Assignment 11

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***Abstract*—This document gives us information about invertibility conditions of a matrix.**

https://github.com/vishalashok98/AI5006

Download latex-tikz codes from

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1 Problem

Consider the matrix A=$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ over the field Q of rationals which of the following matrices are of the form P^tAP for a suitable invertible matrix P over Q? Here P^t denotes transpose of P.

1) $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$
2) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$
3) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
4) $\begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix}$

2 Explanation

A matrix is said to be invertible if its rank is not less then number of rows or its a full rank matrix.

3 Solution

Let P be a invertible matrix given by P=$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

For P to be invertible it must be full rank martrix.

$$P = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{3.0.1}$$

Performing row operations to reduce it to echolon form.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \xleftrightarrow{R_2 = R_2 - \frac{cR_1}{b}} \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & d - \frac{cb}{a} \end{pmatrix} \tag{3.0.2}$$

For P to be full rank matrix rows must be independent. So last row of echolon form must not be zero.

$$d - \frac{bc}{a} \neq 0 \tag{3.0.3}$$

$$P^tAP = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{3.0.4}$$

$$P^tAP = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} c & d \\ a & b \end{pmatrix} \tag{3.0.5}$$

$$P^tAP = \begin{pmatrix} 2ac & ad+bc \\ ad+bc & bd \end{pmatrix} \tag{3.0.6}$$

Suppose among the given options if option 1 is correct

$$P^tAP = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \tag{3.0.7}$$

$$\begin{pmatrix} 2ac & ad+bc \\ ad+bc & bd \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \tag{3.0.8}$$

$$2ac = 2 \tag{3.0.9}$$
$$ad + bc = 0 \tag{3.0.10}$$
$$2bd = -2 \tag{3.0.11}$$
$$dc = -1 \tag{3.0.12}$$
$$ad + bc = 0 \tag{3.0.13}$$
$$bd = -1 \tag{3.0.14}$$

For invertibility of P , det(P) should not be zero

$$det(P) = ad - bc \neq 0 \tag{3.0.15}$$

$$(ad-bc)^2 = (ad+bc)^2 - 4adbc \tag{3.0.16}$$
$$(ad-bc)^2 = 0^2 - 4(1)(-1) \tag{3.0.17}$$
$$(ad-bc)^2 = 4 \tag{3.0.18}$$
$$ad - bc \neq 0 \tag{3.0.19}$$

So matrix $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ is the right option