Assignment 11

Vishal Ashok

***Abstract*—This document gives us information to find transformation matrix for a given function.**

https://github.com/vishalashok98/AI5006

Download latex-tikz codes from

https://github.com/vishalashok98/AI5006

1 Problem

Consider the vector space P_n of real polynomials in x of degree less than or equal to n. Define $T : P_2 \to P_3$ by $(Tf)(x) = \int_0^x f(t)dt + f'(x)$.Write the matrix representation of T with respect to the bases $1, x, X^2$ and $1, x, x^2, x^3$.

2 Explanation

Transformation matrix gives the co-ordinates of images of vectors which are mapped from domain to co-domain.

3 Solution

$1, x, x^2$ is basis for P_2 which is domain space.
$(Tf)(x) = \int_0^x f(t)dt + f'(x)$

We should the image of basis vectors of domain space in co-domain space with respect to T.
Base vector 1:$f(x) = 1$

$f'(x) = 0$ $T(1) = \int_0^x 1dt + 0 = t\Big|_0^x$

It can be written as linear combination of basis vectors of co-domain space.
$x = 0 + 1.x + 0.x^2 + 0.x^3$
So coordinates of $T(1)$ with respect to co-domain basis is given by $[T(1)]=\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$

Base vector 2: $f(x) = x$
$f'(x) = 1$
$T(x) = \int_0^x tdt + 1 = \frac{t^2}{2}\Big|_0^x + 1 = \frac{x^2}{2} + 1$
It can be written as linear combination of basis vectors of co-domain space.
$\frac{1}{2}x^2 + 1 = 1 + 0.x + \frac{1}{2}.x^2 + 0.x^3$
So coordinates of $T(1)$ with respect to co-domain basis is given by

$[T(x)]=\begin{bmatrix}1\\0\\\frac{1}{2}\\0\end{bmatrix}$

Base vector 3:$f(x) = x^2$
$f'(x) = 2x$
$T(x) = \int_0^x t^2dt + 2x = \frac{t^3}{3}\Big|_0^x + 2x = \frac{x^3}{3} + 2x$
It can be written as linear combination of basis vectors of co-domain space.
$2x + \frac{1}{3}x^3 = 0 + 2.x + 0.x^2 + \frac{1}{3}.x^3$
So coordinates of $T(1)$ with respect to co-domain basis is given by

$[T(x^2)]=\begin{bmatrix}0\\2\\0\\\frac{1}{3}\end{bmatrix}$

Basis Vectors	Image of basis vector	Coordinates
1	x	$\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$
x	$\frac{1}{2}x^2 + 1$	$\begin{bmatrix}1\\0\\\frac{1}{2}\\0\end{bmatrix}$
x^2	$2x + \frac{1}{3}x^3$	$\begin{bmatrix}0\\2\\0\\\frac{1}{3}\end{bmatrix}$

Matrix associated with linear transformation is coordinates of images of basis vectors of domain with respect to basis vectors of co domain space.
$[T] = [[T(1)][T(x)][T(x^2)]]$

$$[T(x)]=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 2\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{3}\end{bmatrix}$$