

IOWA STATE UNIVERSITY

DEPARTMENT OF ELECTRICAL AND COMPUTER  
ENGINEERING

DEEP MACHINE LEARNING: THEORY AND PRACTICE

EE 526X

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## Homework 2

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## 1 Problem 1

We have a softplus function,

$$f(z) = \log(1 + e^z) \quad (1)$$

Taking first derivative of the function:

$$f'(z) = \frac{1}{(1 + e^z)} e^z \quad (2)$$

multiplying and dividing by  $e^{-z}$ ,

$$f'(z) = \frac{e^z \cdot e^{-z}}{(1 \cdot e^{-z} + e^z \cdot e^{-z})} = \frac{1}{(1 + e^{-z})} \quad (3)$$

Now taking second derivative of softplus function,

$$f''(z) = \frac{(1 + e^{-z}) \cdot 0 + e^{-z}}{(1 + e^{-z})^2} \quad (4)$$

$$f''(z) = \frac{e^{-z}}{(1 + e^{-z})^2} \quad (5)$$

From eq. (5), numerator is always positive because exponential is a positive number and any power to a positive number is always a positive number. The denominator is always positive because it is squared. Therefore we can say that second derivative of a softplus function is always positive for every value of  $z$ . This implies that a softplus function is convex in  $z$ .