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ASSIGNMENT-1

Q1 Suppose $\Omega = \{0,1,---,15\}$, $A=\{0,8\}$, $B=\{1,2,3,5,8,10,12\}$, $C=\{0,4,9,15\}$, Determine $A \cap B$, $B \cap C$, $C \cap A$, $A \cup C$, C/A, $\Omega/(B \cup A \cup C)$

Sol^n
(i) $A \cap B = \{8\}$
(ii) $B \cap C = \{\ \phi$
(iii) $A \cup C = \{0,4,8,9,15\}$
(iv) $C/A = \{4,9,15\}$.
(v) $\Omega/B \cup A \cup C = \{6,7,11,10,14\}$.

Q2 Now consider three pair wise disjoint events E, F and G with $\Omega = E \cup F \cup G$ and $P(E) = 0.2$ and $P(F) = 0.5$. Calculate $P(\bar{F})$, $P(G)$, $P(E \cap G)$, $P(E/E)$, and $P(E \cup F)$

Sol^n
(i) $P(\bar{F}) = 1 - P(F) = 0.5$
(ii) $P(E \cup F \cup G) = P(E) + P(F) + P(G) + P(E \cap F \cap G)$
$P(E \cup F \cup G) = P(E) + P(F) + P(G)$
$1 = 0.5 + 0.2 + P(G)$
$P(G) = 0.3$
(iii) $P(E \cap G) = 0$ [They are pairwise distinct]

(iv) $P(E/E) = 0$

(v) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$P(E \cup F) = 0.7$

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Q3 A driving licence examination consists of two parts which are based on a theoratical and a practical examination. Suppose 25% of people fail the practical examination, 15% of people fail the theoratical examination, and 10% of people fail both the examinations. If a person is randomly choosen, then what is the prob that this person.

ⓐ Fails at least one of the examinations.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{25}{100} + \frac{15}{100} \cancel{+} - \frac{10}{100}.$$

$$P(A \cup B) = 0.3 \text{ or } 30\%$$

ⓑ Only fails the practical examination, but not the theoratical examination

$$P(\text{Only } A) = P(A) - P(A \cap B)$$

$$= 0.15 \text{ or } 15\%$$

ⓒ Successfully passes both the tests

$$\text{Passes both tests} = 1 - P(A \cup B)$$

$$= 0.7 \text{ or } 70\%.$$

ⓓ Fails any of the two examinations

$$= P(A) + P(B) - 2(A \cup B)$$

$$= \frac{25}{100} + \frac{25}{100} \cancel{+} - \frac{20}{100}.$$

$$= 0.2 \text{ or } 20\%$$

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Q4 A new board game uses a twelve-sides die. Suppose the die is rolled once, what is the probablity of getting

(a) an even number

$$P(\text{even no}) = \frac{1}{2}.$$

[~~as~~ from 1 to 12 even has occur 6 times which is exactly half of 12]

(b) a no greater than 9

$$P(\text{no greater than } 9) = \frac{3}{12} = \frac{1}{4}$$

(c) an even no greater than 9

$$P(\text{even no greater than } 9) = \frac{2}{12} = 1/6$$

(d) an even no or no greater than 9

$$P(\text{even no or no greater than } 9) = P(E) + P(>9) - P(E \cap >9)$$
$$= \frac{6}{12} + \frac{3}{12} - \frac{2}{12}.$$
$$= \frac{7}{12}$$

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Q5 A football practice target is a portable wall with two holes (which are the target) in it for training shots. Suppose there are two players A and B. The probablities of hitting the target by A and B are 0.4 and 0.5 respectively

(a) Probablity that at least one of the players succeeds with his shot?

$$P(\text{atleast one succeed}) = P(\text{A succeed}) + P(\text{B succeed}) + P(\text{Both succeed})$$
$$= P_A \cdot \overline{P_B} + \overline{P_A} \cdot P_B + P_A P_B.$$
$$= 0.4 \times 0.5 + 0.6 \times 0.5 + 0.4 \times 0.5$$
$$= 0.5 + 0.2$$
$$= 0.7$$

(b) Probablity that exactly one of the players hit the target

$$P(\text{exactly one}) = P_A \overline{P_B} + P_B \cdot \overline{P_A}$$
$$= 0.4 \times 0.5 + 0.6 \times 0.5$$
$$= 0.5$$

(c) Only B scores

$$P(\text{only B scores}) = P_B \cdot \overline{P_A}$$
$$= 0.5 \times 0.6$$
$$= 0.3$$

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Q6 Classify the following random variables as discrete or continous.

X: The no of automobiles accidents per years in Virginia
Y: length of time to play 18 holes of golf
~~Z~~ N: no of building permits issued each months in a certain city
Q: weight of grain produced per acre.

Sol^n X → Discrete.
Y → Continous
N → Discrete
Q → Continous

Q7 An overseas shipment of 5 foreign automobiles contains 2 that has slight paint blemishes. If an agency receives 3 of these automobiles at random. List the elements of the sample S, using the letters B and N for blemished and nonblemished, respectively. Then to each sample point assign a value x of the random variable ~~x~~ X representing the no of automobiles with paints blemishes purchased by the agency.

Sol^n automobile = 5 buy 3 at random
blemishes (B) = 2.
X → purchase of blemished automobiles.

$X=0$ $P(X) = \frac{{}^3C_3 \cdot {}^2C_0}{{}^5C_3} = \frac{1}{10}$

$X=1$ $P(X) = \frac{{}^3C_2 \cdot {}^2C_1}{{}^5C_3} = \frac{6}{10}$

$X=2$ $P(X) = \frac{{}^3C_1 \cdot {}^2C_2}{{}^5C_3} = \frac{3}{10}$

X	0	1	2
P	1/10	6/10	3/10

Q8 Let w be a random variable giving the no of heads minus the no of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value w of W.

Solⁿ W = head tails.

Possibilities

HHH. W=3
TTT. W=0
HTH. W=1
THT. W=-1
HHT. W=1
THH. W=1
TTH. W=-1
HTT W=-1

sample space
$S=\{-1,0,1,3\}$

$P(W=-1) = 3/8$
$P(W=0) = 1/8$
$P(W=1) = 3/8$
$P(W=3) = 1/8$.

W	-1	0	1	3
P	3/8	1/8	3/8	1/8

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Q9 Consider a random experiment of tossing a coin three times. Let X be the r.v giving the numbers of heads obtained. We assume that the tosses are independent and the probability of a head is p

(a) What is range of X?
(b) Find the probablities $P(X=0)$, $P(X=1)$, and $P(X=3)$.

Sol^n $X \rightarrow$ no of heads

Sample space $\rightarrow S = \{0, 1, 2, 3\} = 4$

$P(X=0) = 1/8$
$P(X=1) = 3/8$
$P(X=2) = 3/8$
$P(X=3) = 1/8$

X	0	1	2	3
P	1/8	3/8	3/8	1/8

Q10 Consider the experiment of throwing a dart onto a circular plate with unit radius. Let X be the r.v. represent the distance of the point where the dart lands from the origin of the plate. Assume that the dart always lands on the plate and that the dart is equally likely to land anywhere on the plate.

(a) What is range of X?
(b) Find (i) $P(X<a)$ (ii) $P(a<X<b)$, where $a<b\leq 1$.

Sol^n Continous random variable

range of X is $0<X<1$

for any $X \in [0,1]$

$$f(x) = \frac{\text{area}(v=x)}{\text{area}(v=1)} = \frac{x^2}{1}.$$

(i) $P(X < a) = a^2$

(ii) $P(a < x < b) = \frac{\pi b^2 - \pi a^2}{\pi} = b^2 - a^2$

Q11 An information source generates symbols at random from a four letter alphabet $\{a, b, c, d\}$ with probablities $P(a) = 1/2$, $P(b) = 1/4$ and $P(c) = P(d) = 1/8$. A coding scheme encodes these symbols into binary codes as follows.

a	1
a	0
b	10
c	110
d	111

Let X be the r.v. denoting the length of the code, that is the no of binary symbols

(a) What is the range of X?

(b) Assuming that the generation of symbols are independent find the probablities $P(X=1)$, $P(X=2)$, $P(X=3)$, $P(X>3)$

Soln a, b, c, d

$P(a) = 1/2$ $P(b) = 1/4$ $P(c) = P(d) = 1/8$

$X \rightarrow$ length of code.

$\therefore$ range $= 1 \leq x \leq 3$

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$P(X=1) = P(a) = 1/2$
$P(X=2) = P(b) = 1/4$
$P(X=3) = P(d) + P(c) = 2/8 = 1/4$

X	1	2	3
P	1/2	1/4	1/4

Range of X be [1,3]

Q12 Let X be the r.v defined in Question 3 above.

(a) Sketch the cdf $F_X(x)$ of X and specify the type of X

Sol^n (a) Cdf of Q11

X	P(x)	F(x)
1	1/2	1/2
2	1/4	3/4
3	1/4	1

(b) Find (i) $P(X \le 1)$ (ii) $P(1 < x \le 2)$ (iii) $P(x > 1)$ and (iv) $P(1 \le x \le 2)$

Sol^n (i) $P(x \le 1) = 1/2$

(ii) $P(1 < x \le 2) = \frac{3}{4} - \frac{1}{2} = 1/4$

(iii) $P(X > 1) = 1/4 + 1/4 = 1/2$

(iv) ~~P(x<2)~~ $= P(1 \le x \le 2) = \frac{1}{4} + \frac{2}{4} = 3/4$.

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Q13 Let X be a continous r.v X with pdf :

$$f_X(x) = \begin{cases} kx & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

where k is constant.

ⓐ Determine the value of k and sketch $f_X(x)$

ⓑ find and sketch the corresponding cdf $F_X(x)$

ⓒ find $P(1/4 \le x \le 2)$

Sol^n

$$f(x) = \begin{cases} kx & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^1 kx + 0 = 1$$

$$\boxed{k=2}$$

$$f(x) = \begin{cases} 2x & \text{if } 0\le x<1 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \int_{-\infty}^{a} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_0^a f(x)dx$$

$$= \int_{-\infty}^{0} 0 \cdot dx + \int_0^a 2x\,dx$$

$$= a^2 \text{ or } x^2$$

$$f(x) = \begin{cases} x^2 & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

$$P(1/4 \le x \le 2) = \int_{1/4}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$$

$$= \int_{1/4}^{1} 2x\,dx + \int_{1}^{2} 0 \cdot dx$$

$$= x^2 \Big|_{1/4}^{1} = 1 - \frac{1}{16} = \boxed{\frac{15}{16}}$$

Q14 Consider a discrete r.v X whose pmf is given by

$P_X(x) = 1/3$ if $x = -1, 0, 1$

$= 0$ otherwise.

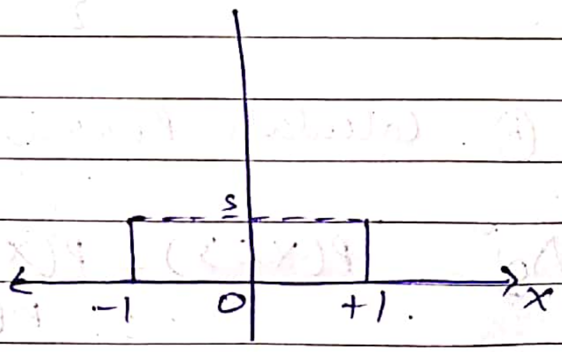
Plot $P_X(x)$ and find the mean and variance of X

Soln: $P(x) = 1/3$ if $x = -1, 0, 1$

$= 0$ otherwise.

Plot $P(x)$.

x	-1	0	1
P	$1/3$	$1/3$	$1/3$



$E(x) = \sum x_i P_i$

$= -1(1/3) + 0(1/3) + 1(1/3)$.

$= 0$.

Variance $= \Sigma(x^2) - [E(x)]^2$

$$= \left[(-1)^2 \tfrac{1}{3} + (1)^2 \tfrac{1}{3} \right] - 0$$

$$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Q15 Consider the following cumulative distribution function of a random variable X:

$$F(x) = \begin{cases} 0 & \text{if } x < 2 \\ -\frac{1}{4}x^2 + 2x - 3 & \text{if } 2 \le x < 4 \\ 1 & \text{if } x > 4 \end{cases}$$

(a) What is PDF of X?

Sol^n

$$f(x) = \begin{cases} 0 & x < 2 \\ -\frac{1}{2}x + 2 & 2 \le x < 4 \\ 0 & x > 4 \end{cases}$$

(b) Calculate $P(X<3)$ and $P(X=4)$

Sol^n

$$P(X<3) = P(X \le 3) - P(X=3)$$
$$= P(3) - 0$$
$$= -\frac{9}{4} + 6 - 3 = 0.75$$

$P(X=4)$ for continous variable $P(X=x_0)=0$

$\therefore P(x=4)=0$

(c) Determine $E(X)$ and $Var(X)$.

Sol^n $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$.

$E(x) = \int_{-\infty}^{2} x(0)\, dx + \int_{2}^{4} x\left(-\tfrac{1}{2}x+2\right) dx + \int_{4}^{\infty} (x \cdot 0)\, dx$.

$E(x) = 0 + \int_{2}^{4} \left(-\tfrac{1}{2}x^2 + 2x\right) dx + 0$

$E(x) = \left[-\frac{x^3}{6} + x^2 \right|_{2}^{4} = \frac{8}{3}.$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \int_{2}^{4} x^2\left(\frac{-1}{2}x+2\right) dx$.

$E(x^2) = \int_{2}^{4} \left(\frac{-1}{2}x^2 + 2x^2\right) dx$

$E(x^2) = \left[\frac{-x^4}{4} + \frac{2}{3}x^3 \right|_{2}^{4}$

$E(x^2) = \frac{22}{3}$

Variance $(x) = \sigma^2 = E(x^2) - (E(x))^2$

$$= \frac{22}{3} - \left(\frac{8}{3}\right)^2$$

$$= \frac{2}{9}$$