

Assignment 8

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Abstract—This document uses Gram-Schmidt process to perform QR decomposition of a matrix.

Download Python code from

https://github.com/vishalashok98/AI5106/tree/main/Assignment_1

Download the latex-tikz codes from

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1 PROBLEM

Trace the curve

$$35x^2 + 30y^2 + 32x - 108y - 12xy + 59 = 0 \quad (1.0.1)$$

Express this conic section in the form

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1.0.2)$$

and perform QR decomposition for the matrix \mathbf{V} .

2 SOLUTION

$$\mathbf{V} = \begin{pmatrix} 35 & -6 \\ -6 & 30 \end{pmatrix} \quad (2.0.1)$$

If, $\mathbf{V} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ Consider, $\mathbf{V} = (\mathbf{a} \ \mathbf{b})$

$$\text{Where, } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad (2.0.2)$$

Then, $\mathbf{u}_1 = \mathbf{a}$, $\mathbf{e}_1 = \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|}$

$$\mathbf{u}_2 = \mathbf{b} - (\mathbf{b}^T \mathbf{e}_1) \mathbf{e}_1, \mathbf{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} \quad (2.0.3)$$

$$\text{and, } \mathbf{V} = (\mathbf{e}_1 \ \mathbf{e}_2) \begin{pmatrix} \mathbf{a}^T \mathbf{e}_1 & \mathbf{b}^T \mathbf{e}_1 \\ 0 & \mathbf{b}^T \mathbf{e}_2 \end{pmatrix} = \mathbf{Q} \mathbf{R} \quad (2.0.4)$$

Performing QR decomposition on \mathbf{V} we get,

$$\mathbf{e}_1 = \frac{1}{\sqrt{1261}} \begin{pmatrix} 35 \\ -6 \end{pmatrix} \quad (2.0.5)$$

$$\mathbf{e}_2 = \frac{1}{380} \begin{pmatrix} \frac{37839}{100} \\ -\frac{35895}{1000} \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{Q} = (\mathbf{e}_1 \ \mathbf{e}_2) \quad (2.0.7)$$

$$\mathbf{Q} = \begin{pmatrix} \frac{35}{\sqrt{1261}} & \frac{37839}{38000} \\ \frac{-6}{\sqrt{1261}} & -\frac{35895}{380000} \end{pmatrix} \quad (2.0.8)$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{1261} & \frac{30\sqrt{13}}{\sqrt{97}} \\ 0 & \frac{-8805}{1000} \end{pmatrix} \quad (2.0.9)$$

It is easy to verify that $\mathbf{QR} = \mathbf{V}$ and $\mathbf{Q}^T \mathbf{Q} = \mathbf{I}$. Thus, \mathbf{V} is decomposed into an orthogonal matrix and an upper triangular matrix.