Indian Institute of Engineering Science and Technology, Shibpur.

Department of Computer Science and Technology.

Problem Set on Symbolic Logic

1. a) Prove that the binary connective ($\alpha \mid \beta$) ("not both …..and ") called the Sheffer Stroke whose truth table is given by

α	β	$\alpha \mid \beta$
T	T	F
T	F	T
F	T	T
F	F	T

is adequate, i. e., Sheffer Stroke can express $\neg$, $\wedge$ and $\vee$.

b) Obtain the truth table for joint denial (neither α nor β),written as $\alpha \downarrow \beta$, and prove that it is also adequate.

2. Given that if the Board of Directors refuses to enact new laws, then the strike will not be over unless it lasts more than one year and the Chairman of the firm resigns. The Board of Directors refuses to act. The strike just starts. Prove that the strike will not be over.

3. Consider the following statements:

   A1 ≡ If the maid stole the jewelry then the butler wasn't guilty.
   A2 ≡ Either the maid stole the jewelry or she milked the cow.
   A3 ≡ If the maid milked the cow then the butler got his cream.
   C ≡ Therefore, if the butler was guilty then he got his cream.

   Demonstrate that the conclusion C is valid using Propositional Logic

4. One of the successful applications of expert systems has been analyzing the problem of which chemical syntheses are possible. Consider the following example of such a problem.

   We know we can perform the following chemical reactions:

   $MgO + H_2 \rightarrow Mg + H_2O$

   $C + O_2 \rightarrow CO_2$

   $CO_2 + H_2O \rightarrow H_2CO_3$

   i) Represent these rules and the assumptions that we have some MgO, H_2, O_2 and C by propositional logic formulae.

   ii) Describe the state of affairs in clausal form and as a PROLOG program.

   iii) Give a resolution proof that we can get some H_2CO_3

5. If taxes are raised, then revenue will be generated. However, if unemployment increases, then people will be unhappy. Use resolution in Propositional Logic to prove that if taxes are raised and unemployment increases, then revenue will be generated but people will be unhappy.

6. Given the following facts,

   "One of Tinker, Tailor, Soldier, or Spy is the culprit. The culprit stole the document. Tinker and Soldier did not steal the document. If Tailor or Spy is the culprit, then the document must be in Paris. The culprit was wearing a red shirt. Both Tailor and Soldier were not wearing a red shirt."

   Show, after encoding the facts in propositional calculus, and using resolution refutation, the proof of the following statement,

   "Spy was wearing a red shirt and the document is in Paris."

7. Heads I win. Tails you lose. Use resolution in PL to prove: I win.

8. Show whether following arguments are valid using PL:

   i) If the band performs, then the hall will be full provided that the tickets are not too costly. However, if the band performs, the tickets will not be too costly. Therefore, if the band performs, then the hall will be full.

   ii) If the initialization is correct and the loop terminates, then the required output is obtained. The output has been obtained. Therefore, if the initialization is correct, the loop must terminate.

9. Prove the following using FOPL:

No used-car dealer buys used car for their families. Some people who buy used car for their families are absolutely dishonest. Therefore, some absolutely dishonest people are not used-car dealer.

10. Prove the following using FOPL:

Some residents of Kerala are Christians.

All residents of Kerala are Indians.

Prove that some Indians are. Christians.

11. All numbers divisible by 2 are even numbers. Number 25 is not an even number. Prove that 25 is not divisible by 2.

12. Consider the following sentences and prove that Y killed the cat using resolution with refutation:
X owns a dog. Every dog owner is an animal lover. No animal owner kills an animal. Either X or Y killed the cat who is named Tom. Did Y kill the cat?

13. Some people like all doctors. No people like any quack. Prove using resolution-refutation in FOPL that no doctor is a quack.

14. No friend of mine lives in an army base. Aniruddha lives in an army base. Therefore, prove that Aniruddha is not a friend of mine.

15. Students are citizens. Prove using resolution-refutation that students' votes are citizens' votes.

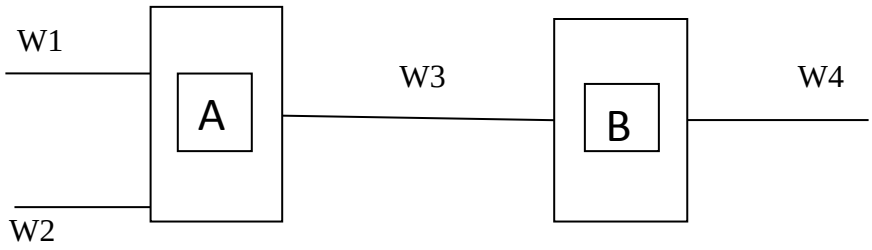
16. Premises: The custom officials searched everyone who entered this country who was not a VIP. Some of the drug pushers entered this country and they were only searched by drug pushers. No drug pusher was a VIP.
Conclusion: Some of the officials were drug pushers.
Prove the conclusion using Resolution in First Order Predicate Logic.

17. Use resolution refutation on a set of clauses to prove that there is a green object if we are given:

- If pushable objects are blue, then nonpushable ones are green.
- All objects are either blue or green but not both.
- If there is a nonpushable object, then all pushable ones are blue.
- Object O1 is pushable.
- Object O2 is not pushable.

i) Convert these statements to expressions in FOPL.
ii) Convert the preceding FOPL expressions to clause form.
iii) Combine the preceding clause form expressions with the clause form of the negation of the statement to be proved, and then show the steps used in obtaining a resolution refutation proof.

18. The following logic circuit has four wires, W1, W2, W3 and W4. It has an "and gate" A, an "inverter" B. The input wires, W1 and W2, can be either "on" or not. If the input and gate, A, is functioning properly (OK), wire W3 is "on" if and only if wires W1 and W2 both are "on". If the inverter, B, is functioning properly (OK), wire W4 is on if and only if wire W3 is not "on".



i) Use expressions like OK(A), ON(W1), and so on to describe the functioning of this circuit as defined.
ii) Using the formulae describing the functioning of the circuit, and assuming that all components are functioning properly and wires W1 and W2 are "on", use resolution to prove that wire W4 is not "on".
iii) Again, using the formulae describing the functioning of the circuit, and given that wires W1 and W2 are "on", but that wire W4 is also "on", use resolution to show that either the and gate or the inverter is not functioning properly.

19. Consider the following sentences:

Mary likes all kinds of food.
Pizza is a food.
Apple is a food.
Anything anyone eats is a food.
John eats chicken.
Ana eats everything Mary eats.

Use Answer Extraction in First Order Predicate Logic to answer the following question:

What food does Ana eat?

20. Consider the following piece of knowledge:

Amar, Akbar and Antony belong to the Himalayan Mountaineering Institute. Every member of the Mountaineering Institute who is not a skier is a mountain climber. Mountain climbers do not like rain, and anyone who does not like snow is not a skier. Akbar dislikes whatever Amar likes and likes whatever Amar dislikes. Amar likes rain and snow.

Represent this knowledge as a set of predicate calculus statements and use answer extraction to obtain the answer to the query " Is there a member of the Himalayan Mountaineering Institute who is a mountain climber but not a skier?"

21. Victor has been murdered, and Arthur, Bertram, and Carleton are suspects. Arthur says he did not do it. He says that Bertram was the victim's friend but Carleton hated the victim. Bertram says he was out of town on the day of murder, and besides he didn't even know the victim. Carleton says he is innocent and he saw Arthur and Bertram with the victim just before the murder.

Assuming that everyone – except possibly the murderer – is telling the truth, use answer extraction to solve the crime.