Resource Allocation and Binding

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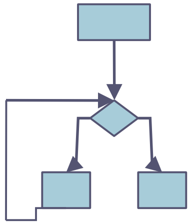
High-level Synthesis Steps

```
while (x_var < a_var) loop
    t1 := u_var * dx_var;
    t2 := 3 * x_var;
    t3 := 3 * y_var;
    t4 := t1 * t2;
    t5 := dx_var * t3;
    t6 := u_var - t4;
    u_var := t6 - t5;
    y1 := u_var * dx_var;
    y_var := y_var + y1;
    x_var := x_var + dx_var;
 end loop;
X <= x_var;
Y <= y_var;
U <= u_var;
```

pre-processing



Input behaviour

scheduling

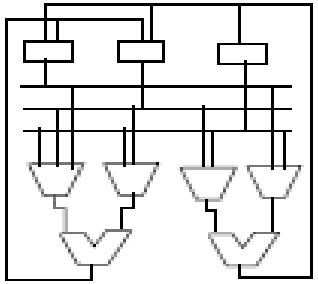
1.	< 1 *>		
2.		<2 + >	<0 * >
3.		<4 * >	| * |
4.		| * |	<3 *>
5.	<7 *>	<6 *>	| * |
6.	| * |	| * |	<5 - >
7.		<9 +>	<8 - >

Allocation & binding

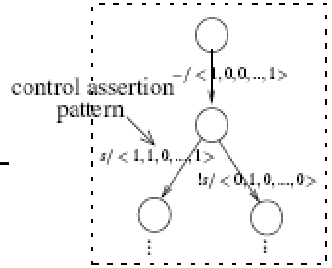
R1 : 3, v1
R2 : x u, v5
R3 : v0, v6
R4 : v3

FU1: op1, on3. ..
FU2: op2, op5, …
FU3: …

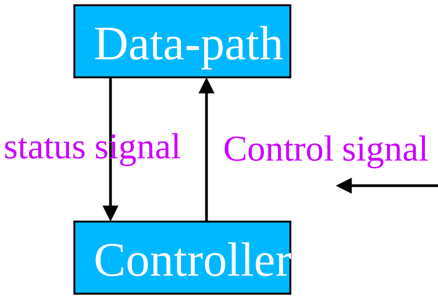
Data-path generation



Controller generation



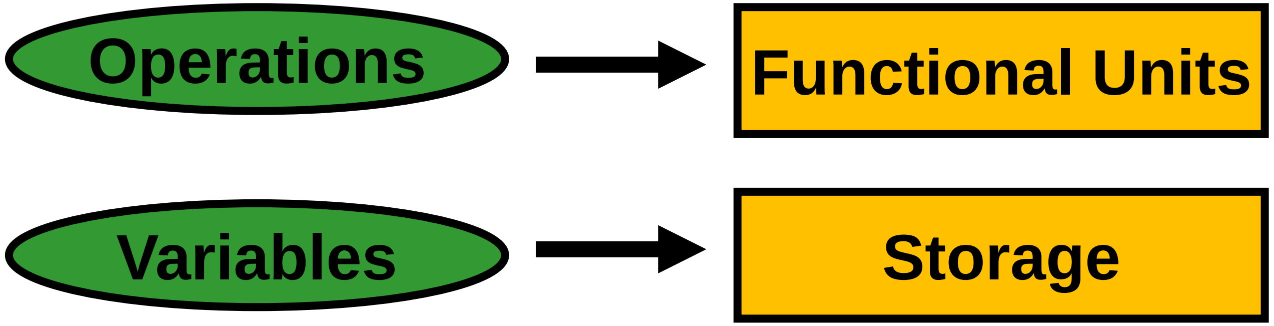
Controller FSM



RTL behaviour

Allocation and Binding

- **Objectives: Maximize Resource sharing; hence, minimize resource usage**



Subtasks:

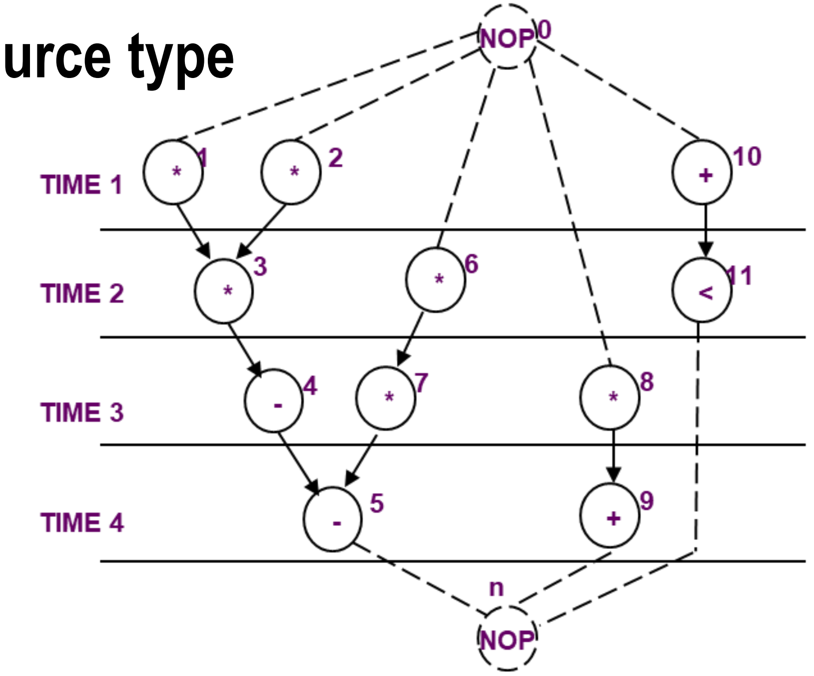
1. **FU allocation & Binding**
2. **Register Allocation & Binding**

Allocation and Binding

- Allocation:
  - Identify the minimum resources or Use the available resources
- Binding:
  - Map the operations to FUs and
  - Variables to registers
- Sharing:
  - Many-to-one relation
- Optimum binding/sharing:
  - Minimize the resource usage

Optimum sharing problem

- Scheduled sequencing graphs
  - Operation concurrency is well defined
- Consider *operation types* independently
  - Problem decomposition
  - Perform analysis for each resource type

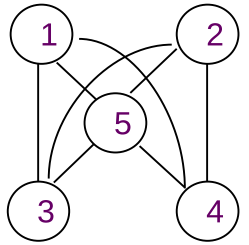


Compatibly and conflicts

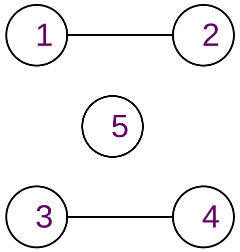
- **Operation compatibility:**
  - Same type
  - Non concurrent
- ***Compatibility* graph:**
  - Vertices: operations
  - Edges: compatibility relation
- ***Conflict* graph:**
  - Complement of compatibility graph

t1	x=a+b	y=c+d	1	2
t2	s=x+y	t=x-y	3	4
t3	z=a+t		5	

Compatibility graph

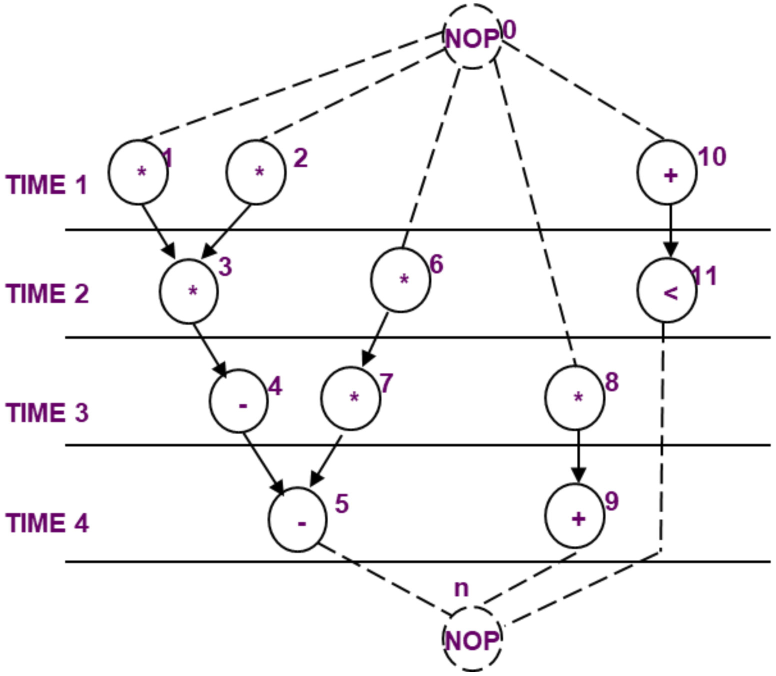


Conflict graph



Compatibility Graph

Definition 6.2.1. The **resource compatibility graph** $G_+(V, E)$ is a graph whose vertex set $V = \{v_i, i = 1, 2, \ldots, n_{ops}\}$ is in one-to-one correspondence with the operations and whose edge set $E = \{\{v_i, v_j\}\ i, j = 1, 2, \ldots, n_{ops}\}$ denotes the compatible operation pairs.



Resource Optimization as Clique Cover Problem Compatible

- A group of mutually compatible operations corresponds to a subset of vertices that are all mutually connected by edges, i.e., to a clique.
- Therefore a ***maximal*** set of mutually compatible operations is represented by a ***maximal clique*** in the compatibility graph.
- An optimum resource sharing is one that minimizes the number of required resource instances.
- We can associate a resource instance to each clique, the problem is equivalent to partitioning the graph into a minimum number of cliques. ***clique cover number*** of G+(V, E), denoted by *k*(G+(V, E)).

Conflict Graph

- Two operations have a conflict when they are not compatible
- The conflict graph is the ***complement*** of the compatibility graph

Definition 6.2.2. The **resource conflict graph** $G_-(V, E)$ is a graph whose vertex set $V = \{v_i, i = 1, 2, \ldots, n_{ops}\}$ is in one-to-one correspondence with the operations and whose edge set $E = \{\{v_i, v_j\}\ i, j = 1, 2, \ldots, n_{ops}\}$ denotes the conflicting operation pairs.

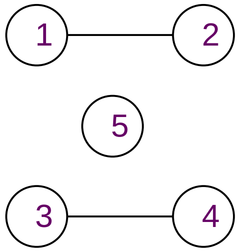
Resource Sharing as Graph Coloring Problem

- A set of mutually compatible operations corresponds to a subset of vertices that are not connected by edges, also called the ***independent set***
- A proper vertex coloring of the conflict graph provides a solution to the sharing problem
- Each color corresponds to a resource instance
- An optimum resource sharing corresponds to a vertex coloring with a minimum number of colors. Such a number is the chromatic number of G-(V, ***E)***

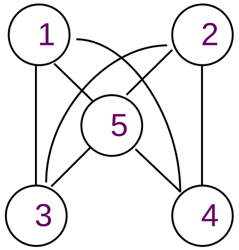
Example

t1	x=a+b	y=c+d	1	2
t2	s=x+y	t=x-y	3	4
t3	z=a+t		5	

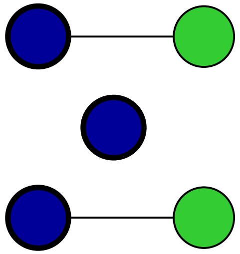
Conflict



Compatibility



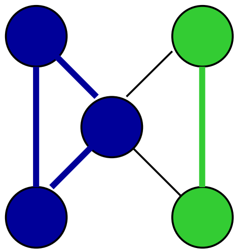
Coloring



ALU1: 1,3,5

ALU2: 2,4

Partitioning



Compatibility and conflicts

- Compatibility graph:
  - Partition the graph into a minimum number of cliques
  - Find clique cover number $k (G_+)$
- Conflict graph:
  - Color the vertices by a minimum number of colors.
  - Find the chromatic number $x (G_-)$
- NP-complete problems
  - Heuristic algorithms

Perfect Graph

- The first number we consider is he ***clique number* ω(G)**, which is the cardinality of its largest clique, called ***maximum clique***
- A ***stable set*** or ***independent set, α(G)*** is a subset of vertices with the property that no two vertices in the stable set are adjacent.
- Relation Between ***α(G)*** ***and*** **clique cover number** k **(G_+)?**
- Relation Between **ω(G) and chromatic number** χ **(G_)?**

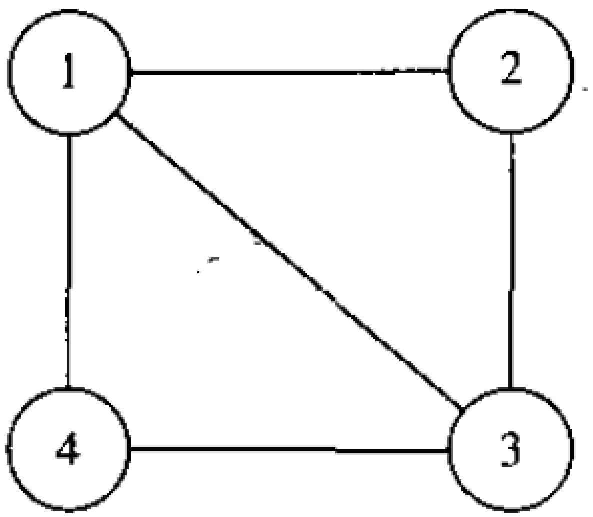
Perfect Graph

- The size of the maximum clique is a lower bound for the chromatic number, because all vertices in that clique must be colored differently
- the stability number is a lower bour $\omega(G) \le \chi(G)$ cover number, since each vertex of the stable set must belong to a different clique of a clique cover
- A graph is ***perfect*** when the inequalities can be replaced by equalities

$$\alpha(G) \le \kappa(G)$$

Chordal Graph (A special set of Perfect Graph)

- A graph is said to be ***chordal,*** or ***triangulated,*** if every cycle with more than three edges possesses a ***chord,*** i.e., an edge joining two non-consecutive vertices in the cycle

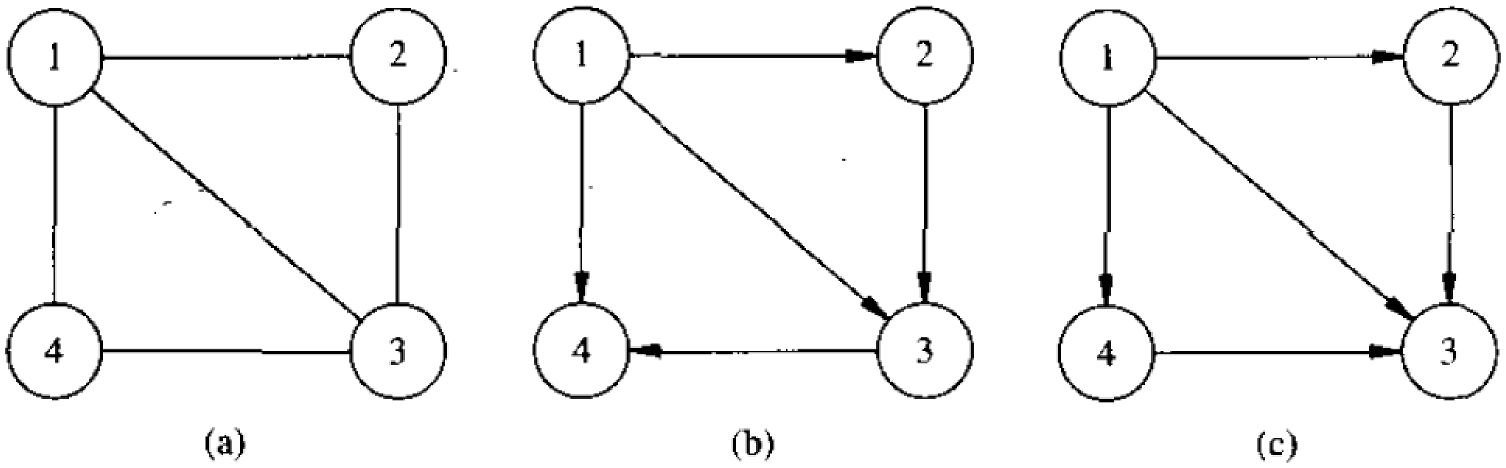


Interval Graph

- A subclass of chordal graphs is the one of ***interval graphs.***
- An interval graph is a graph whose vertices can be put in one-to-one correspondence with a set of ***intervals,*** so that two vertices are adjacent if and only if the corresponding intervals intersect.
- Advantage: The graph coloring problem is polynomial time solvable
- How to identify a graph is an interval graph?

Comparability graph

- A graph ***G(V, F)*** is a ***comparability graph*** if it satisfies the ***transitive orientation property,*** i.e., if it has an orientation such that in the resulting directed graph **G(V, E*)*,** **((vi, vj) in E** and **(vj, vk) in E implies (vi, vk) in E.**



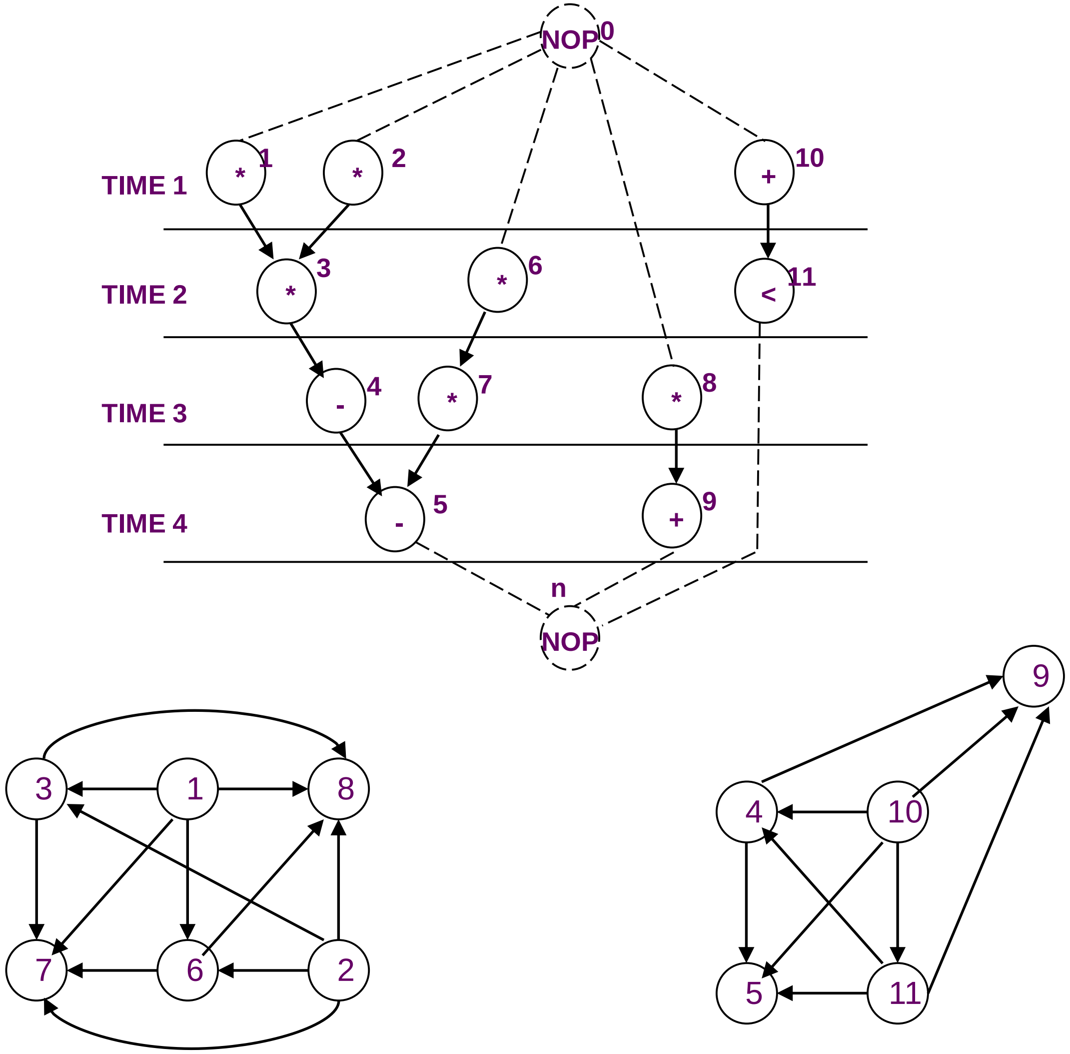
Gilmore Theorem

- An important theorem, by Gilmore and Hoffman, relates comparability to interval graphs
- **Theorem:** An undirected graph is an interval graph if and only if it is chordal and its complement is a comparability graph

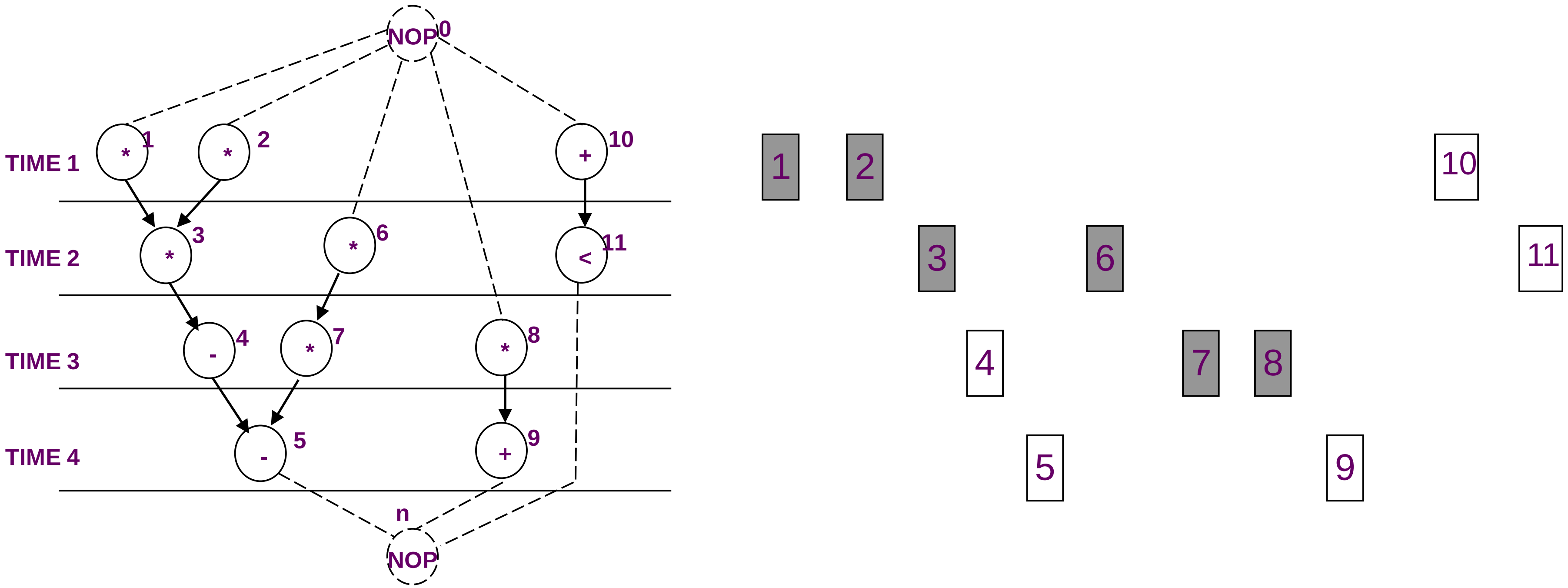
Data-flow graphs
(flat sequencing graphs)

- The compatibility/conflict graphs have special properties:
  - Compatibility
    - Comparability graph
  - Conflict
    - Chordal
  - So they are perfect graph satisfying Gilmore's theorem
- Polynomial time solutions for Interval Graph
  - Left-edge algorithm

Example


NOP 0
TIME 1
TIME 2
TIME 3
TIME 4
* 1
* 2
+ 10
* 3
* 6
< 11
- 4
* 7
* 8
- 5
+ 9
n
NOP
3
1
8
7
6
2
9
4
10
5
11

Example



Thank You