

Assignment 1

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Abstract—This document explains how to find the point of intersection of a line and a plane.

Download the python code from

<https://github.com/vishalashok98/AI5006/tree/master/Assignment1>

and latex-tikz codes from

<https://github.com/vishalashok98/AI5006/tree/master/Assignment1>

1 PROBLEM

Find the co ordinates of the point when the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane $[2 \ 1 \ 1]x=7$ and perpendicular to the two lines

2 EXPLANATION

Equation of a line passing through the point $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$

and $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ is given by:

$$\frac{x-3}{2} = \frac{y+4}{-1} = \frac{z+5}{-6}$$

A line can be considered as intersection of two planes ,from equation 1 equations of two planes can be given as

$$\frac{x-3}{2} = \frac{y+4}{-1}$$

and

$$\frac{x-3}{2} = \frac{y+4}{-1}$$

So there are three set of equations which should be solved to find the point of intersection.

3 SOLUTION

There are three equations

$$2x + y + z = 7 \quad (3.0.1)$$

$$x + 2y + 0 = -5 \quad (3.0.2)$$

$$3x + 0 + z = 8 \quad (3.0.3)$$

These equations can be represented in matrix form as

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \\ 8 \end{pmatrix} \quad (3.0.4)$$

Writing it in the form of augmented matrix and reducing it to echolon form we get

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 1 & 2 & 0 & : & -5 \\ 3 & 0 & 1 & : & 8 \end{pmatrix} \xrightarrow{R_2=R_2-R_1/2} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & -0.5 & : & -8.5 \\ 3 & 0 & 1 & : & 8 \end{pmatrix} \quad (3.0.5)$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & -0.5 & : & -8.5 \\ 3 & 0 & 1 & : & 8 \end{pmatrix} \xrightarrow{R_3=R_3-1.5R_1} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & -0.5 & : & -8.5 \\ 0.5 & -1.5 & -0.5 & : & -2.5 \end{pmatrix} \quad (3.0.6)$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & -0.5 & : & -8.5 \\ 0.5 & -1.5 & -0.5 & : & -2.5 \end{pmatrix} \xrightarrow{R_3=R_3+R_2} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & -0.5 & : & -8.5 \\ 0 & 0 & -1 & : & -11 \end{pmatrix} \quad (3.0.7)$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & -0.5 & : & -8.5 \\ 0 & 0 & -1 & : & -11 \end{pmatrix} \xrightarrow{R_2=R_2+0.5R_3} \begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & 0 & : & -3 \\ 0 & 0 & -1 & : & -11 \end{pmatrix} \quad (3.0.8)$$

$$\begin{pmatrix} 2 & 1 & 1 & : & 7 \\ 0 & 1.5 & 0 & : & -3 \\ 0 & 0 & -1 & : & -11 \end{pmatrix} \xleftrightarrow{R_1=R_1+R_3-0.666R_2} \begin{pmatrix} 2 & 0 & 0 & : & -2 \\ 0 & 1.5 & 0 & : & -3 \\ 0 & 0 & -1 & : & -11 \end{pmatrix}$$

(3.0.9)

So from the Row reduced echolon form of matrix we know that point of intersection is $[-1, -2, 11]$

4 PLOT

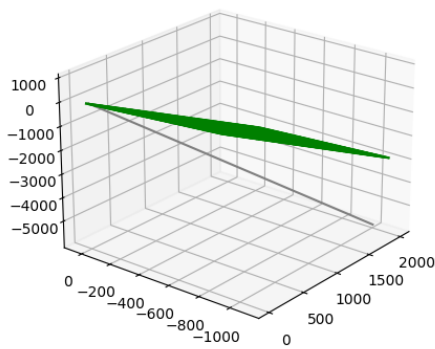


Fig. 0: Intersection of Plane and Line