

# Assignment 8

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**Abstract—This document uses Gram-Schmidt process to perform QR decomposition of a matrix.**

Download Python code from

[https://github.com/vishalashok98/AI5106/tree/main/Assignment\\_1](https://github.com/vishalashok98/AI5106/tree/main/Assignment_1)

Download the latex-tikz codes from

[https://github.com/vishalashok98/AI5106/tree/main/Assignment\\_1](https://github.com/vishalashok98/AI5106/tree/main/Assignment_1)

## 1 PROBLEM

Trace the curve

$$35x^2 + 30y^2 + 32x - 108y - 12xy + 59 = 0 \quad (1.0.1)$$

Express this conic section in the form

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1.0.2)$$

and perform QR decomposition for the matrix  $\mathbf{V}$ .

## 2 EXPLANATION

The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.1)$$

and can be expressed as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.2)$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.3)$$

The QR decomposition of a matrix is a factorization of the matrix into an orthogonal matrix and a triangular matrix. A QR decomposition of a real square matrix  $\mathbf{V}$  is given by

$$\mathbf{V} = \mathbf{Q} \mathbf{R} \quad (2.0.4)$$

where  $\mathbf{Q}$  is an orthogonal matrix ( $\mathbf{Q}^T \mathbf{Q} = \mathbf{I}$ ) and  $\mathbf{R}$  is an upper triangular matrix.

## 3 SOLUTION

Comparing (1.0.1) with (2.0.1), we get

$$\mathbf{V} = \begin{pmatrix} 35 & -6 \\ -6 & 30 \end{pmatrix} \quad (3.0.1)$$

If,  $\mathbf{V} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$  Consider,  $\mathbf{V} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}$

Where,  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$  (3.0.2)

Then,  $\mathbf{u}_1 = \mathbf{a}$ ,  $\mathbf{e}_1 = \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|}$

$$\mathbf{u}_2 = \mathbf{b} - (\mathbf{b} \cdot \mathbf{e}_1) \mathbf{e}_1, \mathbf{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} \quad (3.0.3)$$

and,  $\mathbf{V} = (\mathbf{e}_1 \ \mathbf{e}_2) \begin{pmatrix} \mathbf{a} \cdot \mathbf{e}_1 & \mathbf{b} \cdot \mathbf{e}_1 \\ 0 & \mathbf{b} \cdot \mathbf{e}_2 \end{pmatrix} = \mathbf{Q} \mathbf{R}$  (3.0.4)

Performing QR decomposition on  $\mathbf{V}$  we get,

$$\mathbf{e}_1 = \frac{1}{\sqrt{1261}} \begin{pmatrix} 35 \\ -6 \end{pmatrix} \quad (3.0.5)$$

$$\mathbf{e}_2 = \frac{1}{380} \begin{pmatrix} \frac{37839}{100} \\ -\frac{35895}{1000} \end{pmatrix} \quad (3.0.6)$$

$$\mathbf{Q} = (\mathbf{e}_1 \ \mathbf{e}_2) \quad (3.0.7)$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{1261} & \frac{30\sqrt{13}}{\sqrt{97}} \\ 0 & \frac{-8805}{1000} \end{pmatrix} \quad (3.0.8)$$

It is easy to verify that  $\mathbf{Q} \mathbf{R} = \mathbf{V}$  and  $\mathbf{Q}^T \mathbf{Q} = \mathbf{I}$ . Thus,  $\mathbf{V}$  is decomposed into an orthogonal matrix and an upper triangular matrix.