Assignment 8

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***Abstract*—This document uses Gram-Schmidt process to perform QR decomposition of a matrix.**

Download Python code from

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https://github.com/vishalashok98/AI5106/tree/main/Assignment_1
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Download the latex-tikz codes from

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1 Problem

Trace the curve

$$35x^2 + 30y^2 + 32x - 108y - 12xy + 59 = 0 \quad (1.0.1)$$

Express this conic section in the from

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \quad (1.0.2)$$

and perform QR decomposition for the matrix V.

2 Explanation

The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.1)$$

and can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \quad (2.0.2)$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.3)$$

The QR decomposition of a matrix is a factorization of the matrix into an orthogonal matrix and a triangular matrix. A QR decomposition of a real square matrix $\mathbf{V}$ is given by

$$\mathbf{V} = \mathbf{QR} \quad (2.0.4)$$

where $\mathbf{Q}$ is an orthogonal matrix $\left(\mathbf{Q}^T\mathbf{Q} = \mathbf{I}\right)$ and $\mathbf{R}$ is an upper triangular matrix.

3 Solution

Comparing (1.0.1) with (2.0.1), we get

$$\mathbf{V} = \begin{pmatrix} 35 & -6 \\ -6 & 30 \end{pmatrix} \quad (3.0.1)$$

$$\text{If, } \mathbf{V} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \text{ Consider, } \mathbf{V} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}$$

$$\text{Where, } \mathbf{a} = \begin{pmatrix} a1 \\ a2 \end{pmatrix} \text{and } \mathbf{b} = \begin{pmatrix} b1 \\ b2 \end{pmatrix} \quad (3.0.2)$$

$$\text{Then, } \mathbf{u_1} = \mathbf{a} \text{ , } \mathbf{e_1} = \frac{\mathbf{u_1}}{||\mathbf{u_1}||}$$

$$\mathbf{u_2} = \mathbf{b} - \left(\mathbf{b}^T\mathbf{e_1}\right)\mathbf{e_1} \text{ , } \mathbf{e_2} = \frac{\mathbf{u_2}}{||\mathbf{u_2}||} \quad (3.0.3)$$

$$\text{and, } \mathbf{V} = \begin{pmatrix} \mathbf{e_1} & \mathbf{e_2} \end{pmatrix} \begin{pmatrix} \mathbf{a}^T\mathbf{e_1} & \mathbf{b}^T\mathbf{e_1} \\ 0 & \mathbf{b}^T\mathbf{e_2} \end{pmatrix} = \mathbf{QR} \quad (3.0.4)$$

Performing QR decomposition on $\mathbf{V}$ we get,

$$\mathbf{e_1} = \frac{1}{\sqrt{1261}} \begin{pmatrix} 35 \\ -6 \end{pmatrix} \quad (3.0.5)$$

$$\mathbf{e_2} = \frac{1}{380} \begin{pmatrix} \frac{37839}{100} \\ -\frac{35895}{1000} \end{pmatrix} \quad (3.0.6)$$

$$\mathbf{Q} = \begin{pmatrix} \mathbf{e_1} & \mathbf{e_2} \end{pmatrix} \quad (3.0.7)$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{1261} & \frac{30\sqrt{13}}{\sqrt{97}} \\ 0 & \frac{-8805}{1000} \end{pmatrix} \quad (3.0.8)$$

It is easy to verify that $\mathbf{QR} = \mathbf{V}$ and $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$. Thus, $\mathbf{V}$ is decomposed into an orthogonal matrix and an upper triangular matrix.