AI Lab CS236 Lab2

1. Create two matrics, P and Q, each of size 10^6 X 10^4 with random values. Perform the following.
    a. Matrix multiplication $P.Q^T$ using loops in Python.
    b. Vectorized matrix multiplication to compute $P.Q^T$.
    c. Calculate the speedup for operations a) and b):

        Speed up $= t1 \,/\, t2$

2. Assume any two vectors P and Q with random values. Do the following.
    a. Compute the euclidean distance between them.
       Hint: Euclidean distance between two vectors p and q are as follows:

$$d(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=1}^{n} (q_i - p_i)^2}$$

    b. Compute the Pearson correlation coefficient (PCC) between them.
       Hint: PCC between two vectors x and y can be computed using:

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

       where $\bar{x}$ and $\bar{y}$ are the mean of x and y.

3. Write a program to find the angle between the vectors u = (2, 1, 2) and v = (1, 1, 1) using the cosine formula - $cos\,\theta = \frac{u.v}{||u||\,||v||}$ and convert u and v into unit vectors.

4. Obtain two random matrices of integers, A and B. Find $\mathbf{A^TB}$ and eigenvalues of **AB** and **BA.**

    Hint: For a given matrix A,

$$(A - \lambda I)\mathbf{X} = \mathbf{0},$$

    Where λ is an Eigenvalue and X is an eigenvector.

5.
    a. Assume any two vectors X and Y with random values. Compute the Manhattan Distance (L1 distance) between them.
       Hint: The Manhattan Distance can be calculated as follows:

$$D(x, y) = \sum_{i=1}^{k} |x_i - y_i|$$

    b. Given an m x n matrix, return all matrix elements in spiral order.
       Hint: Input: matrix = [[1,2,3],[4,5,6],[7,8,9]]
       Output: [1,2,3,6,9,8,7,4,5]

6. Given a random variable x with n size sample space, Plot the probability distribution.
   Hint: The probability density function (pdf) for Normal Distribution can be calculated using

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi * \sigma^2}} * e^{-\frac{1}{2} * (\frac{x-\mu}{\sigma})^2}$$

Probability Density Function Of Normal Distribution

where, μ = Mean , σ = Standard deviation , x = input value.

7.
   a. Create a 1D array M with random values.
   b. Then calculate the Standard Deviation using the following formula:

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}$$

Here, x_i = Each value from the population, μ and Nis the Population Mean and N is the size of the population respectively.

8.
   a. Create two matrices X and Y with random values.
   b. Calculate the value of $X^{-1}Y$, X^TX and X^TY.

9. Create a random list of 15 variables and by using statistical formulas calculate Mean, Median, and Variance Standard deviation and verify your output using inbuilt python functions.
   Hint: $Mean(\bar{x}) = \frac{\Sigma x}{n}$
   $Median(x) = \frac{n}{2}, \frac{n}{2} + 1$, for even and odd numbers respectively.
   $Variance(\sigma^2) = \frac{\Sigma(x-\mu^2)}{N}$ μ = Mean, N = number of terms
   Standard deviation: formula from Q7.

10. Consider a 2-dimensional space having three points P1 (X1, Y1), P2 (X2, Y2), and P3 (X3, Y3), Find the Minkowski distance for p = 1, 2, 3, 4

    Hint: for given p, Minkowski distance can be calculated as follows:

    (|X1 – Y1|^p + |X2 – Y2|^p + |X3 – Y|^p) ^ 1/p

11. Implement normalizeRows() function to normalize the rows of a matrix. After applying this function to an input matrix x, each row of x should be a vector of unit length.

12. For the following two vectors.

    x_1 = [9, 2, 5, 0, 0, 7, 5, 0, 0, 0, 9, 2, 5, 0, 0]
    x_2 = [9, 2, 2, 9, 0, 9, 2, 5, 0, 0, 9, 2, 5, 0, 0]

    Find the following
    a. Vectorized dot product of vectors
    b. Vectorized outer product
    c. Vectorized elementwise multiplication
    d. Vectorized general dot product