

Assignment 1

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Abstract—This document explains how to find the point of intersection of a line and a plane.

Download the python code from

<https://github.com/vishalashok98/AI5006/tree/master/Assignment1>

and latex-tikz codes from

<https://github.com/vishalashok98/AI5006/tree/master/Assignment1>

1 PROBLEM

Find the co ordinates of the point when the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}^T \mathbf{x} = 7$ and perpendicular to the two lines

2 EXPLANATION

Equation of the plane is

$$\begin{pmatrix} 2 & -3 & 1 \end{pmatrix} \mathbf{x} = 7 \quad (2.0.1)$$

Direction vector of line passing through points

$\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ is given by

$$\mathbf{m} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}$$

Equation of a line passing through the point \mathbf{a} and having direction vector \mathbf{m} is given by:

$$\mathbf{x} = \mathbf{a} + \lambda \mathbf{m} \quad (2.0.2)$$

where λ is some constant. Parametric equations of

line passing through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ is

$$\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}$$

3 SOLUTION

Since the line intersects plane, by substituting parametric equation of line in equation of plane we get

$$\begin{pmatrix} 2 & -3 & 1 \end{pmatrix} (\mathbf{a} + \lambda \mathbf{m}) = 7 \quad (3.0.1)$$

$$\begin{pmatrix} 2 & -3 & 1 \end{pmatrix} \mathbf{a} + \begin{pmatrix} 2 & -3 & 1 \end{pmatrix} \lambda \mathbf{m} = 7 \quad (3.0.2)$$

$$\begin{pmatrix} 2 & -3 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix} \lambda = 7 \quad (3.0.3)$$

$$\lambda = -1 \quad (3.0.4)$$

Substituting the value of λ in parametric equations

we get point of intersection as $\begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}$

4 PLOT

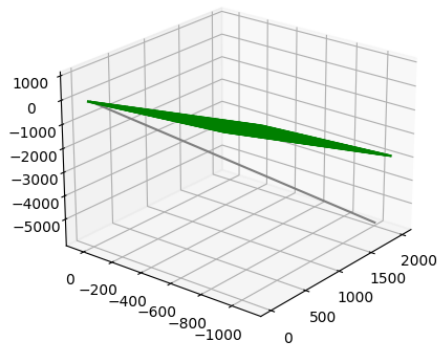


Fig. 0: Intersection of Plane and Line