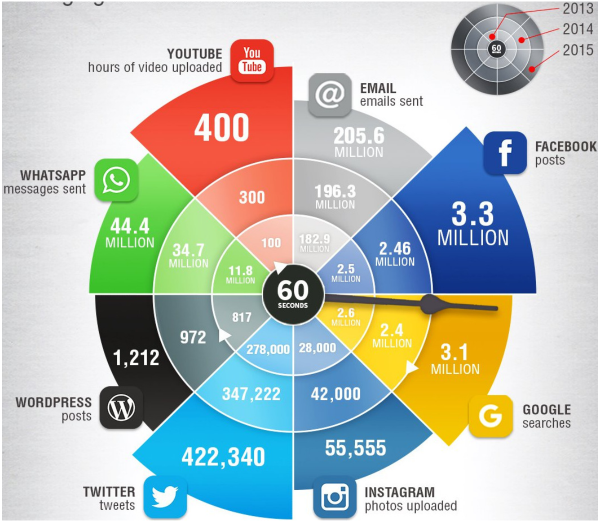
# What happens online in a single minute?




Source: Smart Insights

# Basics

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- **Volume:** Volume refers to the size of the data to be analyzed.
- **Variety:** Variety refers to the different forms of the data.
- **Veracity:** Veracity refers to the trustworthiness of the data.
- **Velocity:** Velocity is the frequency of incoming data that needs to be processed.

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**Note**: The **Value** of data is also an important issue.

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4. Studying dynamically changing health conditions
   - A wearable device aims to keep track of all the body parameters (that keep changing) in real-time.

# Data stream models

## Definition (Data stream model)

A *data stream model* defines an input stream $\mathcal{S} =< s_1, s_2, \cdots >$ arriving sequentially, item by item, and describes an underlying signal $S$, where $S : [1 \ldots N] \rightarrow \mathbb{R}$ is a one-dimensional function.

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The data stream models can be of the following three types [Muthukrishnan, 2005]:

- Time Series Model
- Cash Register Model
- Turnstile Model

# Different data stream models

The models are classified based on the information about how the input data elements stream in.

1. Time Series Model
   - Each $s_i$ equals $S[i]$ and they appear in increasing order of $i$.
   - Observing the traffic at an IP link for each 5 min, or volume estimation of share trading in every 10 min, etc.
2. Cash Register Model
   - Perhaps the most popular data model.
   - Here $s_i$'s are increments to $S[j]$'s.
   - Monitoring IP addresses that access a web server.
3. Turnstile Model
   - This is the most general model.
   - Here $s_i$'s are updates to $S[j]$'s.
   - Monitoring the stock values of a company.

# *Processing streaming data*

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*Consider a stream of integers. Further suppose, we want to find out the average of the integers in a window of size N.*

For the first $N$ inputs, we can sum and count the integers and calculate the average. After that, for each of the new input $i$ received, add $\frac{(i-j)}{N}$ to the previous average value, where $j$ is the oldest integer in the window.

# *Streaming algorithms*

In streaming algorithms, the data is available as a stream and we would like –

- the per-item processing time
- storage and
- overall computing time

to be simultaneously $O(N, t)$, preferably $O(polylog(N, t))$, at any time instant $t$ in the data stream.

**Note:** $O(polylog(N, t))$, often written as $polylog(N, t)$, means $O((\log n)^k)$ or $O(\log^k n)$ for some $k$.

# Developments on streaming algorithms

- The limited earlier contributions before 2005 have been well reviewed in [Muthukrishnan, 2005].
- Diverse efforts were made to revisit and solve a number of problems in a streaming setting. There were studies on matrix approximation, matrix decomposition, low rank approximation, $\ell_p$ regression, etc. [Halko, 2010, Kannan, 2009, Mahoney, 2011].
- There has been an influential line of work on computing a low-rank approximation of a given matrix, starting with the works of [Frieze, 2004, Papadimitriou, 1998].
- Very recently, the $\ell_1$ and $\ell_2$ heavy eigen-hitter problems have been estimated in the streaming model in a lower dimension [Andoni, 2013]. Andoni and Huy achieved a success probability of $\frac{5}{9}$ [Andoni, 2013]. They also estimated the residual error with the same probabilistic accuracy.

# Streaming graph

**Definition (Streaming graph)**

A *streaming graph* is a simple graph on $n$ vertices $V = \{v_1, v_2, \ldots, v_n\}$ with edges $E = \{(v_i, v_j) : s_k = (i, j) \text{ for some } k \in [m]\}$, where the data items $s_k \in [n] \times [n]$ are available as an input stream $\mathcal{S} =< s_1, s_2, \ldots, s_m >$.

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| Input stream | Interpretation in terms of a graph |
|---|---|
| (1, 2) | Two new vertices and a new edge are included |
| (1, 3) | A new vertex and a new edge are included |
| (2, 3) | A new edge is included |
| ... | ... |

# Basics

Sampling in a data stream refers to retaining only a subset of items (at most polylogarithmic sized), even though every input/update is seen.

The sampling methods can be of different types [Muthukrishnan, 2005]:

- Domain sampling
- Universe sampling
- Reservoir sampling
- Priority sampling
- Distinct sampling
- etc.

# *Finding missing numbers*

**Problem statement:** Let $\pi$ be a permutation of $\{1, \ldots, n\}$. Further, let $\pi_{-1}$ be $\pi$ with one element missing. Given the stream of elements $\pi_{-1}[i]$ in increasing order $i$, one by one, determine the missing integer.

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**Note:** This was actually posed as a puzzle by Paul Erdos.

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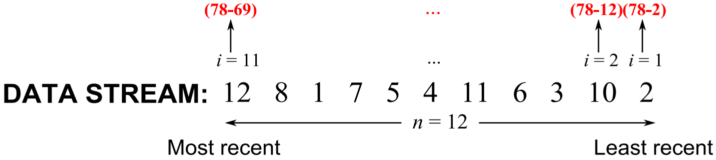

# *Finding missing numbers*

(78-69) ... (78-12)(78-2)

$i = 11$ ... $i = 2$ $i = 1$

**DATA STREAM:** 12 8 1 7 5 4 11 6 3 10 2

$\longleftarrow n = 12 \longrightarrow$

Most recent Least recent

**Note:** This consumes exactly $2 \log n$ bits of space in the worst case.

# *Finding missing numbers*

Smarter approach: For each $i$, store the parity sum of the $i^{th}$ bits of all numbers seen thus far. The final parity sum bits are the bits of the missing number. It will consume $\log n$ bits of space in the worst case.

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**Note:** A similar solution will work even if $n$ is unknown, for example by letting $n = \max_{j \leq i} \pi_{-1}[j]$ in each pass.

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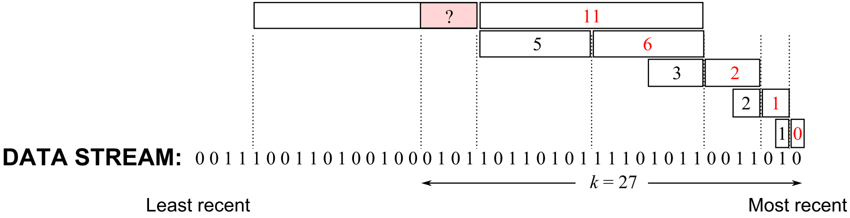
**Note:** Counting the number of 1's in a stream of binary digits (bitstream) helps to compute the entropy of the stream.

# *Counting 1's*

We need no more than two blocks of any size as explained below.

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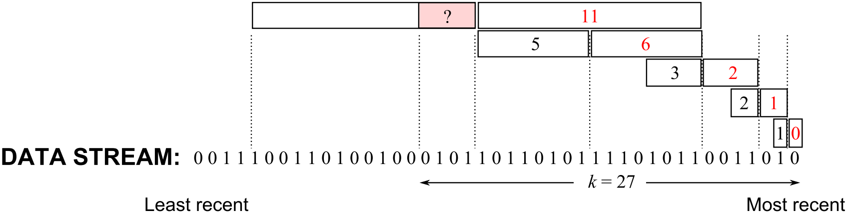


The count of 1's is: $(0 + 1 + 2 + 6 + 11) + \frac{7}{16}.4 \simeq 22$.

**<u>Note</u>:** Error in count is due to the unknown area in the most recent block. Hence, it is unbounded.

# *Counting 1's*

Smarter approach (DGIM) [Datar, 2002]: Store the count of 1's in *buckets* (of the stream) with variable number of elements but having exponentially increasing number of 1's (size), in the reverse order. It will consume $O(\log^2 N)$ space.

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A *bucket* is a segment of the window having the following properties:

- The size of a bucket (number of 1's in it) is in the form of $2^i$.
- Each bucket contains the timestamp of its end bit (requires $O(\log N)$ bits) and its size (requires $O(\log \log N)$ bits).

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**Note:** Each bit in the stream has a *timestamp*, which is defined with a $(\mod N)$ function (to map everything within the window).

# *Counting 1's*

The bitstream is represented with a collection of buckets as follows:

- There can be either one or two buckets having the same size.
- Buckets are sorted by size.
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- Buckets are removed as and when their end-time is more than $N$ time units in the past.

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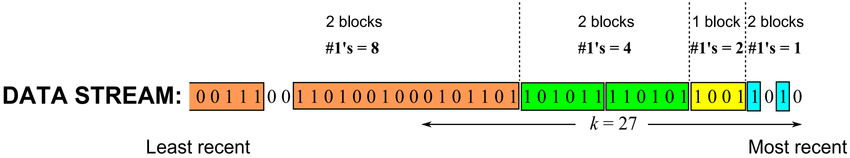

# Counting 1's

The buckets are updated as follows:

- When a new bit comes in, drop the oldest bucket if its end-time is prior to $N$ time units before the current time.
- No changes are required if the new bit is 0. Otherwise, do the following:
    1. Create a new bucket of size 1 containing the new bit.
    2. Define the timestamp of new bucket with the current time.
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**Note:** While combining two buckets into a new one, timestamp of the newest bucket becomes the timestamp of the new bucket.

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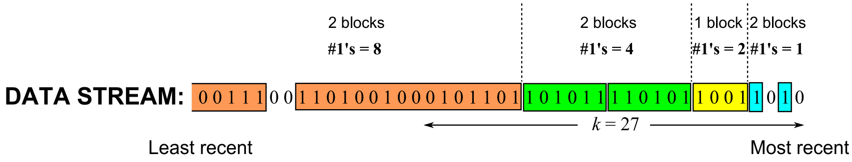
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Least recent
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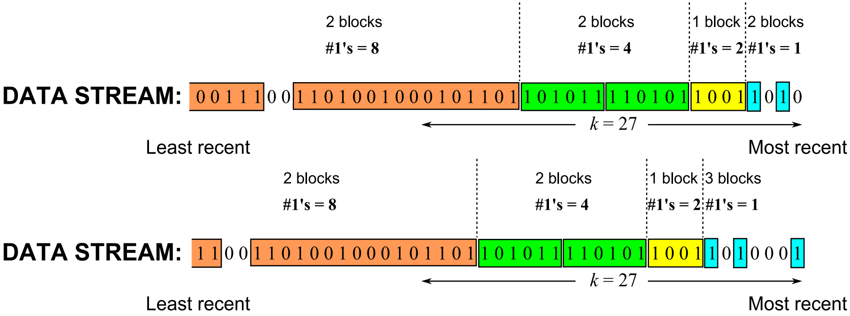
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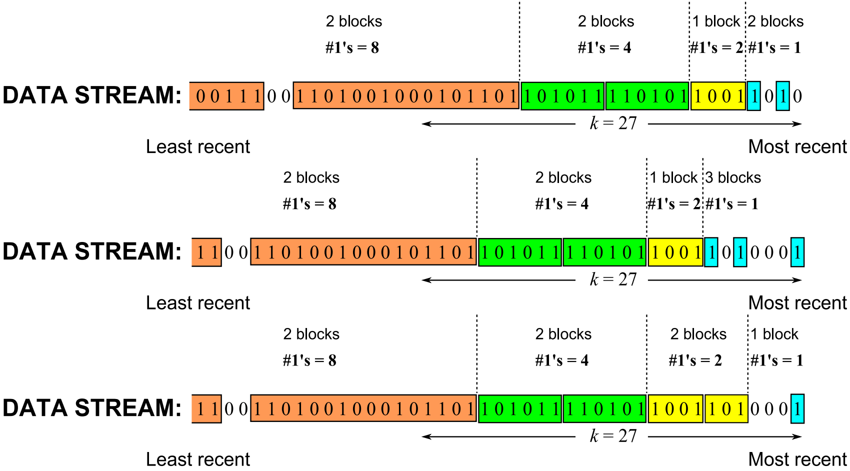
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