Assignment 1

Vishal Ashok

***Abstract*—This document explains how to find the point of intersection of a line and a plane.**

Download the python code from

```
https://github.com/vishalashok98/AI5006/tree/
    master/Assignment1
```

and latex-tikz codes from

```
https://github.com/vishalashok98/AI5006/tree/
    master/Assignment1
```

1 Problem

Find the co ordinates of the point when the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane [2 1 1]x=7 and perpendicular to the two lines

2 Explanation

Equation of a line passing through the point $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ is given by:

$$\tfrac{x-3}{1} = \tfrac{y+4}{-1} = \tfrac{z+5}{-6}$$

A line can be considered as intersection of two planes ,from equation 1 equations of two planes can be given as

$$\tfrac{x-3}{1} = \tfrac{y+4}{-1}$$

and

$$\tfrac{x-3}{1} = \tfrac{z+5}{-6}$$

So there are three set of equations which should be solved to find the point of intersection.

3 Solution

There are three equations

$$2x + y + z = 7 \tag{3.0.1}$$

$$x + y + 0 = -5 \tag{3.0.2}$$

$$6x + 0 + z = 13 \tag{3.0.3}$$

These equations can be represented in matrix form as

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 6 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \\ 13 \end{pmatrix} \tag{3.0.4}$$

Writing it in the form of augumented matrix and reducing it to echolon form we get

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 1 & 1 & 0 & : & -5 \\ 6 & 0 & 1 & : & 13 \end{pmatrix} \xleftrightarrow{R_2=R_2-R_1/2} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & -0.5 & : & -8.5 \\ 6 & 0 & 1 & : & 13 \end{pmatrix} \tag{3.0.5}$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & -0.5 & : & -8.5 \\ 6 & 0 & 1 & : & 13 \end{pmatrix} \xleftrightarrow{R_3=R_3-3R_1} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & -0.5 & : & -8.5 \\ 0 & -3 & -2 & : & -8 \end{pmatrix} \tag{3.0.6}$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & -0.5 & : & -8.5 \\ 0 & -3 & -2 & : & -8 \end{pmatrix} \xleftrightarrow{R_3=R_3+6R_2} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & -0.5 & : & -8.5 \\ 0 & 0 & -5 & : & -59 \end{pmatrix} \tag{3.0.7}$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & -0.5 & : & -8.5 \\ 0 & 0 & -5 & : & -59 \end{pmatrix} \xleftrightarrow{R_2=R_2-0.1R_3} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & 0 & : & -2.6 \\ 0 & 0 & -5 & : & -59 \end{pmatrix} \tag{3.0.8}$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 0.5 & 0 & : & -2.6 \\ 0 & 0 & -5 & : & -59 \end{pmatrix} \xleftrightarrow{R_1=R_1+0.2R_3-2R_2} \begin{pmatrix} 2 & 0 & 0 & : & 0.4 \\ 0 & 0.5 & 0 & : & -2.6 \\ 0 & 0 & -5 & : & -59 \end{pmatrix} \quad (3.0.9)$$

So from the Row reduced echolon form of matrix we know that point of intersection is $[0.2, -2, 11.8]$

4 Plot

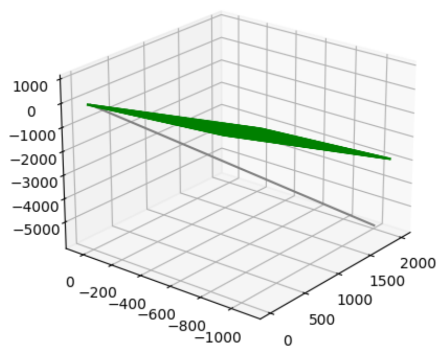

Fig. 0: Intersection of Plane and Line