This is the standandard LSTM RNN

LSTM

$$
\begin{aligned}
i_t &= \sigma_{in}(U_i h_{t-1} + W_i x_t + b_i) \\
f_t &= \sigma_{in}(U_f h_{t-1} + W_f x_t + b_f) \\
o_t &= \sigma_{in}(U_o h_{t-1} + W_o x_t + b_o) \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{0}
$$

Gate equations parameter-reductions: The following considers paramter redcutions applied to the gate equations (uniformlly, for simplicity) We lable the gate-equations reductions as follows:

LSTM1

$$
\begin{aligned}
i_t &= \sigma_{in}(U_i h_{t-1} + b_i) \\
f_t &= \sigma_{in}(U_f h_{t-1} + b_f) \\
o_t &= \sigma_{in}(U_o h_{t-1} + b_o) \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{1}
$$

LSTM2

$$
\begin{aligned}
i_t &= \sigma_{in}(U_i h_{t-1}) \\
f_t &= \sigma_{in}(U_f h_{t-1}) \\
o_t &= \sigma_{in}(U_o h_{t-1}) \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{2}
$$

LSTM3

$$
\begin{aligned}
i_t &= \sigma_{in}(b_i) \\
f_t &= \sigma_{in}(b_f) \\
o_t &= \sigma_{in}(b_o) \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{3}
$$

LSTM4

$$
\begin{aligned}
i_t &= \sigma_{in}(u_i \odot h_{t-1}) \\
f_t &= \sigma_{in}(u_f \odot h_{t-1}) \\
o_t &= \sigma_{in}(u_o \odot h_{t-1}) \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{4}
$$

LSTM4a

$$
\begin{aligned}
i_t &= \sigma_{in}(u_i \odot h_{t-1}) \\
f_t &= \alpha, -1 < \alpha < 1 \text{ (default=0.96)} \\
o_t &= 1.0 \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{4a}
$$

LSTM5

$$
\begin{aligned}
i_t &= \sigma_{in}(u_i \odot h_{t-1} + b_i) \\
f_t &= \sigma_{in}(u_f \odot h_{t-1} + b_f) \\
o_t &= \sigma_{in}(u_o \odot h_{t-1} + b_o) \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{5}
$$

LSTM5a

$$
\begin{aligned}
i_t &= \sigma_{in}(u_i \odot h_{t-1} + b_i) \\
f_t &= \alpha, \ \ -1 < \alpha < 1 \text{ (default=0.96)} \\
o_t &= 1.0 \\
\tilde{c_t} &= \sigma(U_c h_{t-1} + W_c x_t + b_c) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
h_t &= o_t \odot \sigma(c_t)
\end{aligned}
\tag{5a}
$$

LSTM6

$$
\begin{aligned}
&i_t = 1.0 \\
&f_t = \alpha, -1 < \alpha < 1 \text{ (default=0.59)} \\
&o_t = 1.0 \\
&\tilde{c_t} = \sigma(W_c x_t + U_c h_{t-1} + W_c x_t + b_c) \\
&c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
&h_t = o_t \odot \sigma(c_t)
\end{aligned}
\tag{6}
$$

Memory Cell equations parameter-reductions: Note that the memory cell can be combined with any gate-equations reductions. One can also reduce the memory cell equation itself. Example combinations are as follows:

LSTM10

$$
\begin{aligned}
&i_t = \sigma_{in}(u_i \odot h_{t-1}) \\
&f_t = \sigma_{in}(u_f \odot h_{t-1}) \\
&o_t = \sigma_{in}(u_o \odot h_{t-1}) \\
&\tilde{c_t} = \sigma(u_c \odot h_{t-1} + W_c x_t + b_c) \\
&c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
&h_t = o_t \odot \sigma(c_t)
\end{aligned}
\tag{10}
$$

LSTM11

$$
\begin{aligned}
&i_t = \sigma_{in}(u_i \odot h_{t-1} + b_i) \\
&f_t = \sigma_{in}(u_f \odot h_{t-1} + b_f) \\
&o_t = \sigma_{in}(u_o \odot h_{t-1} + b_o) \\
&\tilde{c_t} = \sigma(u_c \odot h_{t-1} + W_c x_t + b_c) \\
&c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c_t} \\
&h_t = o_t \odot \sigma(c_t)
\end{aligned}
\tag{11}
$$