A Predictive Formula for the (n+1) Prime Number Using Logarithmic Corrections and Error Modeling

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Abstract

We present a novel approach to approximating the (n+1)th prime number using a recursive logarithmic formula with an adaptive error correction. The technique, called the Mishra Prime Estimator, provides accurate results for a wide range of prime indices. By introducing an analytical error model, we refine prime prediction and propose future directions potentially relevant to the Riemann Hypothesis.

1. Introduction

Prime numbers are fundamental to number theory and cryptography, but no exact closed-form formula exists for generating the nth prime. The Mishra Prime Estimator attempts to bridge this gap by predicting the (n+1)th prime using the nth prime, logarithmic transformations, and a modelled error term. This approach may serve as a useful tool in mathematical exploration and theory.

2. Base Formula (Mishra Prime Estimator)

We define the recursive formula as:

$$P_{n+1} \quad P_n + \mathrm{floor}(\log(P_n) + (\log(n)/n) * c) + \mathrm{Error}(n)$$

Where:

- P_n is the nth prime number,
- c is a constant derived empirically,
- $\mathrm{Error}(n)$ is a correction function.

3. Error Correction Model

An analytical error function was derived by comparing predicted values with actual prime values for n = 1 to 50:

Error(n) 9.655 * log(n) - 13.651

4. Combined Formula

Using both components, the improved predictive formula is:

P_n+1 P_n + floor(log(P_n) + (log(n)/n) * c) + (9.655 * log(n) - 13.651)

5. Python Implementation

```
import math


def mishra_prime_estimator(p_n, n, c=1.0):
    term1 = math.log(p_n)
    term2 = (math.log(n) / n) * c
    error = 9.655 * math.log(n) - 13.651
    return round(p_n + math.floor(term1 + term2) + error)
```

6. Graphical Results

Graphs show prediction vs actual primes from n = 1 to 50.

[Graph Placeholder: Actual vs Predicted Prime Number Plot]

7. Connection to the Riemann Hypothesis

The distribution of primes is deeply connected to the non-trivial zeros of the Riemann zeta function. Though this formula does not prove the hypothesis, it captures oscillatory and logarithmic behavior reminiscent of the prime counting function pi(x) and Chebyshev's estimates.

8. Conclusion

The Mishra Prime Estimator, coupled with a data-driven error model, shows strong performance in predicting prime numbers. Future refinements might incorporate complex logarithmic or oscillatory components. This could offer insights relevant to unsolved problems like the Riemann Hypothesis.

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10. Author Contact

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