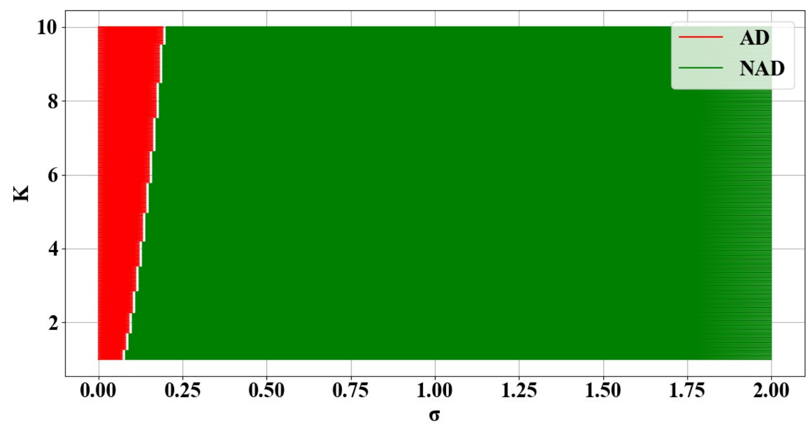

Fig 16 K vs σ for a = -0.5 and η = 0.495.

4.3 Analytical determination of cutoff value a_c

From the previous section, we saw the dynamics of aging transition w.r.t the coupling strength and standard deviation of a_j, when the value of α and η are varied within certain limits. Also, we came to a pretty obvious conclusion that the variation of mean value can also cause some changes in the behaviour of aging transition. Now, it seems legitimate to see how the behaviour of the array of globally coupled oscillators vary w.r.t the coupling strength, the mean of a_j's and the amplitude of the centroid of the array.

4.3.1 Two parameter plot for $K\ vs\ a$

As it is evident from Fig 4-7, changing the mean value causes a large change in the AD state of the array of the oscillators. But, as pointed out in the very section earlier, there was no aprroximate value of the mean which can be used to describe the dynamics of the aging transition of the array of oscillators. Emphasizing on that point, we need to see the variation of the coupling strength w.r.t the mean value for different value of deviation. Over here, the third parameter is selected as deviation because we intend to study the parameters of the distribution from which the values of the paramters specifying the distance from the HB are drawn. Fig 4-7 showed the dynamical behaviour of aging transition w.r.t K and σ with variable means. In this very case, the behaviour of aging tranisiton is analysed w.r.t K and a with variable standard deviation σ. It is evident form the same set of figues that as the standard deviation increases, for higher values of coupling, the cutoff value of the mean(a), i.e. after which there is no AD state, gets reduced. Also, standard deviation is increased, the AD region for lower values of coupling vanishes for any value of a >= -0.5.

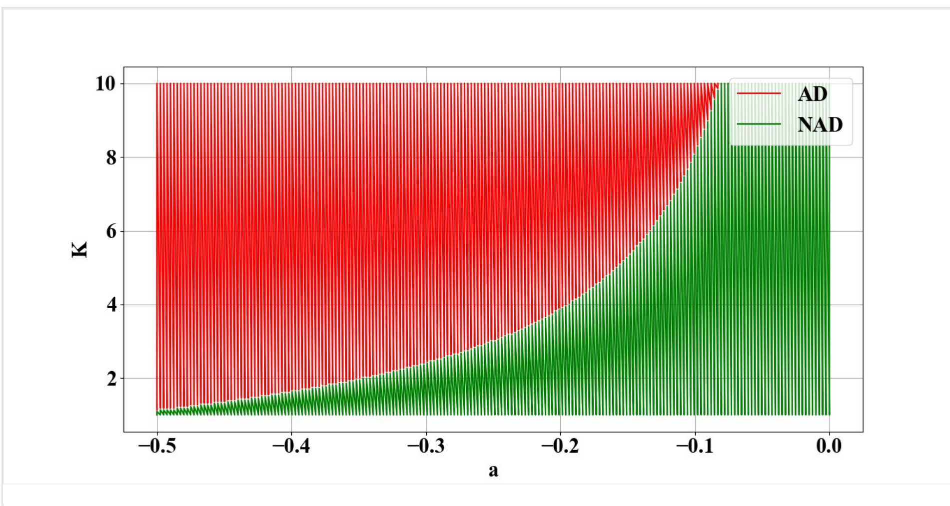

Fig 17 K vs a plot for σ = 0.9. For higher coupling values, the AD state vanishes at a = -0.1.

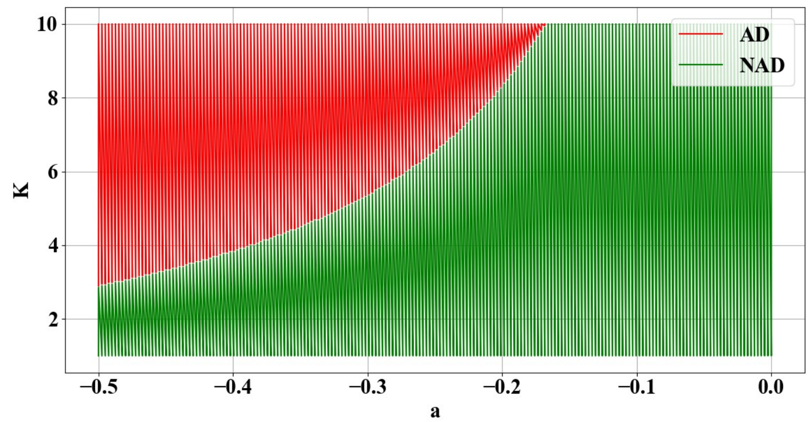

Fig 18 K vs a plot for σ = 1.3. For higher coupling values, the AD transits to NAD state around a higher than -0.2.

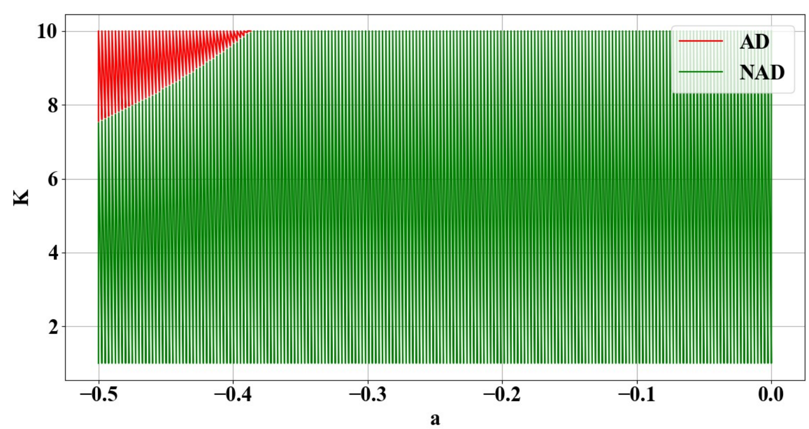

Fig 19 K vs a plot for σ = 2. Now, the AD state only remains for very higher values of coupling and low values of the mean. The region boundary looks linear but actually is a non-linear curve, when seen closely.

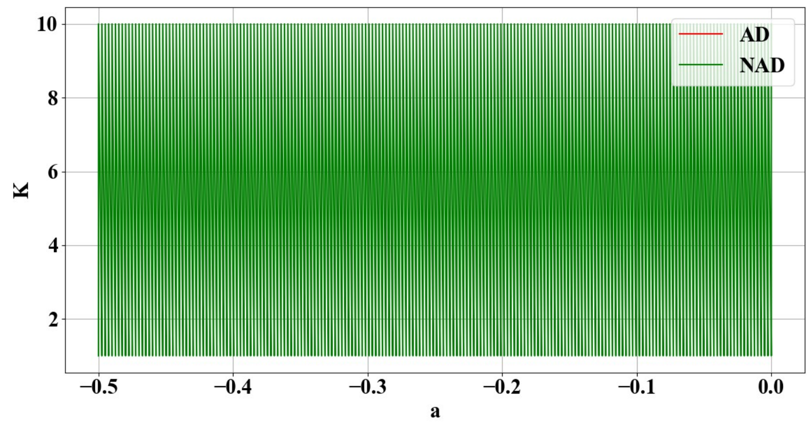

Fig 20 K vs a plot for σ = 2.3. It can be easily seen that the AD region is now vanished, which means there is no further aging transition in the array of globally coupled SL oscillators.

4.3.2 $(r\ ,a)$:Variation of mean field amplitude as a function of the mean of the distribution

In the previous sections, it was pretty easy to observe the dynamics of aging transition of the array of oscillators by changing the order parameters along with the mean, self feedback and mean filed feedback. The results were promising in the sense that the behaviour could be changed by variation the above mentioned factors. the values of the mean of the distribution are continuous. Hence, it is impossible to pinpoint a particular value which suffices a condition of aging. Rather, it would be more precise to obtain a range, to work in a better way. Now, as mentioned earlier, we are keen to obtain a range of value of **a** i.e. the mean of the distribution, between which there is a point where the transition occurs. It is very cumbersome to deduce this range from the two parameter plot of the order paramters coupling strength and deviation. The other way round to obatin this range analytically is through a plot of the mean field r as a fucntion of the mean. Over here, the plot is drawn for different values of the coupling strength K. From FIg. 21, it can be deduced that as the value of coupling strength is increased, the cutoff value of mean, i.e. a_c for aging transition of the array of oscillators increases. It basically means that as the global coupling of the array of oscillators is increased, the value of the mean for which the aging transition occurs increases. As the mean increases, the value of α_j's also increases, meaning the values can now be drawn from a much greater distance from the HB .

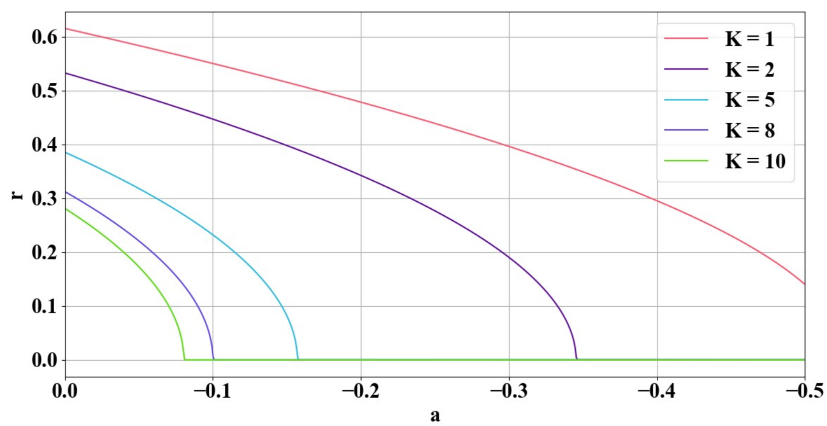

Fig 21 mean field r as a function of time for σ = 0.9.

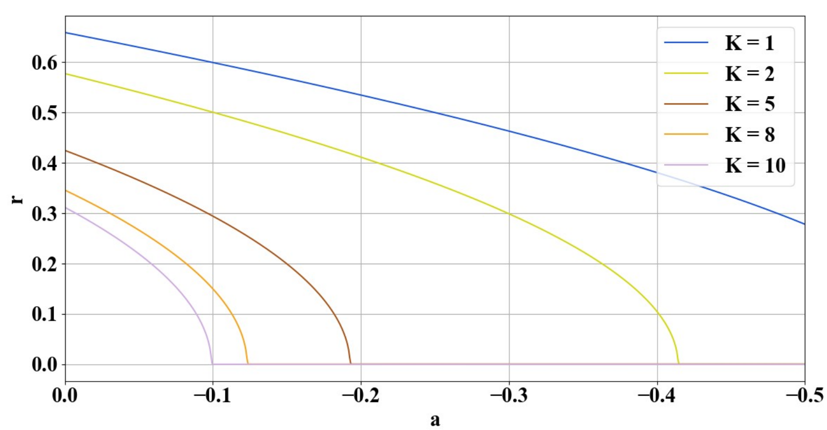

Fig 22 mean field r as a function of time for σ = 1.

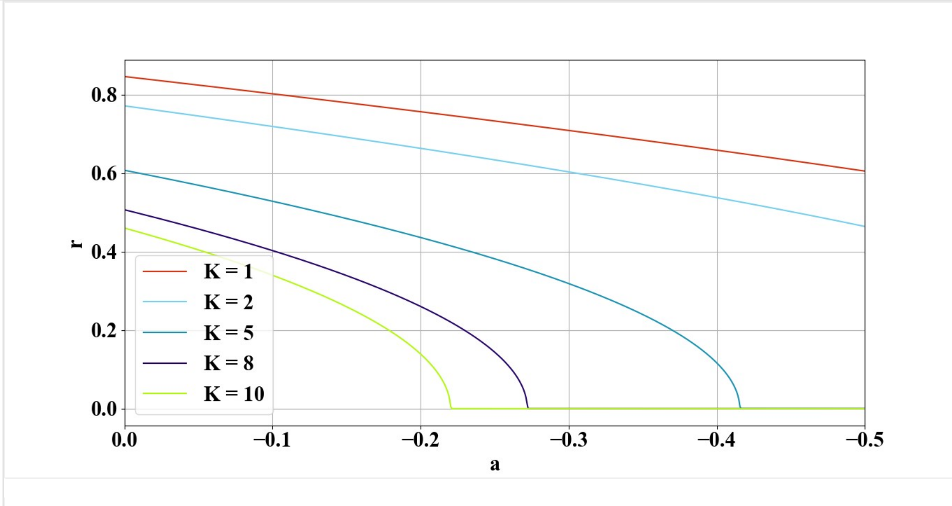

Fig 23 mean field r as a function of time for σ = 1.5.

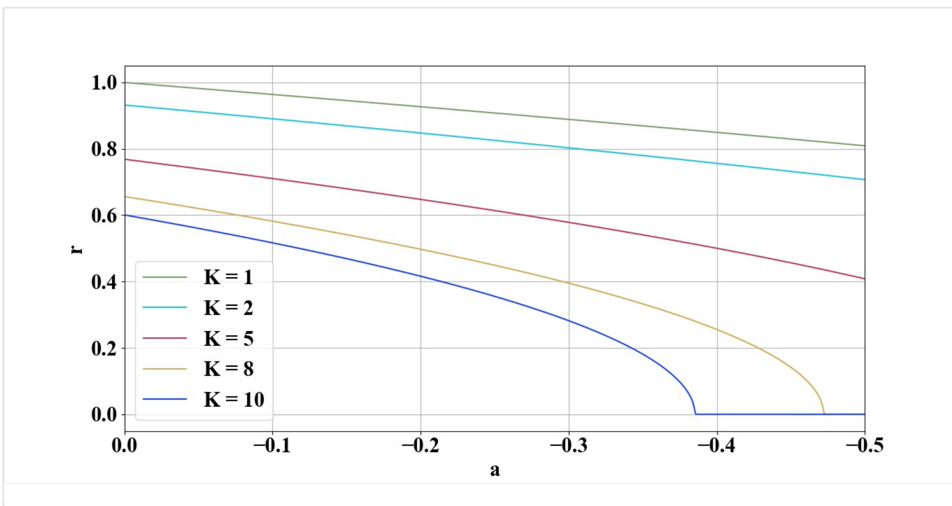

Fig 24 mean field r as a function of time for σ = 2.

It is pretty clear from the Fig. 21, 22, 23 and 24 that as the value of standard deviation of α_j's is increased, the magnitude of the cutoff value a_c is also increased for any coupling value and vice versa. It can be clearly seen that we have obtained an analytical range of mean inside which the aging transition takes place. As the deviation and the coupling strength is increased, this range also increases.

5 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

We have successfully found out analytically, the range of cutoff value of mean distance parameter from HB (a) i.e. a_c for different values of deviations (σ). It has been analytically shown that the resultant of amplitude of an array of SL oscillators, coupled globally, die out for any value of coupling for lower values of deviation, for any N. If the coupling of the oscillators is increased, keeping the standard deviation fixed, the cutoff value of mean distance from HB (a_c) is further decreased.

These results can further be used to study the nature of the globally coupled oscillators in an invariant manifold. Also, more deep probing could be done on different states like chimera, cluster and syncronisation states. Moreover, similar to the cutoff value of the mean, simulations can be used to study the nature of self feedback term α and the mean field feedback term η.

In this very paper, the main focus was to study the statistical parameters, mean and variance, of the distribution of α_j's, which signify the distance of j^{th} oscillator form HB . Increasing the deviation value increases the cutoff value of distance parameter for all coupling values. The results also reveal that the value of α also has an effect in aging. The parameter α basically controls the feedback applied to individual oscillators, hence the name self-feedback. Reducing the value of self-feedback basically increases the factor $(1 - \alpha)$ and thus, the death region is further decreased. As argued earlier, the facter $(1 - \alpha)$ is multiplies with K, hence decreasing the value of α increases $(1 - \alpha)*K$, hence the overall rate of change of mean amplitude is also increased.

In this paper, we have used the mean field approach to expand (1) in terms of the mean field Z. Moreover, we have introduced another term W, namely shape parameter, which measures the collection of mismatch from each oscillator and used the mean field theory to evaluate the term. Lastly, we provide an expression for magnitudes of the resultant amplitude of the array of non-identical globally coupled SL oscillators and the shape parameter using polar substitution along with expressions for phase of mean field Z and shape parameter W, namely, Φ and Θ. Using numerical aprroximations, the graph of r, w and $\phi - \theta$ is plotted vs t. This graph shows amplitude death when $\phi - \theta$ tends to 0, which means the phase of the centroid gets more closer to the phase of the shape parameter.

5.2 Recommendations

The numerical simulations are made with the help of Adam Bashforth method. If any different method is used, the accuracy of the results may vary. Also, the expressions in (6) are obtained by neglecting higher order terms of the order of $O(< \epsilon_j^2 >)$ and $O < (a_j - a_0)^2 \epsilon_j >$.

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