Robotics [IIT-Jodhpur]

Practical 3: Forward kinematics of *n* Link Robot

1 Write a program for forward kinematics of an *n*-link serial robotic system. Sketch the robot configuration.

Note:

a) Input to the program would be

DH parameters and $\boldsymbol{a}_{nE}^{n}$

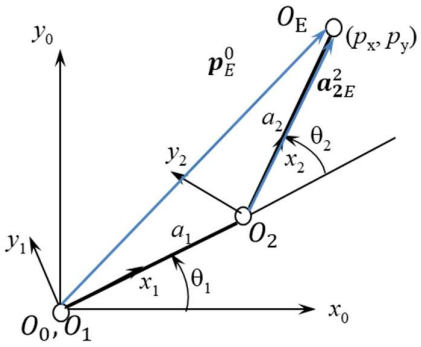
b) Output of the program would be

$\boldsymbol{T}_n^0$ ($\boldsymbol{R}_n^0$,$\boldsymbol{o}_n^0$) and $\boldsymbol{p}_E^0$

Hint:

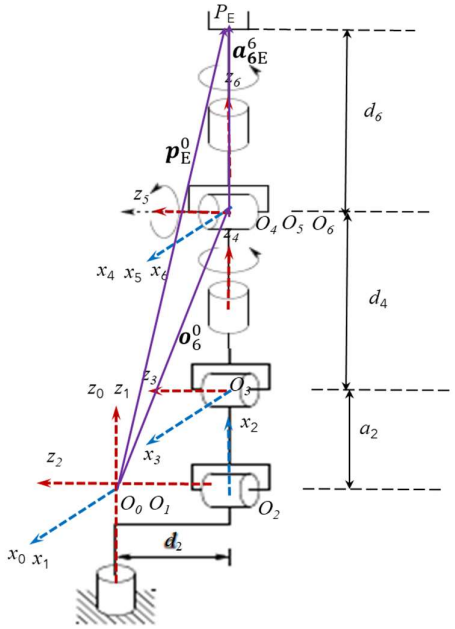
$$\boldsymbol{T}_i^{i-1} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & a_{i-1} \\ s_{\theta_i}c_{\alpha_{i-1}} & c_{\theta_i}c_{\alpha_{i-1}} & -s_{\alpha_{i-1}} & -d_i s_{\alpha_{i-1}} \\ s_{\theta_i}s_{\alpha_{i-1}} & c_{\theta_i}s_{\alpha_{i-1}} & c_{\alpha_{i-1}} & d_i c_{\alpha_{i-1}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} \boldsymbol{p}_E^0 \\ 1 \end{bmatrix} = \boldsymbol{T}_n^0 \begin{bmatrix} \boldsymbol{a}_{nE}^n \\ 1 \end{bmatrix}$$

c) Generate results for 2-link robot shown below. Note: a_1= a_2=1 m and DH parameters are given below:



link	α_i	a_i	d_i	θ_i
1	0	0	0	θ_1
2	0	a_1	0	θ_2

2 Using the above code solve forward kinematics of the robot shown in Fig. 1, for the given configuration. Find end effector position $\boldsymbol{p}_E^0$ and orientation $\boldsymbol{R}_n^0$. Sketch configuration of the robot. Note: a_2= d_2=0.5 m; d_4= d_6=1 m



link	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	$\theta_1(0)$
2	90	0	- d_2	θ_2 (90)
3	0	a_2	0	θ_3(-90)
4	-90	0	d_4	$\theta_4(0)$
5	90	0	0	$\theta_5(0)$
6	- 90	0	0	θ_6 (0)

$$\boldsymbol{T}_6^0 = \begin{bmatrix} \boldsymbol{R}_6^0 & \boldsymbol{o}_6^0 \\ \boldsymbol{0}^T & 1 \end{bmatrix} = \boldsymbol{T}_1^0 \boldsymbol{T}_2^1 \boldsymbol{T}_3^2 \boldsymbol{T}_4^3 \boldsymbol{T}_5^4 \boldsymbol{T}_6^5$$

$$\begin{bmatrix} \boldsymbol{p}_E^0 \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_6^0 & \boldsymbol{o}_6^0 \\ \boldsymbol{0}^T & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_{6E}^6 \\ 1 \end{bmatrix}$$

Additional Task (Not to be graded)

m Find ZYZ Euler angles corresponding to orientation ($\boldsymbol{R}_n^0$) of the end effector.
Hint:

If $\boldsymbol{R}_n^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$r_{33} \neq \pm 1$	$c\theta = r_{33}, \quad s\theta = \pm\sqrt{1 - {r_{33}}^2}$ $\theta = atan2\left(\pm\sqrt{1 - {r_{33}}^2}, r_{33}\right)$
$s\theta > 0$	$\phi = atan2(r_{23}, r_{13})$ $\psi = atan2(r_{32}, -r_{31})$
$s\theta < 0$	$\phi = atan2(-r_{23}, -r_{13})$ $\psi = atan2(-r_{32}, r_{31})$