

# Assignment 8

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**Abstract**—This document uses the concepts of singular value decomposition in solving a problem.

<https://github.com/vishalashok98/AI5006>

Download latex-tikz codes from

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each pair of vectors  $\alpha, \beta$  in  $W$  and each scalar  $c$  in  $F$  the vector  $c\alpha + \beta$  is again in  $W$ .

So given vector space is a subspace.

## 1 PROBLEM

Let  $V$  be the (real) vector space of all functions  $f$  from  $\mathbb{R}$  into  $\mathbb{R}$ . Whether the set containing all functions which are continuous is subspace of  $V$

## 2 EXPLANATION

If  $V$  is a vector Space over field  $F$ . A subspace of  $V$  is a subset  $W$  of  $V$  which is itself a vector space over  $F$  with the operations of vector addition and scalar multiplication on  $V$

## 3 SOLUTION

Let  $f$  and  $g$  be any continuous functions from  $\mathbb{R} \rightarrow \mathbb{R}$

and let  $c$  be any scalar  $\in \mathbb{R}$

From real analysis we know that sum and product of continuous functions is continuous. So  $cf + g$  is also a continuous function.

Proof:  $f$  and  $g$  are continuous at  $a$ , condition for continuity will be satisfied

$$\lim_{x \rightarrow a} f(x) = k_1 \quad (3.0.1)$$

$$\lim_{x \rightarrow a} g(x) = k_2 \quad (3.0.2)$$

Applying limits to  $cf + g$

$$\lim_{x \rightarrow a} cf(x) + g(x) = \lim_{x \rightarrow a} g(x) + \lim_{x \rightarrow a} f(x) = k_1 + k_2 \quad (3.0.3)$$

So  $cf + g$  is also continuous at  $a$

We know from theorem that any non-empty subset  $W$  of  $V$  is a subspace of  $V$  if and only if for