Subir Chowdhury School of Quality and Reliability

Introduction to Quality and SPC Tools

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Quality Control Tools

Introduction

- If a product is to meet or exceed customer expectations, generally it should be produced by a process that is stable or repeatable. More precisely, the process must be capable of operating with little variability around the target or nominal dimensions of the product's quality characteristics.
- Statistical process control (SPC) is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability.
- SPC is one of the greatest technological developments of the twentieth century because it is based on sound underlying principles, is easy to use, has significant impact, and can be applied to any process.
- Its seven major tools are these:

Introduction

1. Histogram or stem-and-leaf plot, Box-plot, Flow Charts
2. Check sheet
3. Pareto chart
4. Cause-and-effect diagram
5. Defect concentration diagram .
6. Scatter diagram , Multivariate charts, Matrix and 3-D Plots
7. Control chart

The Stem-and- Leaf Plot

- Statistical data, generated in large masses, can be very useful for studying the behavior of the distribution if presented in a combined tabular and graphic display called a stem-and-leaf plot.

- To illustrate the construction of a stem-and-leaf plot, consider the data of **Table 1**, which specifies the “life” of 40 similar car batteries recorded to the nearest tenth of a year. The batteries are guaranteed to last 3 years.

Table 1. Car Battery Life

2.2	4.1	3.5	4.5	3.2	3.7	3.0	2.6
3.4	1.6	3.1	3.3	3.8	3.1	4.7	3.7
2.5	4.3	3.4	3.6	2.9	3.3	3.9	3.1
3.3	3.1	3.7	4.4	3.2	4.1	1.9	3.4
4.7	3.8	3.2	2.6	3.9	3.0	4.2	3.5

The Stem-and- Leaf Plot

- First, split each observation into two parts consisting of a stem and a leaf such that the stem represents the digit preceding the decimal and the leaf corresponds to the decimal part of the number.
- In other words, for the number 3.7, the digit 3 is designated the stem and the digit 7 is the leaf.
- The four stems 1, 2, 3, and 4 for our data are listed vertically on the left side in **Table 2**; the leaves are recorded on the right side opposite the appropriate stem value.
- Thus, the leaf 6 of the number 1.6 is recorded opposite the stem 1; the leaf 5 of the number 2.5 is recorded opposite the stem 2; and so forth.
- The number of leaves recorded opposite each stem is summarized under the frequency column.

The Stem-and- Leaf Plot

Table 2. Stem-and Leaf Plot of Battery Life

Stem	Leaf	Frequency
1	69	2
2	25669	5
3	0011112223334445567778899	25
4	11234577	8

- The stem-and-leaf plot of **Table 2** contains only four stems and consequently does not provide an adequate picture of the distribution.
- To remedy this problem, we need to increase the number of stems in our plot.
- One simple way to accomplish this is to write each stem value twice and then record the leaves 0, 1, 2, 3, and 4 opposite the appropriate stem value where it appears for the first time, and the leaves 5, 6, 7, 8, and 9 opposite this same stem value where it appears for the second time.

The Stem-and- Leaf Plot

This modified double-stem-and-leaf plot is illustrated in **Table 3**, where the stems corresponding to leaves 0 through 4 have been coded by the symbol * and the stems corresponding to leaves 5 through 9 by the symbol ▪

Table 3. Double Stem-and Leaf Plot of Battery Life

Stem	Leaf	Frequency
1·	69	2
2⋆	2	1
2·	5669	4
3⋆	001111222333444	15
3·	5567778899	10
4⋆	11234	5
4·	577	3

The Stem-and- Leaf Plot

- The stem-and-leaf plot represents an effective way to summarize data.
- Another way is through the use of the frequency distribution, where the data, grouped into different classes or intervals, can be constructed by counting the leaves belonging to each stem and noting that each stem defines a class interval.
- In **Table 2**, the stem 1 with 2 leaves defines the interval 1.0–1.9 containing 2 observations; the stem 2 with 5 leaves defines the interval 2.0–2.9 containing 5 observations; the stem 3 with 25 leaves defines the interval 3.0–3.9 with 25 observations; and the stem 4 with 8 leaves defines the interval 4.0–4.9 containing 8 observations.
- For the double-stem-and-leaf plot of **Table 3**, the stems define the seven class intervals 1.5–1.9, 2.0–2.4, 2.5–2.9, 3.0–3.4, 3.5–3.9, 4.0–4.4, and 4.5–4.9 with frequencies 2, 1, 4, 15, 10, 5, and 3, respectively.

The Histogram

- Dividing each class frequency by the total number of observations, we obtain the proportion of the set of observations in each of the classes. A table listing relative frequencies is called a relative frequency distribution.

- The relative frequency distribution for the data of **Table 1**, showing the midpoint of each class interval, is given in **Table 4**.

Table 4. Relative Frequency Distribution of Battery Life of Battery Life

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
1.5–1.9	1.7	2	0.050
2.0–2.4	2.2	1	0.025
2.5–2.9	2.7	4	0.100
3.0–3.4	3.2	15	0.375
3.5–3.9	3.7	10	0.250
4.0–4.4	4.2	5	0.125
4.5–4.9	4.7	3	0.075

The Histogram

Using the midpoint of each interval and the corresponding relative frequency, we construct a relative frequency histogram **(Figure 1).**

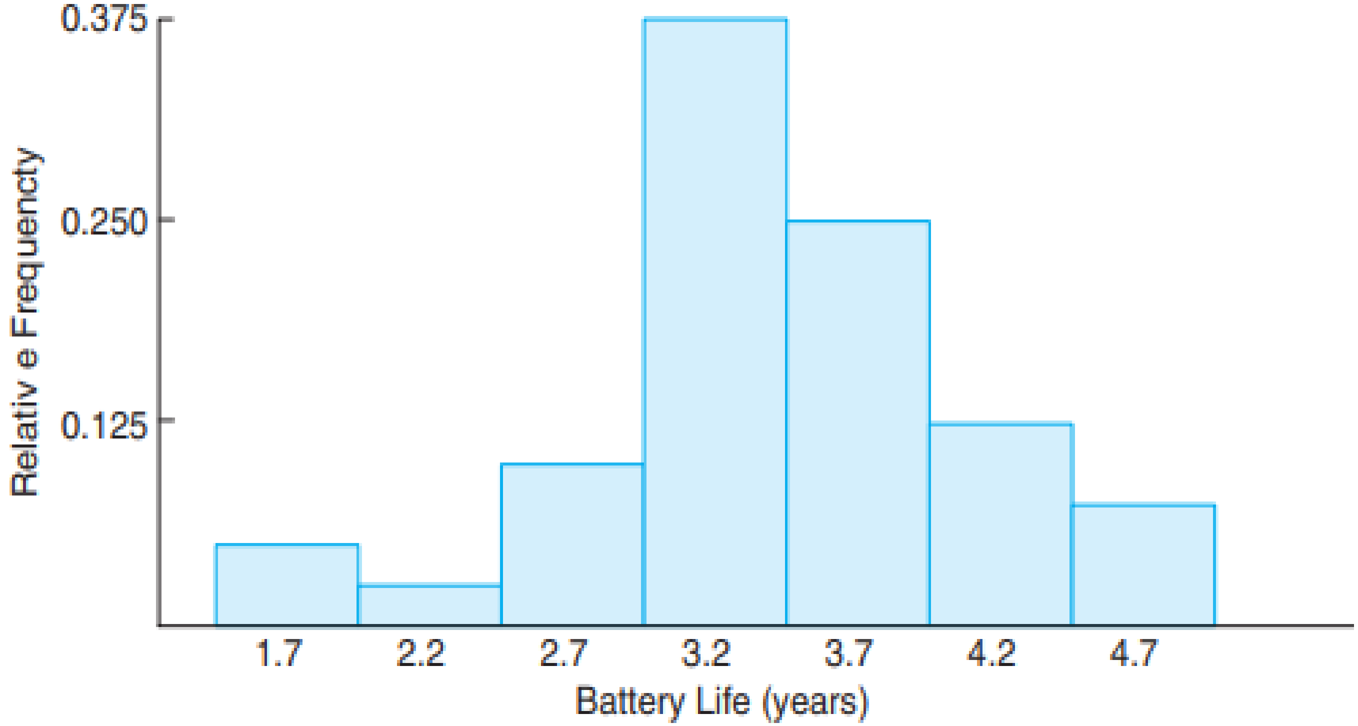


Figure 1: Relative frequency histogram

The Histogram

Many continuous frequency distributions can be represented graphically by the characteristic bell-shaped curve of **Figure 2**. Graphical tools such as what we see in **Figures 1** and **2** aid in the characterization of the nature of the population.

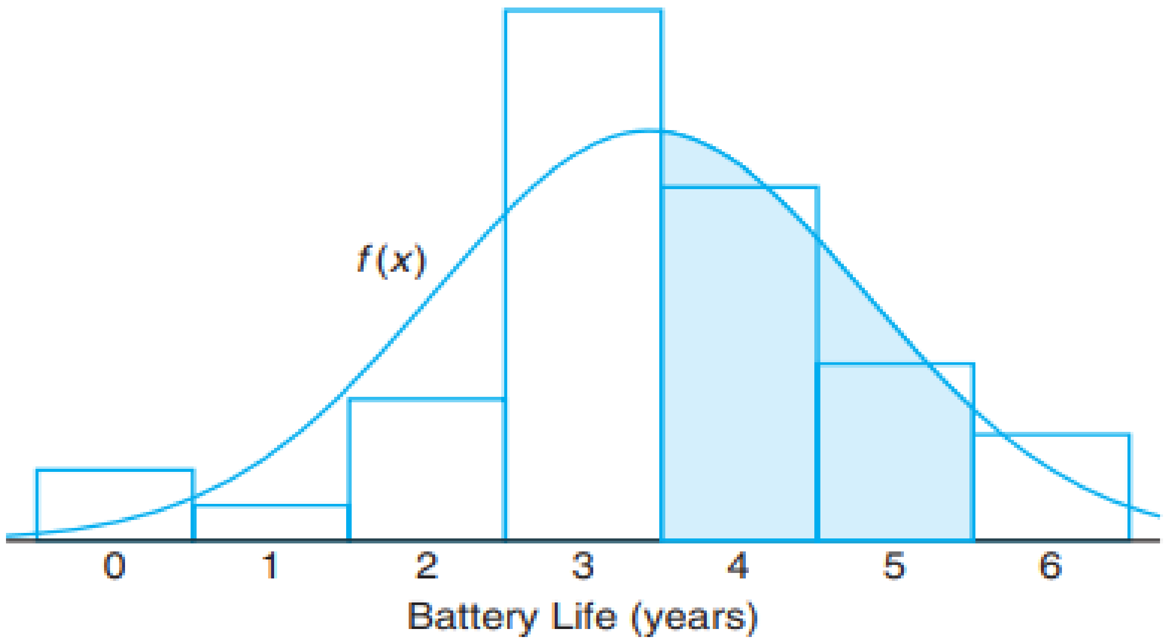


Figure 2: Estimating frequency distribution

Box-and-Whisker Plot or Box Plot

- The box plot is a graphical display that simultaneously displays several important features of the data, such as location or central tendency, spread or variability, departure from symmetry, and identification of observations that lie unusually far from the bulk of the data (these observations are often called “outliers”)

- A box plot displays the three quartiles, the minimum, and the maximum of the data on a rectangular box.

- The box encloses the interquartile range with the left (or lower) line at the first quartile Q1 and the right (or upper) line at the third quartile Q3.

- A line is drawn through the box at the second quartile (which is the fiftieth percentile or the median).

- A line at either end extends to the extreme values. These lines are usually called whiskers.

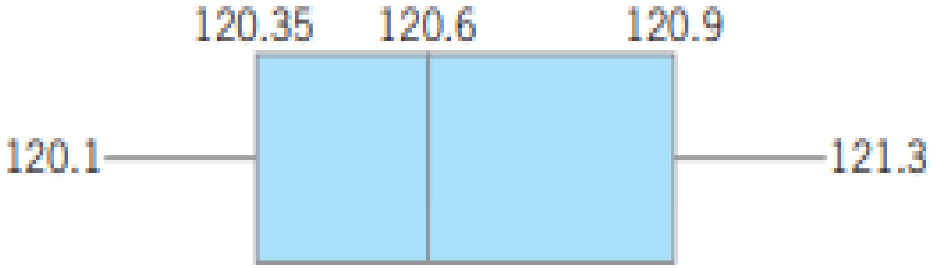
Box-and-Whisker Plot or Box Plot

The data in **Table 5** are diameters (in mm) of holes in a group of 12 wing leading edge ribs for a commercial transport airplane. Construct and interpret a box plot of those data.

Table 5. Hole Diameters (in mm) in Wing Leading Edge Ribs

120.5	120.4	120.7
120.9	120.2	121.1
120.3	120.1	120.9
121.3	120.5	120.8

The box plot is shown in **Figure 3**.



Box plot for the aircraft wing leading edge hole diameter data in Table 5.

Box-and-Whisker Plot or Box Plot

Note that the median of the sample is halfway between the sixth and seventh rank ordered observation, or (120.5 + 120.7)/2 = 120.6.

The quartiles are Q1 = 120.35 and Q3 = 120.9.

The box plot indicates that the hole diameter distribution is not exactly symmetric around a central value, because the left and right whiskers and the left and right boxes around the median are not the same lengths.

Box plots are very useful in graphical comparisons among data sets, because they have visual impact and are easy to understand.

For example, **Figure 4** shows the comparative box plots for a manufacturing quality index on products at three manufacturing plants. Inspection of this display reveals that there is too much variability at plant 2, and that plants 2 and 3 need to raise their quality index performance

Box-and-Whisker Plot or Box Plot

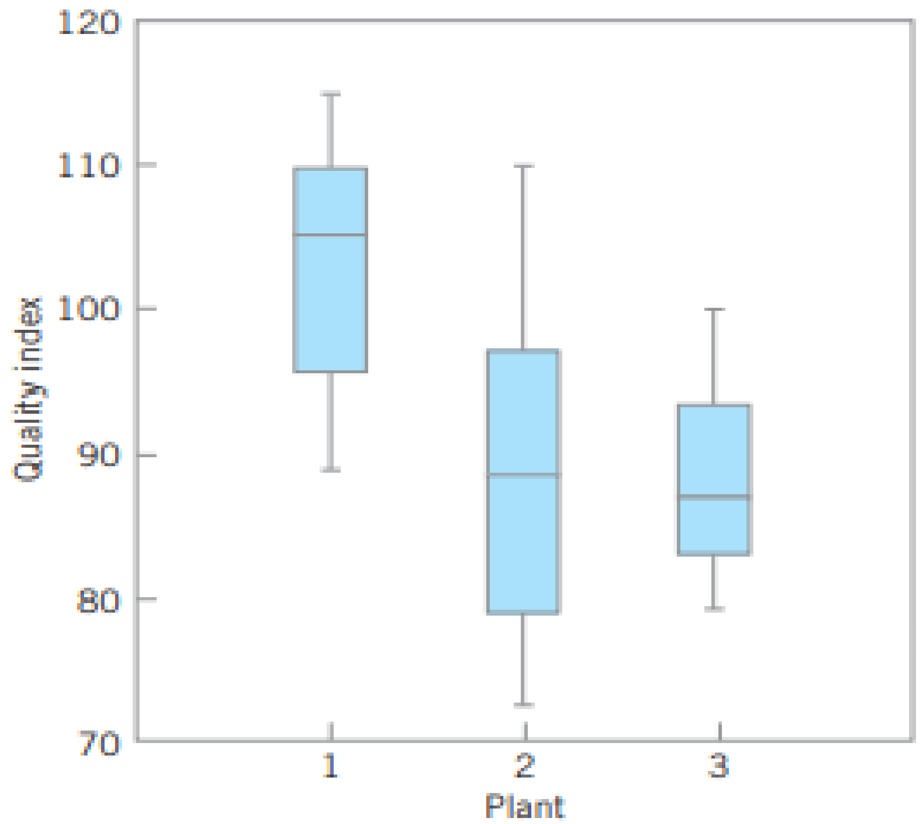


Figure 4: Comparative box plots of a quality index for products produced at three plants.

Flow Charts

- Flowcharts, which show the sequence of events in a process, are used for manufacturing and service operations. They are often used to diagram operational procedures to simplify a system, as they can identify bottlenecks, redundant steps, and non-value-added activities.

- A realistic flow chart can be constructed by using the knowledge of the personnel who are directly involved in the particular process. Valuable process information is usually gained through the construction of flow charts.

- **Figure 5** shows a flow chart for patients reporting to the emergency department in a hospital. The chart identifies where delays can occur: for example, in several steps that involve waiting. A more detailed flow chart would allow pinpointing of key problem areas that contribute to lengthening waiting time.

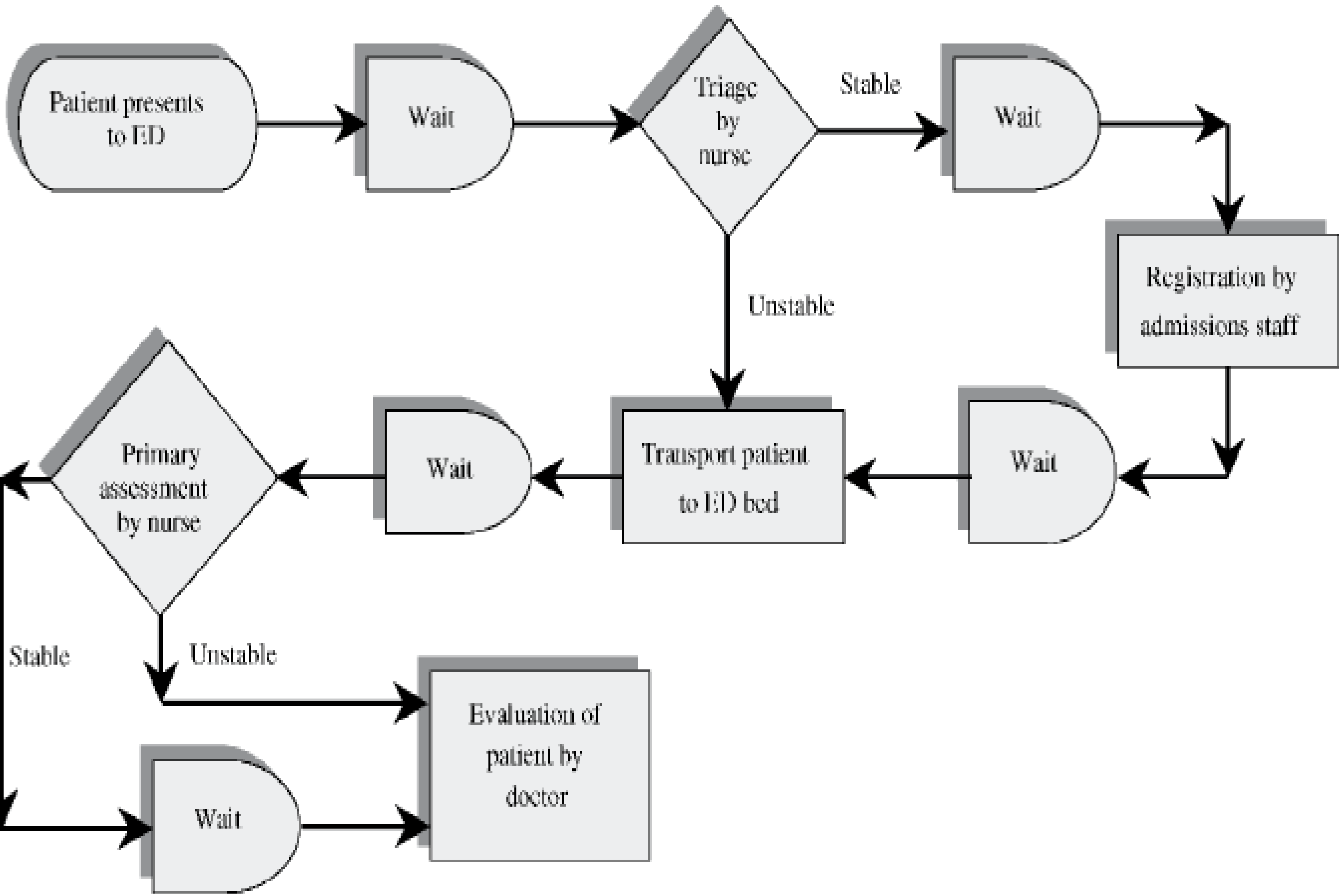


Figure 5: Flow chart for patients in an emergency department(ED).

Check Sheets

- In the early stages of process improvement, it will often become necessary to collect either historical or current operating data about the process under investigation.

- This is a common activity in the measure step of DMAIC.

- A check sheet can be very useful in this data collection activity. The check sheet shown in **Figure 6** was developed by an aerospace firm engineer who was investigating defects that occurred on one of the firm’s tanks.

- The engineer designed the check sheet to help summarize all the historical defect data available on the tanks.

- Because only a few tanks were manufactured each month, it seemed appropriate to summarize the data monthly and to identify as many different types of defects as possible.

CHECK SHEET
DEFECT DATA FOR 2002–2003 YTD

Part No.: TAX-41
Location: Bellevue
Study Date: 6/5/03
Analyst: TCB

	2002												2003					
Defect	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	Total
Parts damaged		1		3	1	2		1		10	3		2	2	7	2		34
Machining problems			3	3				1	8		3		8	3				29
Supplied parts rusted			1	1		2	9											13
Masking insufficient		3	6	4	3	1												17
Misaligned weld	2																	2
Processing out of order	2															2		4
Wrong part issued		1						2										3
Unfinished fairing			3															3
Adhesive failure				1							1		2			1	1	6
Powdery alodine					1													1
Paint out of limits						1								1				2
Paint damaged by etching			1															1
Film on parts						3		1	1									5
Primer cans damaged								1										1
Voids in casting									1	1								2
Delaminated composite										2								2
Incorrect dimensions											13	7	13	1		1	1	36
Improper test procedure										1								1
Salt-spray failure													4			2		4
TOTAL	4	5	14	12	5	9	9	6	10	14	20	7	29	7	7	6	2	166

Figure 6. A check sheet to record defects on a tank used in an aerospace application.

Check Sheets

- The time-oriented summary is particularly valuable in looking for trends or other meaningful patterns. For example, if many defects occur during the summer, one possible cause might be the use of temporary workers during a heavy vacation period.

- When designing a check sheet, it is important to clearly specify the type of data to be collected, the part or operation number, the date, the analyst, and any other information useful in diagnosing the cause of poor performance.

- If the check sheet is the basis for performing further calculations or is used as a worksheet for data entry into a computer, then it is important to be sure that the check sheet will be adequate for this purpose.

- In some cases, a trial run to validate the check sheet layout and design may be helpful.

Pareto Chart

- The Pareto chart is simply a frequency distribution (or histogram) of attribute data arranged by category.
- Pareto charts are often used in both the Measure and Analyze steps of DMAIC.
- The Pareto principle also lends support to the 80/20 rule, which states that 80% of problems (non conformities or defects) are created by 20% of causes.
- Pareto diagrams help prioritize problems by arranging them in decreasing order of importance.
- In an environment of limited resources, these diagrams help companies decide the order in which they should address problems.
- To illustrate a Pareto chart, consider the tank defect data presented in **Figure 6.**

Pareto Chart

- Plotting the total frequency of occurrence of each defect type (the last column of the table in **Figure 6**) against the various defect types will produce **Figure 7**, which is called a Pareto chart.

- Through this chart, the user can quickly and visually identify the most frequently occurring types of defects.

- For example, **Figure 7** indicates that incorrect dimensions, parts damaged, and machining are the most commonly encountered defects. Thus the causes of these defect types probably should be identified and attacked first.

- There are many variations of the basic Pareto chart. **Figure 8a** shows a Pareto chart applied to an electronics assembly process using surface-mount components. The vertical axis is the percentage of components incorrectly located, and the horizontal axis is the component number, a code that locates the device on the printed circuit board.

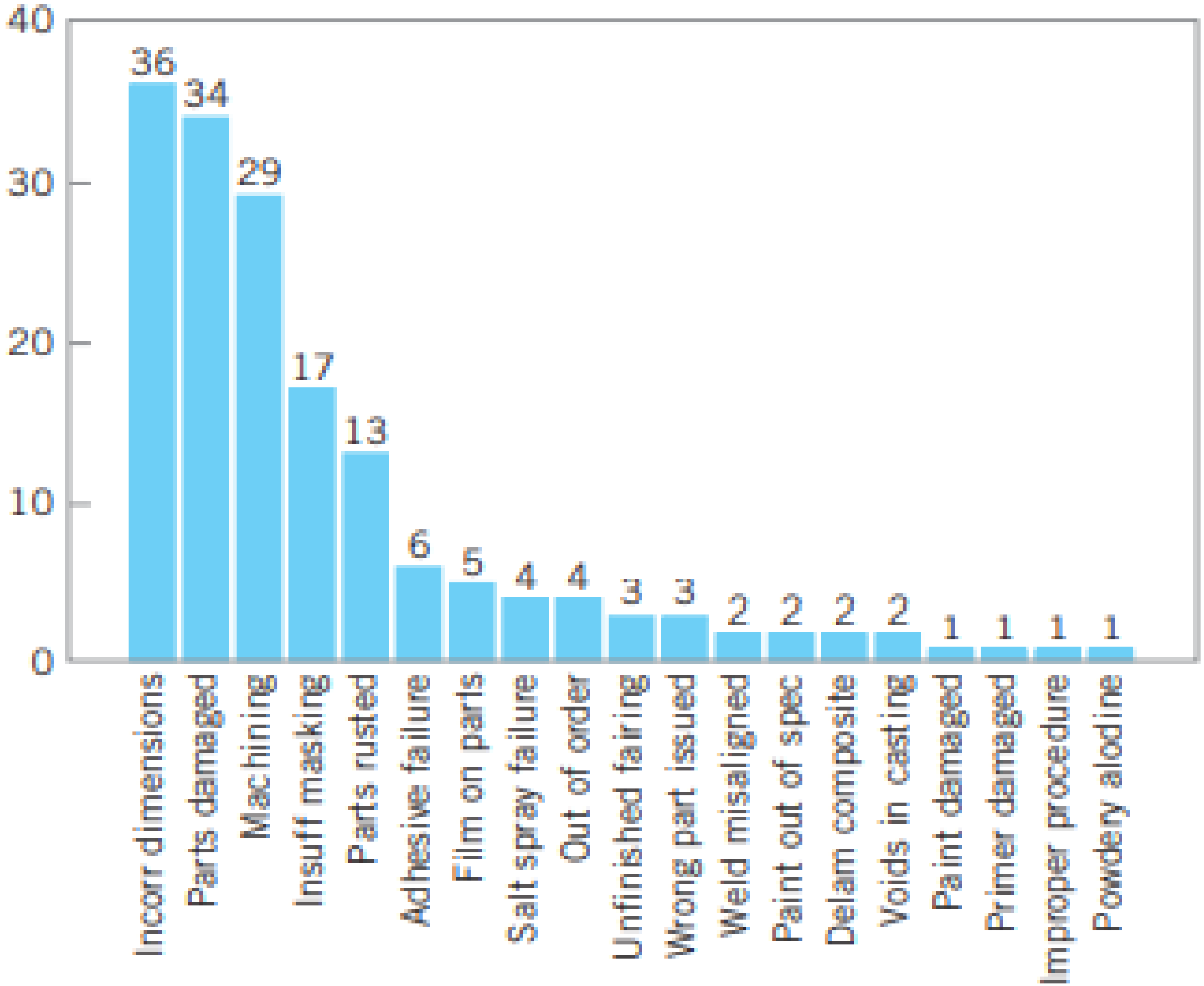
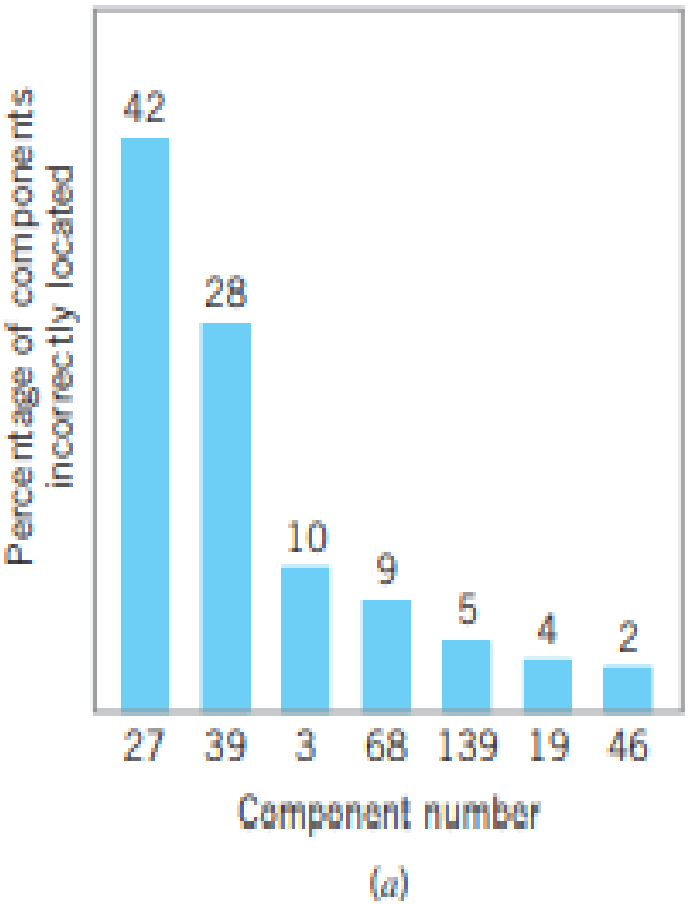


Figure 7. Pareto chart of the tank defect data



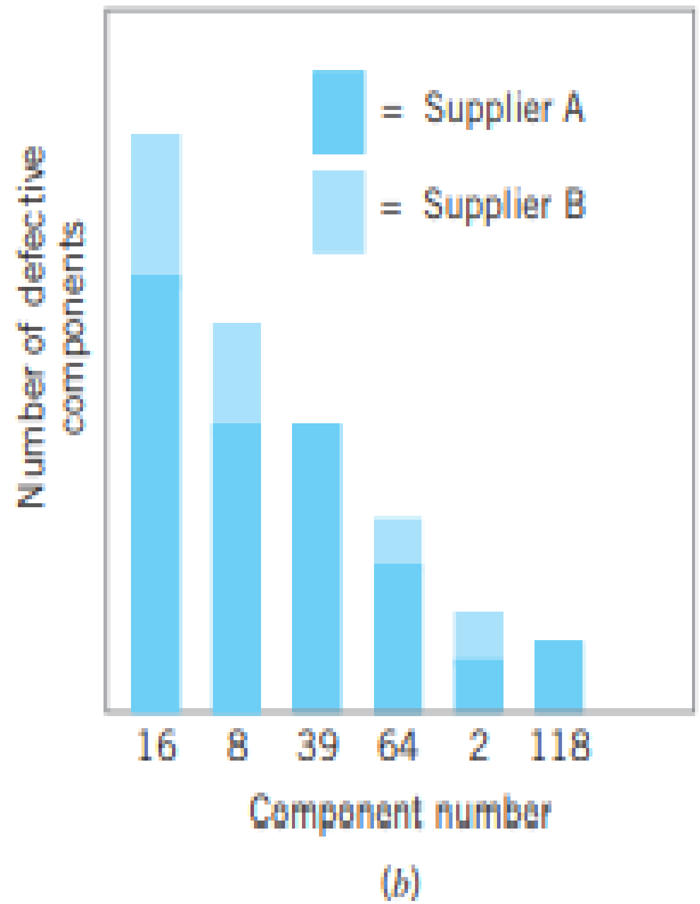


Figure 8. Examples of Pareto charts.

Pareto Chart

- Note that locations 27 and 39 account for 70% of the errors. This may be the result of the type or size of components at these locations, or of where these locations are on the board layout.

- **Figure 8b** presents another Pareto chart from the electronics industry. The vertical axis is the number of defective components, and the horizontal axis is the component number. Note that each vertical bar has been broken down by supplier to produce a stacked Pareto chart. This analysis clearly indicates that supplier A provides a disproportionately large share of the defective components.

- Pareto charts are widely used in nonmanufacturing applications of quality improvement methods. A Pareto chart used by a quality improvement team in a procurement organization is shown in **Figure 8c**. The team was investigating errors on purchase orders in an effort to reduce the organization’s number of purchase order changes.

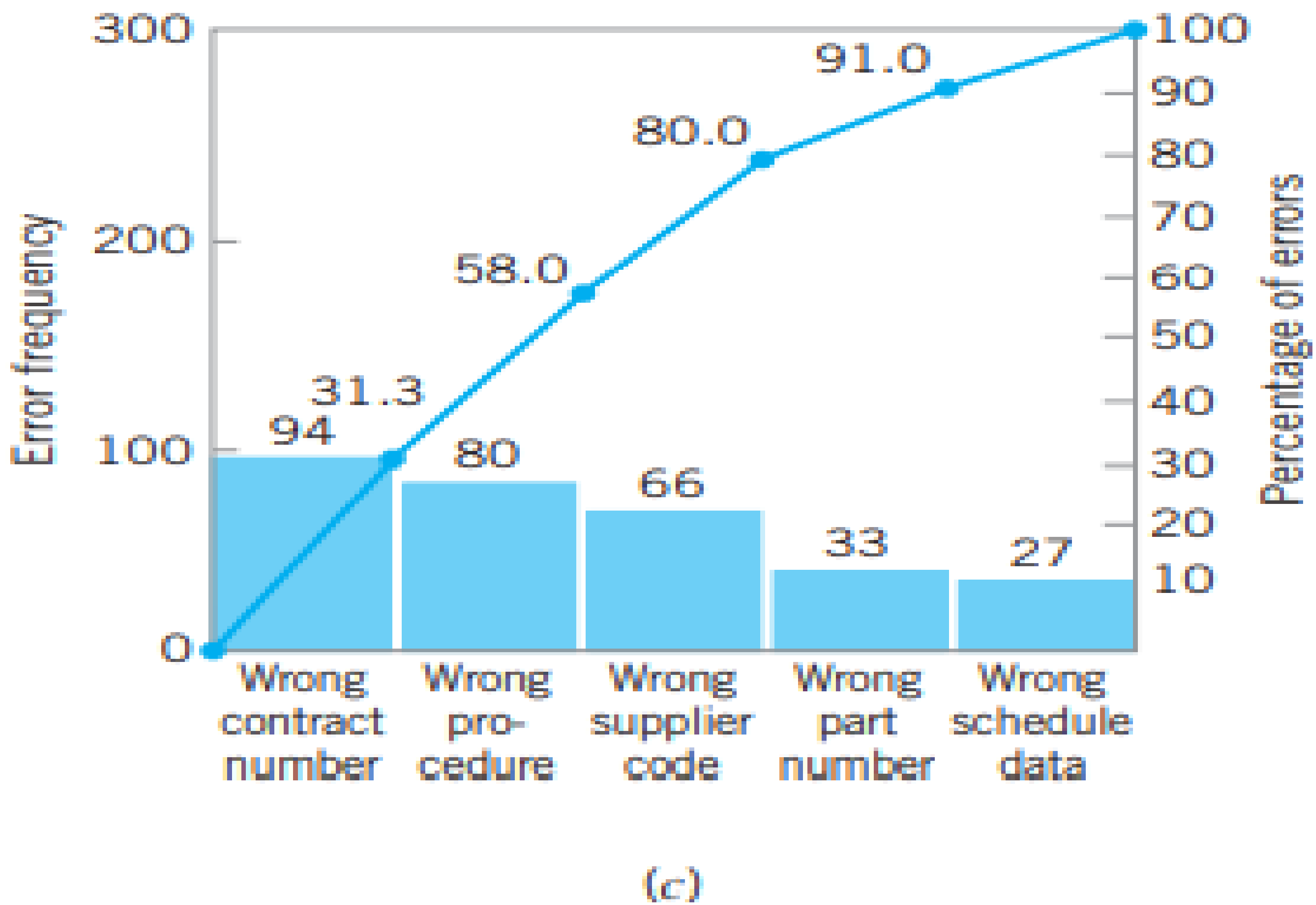


Figure 8. Examples of Pareto charts.

Pareto Chart

- (Each change typically cost between \$100 and \$500, and the organization issued several hundred purchase order changes each month.)
- This Pareto chart has two scales: one for the actual error frequency and another for the percentage of errors.
- **Table 6 and Figure 9** shows a Pareto diagram of reasons for airline customer dissatisfaction.
- Delays in arrival is the major reason, as indicated by 40% of customers. Thus, this is the problem that the airlines should address first.

Table 6. Customer dissatisfaction in Airlines

Reasons	Count
Lost baggage	15
Delay in arrival	40
Quality of meals	20
Attitude of attendant	25

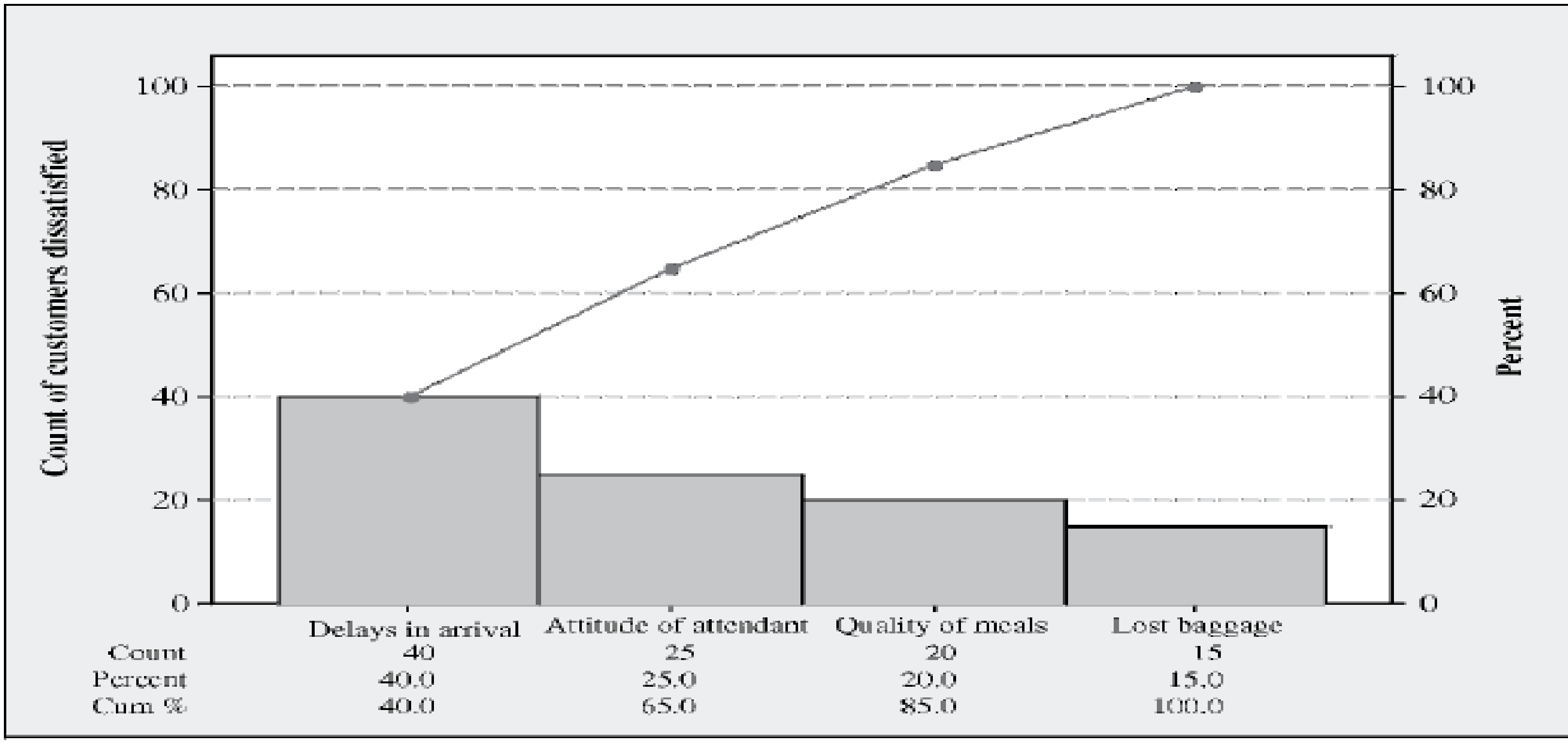


Figure 9. Pareto diagram for dissatisfied airline customers.

Cause-and-Effect Diagram

- Once a defect, error, or problem has been identified and isolated for further study, we must begin to analyze potential causes of this undesirable effect.

- In situations where causes are not obvious (sometimes they are), the cause-and-effect diagram is a formal tool frequently useful in unlayering potential causes.

- The cause-and-effect diagram is very useful in the Analyze and Improve steps of DMAIC.

- The cause-and-effect diagram constructed by a quality improvement team assigned to identify potential problem in the tank manufacturing process mentioned earlier is shown in **Figure 10**.

- The steps in constructing the cause-and-effect diagram are as follows:

Cause-and-Effect Diagram

How to Construct a Cause-and-Effect Diagram

1. Define the problem or effect to be analyzed.
2. Form the team to perform the analysis. Often the team will uncover potential causes through brainstorming.
3. Draw the effect box and the center line.
4. Specify the major potential cause categories and join them as boxes connected to the center line.
5. Identify the possible causes and classify them into the categories in step 4. Create new categories, if necessary.
6. Rank order the causes to identify those that seem most likely to impact the problem.
7. Take corrective action.

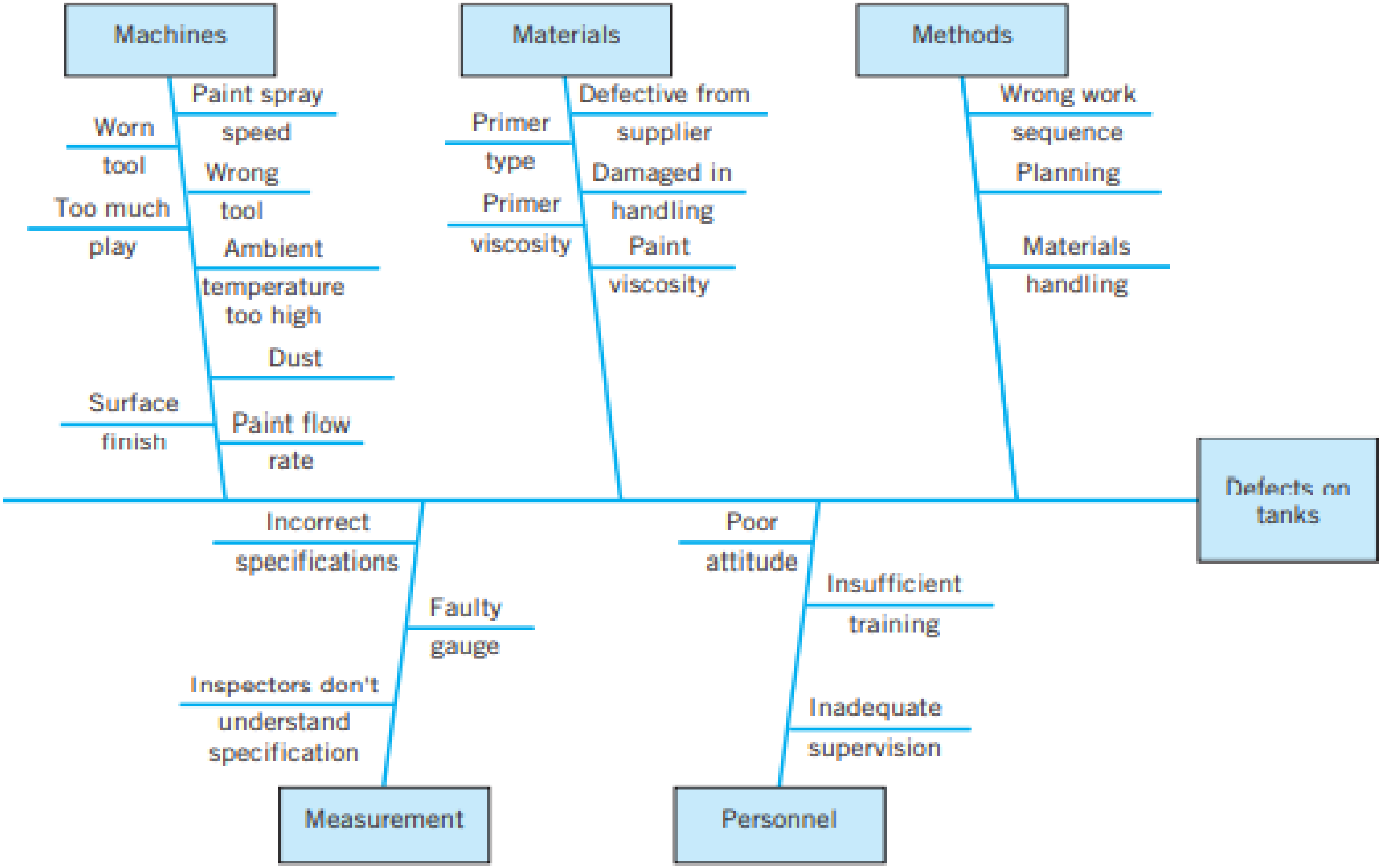


Figure 10. Cause-and-effect diagram for the tank defect problem.

Cause-and-Effect Diagram

- In analyzing the tank defect problem, the team elected to lay out the major categories of tank defects as machines, materials, methods, personnel, measurement, and environment.

- A brainstorming session ensued to identify the various sub causes in each of these major categories and to prepare the diagram in **Figure 10.** Then through discussion and the process of elimination, the group decided that materials and methods contained the most likely causes categories.

- Cause-and-effect analysis is an extremely powerful tool. A highly detailed cause-and effect diagram can serve as an effective troubleshooting aid.

- Furthermore, the construction of a cause-and-effect diagram as a team experience tends to get people involved in attacking a problem rather than in affixing blame.

Cause-and-Effect Diagram

Example .

One of the quality characteristics of interest in automobile tires is the bore size, which should be within certain specifications.

In a cause-and-effect diagram, the final bore size is the effect.

Some of the main causes that influence the bore size are the incoming material, mixing process, tubing operation, splicing, press operation, operator, and measuring equipment.

For each main cause, sub causes are identified and listed. For the raw material category, the incoming quality is affected by such sub causes as vendor selection process(e.g., is the vendor certified?), the content of scrap tire in the raw material, the density, and the ash content.

Construct a Cause-and-Effect Diagram.

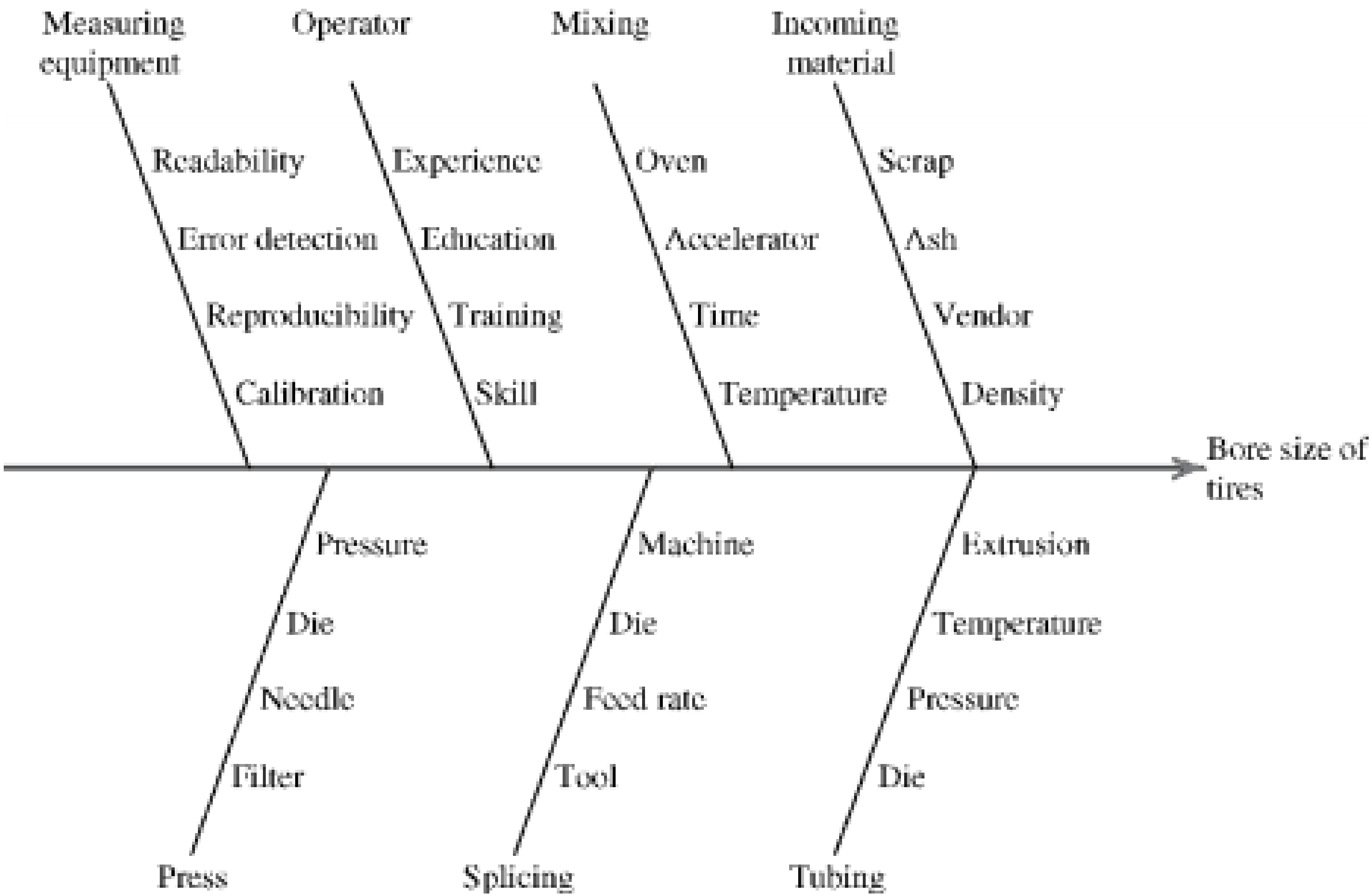


Figure 11. Cause-and-effect diagram for the bore size of Tyres.

Defect Concentration Diagram

- A defect concentration diagram is a picture of the unit, showing all relevant views.

- Then the various types of defects are drawn on the picture, and the diagram is analyzed to determine whether the location of the defects on the unit conveys any useful information about the potential causes of the defects.

- Defect concentration diagrams are very useful in the analyze step of DMAIC.

- **Figure 12** presents a defect concentration diagram for the final assembly stage of a refrigerator manufacturing process.

- Surface-finish defects are identified by the dark shaded areas on the refrigerator.

- From inspection of the diagram it seems clear that materials handling is responsible for the majority of these defects.

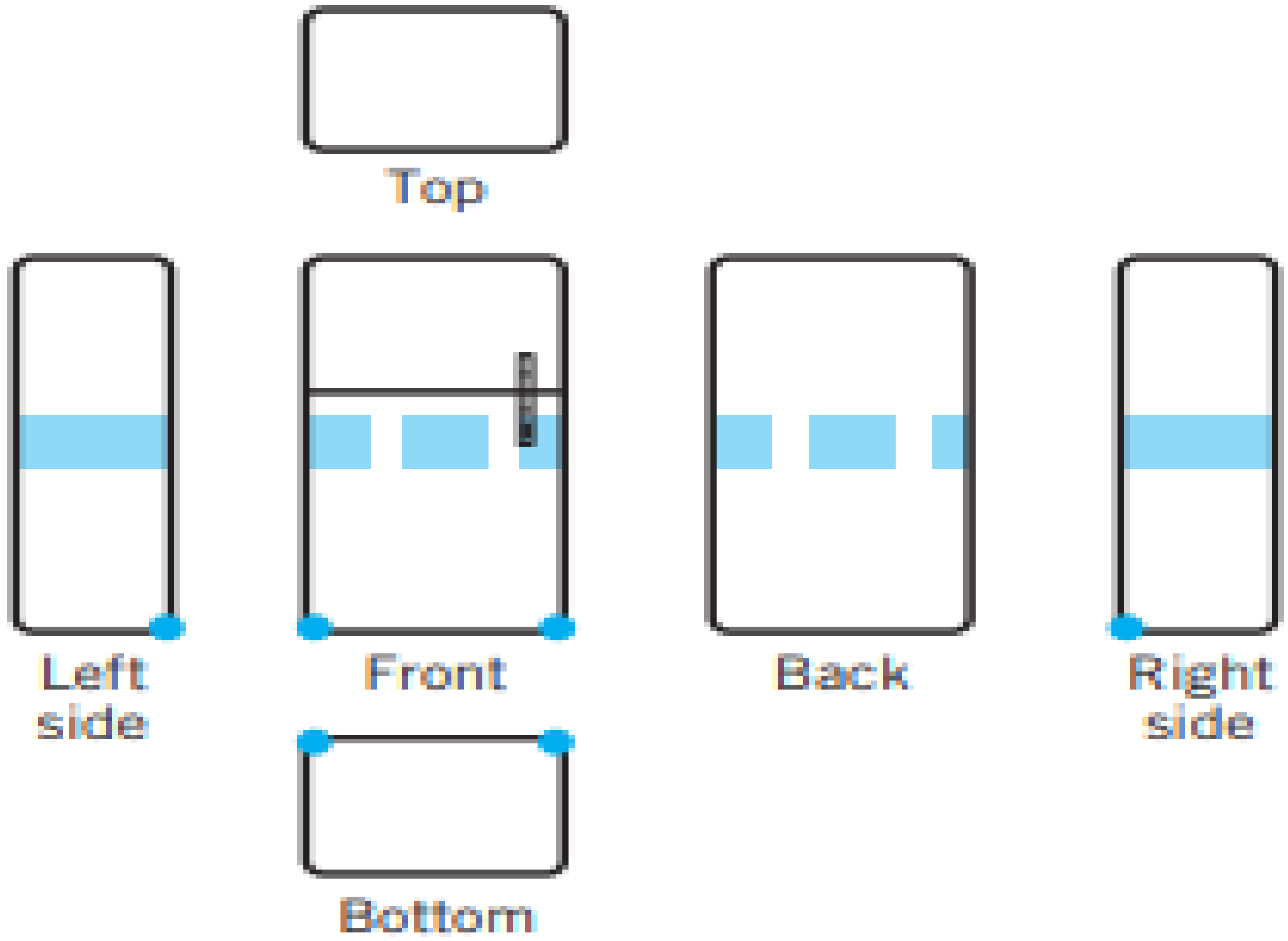


Figure 12. Surface-finish defects on a refrigerator.

Defect Concentration Diagram

- The unit is being moved by securing a belt around the middle, and this belt is either too loose or tight, worn out, made of abrasive material, or too narrow. Furthermore, when the unit is moved the corners are being damaged.
- It is possible that worker fatigue is a factor.
- In any event, proper work methods and improved materials handling will likely improve this process dramatically.
- **Figure 13** shows the defect concentration diagram for the tank problem mentioned earlier. Note that this diagram shows several different broad categories of defects, each identified with a specific code. Often different colors are used to indicate different types of defects.

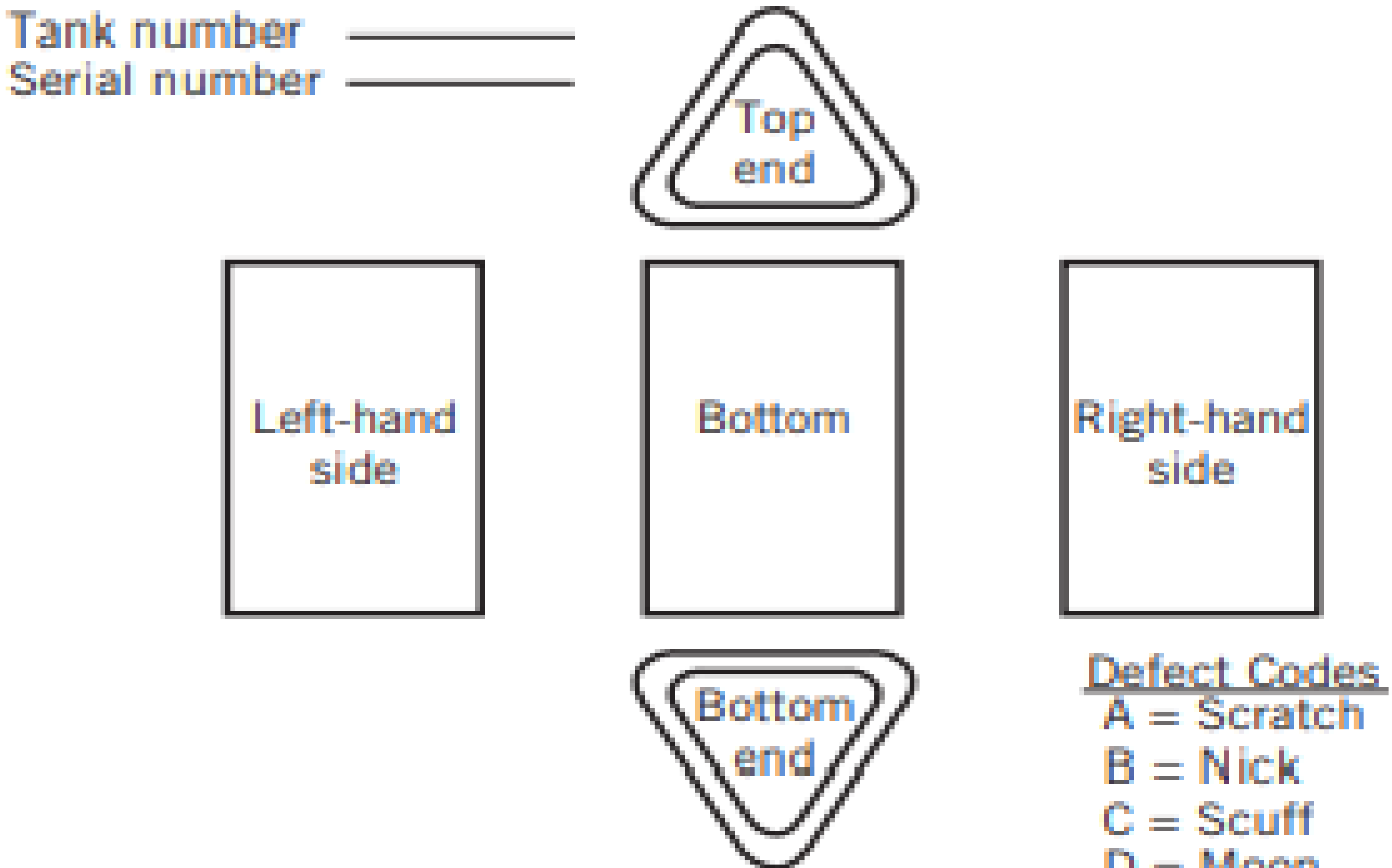


Figure 13. Defect Concentration Diagram for the Tank

Defect Concentration Diagram

- When defect data are portrayed on a defect concentration diagram over a sufficient number of units, patterns frequently emerge, and the location of these patterns often contains much information about the causes of the defects.

- We have found defect concentration diagrams to be important problem-solving tools in many industries, including plating, painting and coating, casting and foundry operations, machining, and electronics assembly.

Scatter Diagram

- The scatter diagram is a useful plot for identifying a potential relationship between two variables.

- Data are collected in pairs on the two variables—say, (y_i, x_i)—for i = 1, 2, . . . , n. Then y_i is plotted against the corresponding x_i .

- The shape of the scatter diagram often indicates what type of relationship may exist between the two variables.

- **Figure 14** shows a scatter diagram relating metal recovery (in percent) from a magnathermic smelting process for magnesium against corresponding values of the amount of reclaim flux added to the crucible.

- The scatter diagram indicates a strong positive correlation between metal recovery and flux amount; that is, as the amount of flux added is increased, the metal recovery also increases.

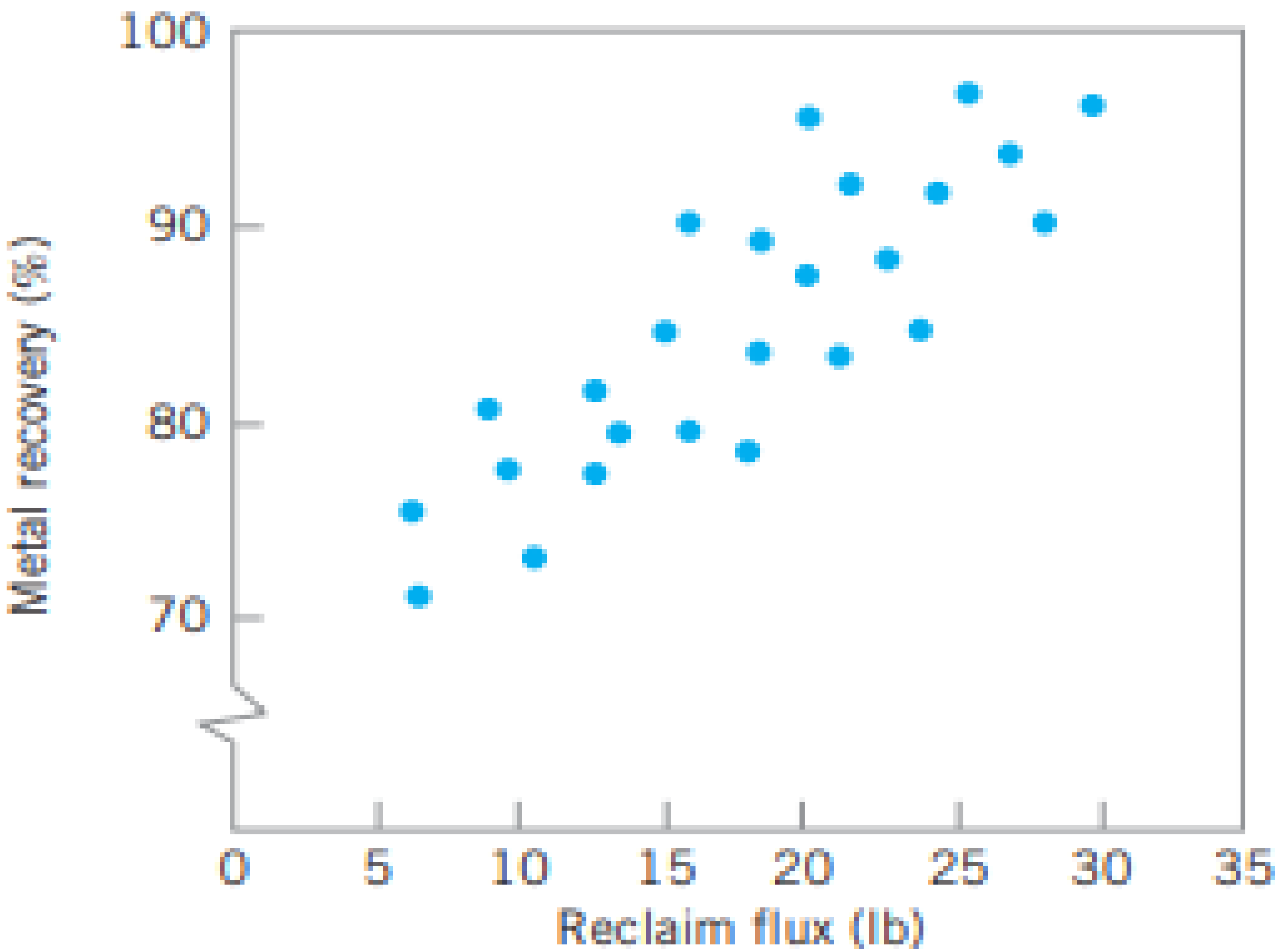


Figure 14. A Scatter Diagram

Scatter Diagram

- It is tempting to conclude that the relationship is one based on cause and effect: By increasing the amount of reclaim flux used, we can always ensure high metal recovery.
- This thinking is potentially dangerous, because correlation does not necessarily imply causality. This apparent relationship could be caused by something quite different. For example, both variables could be related to a third one, such as the temperature of the metal prior to the reclaim pouring operation, and this relationship could be responsible for what we see in **Figure 14**.
- If higher temperatures lead to higher metal recovery and the practice is to add reclaim flux in proportion to temperature, adding more flux when the process is running at low temperature will do nothing to enhance yield.
- The scatter diagram is useful for identifying potential relationships. Designed experiments must be used to verify causality.

Scatter Diagram

Example

Suppose that we are interested in determining the relationship between the depth of cut in a milling operation and the amount of tool wear.

We take 40 observations from the process such that the depth of cut (in millimeters) is varied over a range of values and the corresponding amount of tool wear (also in millimeters) over 40 operation cycles is noted.

The data values are shown in **Table 7**.

Construct a Scatter Plot for the data provided at **Table 7**.

Table 7. Data on Depth of Cut and Tool Wear

Observation	Depth of Cut (mm)	Tool Wear (mm)	Observation	Depth of Cut (mm)	Tool Wear (mm)
1	2.1	0.035	21	5.6	0.073
2	4.2	0.041	22	4.7	0.064
3	1.5	0.031	23	1.9	0.030
4	1.8	0.027	24	2.4	0.029
5	2.3	0.033	25	3.2	0.039
6	3.8	0.045	26	3.4	0.038
7	2.6	0.038	27	2.8	0.040
8	4.3	0.047	28	2.2	0.031
9	3.4	0.040	29	2.0	0.033
10	4.5	0.058	30	2.9	0.035
11	2.6	0.039	31	3.0	0.032
12	5.2	0.056	32	3.6	0.038
13	4.1	0.048	33	1.9	0.032
14	3.0	0.037	34	5.1	0.052
15	2.2	0.028	35	4.7	0.050
16	4.6	0.057	36	5.2	0.058
17	4.8	0.060	37	4.1	0.048
18	5.3	0.068	38	4.3	0.049
19	3.9	0.048	39	3.8	0.042
20	3.5	0.036	40	3.6	0.045

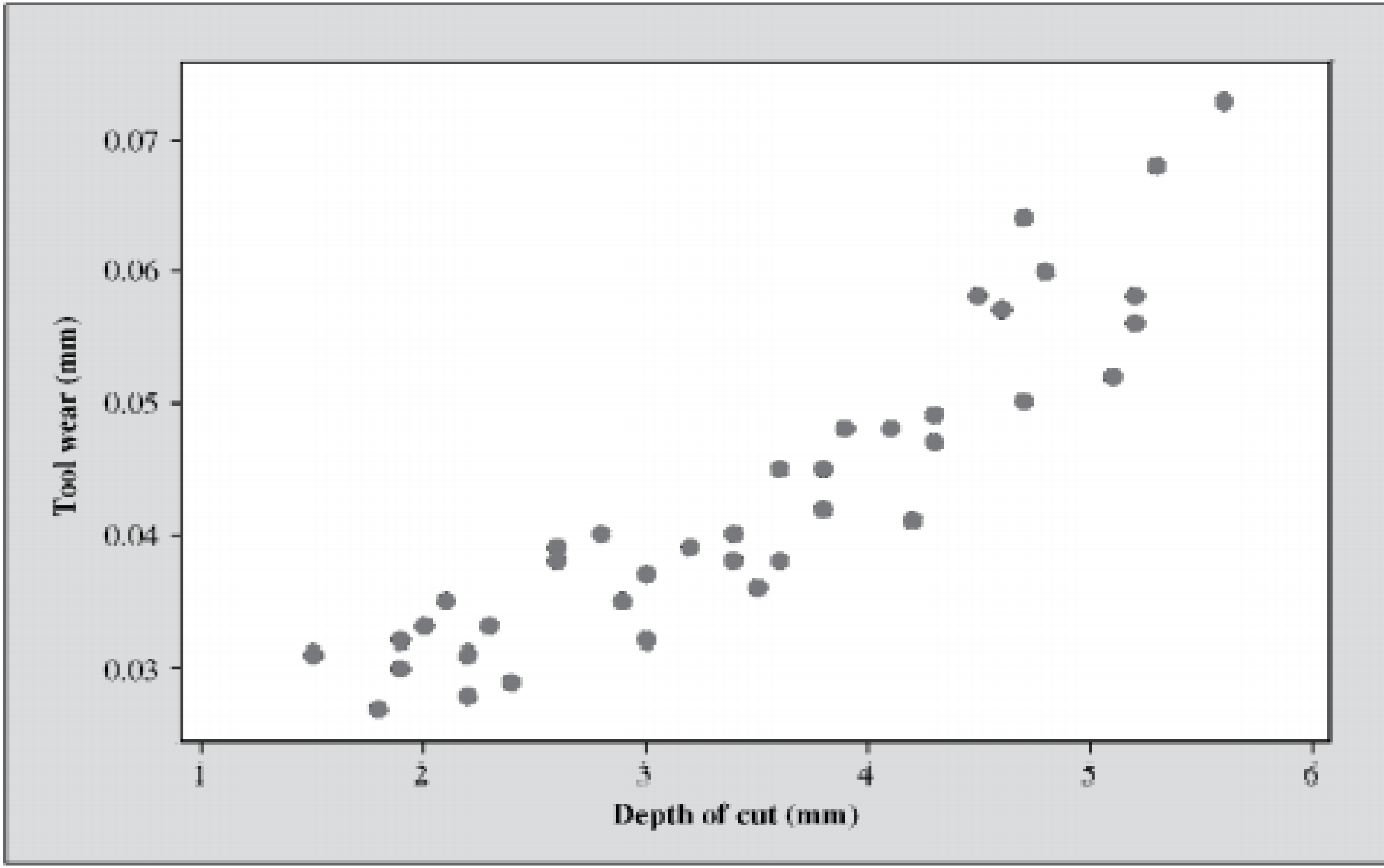


Figure 15. Scatterplot of tool wear versus depth of cut.

Scatter Diagram

- **Figure 15** gives us an idea of the relationship that exists between depth of cut and amount of tool wear.

- In this case the relationship is generally nonlinear. For depth-of-cut values of less than 3.0 mm, the tool wear rate seems to be constant, whereas with increases in depth of cut, tool wear starts increasing at an increasing rate.

- For depth-of-cut values above 4.5 mm, tool wear appears to increase drastically.

- This information will help us determine the depth of cut to use to minimize downtime due to tool changes

Multivariate Charts

- In most manufacturing or service operations, there are usually several variables or attributes that affect product or service quality. Since realistic problems usually have more than two variables, multivariable charts are useful means of displaying collective information.

- Several types of multivariate charts are available .One of these is known as a radial plot, or star, for which the variables of interest correspond to different rays emanating from a star. The length of each ray represents the magnitude of the variable.

Multivariate Charts

- Suppose that the controllable variables in a process are temperature, pressure, manganese content, and silicon content.
- **Figure 16** shows radial plots, or stars, for two samples of size 10 taken an hour apart.
- The sample means for the respective variables are calculated.
- These are represented by the length of the rays. A relative measure of quality performance is used to locate the center of a star vertically (in this case, the percentage of nonconforming product), while the horizontal axis represents the two sampling times.

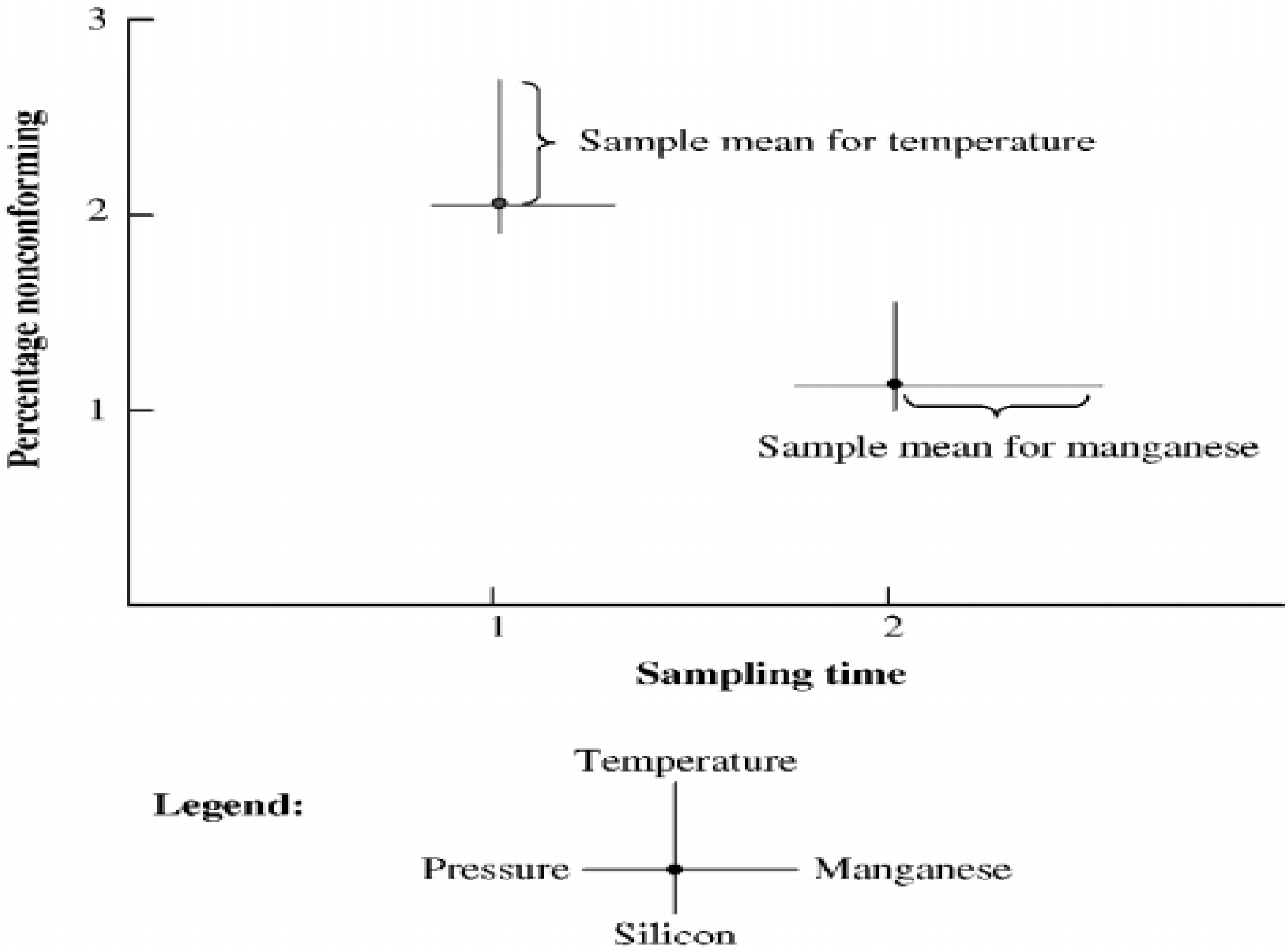


Figure 16. Radial plot of multiple variables.

Multivariate Charts

- Several process characteristics can be observed from the Figure.
- First, from time 1 to time 2, an improvement in the process performance is seen, as indicated by a decline in the percentage nonconforming.
- Next, we can examine what changes in the controllable variables led to this improvement. We see that a decrease in temperature, an increase in both pressure and manganese content, and a basically constant level of silicon caused this reduction in the percentage nonconforming.
- Various other forms of multivariable plots.

Matrix Plots

- The matrix plot is a graphical option for situations with more than two variables. This plot depicts two-variable relationships between a number of variables all in one plot. As a two-dimensional matrix of separate plots, it enables us to conceptualize relationships among the variables.

- Consider the data shown in **Table 8** on temperature, pressure, and seal strength of plastic packages. Since temperature and pressure are process variables, we want to investigate their impact on seal strength, a product characteristic.

Table 8. Data on Temperature, Pressure, and Seal Strength for Plastic Packages

Observation	Temperature	Pressure	Seal Strength	Observation	Temperature	Pressure	Seal Strength
1	180	80	8.5	16	220	40	11.5
2	190	60	9.5	17	250	30	10.8
3	160	80	8.0	18	180	70	9.3
4	200	40	10.5	19	190	75	9.6
5	210	45	10.3	20	200	65	9.9
6	190	50	9.0	21	210	55	10.1
7	220	50	11.4	22	230	50	11.3
8	240	35	10.2	23	200	40	10.8
9	220	50	11.0	24	240	40	10.9
10	210	40	10.6	25	250	35	10.8
11	190	60	8.8	26	230	45	11.5
12	200	70	9.8	27	220	40	11.3
13	230	50	10.4	28	180	70	9.6
14	240	45	10.0	29	210	60	10.1
15	240	30	11.2	30	220	55	11.1

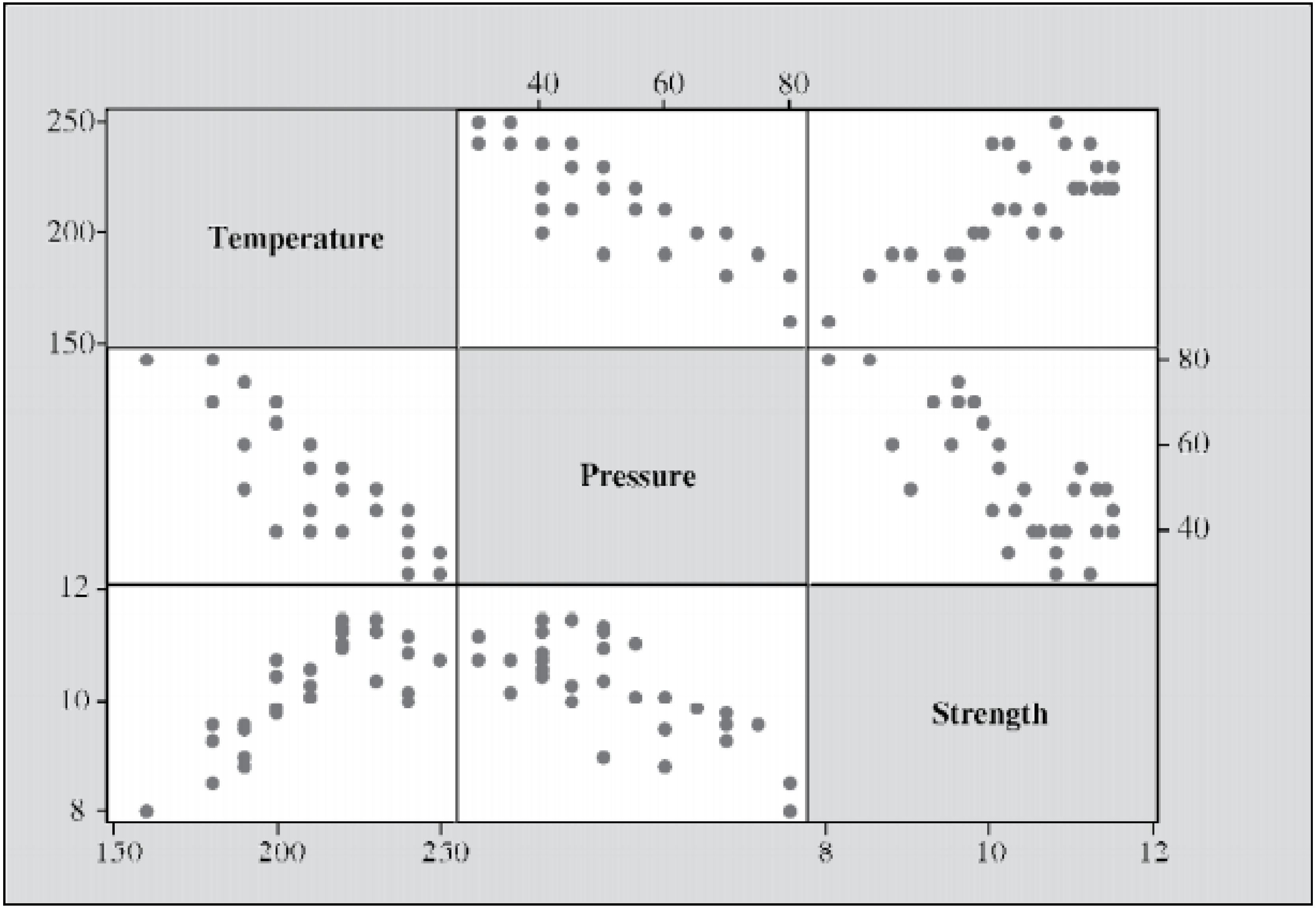


Figure 17. Matrix plot of strength, temperature, and pressure of plastic packages

Matrix Plots

- The seal strength tends to increase linearly with temperature up to a certain point, which is about 210°C. Beyond 210°C, seal strength tends to decrease.
- The relationship between seal strength and pressure decreases with pressure.
- Also, the existing process conditions exhibit a relationship between temperature and pressure that decreases with pressure.
- Such graphical aids provide us with some insight on the relationship between the variables, taken two at a time.

Note: The variables (Temperature, Pressure and Strength are on Y-axis on that particular row and on X-axis on that particular column.)

Control Charts

- A control chart is a graphical tool for monitoring the activity of an ongoing process.

- The values of the quality characteristic are plotted along the vertical axis, and the horizontal axis represents the samples, or subgroups (in order of time), from which the quality characteristic is found **(Figure 18)**

- Examples of quality characteristics include average length, average diameter, average tensile strength, average resistance, and average service time. These characteristics are variables, and numerical values can be obtained for each.

- The term attribute applies to such quality characteristics as the proportion of nonconforming items, the number of nonconformities in a unit, and the number of demerits per unit.

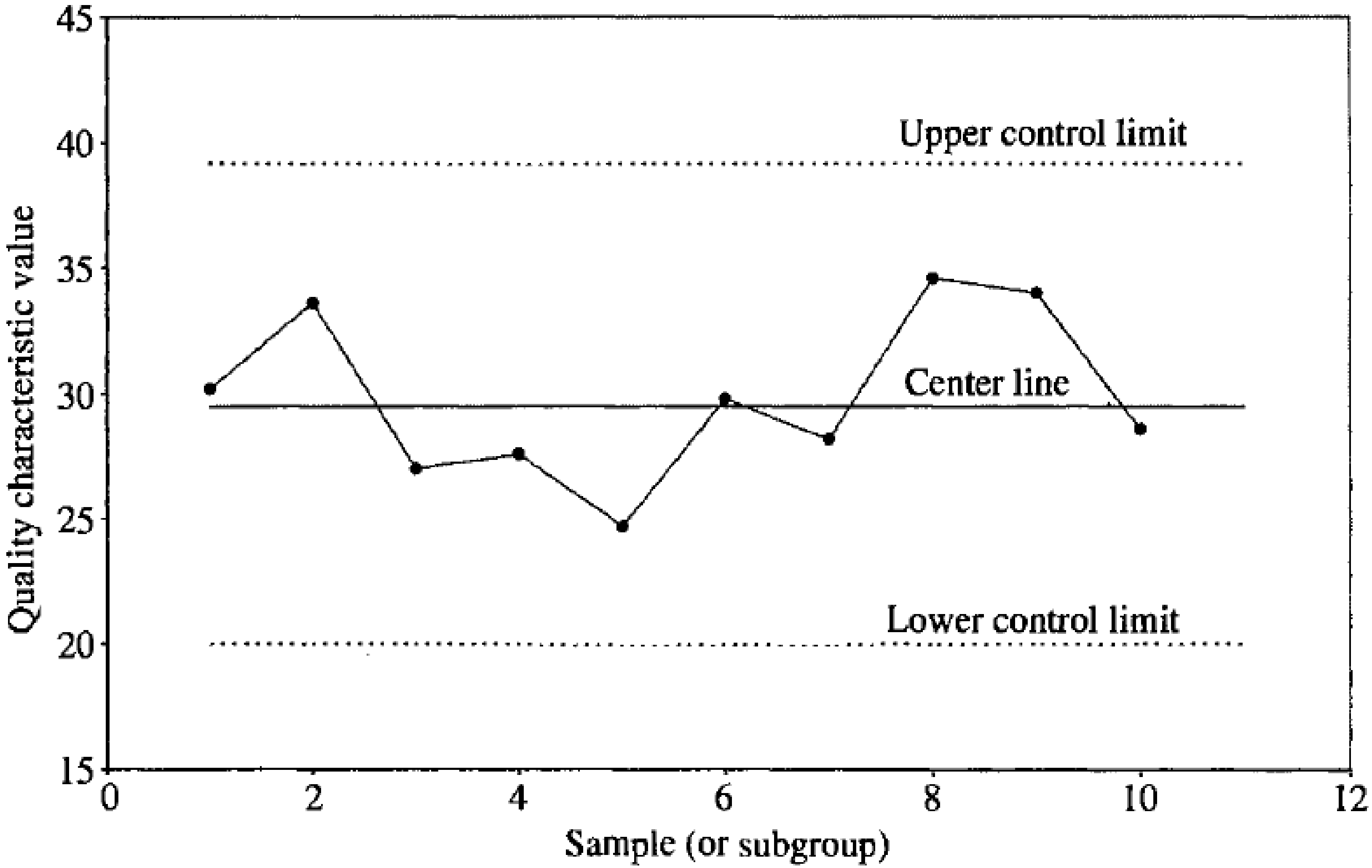


Figure 18. Typical Control chart.

Control Charts

- Three lines are indicated on the control chart. The centerline, which typically represents the average value of the characteristic being plotted, is an indication of where the process is centered.

- Two limits, the upper control limit and the lower control limit, are used to make decisions regarding the process.

- If the points plot within the control limits and do not exhibit any identifiable pattern, the process is said to be in statistical control.

- If a point plots outside the control limits or if an identifiable nonrandom pattern exists (such as 12 out of 14 successive points plotting above the centerline), the process is said to be out of statistical control.

Control Charts indicate the following:

1. **When to take corrective action.** A control chart indicates when to initiate corrective actions .

2. **Type of remedial action necessary.** The pattern of the plot on a control chart diagnoses possible causes and hence indicates possible remedial actions.

3. **When to leave a process alone.** Variation is part of any process. A control chart shows when an exhibited variability is normal and inherent such that no corrective action is necessary. Inappropriate over control through frequent adjustments only increases process variability.

4.Process capability. If the control chart shows a process to be in statistical control, we can estimate the capability of the process and hence its ability to meet customer requirements. This helps product and process design.

5.Possible means of quality improvement. The control chart provides a baseline for instituting and measuring quality improvement. Control charts also provide useful information regarding actions to be taken for quality improvement.

Control Charts

Variability

- Variability is a part of any process, no matter how sophisticated.
- Several factors over which we have some control, such as methods, equipment, people, materials, and policies, influence variability. Environmental factors also contribute to variability.
- The causes of variation can be subdivided into two groups: common causes and special causes.
- Control of a process is achieved through the elimination of special causes. Improvement of a process is accomplished through the reduction of common causes.

Special Causes

- Variability caused by special or assignable causes is something that is not inherent in the process.
- Special causes can be the use of a wrong tool, an improper raw material, or an incorrect procedure.
- If an observation falls outside the control limits or a nonrandom pattern is exhibited, special causes are assumed to exist, and the process is said to be out of control.

Special Causes

- One objective of a control chart is to detect the presence of special causes as soon as possible to allow appropriate corrective action.
- Once the special causes are eliminated through remedial actions, the process is again brought to a state of statistical control.

Common Causes

- Variability due to common or chance causes is something inherent to a process. It exists as long as the process is not changed and is referred to as the natural variation in a process.
- It is an inherent part of the process design and effects all items. This variation is the effect of many small causes and cannot be totally eliminated.
- When this variation is random, we have what is known as a stable system of common causes. A process operating under a stable system of common causes is said to be in statistical control.

Control Charts

Common Causes

- Examples include inherent variation in incoming raw material from a qualified vendor, the vibration of machines, and fluctuations in working conditions.

- In a control chart, if quality characteristic values are within control limits and no nonrandom pattern is visible, it is assumed that a system of common causes exists and that the process is in a state of statistical control.

Statistical Basis for Control Charts

- A control chart has a centerline and lower and upper control limits.

- The centerline is usually found in accordance with the data in the samples. It is an indication of the mean of a process and is usually found by taking the average of the values in the sample. However, the centerline can also be a desirable target or standard value.

- Normal distributions play an important role in the use of control charts . The values of the statistic plotted on a control chart (e.g., average diameter) are assumed to have an approximately normal distribution.

Statistical Basis for Control Charts

- The control limits are two lines, one above and one below the centerline, that aid in the decision-making process. These limits are chosen so that the probability of the sample points falling between them is almost 1 (usually about 99.7% for 3σ limits) if the process is in statistical control.

- If a point falls outside the control limits, there is a reason to believe that a special cause exists in the system. We must then try to identify the special cause and take corrective action to bring the process back to control.

Selection of Control Limits

- Let θ represent a quality characteristic of interest and $\hat{\theta}$ represent an estimate of θ.

- Let $E\left(\hat{\theta}\right)$ represent the mean, or expected value, and $\sigma\left(\hat{\theta}\right)$ be the standard deviation of the estimator .

- The centerline and control limits for this arrangement are given by:

Selection of Control Limits

$$CL = E\left(\hat{\theta}\right)$$
$$UCL = E\left(\hat{\theta}\right) + k\sigma\left(\hat{\theta}\right)$$
$$LCL = E\left(\hat{\theta}\right) - k\sigma\left(\hat{\theta}\right)$$

- where k represents the number of standard deviations of the sample statistic that the control limits are placed from the centerline.

- Typically, the value of k is chosen to be 3 (hence the name 3σ limits). If the sample statistic is assumed to have an approximately normal distribution, a value of k = 3 implies that there is a probability of only 0.0026 of a sample statistic falling outside the control limits if the process is in control.

Errors in Making Inferences from Control Charts

- Types I and II errors can occur when making inferences from control charts.

Type I Errors

- result from inferring that a process is out of control when it is actually in control. The probability of a type I error is denoted by α.

- Suppose that a process is in control. If a point on the control chart falls outside the control limits, we assume that the process is out of control. However, since the control limits are a finite distance (usually, 3 standard deviations) from the mean, there is a small chance (about 0.0026) of a sample statistic falling outside the control limits. In such instances, inferring that the process is out of control is a wrong conclusion.

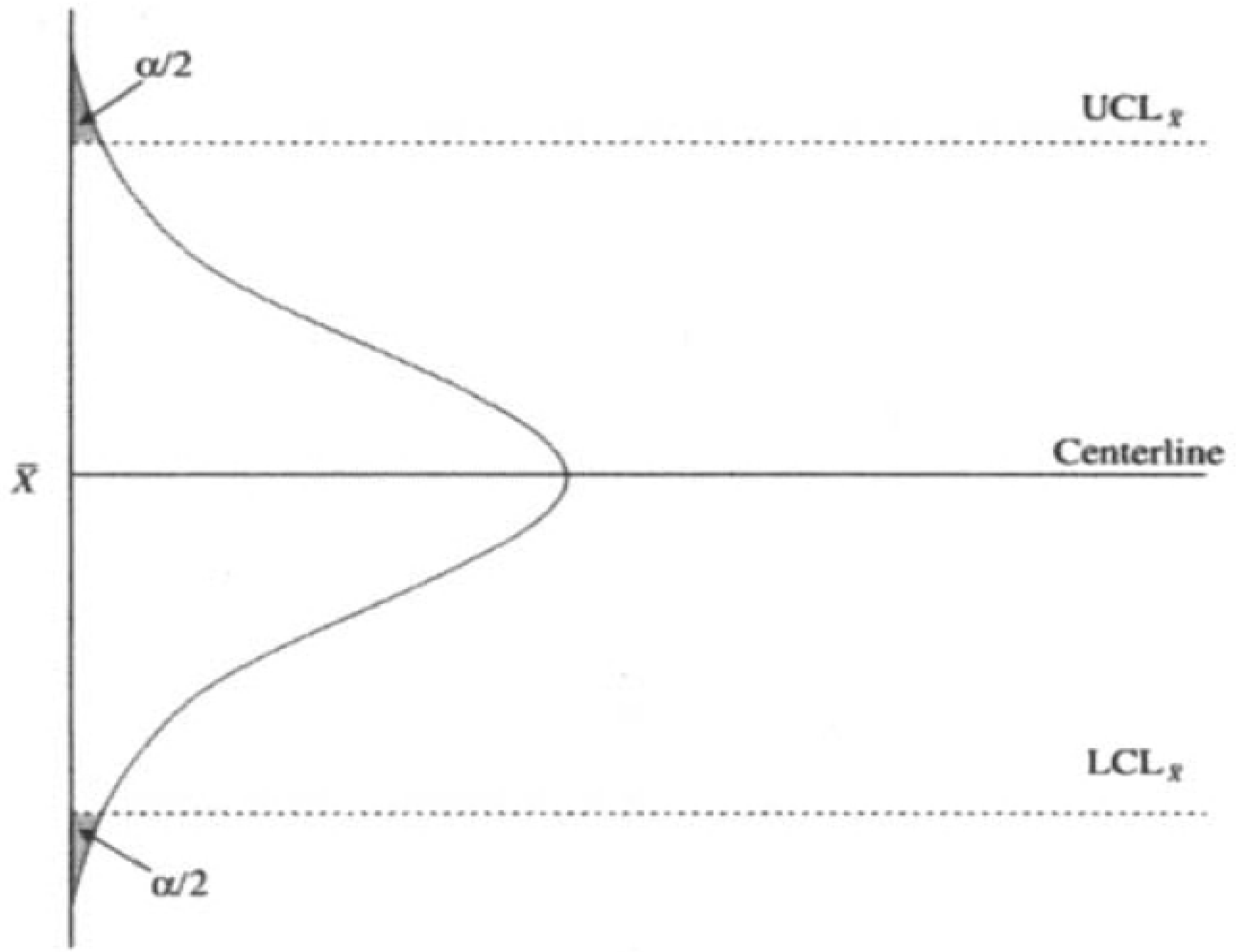


Figure 19. Type I error in control charts.

Type II Errors

- result from inferring that a process is in control when it is really out of control.
- If no observations fall outside the control limits, we conclude that the process is in control.
- Suppose, however, that a process is actually out of control. Perhaps the process mean has changed (say, an operator has inadvertently changed a depth of cut or the quality of raw materials has decreased). Or, the process could go out of control because the process variability has changed (due to the presence of a new operator). Under such circumstances, a sample statistic could fall within the control limits, yet the process would be out of control—this is a type II error.

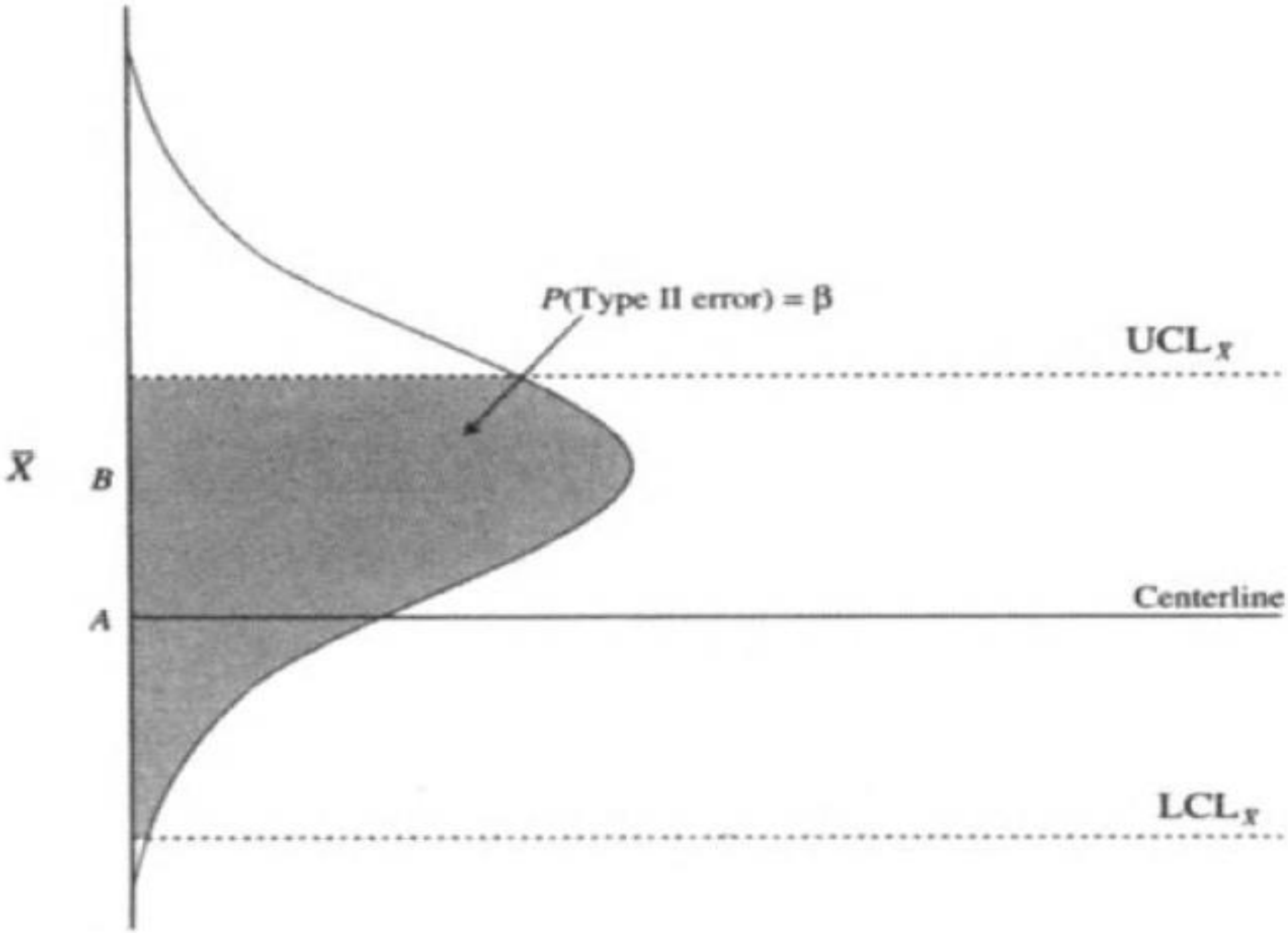


Figure 20. Type II error in control charts.

Analysis of Patterns in Control Charts

- **Criteria other than a plotted point falling outside the control limits are also used to determine whether a process is out of control.**

Some Rules for Identifying an Out-of-Control Process

Rule 1 : A process is assumed to be out of control if a single point plots outside the control limits.

- **This is the most commonly used rule. If the control limits are placed at 3 standard deviations from the mean of the quality characteristic being plotted (assuming a normal distribution), the probability of a point falling outside these limits if the process is in control is very small (about 0.0026). Figure 21 in the next slide depicts this situation.**

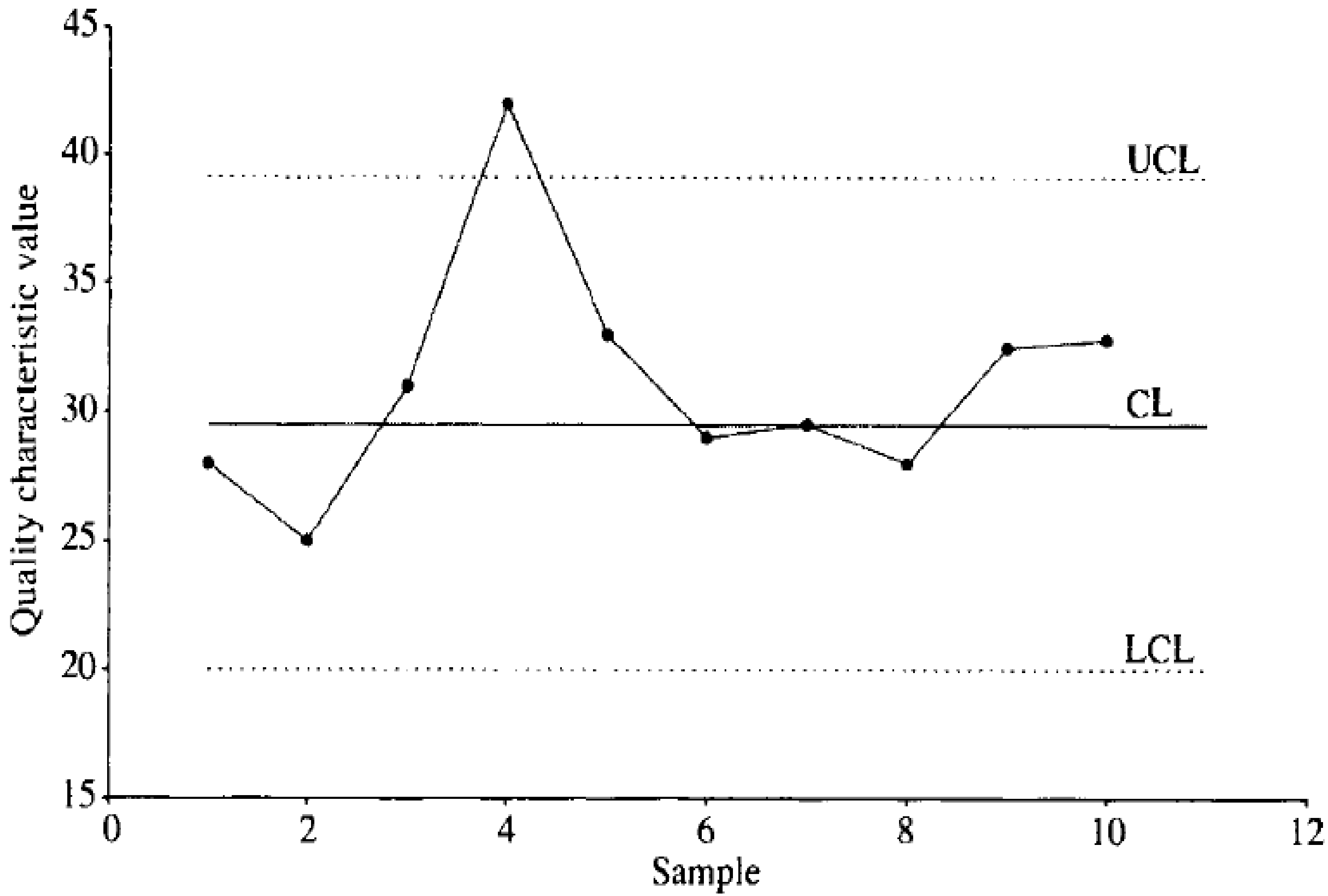


Figure 21. Out-of-control patterns: Rule 1

Rule 2: **A process is assumed to be out of control if two out of three consecutive points fall outside the 2σ warning limits on the same side of the centerline.**

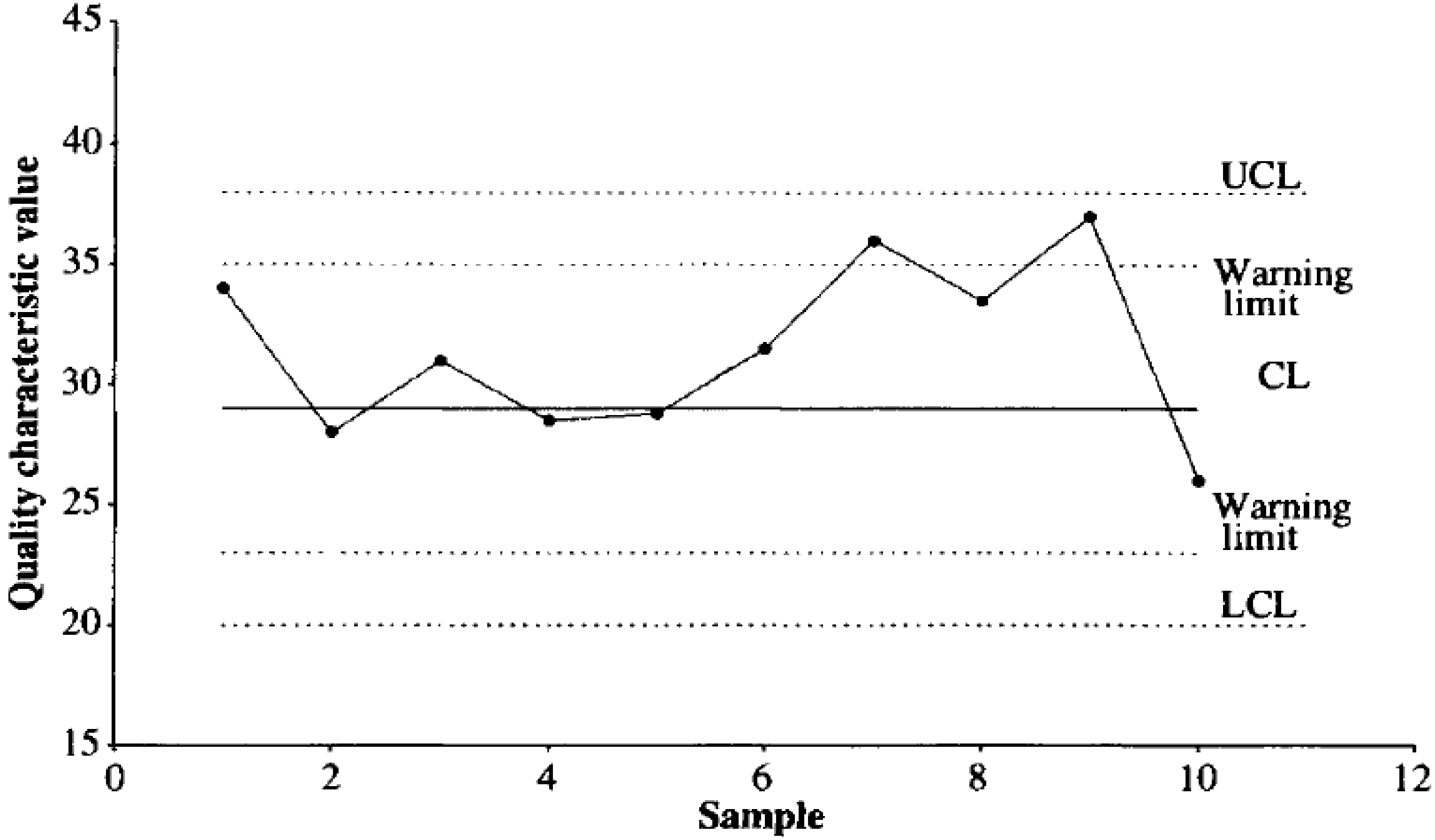


Figure 22. Out-of-control patterns: Rule 2

Rule 3 :A process is assumed to be out of control if four out of five consecutive points fall beyond the 1σ limit on the same side of the centerline.

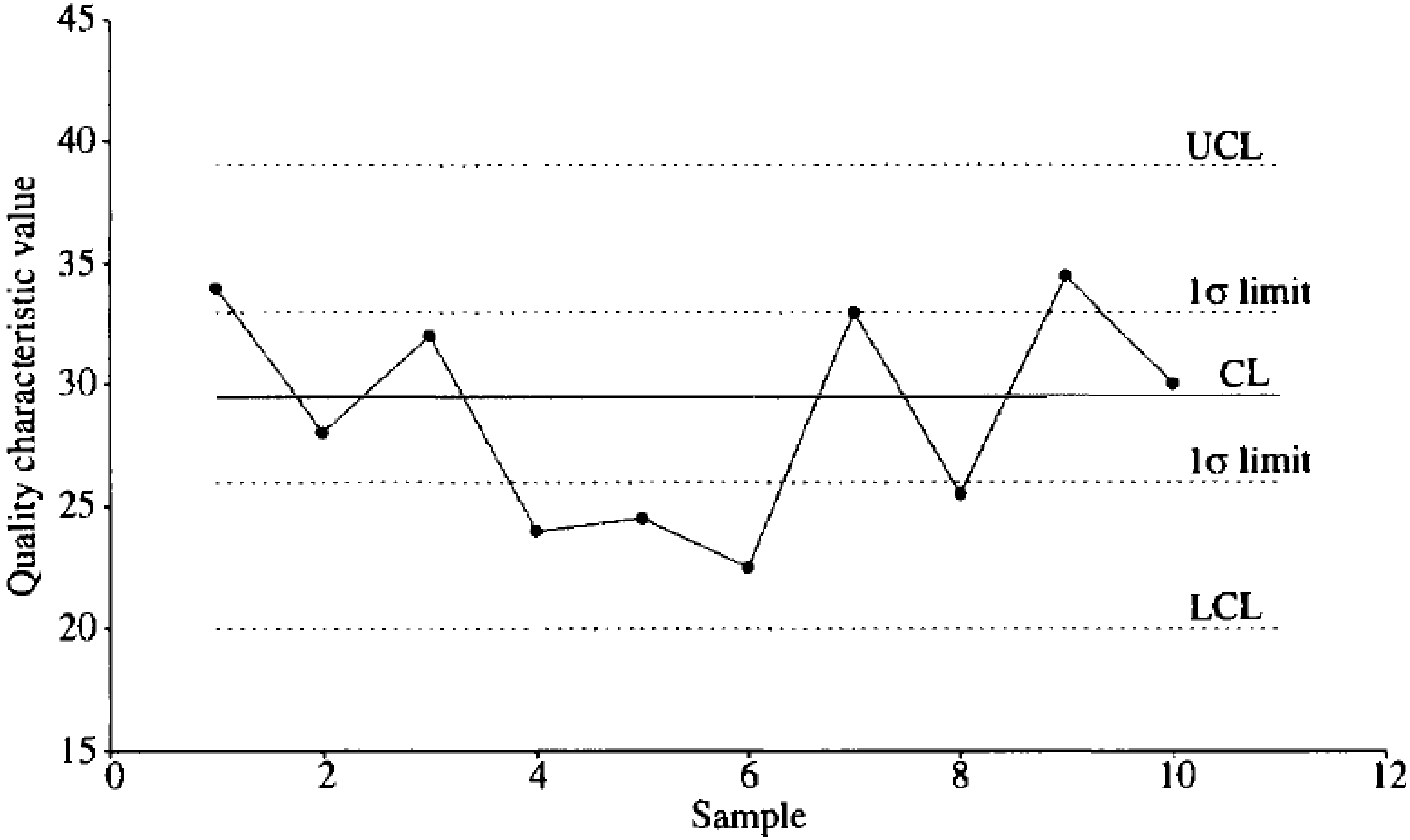


Figure 23. Out-of-control patterns: Rule 3

Rule 4: **A process is assumed to be out of control if nine or more consecutive points fall to one side of the centerline.**

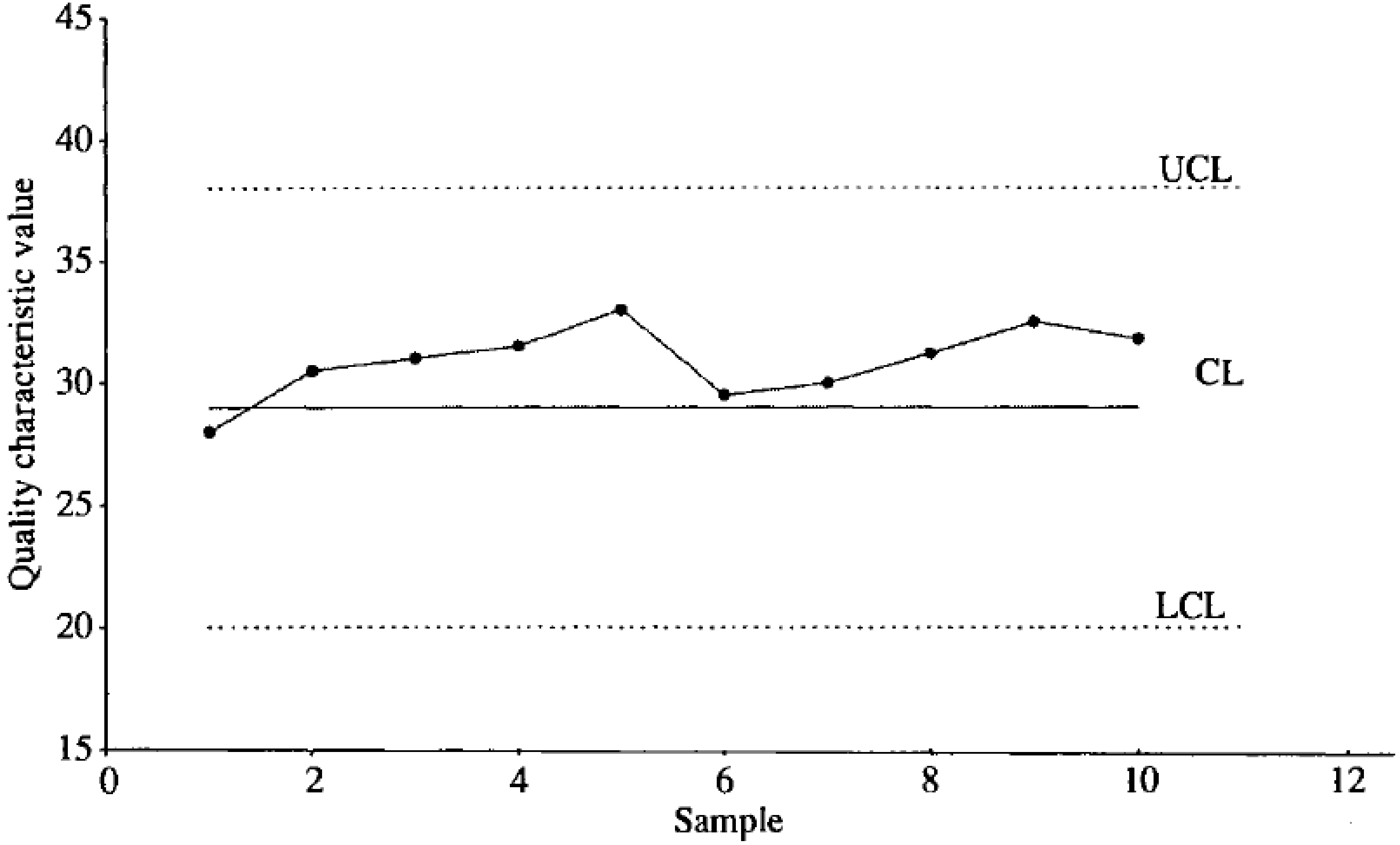


Figure 24. Out-of-control patterns: Rule 4

Rule 5: **A process is assumed to be out of control if there is a run of six or more consecutive points steadily increasing or decreasing.**

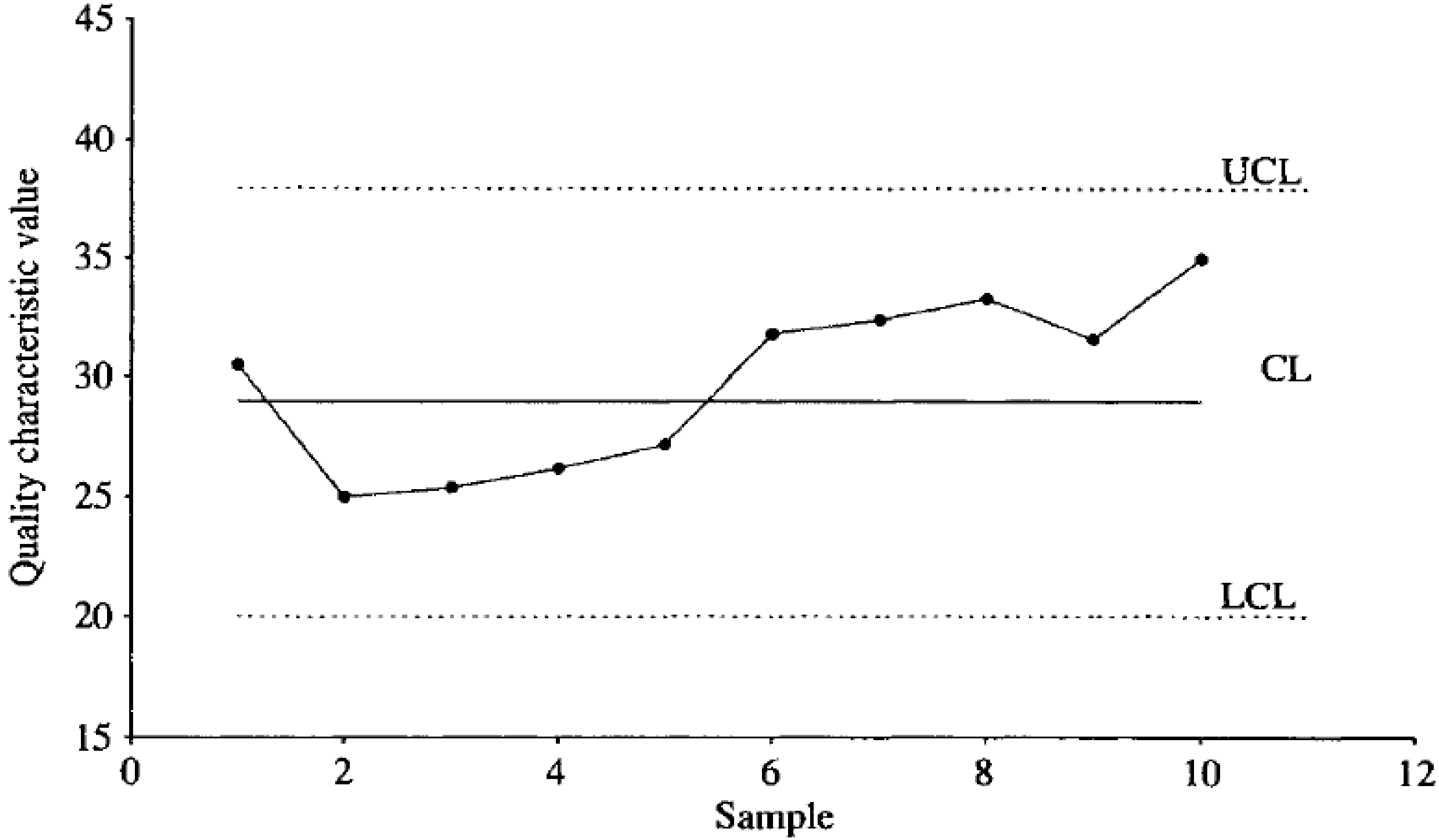


Figure 25. Out-of-control patterns: Rule 5

Recommended Books:

1. D.C. Montgomery, Introduction to statistical Quality Control
2. Amitava Mitra, Fundamentals of Quality Control and Improvement.
3. J Banks, Principles of Quality Control
4. E.L.Grant and R.S. Leavenworth, Statistical Quality Control