

Assignment 2

Vishal Ashok

Abstract—This document explains how to trace a curve given its equation.

Download Python code from

https://github.com/vishalashok98/AI5106/tree/main/Assignment_1

Download the latex-tikz codes from

https://github.com/vishalashok98/AI5106/tree/main/Assignment_1

1 PROBLEM

Trace the curve

$$35x^2 + 30y^2 + 32x - 108y - 12xy + 59 = 0 \quad (1.0.1)$$

2 EXPLANATION

The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.1)$$

and can be expressed as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.2)$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{u}^T = (d \quad e) \quad (2.0.4)$$

3 SOLUTION

Comparing (1.0.1) with (2.0.1), we get

$$\mathbf{V} = \begin{pmatrix} 35 & -6 \\ -6 & 30 \end{pmatrix} \quad (3.0.1)$$

$$\mathbf{u}^T = (16 \quad -54) \quad (3.0.2)$$

If $|\mathbf{V}| > 0$, then (2.0.2) is an ellipse.

$$|\mathbf{V}| = \begin{vmatrix} 35 & -6 \\ -6 & 30 \end{vmatrix} = 1014 > 0 \quad (3.0.3)$$

(2.0.2) can be expressed as

$$\mathbf{y}^T \mathbf{D} \mathbf{y} = \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f \quad |\mathbf{V}| \neq 0 \quad (3.0.4)$$

$$\mathbf{y}^T \mathbf{D} \mathbf{y} = -\eta \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{y} \quad |\mathbf{V}| = 0 \quad (3.0.5)$$

with center as

$$\mathbf{c} = -\mathbf{V}^{-1} \mathbf{u} \quad |\mathbf{V}| \neq 0 \quad (3.0.6)$$

Calculating the center for given curve we get,

$$\mathbf{c} = -\frac{1}{|35 * 30 - 6 * 6|} \begin{pmatrix} 35 & 6 \\ 6 & 30 \end{pmatrix} \begin{pmatrix} 16 \\ -54 \end{pmatrix} \quad (3.0.7)$$

$$= \frac{1}{1014} \begin{pmatrix} 236 \\ -1524 \end{pmatrix} \quad (3.0.8)$$

$$= \begin{pmatrix} \frac{118}{507} \\ \frac{354}{169} \end{pmatrix} \quad (3.0.9)$$

For

$$|\mathbf{V}| > 0, \quad \text{or, } \lambda_1 > 0, \lambda_2 > 0 \quad (3.0.10)$$

(3.0.4) becomes

$$\lambda_1 y_1^2 + \lambda_2 y_2^2 = \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f \quad (3.0.11)$$

which is the equation of an ellipse with major and minor axes parameters

$$\sqrt{\frac{\lambda_1}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}}, \sqrt{\frac{\lambda_2}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}} \quad (3.0.12)$$

The characteristic equation of \mathbf{V} is obtained by evaluating the determinant

$$|\lambda \mathbf{I} - \mathbf{V}| = \begin{vmatrix} \lambda - 35 & 6 \\ 6 & \lambda - 30 \end{vmatrix} = 0 \quad (3.0.13)$$

$$\Rightarrow \lambda^2 - 65\lambda + 1014 = 0 \quad (3.0.14)$$

The eigenvalues are the roots of (3.0.14) given by

$$\lambda_1 = 39, \lambda_2 = 26 \quad (3.0.15)$$

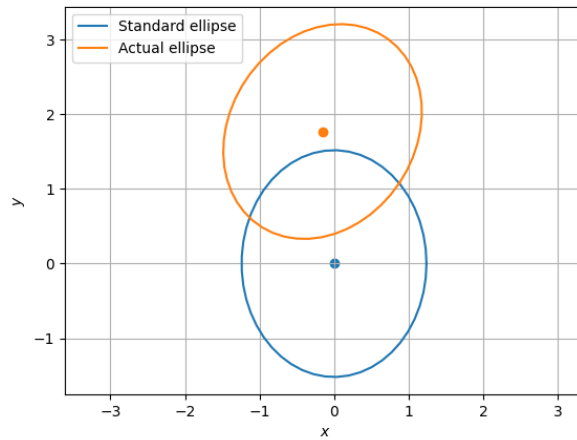
Calculating the major and minor axes lengths

using (3.0.12), we get

$$\begin{aligned}
 \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} &= \\
 &= (16 - 54) \frac{1}{1014} \begin{pmatrix} 35 & 6 \\ 6 & 30 \end{pmatrix} \begin{pmatrix} 16 \\ -54 \end{pmatrix} \\
 &= \frac{1}{1014} (16 \quad -54) \begin{pmatrix} 236 \\ -1524 \end{pmatrix} \\
 &= \frac{43036}{507} \\
 \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f &= \frac{43036}{507} - 59 = \frac{13123}{507} \quad (3.0.16)
 \end{aligned}$$

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = \sqrt{\frac{13123}{19773}} = \frac{814}{1000} \quad (3.0.17)$$

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = \sqrt{\frac{13123}{13182}} = \frac{997}{1000} \quad (3.0.18)$$



Assignment/Figure₂.png

Fig. 0: Circle with center (2 1.5) and having the lines $(1 \ 1)x = 5.08$ and $(1 \ 1)x = 1.92$ as tangents with $(2.79 \ 2.29)$ and $(1.21 \ 0.71)$ as point of contact.