Assignment 1

Vishal Ashok

Abstract—**This document explains how to find the point of intersection of a line and a plane.**

1 Problem

Find the co ordinates of the point when the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane $\begin{pmatrix} 2 & -3 & 1 \end{pmatrix}\mathbf{x}=7$ and perpendicular to the two lines

2 Explanation

Equation of the plane is

$$\begin{pmatrix} 2 & -3 & 1 \end{pmatrix}\mathbf{x} = 7 \qquad (2.0.1)$$

Direction ratios of line passing through points $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ is given by [1,-1,-6]

Equation of a line passing through the point $\mathbf{a}$ and having direction ratios $\mathbf{m}$ is given by:

$$\mathbf{x} = \mathbf{a} + \lambda\mathbf{m} \qquad (2.0.2)$$

where λ is some constant.

Parametric equations of line passing through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ is

$$\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}$$

3 Solution

Since the line intersects plane, by substituting parametric equation of line in equation of plane we get

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix}(\mathbf{a} + \lambda\mathbf{m}) = 7 \qquad (3.0.1)$$

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix}\mathbf{a} + \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}\lambda\mathbf{m} = 7 \qquad (3.0.2)$$

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}\lambda = 7 \qquad (3.0.3)$$

$$\lambda = -1 \qquad (3.0.4)$$

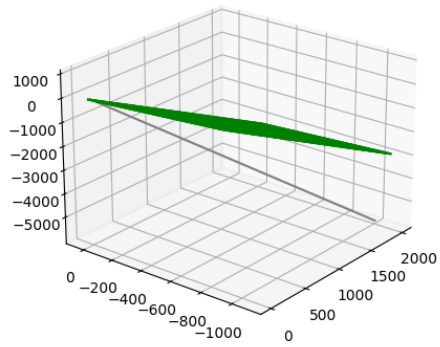Substituting the value of λ in parametric equations we get point of intersection as [1,-2,7]

4 PLOT

Fig. 0: Intersection of Plane and Line