Multiprocessor Scheduling

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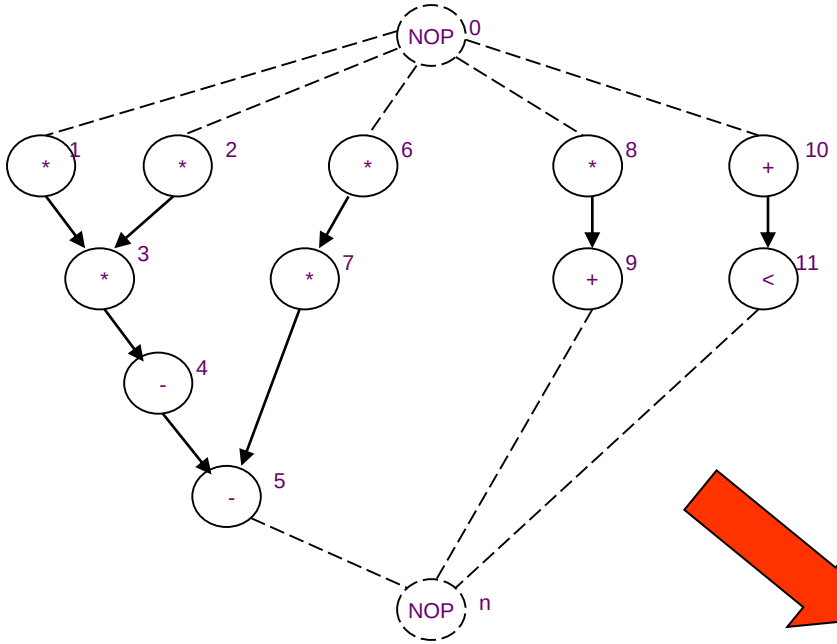
Scheduling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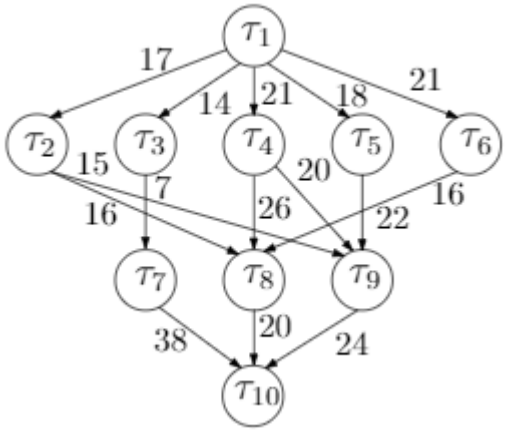
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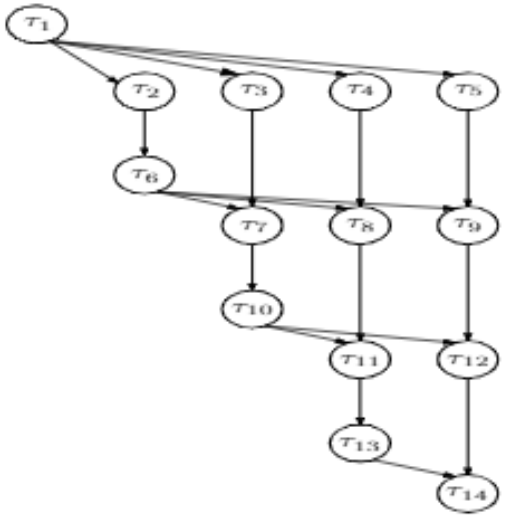
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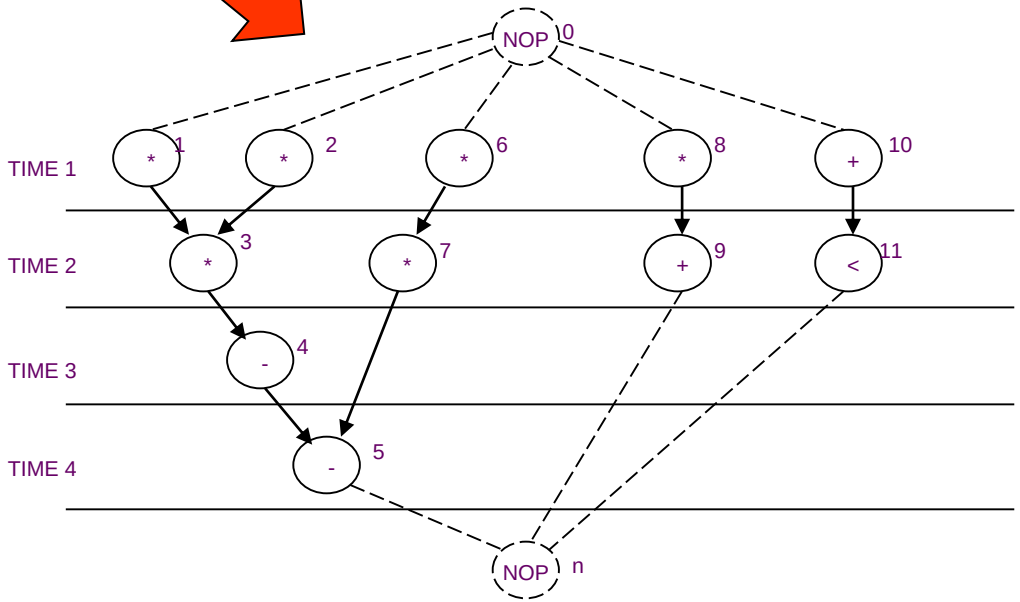
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Multiprocessor Scheduling

- Assumptions:
  - A1: All operations have the same type – a multiprocessor is executing the operations
  - A2: All operations have unit delay

- General Scheduling Problem:
  - Different type of functional units.
  - Operation can have delay more than one.

ILP formulation for Multiprocessor Scheduling - MLRC

Objective function

$$\text{minimize } \mathbf{c}^T\mathbf{t} \text{ such that}$$

Unique start time

$$\sum_l x_{il} = 1, \quad i = 0, 1, \ldots, n$$

$$x_{il} \in \{0, 1\}, \quad i = 0, 1, \ldots, n, \quad l = 1, 2, \ldots, \bar{\lambda} + 1$$

General case:

$$\text{minimize } \mathbf{c}^T\mathbf{t} \text{ such that}$$

$$\sum_l x_{il} = 1, \quad i = 0, 1, \ldots, n$$

$$x_{il} \in \{0, 1\}, \quad i = 0, 1, \ldots, n, \quad l = 1, 2, \ldots, \bar{\lambda} + 1$$

ILP formulation for Multiprocessor Scheduling - MLRC

- Data dependency/precedence constraints

$$\sum_l l \cdot x_{il} - \sum_l l \cdot x_{jl} \geq 1, \quad i, j = 0, 1, \ldots, n \ : (v_j, v_i) \in E$$

General case:

$$\sum_l l \cdot x_{il} - \sum_l l \cdot x_{jl} - d_j \geq 0, \quad i, j = 0, 1, \ldots, n \ : (v_j, v_i) \in E$$

ILP formulation for Multiprocessor Scheduling - MLRC

- Resource constraints

$$\sum_i x_{il} \leq a, \quad l = 1, 2, \ldots, \bar{\lambda} + 1$$

General case

$$\sum_{i:\mathcal{T}(v_i)=k} \sum_{m=l-d_i+1}^{l} x_{im} \leq a_k, \quad k = 1, 2, \ldots, n_{res}, \quad l = 1, 2, \ldots, \bar{\lambda} + 1$$

MLRC formulation can done by similar way

Example 1 (cont'd.)

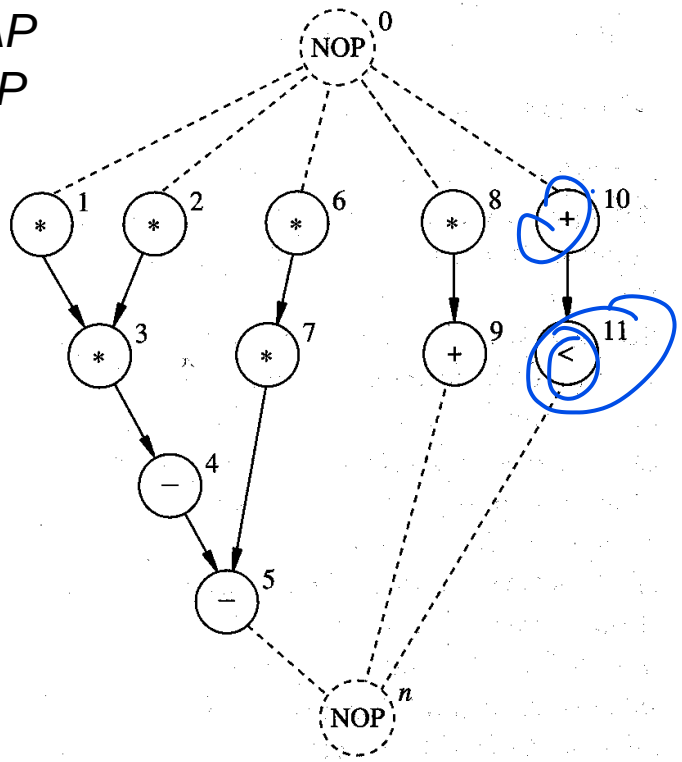
- Start time must be unique

Recall: $\sum_{l} x_{il} = \sum_{l=t_i^S}^{l=t_i^L} x_{il}$

where:

$t_i^S = t_i$ *computed with ASAP*

$t_i^L = t_i$ *computed with ALAP*



$$x_{0,1} = 1$$

$$x_{1,1} = 1$$

$$x_{2,1} = 1$$

$$x_{3,2} = 1$$

$$x_{4,3} = 1$$

$$x_{5,4} = 1$$

$$x_{6,1} + x_{6,2} = 1$$

$$x_{7,2} + x_{7,3} = 1$$

$$x_{8,1} + x_{8,2} + x_{8,3} = 1$$

$$x_{9,2} + x_{9,3} + x_{9,4} = 1$$

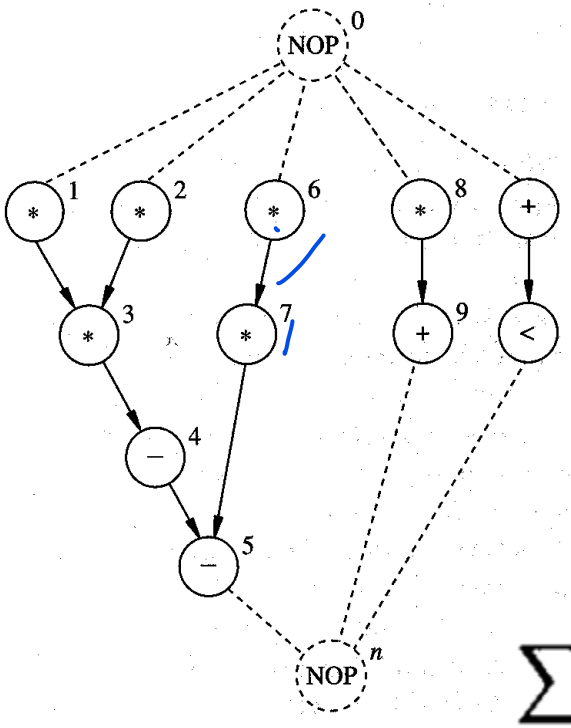
$$x_{10,1} + x_{10,2} + x_{10,3} = 1$$

$$x_{11,2} + x_{11,3} + x_{11,4} = 1$$

$$x_{n,5} = 1$$

Example 1 (cont'd.)

- Precedence constraints
  - Note: only non-trivial ones listed



$$2x_{7,2} + 3x_{7,3} - x_{6,1} - 2x_{6,2} - 1 \geq 0$$

$$2x_{9,2} + 3x_{9,3} + 4x_{9,4} - x_{8,1} - 2x_{8,2} - 3x_{8,3} - 1 \geq 0$$

$$2x_{11,2} + 3x_{11,3} + 4x_{11,4} - x_{10,1} - 2x_{10,2} - 3x_{10,3} - 1 \geq 0$$

$$4x_{5,4} - 2x_{7,2} - 3x_{7,3} - 1 \geq 0$$

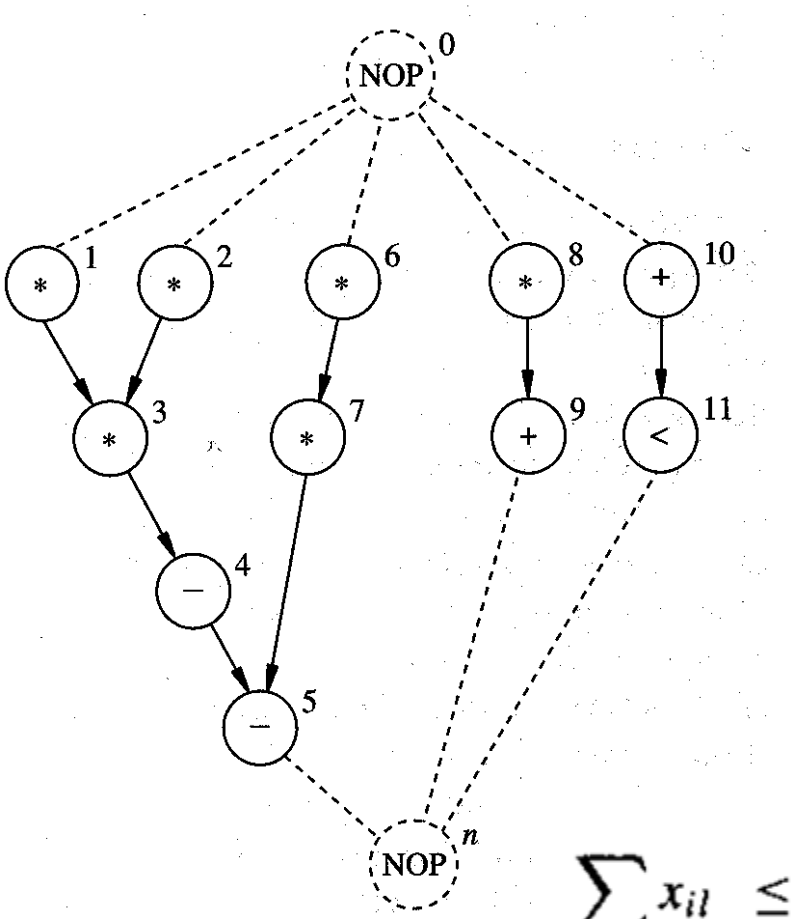
$$5x_{n,5} - 2x_{9,2} - 3x_{9,3} - 4x_{9,4} - 1 \geq 0$$

$$5x_{n,5} - 2x_{11,2} - 3x_{11,3} - 4x_{11,4} - 1 \geq 0$$

$$\sum_l l \cdot x_{il} - \sum_l l \cdot x_{jl} \geq 1, \quad i, j = 0, 1, \ldots, n \ : (v_j, v_i) \in E$$

Example 1 (cont'd.)

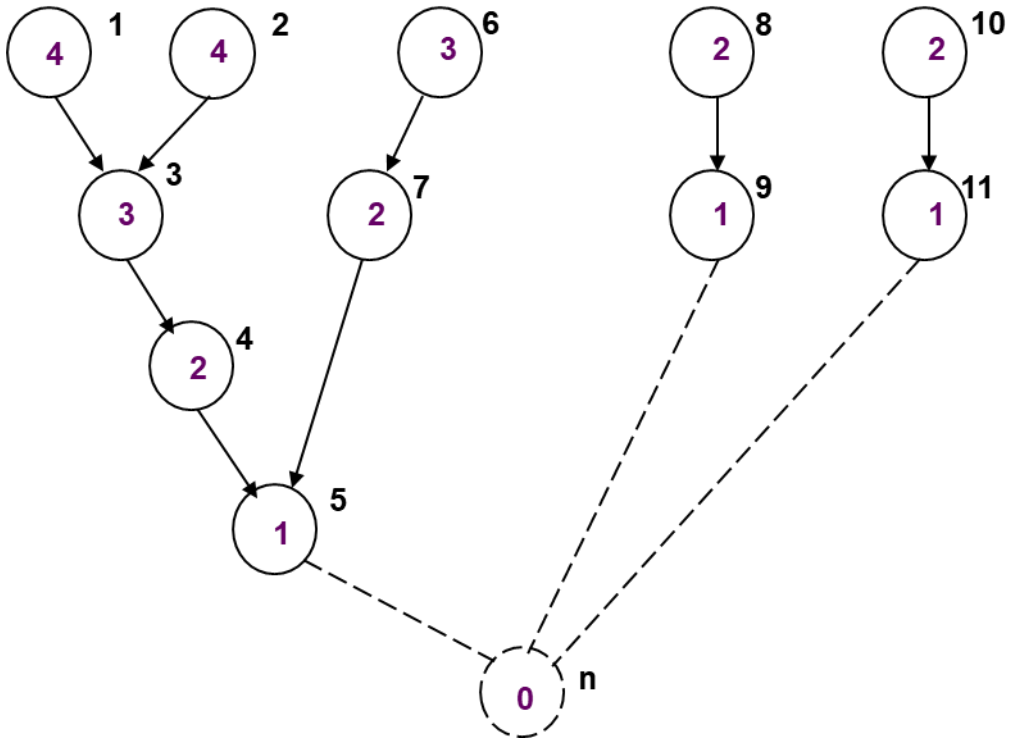
Resource Constraints



$$\sum_i x_{il} \leq a, \quad l = 1, 2, \ldots, \bar{\lambda} + 1$$

Multiprocessor Scheduling and Hu's algorithm

- Assumptions:
  - A1: All operations have the same type – a multiprocessor is executing the operations
  - A2: All operations have unit delay
  - A3: Sequence graph is Tree
    - Implications: There is no parallel paths.
    - have single paths from each vertex to the sink with monotonically unit-wise decreasing labels (path length from the sink node)
- The problem is in P with A1, A2 and A3
  - Greedy strategy
  - Exact solution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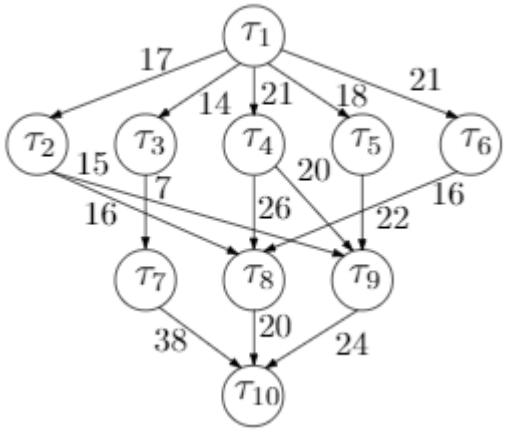
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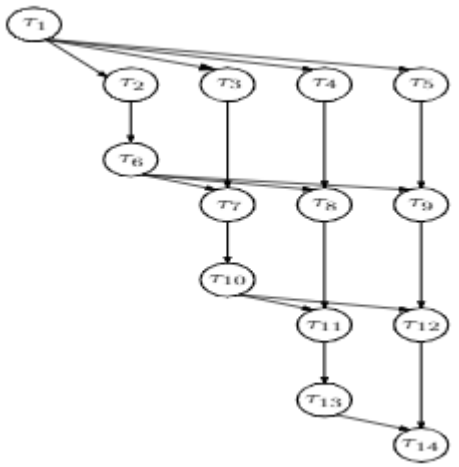
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Thank You