Answer 1.

Axiom 1 :-

$P(A) \geq 0$ for all ~~from~~ $A \subset S$

Probability of an event can range from 0 (min) to ∞ (exclusive). Probability can not be ∞.

Axiom 2 :-

$P(S) = 1$

Probability of entire sample space is 1.

Axiom 3 :-

$P(A \cup B) = P(A) + P(B)$ provided $A \cap B = \phi$

Answer 2 :-

A variable whose possible values are numerical outcome of a random phenomenon. They are two types :-

1. Discrete Random Variable :-

These variable can take only a countable no. of distinct values.

2. Continuous Random Variable :-

These variables can take infinite number of possible values. Random variables are ~~us~~ usually measurements like weight, air pressure, etc.

3. Pdf function of a continuous random variable X with support S is an integral function f(x) satisfying:-

1. f(x) > 0, for all x in S.
2. Area under the curve f(x) in support S is 1.

(4) Discrete distribution :-

1. Binomial distribution
2. Poisson distribution

Continuous distribution :-

1. ~~Dist A random variable~~ an A random no in between 1 and 6.
2. Time taken by a spaceship to reach celestial bodies in space

(5) Expectation E [x], of a continuous random variable X is defined by :-

$$E[x] = \int_{-\infty}^{\infty} x\, f(x)\, dx .$$

Here f(x) is the density function of X.

E [x] is the measure of location or central tendency. It is also known as mean or variance:

(6). Total area under standard normal distribution curve is always 1.

Since every normally distributed random variable has a different shape, we have to transform our variable so it has (μ) mean of 0 and σ of 1 using formula:

$$Z = \frac{X - \mu}{\sigma} .$$

Area can be Calculated using standard normal distribution table thereafter.

(7) Independent Variables are controlled or changed to test effect on dependent Variable.

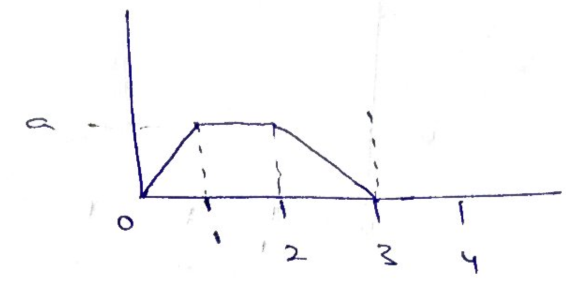
For example :- If a tutor wants to test maths competency of students. Then the difficulty of question is an independent Variables, while marks the score is dependent Variable

(8) Mutually exclusive and collectively exhaustive set Consititutes the entire sample space.

(9)

f(x) graphical representation :-

f(x) :



Total area under f(x) should be 1.

i.e, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

$$\Rightarrow \left[\frac{ax^2}{2}\right]_0^1 + [ax]_1^2 + \left[-\frac{ax^2}{2} + 3ax\right]_2^3 = 1$$

$$\Rightarrow \frac{a}{2} + a + \left[-\frac{9a}{2} + 9a + \frac{4a}{2} - 6a\right] = 1$$

$$\Rightarrow \frac{3a}{2} + \left[-\frac{5a}{2} + 3a\right] = 1$$

$$\Rightarrow \frac{3a}{2} - \frac{5a}{2} + \frac{6a}{2} = 1$$

$$\Rightarrow 4a = 2$$

$$a = \frac{1}{2}$$

(b) Bayes' theorem is way of finding a probability when we know certain other probabilities.

Let us consider two event A and B; their respective probability is P(A) and P(B)

then the probability of A happen given that B happens:

$$P(A|B) = \frac{P(A)\ P(B|A)}{P(B)}$$

(11) (a) $\int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$

$= \int_{-\infty}^{\infty} e^{-\frac{v^2}{2s^2}}\, dv \qquad \left[v = -\mu + x \right.$

$= \int_{-\infty}^{\infty} e^{-\left(\frac{v}{\sqrt{2}\sqrt{s^2}}\right)^2} dv$

$= \int_{-\infty}^{\infty} \sqrt{2}\, e^{-w^2} \sqrt{s^2}\, dw \qquad \left(w = \frac{v}{\sqrt{2}\sqrt{s^2}} \right.$

$= \left[\sqrt{s^2}\sqrt{2} \int_{-\infty}^{\infty} e^{-w^2} dw\right]_{-\infty}^{\infty} = \left[\sqrt{2}\sqrt{s^2}\, \frac{\sqrt{\pi}}{2}\, \mathrm{erf}(w)\right]_{-\infty}^{\infty}$

$= \left[\sqrt{2}\sqrt{s^2}\, \sqrt{\frac{\pi}{2}}\, \mathrm{erf}\left(\frac{-\mu+x}{\sqrt{2}\sqrt{s^2}}\right)\right]_{-\infty}^{\infty}$

$= \left[\sqrt{\frac{\pi}{2}}\sqrt{s^2}\, \mathrm{erf}\left[\frac{x-\mu}{\sqrt{2}\sqrt{s^2}}\right]\right]_{-\infty}^{\infty}$

$= \sqrt{\frac{\pi}{2}}\, |s| - \left(-\sqrt{\frac{\pi}{2}}\, |s|\right)$

$= \sqrt{2\pi}\, |s|.$

(b) $\int_0^\infty \alpha e^{-\alpha x} dx$

~~Substitute~~ Substitute :-

$-\alpha x = m$

$dx = -\frac{1}{\alpha} dm$

$= \int_{\cancel{\infty}}^{\cancel{\infty}} \alpha \left(-\frac{1}{\cancel{\alpha}}\right) e^{m} dm$

$= (-1) \int e^{m} dm = (-1) (e^{m})$

$= (-1) \left[e^{-\alpha x}\right]_0^\infty$

$= (-1) \left[e^{-\infty} - e^{0}\right]$

$= (-1) [0 - 1] = 1$