Assignment 11

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***Abstract*—This document gives us information to find transformation matrix for a given function.**

https://github.com/vishalashok98/AI5006

Download latex-tikz codes from

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1 Problem

Consider the vector space P_n of real polynomials in x of degree less than or equal to n. Define $T : P_2 \to P_3$ by

$$(Tf)(x) = \int_0^x f(t)dt + f'(x) \tag{1.0.1}$$

Then the matrix representation of T with respect to the bases $1, x, x^2$ and $1, x, x^2, x^3$ is

1) $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{1}{2} & 0 \\ 0 & 2 & 0 & \frac{1}{3} \end{pmatrix}$
2) $B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$
3) $C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{3} \end{pmatrix}$
4) $D = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \frac{1}{2} \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

2 Explanation

Transformation matrix gives the co-ordinates of images of vectors which are mapped from domain to co-domain.

3 Solution

$1, x, x^2$ is basis for P_2 which is domain space.

$$(Tf)(x) = \int_0^x f(t)dt + f'(x) \tag{3.0.1}$$

We should the image of basis vectors of domain space in co-domain space with respect to T.
Basis vector is given by

$$V = \begin{pmatrix} 1 \\ x^2 \\ x^3 \end{pmatrix} \tag{3.0.2}$$

Image of basis vector in co-domain space is given by

$$(Tf)(x) = \int_0^x f(t)dt + f'(x) \tag{3.0.3}$$

$$(Tf)(x) = \int_0^x 1 + t + t^2 dt + 1 + 2x \tag{3.0.4}$$

$$(Tf)(x) = t + \frac{t^2}{2} + \frac{t^3}{3}\bigg|_0^x + 1 + 2x \tag{3.0.5}$$

$$(Tf)(x) = 3x + \frac{x^3}{3} + \frac{x^2}{2} + 1 \tag{3.0.6}$$

It can be written as

$$(Tf)(x) = 1 + 3x + x^2(\frac{1}{2}) + x^3(\frac{1}{3}) \tag{3.0.7}$$

So coordinates of image of base vector is given by

$$V = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{3.0.8}$$

So coordinates of $T()$ with respect to co-domain basis are T(V)=$\begin{pmatrix} 1 \\ 3 \\ \frac{1}{2} \\ \frac{1}{3} \end{pmatrix}$

Among the given options matrix which maps coordinates of basis vector from domain space to co-domain space is

$$[T]=\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$