

## Worksheet Week 20

### Problems

**Q1.**  $X$  and  $Y$  are two (binary) random variables. If  $X$  and  $Y$  are independent, then  $P(X, Y) = P(X)P(Y)$

- (a) Give an example of two random variables that are independent.
- (b) Complete the probability table below in such way that the variables  $X$  and  $Y$  are independent.

	$X = 0$	$X = 1$
$Y = 0$		
$Y = 1$		

- (c) Determine the missing entries ( $a$ ,  $b$ ) of the joint distribution in such a way that the variables  $X$  and  $Y$  are again independent.

$$P(Y = 0, X = 0) = 0.1$$

$$P(Y = 0, X = 1) = 0.3$$

$$P(Y = 1, X = 0) = a$$

$$P(Y = 1, X = 1) = b$$

**Q2.** Solve the questions on slides 42 and 44 of the lecture slides.

**Q3.** A patient can have a symptom,  $S$ , that is caused by two different diseases,  $A$  and  $B$ . It is known that the presence of a gene  $G$  is important in the manifestation of disease  $A$ . The Bayes net and conditional probability tables are shown in Figure 2.

- (a) What is the probability that a patient has disease  $A$ ?
- (b) What is the probability that a patient has disease  $A$  if we know that the patient has disease  $B$ ?
- (c) What is the probability that a patient has disease  $A$  if we know that the patient has disease  $B$  AND symptom  $S$ ?

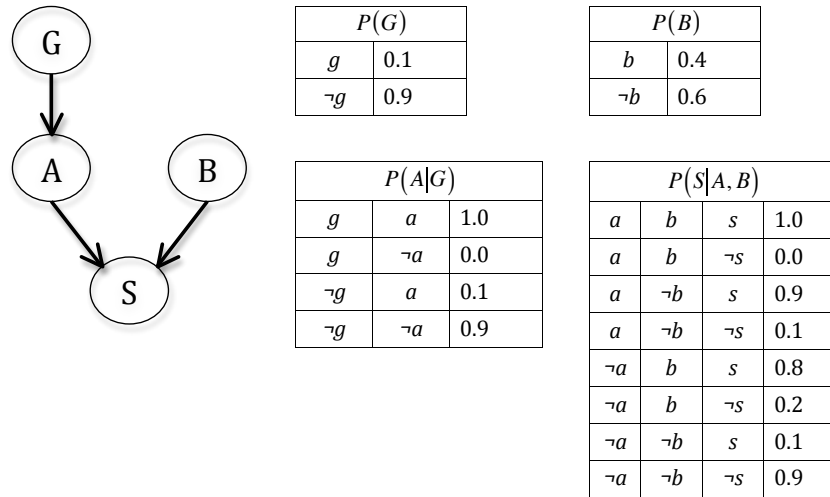
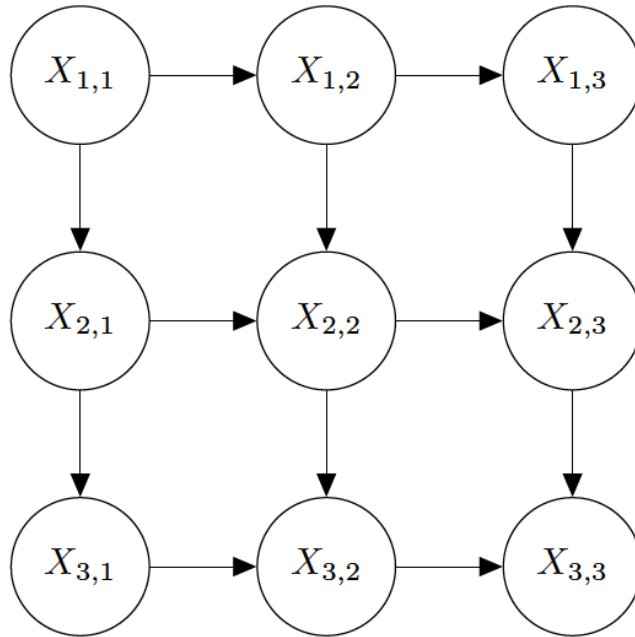


Figure 1: Bayes net and probability tables for Q5

**Q4.** Consider the following Bayesian network:



- Which random variables are independent of  $X_{3,1}$ ?
- Which random variables are independent of  $X_{3,1}$  given  $X_{1,1}$ ?