Modeling the Lorenz Attractor with SINDY

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Abstract

Introduction

System identification techniquesfor recovering a system's governing equations from data collected from the system are useful for a wide range of applications in various fields. The system-identification algorithm proposed in [paper] involves sparse identification of nonlinear dynamical systems (SINDy). As stated in the title, the algorithm relies on the assumption that dynamics of the system depend on only a few linear and nonlinear terms. The algorithm has been

SINDY Algorithm

The algorithm is effective under the assumption that the physical systems that are to be studied with SINDy have dynamics that are determined but only a few terms and are thus sparse in a high-dimensional nonlinear function space. The algorithm applies to systems of the form

$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{f}(\boldsymbol{x}(t)) \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state of the system at time t and $\boldsymbol{f}$ consists of the governing equations of the system. Since $\boldsymbol{f}$ is sparse in the nonlinear function space, the solution to the system can be obtain with convex methods/regression ?

To compute the active nonlinear terms in the dynamics, the system

$$\dot{\boldsymbol{X}} = \boldsymbol{\Theta}(\boldsymbol{X})\boldsymbol{\Xi} \tag{2}$$

where $\boldsymbol{X}$ and $\dot{\boldsymbol{X}}$ are data matrices containing the samples of the state and it's derivative at times $t_1, t_2, \cdots, t_m$ arranged as follows

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}^T(t_1) \\ \boldsymbol{x}^T(t_2) \\ \vdots \\ \boldsymbol{x}^T(t_m) \end{bmatrix} = \begin{bmatrix} x_1(t_1) & x_2(t_1) & \cdots & x_n(t_1) \\ x_1(t_2) & x_2(t_2) & \cdots & x_n(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(t_m) & x_2(t_m) & \cdots & x_n(t_m) \end{bmatrix}$$

$$\dot{\boldsymbol{X}} = \begin{bmatrix} \dot{\boldsymbol{x}}^T(t_1) \\ \dot{\boldsymbol{x}}^T(t_2) \\ \vdots \\ \dot{\boldsymbol{x}}^T(t_m) \end{bmatrix} = \begin{bmatrix} \dot{x}_1(t_1) & \dot{x}_2(t_1) & \cdots & \dot{x}_n(t_1) \\ \dot{x}_1(t_2) & \dot{x}_2(t_2) & \cdots & \dot{x}_n(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ \dot{x}_1(t_m) & \dot{x}_2(t_m) & \cdots & \dot{x}_n(t_m) \end{bmatrix}.$$

The state data are input to the library of candidate functions, $\boldsymbol{\Theta}(\boldsymbol{X})$, which consist of constant, polynomial, and trigonometric terms that are chosen based on hypotheses about the system dynamics. [[More about choosing functions]]. The terms of the candidate functions are arranged in a matrix

$$\boldsymbol{\Theta}(\boldsymbol{X}) = \begin{bmatrix} | & | & | & & | & | & \\ 1 & \boldsymbol{X} & \boldsymbol{X}^{P_2} & \cdots & \sin(\boldsymbol{X}) & \cos(\boldsymbol{X}) & \cdots \\ | & | & | & & | & | & \end{bmatrix} \tag{3}$$

where polynomials of degree i are denoted $\boldsymbol{X}^{P_i}$. For example, $\boldsymbol{X}^{P_i}$ contains quadratic nonlinearities in the state $\boldsymbol{x}$ as follows

$$\boldsymbol{X}^{P_2} = \begin{bmatrix} x_1^2(t_1) & x_1(t_1)x_2(t_1) & \cdots & x_2^2(t_1) & \cdots & x_n^2(t_1) \\ x_1^2(t_2) & x_1(t_2)x_2(t_2) & \cdots & x_2^2(t_2) & \cdots & x_n^2(t_2) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_1^2(t_m) & x_1(t_m)x_2(t_m) & \cdots & x_2^2(t_m) & \cdots & x_n^2(t_m) \end{bmatrix}.$$

The final term in Eq. (2), $\boldsymbol{\Xi} = [\boldsymbol{\xi}_1 \boldsymbol{\xi}_2 \cdots \boldsymbol{\xi}_n]$ contains sparse vectors of coefficients that determine the influence the active nonlinear terms. Solving for these coefficients requires a distinct optimization of each vector,

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{\Xi}^T(\boldsymbol{\Theta}(\boldsymbol{x}^T))^T. \tag{4}$$

SINDyC Algorithm

Lorenz Attractor

Theory

Results

0.1 Further Study

Conclusion

Bibliography

Code

Figures