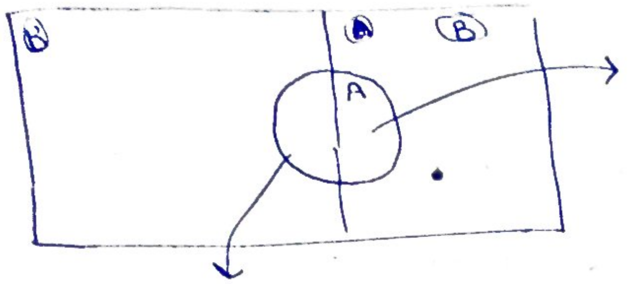
Ans 1. The total probability rule used to find the probability of an event, A, when we don't have enough information to calculate P(A) directly, Instead we use a related ~~event~~ event B to calculate P(A).




$$A = A \cap B + A \cap \bar{B}$$

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$

$$\boxed{P(A) = P(A|B)\,P(B) + P(A|\bar{B})\,P(\bar{B})}$$

| by conditional property

This is called theorem of total probability.

Ans 2 Let event space be

$$S = \{B_1, B_2, B_3, \ldots, B_n\}$$

Now, knowing the fact the A has occured we want to calculate the $P(B_j)$.

By def. of conditional prob. :-

$$P(B_j) = \frac{P(B_j \cap A)}{P(A)}$$

Applying total prob theorem :-

$$P(B_j) = \frac{P(A|B_j)\,P(B_j)}{\sum_i P(A|B_i)\,P(B_i)}.$$

This is called Bayesian theorem

(3) P - Prior (A) = 0.4

P - Prior (B) = 0.6

P (Find in A | A) = 0.25

P (Find in B | B) = 0.15

P(Find in B | A) = 0

P(Find in A | B) = 0

(1) P[Find in A] = P (find in A | A) * A - prior(A) + P (Find A | B)

= 0.25 * 0.4 + 0 = 0.1

P [Find in B] = 0.15 * 0.6 + 0 = ~~[illegible]~~ = 0.096

He Should look in forest A

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(2) P (A & NOT find in A) = A - prior (A) * P (Not Find in A | A)

= 0.4 * (1 - 0.25) = 0.3.

P (Not find in A) = 1 - 0.1 = 0.9

P(A | Not find in A) = P(A AND Not find in A) / P (Not Find in A)

= 0.3 / 0.9 = 1/3

(3) P (looked in A | found dog) = P (found dog | looked in A) * P(looked in A) / P(found dog)

$$= \frac{(.25 \times 0.4 + 0) \times 0.5}{(P(\text{found dog} \mid \text{looked in A}) \times P(\text{looked in A}) + P(\text{found dog} \mid \text{looked in B}) * P(\text{looked in B}))}$$

$$= \frac{0.05}{((0.25 * .4 + 0)\, 0.5 + (.15 \times 0.6 + 0)\, .5)} = .526$$

$$\boxed{= .526}$$