Partial differential Equation (P.D.E)

An Eq^n involving partial derivatives of a function of two or more + independent variables is called P.D.E

Eg $z = f(x, y)$

$$x \frac{\delta z}{\delta x} + y \frac{\delta z}{\delta y} = 1$$

$$\frac{\delta^2 z}{\delta y^2} + \frac{\delta^2 z}{\delta x^2} = x$$

Order of P.D.E - it is defined as the order of the highest order derivative present in P.D.E

Eg $\frac{\delta z}{\delta x} + \frac{\delta^2 z}{\delta x^2} + \frac{\delta z}{\delta y} = 1$ order = 2

Degree of P.D.E - it is defined as the highest power of the highest order derivative in the P.D.E when it is made free from radical + fractional power

Eg 1) $\frac{\delta z}{\delta x} + \left(\frac{\delta z}{\delta y}\right)^2 = z$ order = 1
degree = 2

Eg 2) $\frac{\delta^2 z}{\delta x^2} + \left(\frac{\delta^2 z}{\delta y^2}\right)^3 + \left(\frac{\delta z}{\delta x}\right)^4 = z$ order = 2
degree = 3

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Eg $\left[\left(\frac{\delta^2 z}{\delta x^2}\right)^2\right]^3 = \left[\left(\frac{\delta z}{\delta y}\right)^{1/3}\right]^3$ $O = 2$, $d = 6$

<u>Linear P.D.E</u> - An equation in which independent variable & its partial derivative appear only in the first degree and are not multiplied together is called linear PDE.

Eg (i) $\left(\frac{\delta z}{\delta x}\right)^1 + \left(\frac{\delta z}{\delta y}\right)^1 = z^1$ ✓ LPDE

(ii) $\frac{\delta z}{\delta x} + z\frac{\delta z}{\delta y} = x$ × LPDE

(iii) $\frac{\delta^2 z}{\delta x^2} + \frac{\delta z}{\delta y} = z$ ✓ LPDE

(iv) $\frac{\delta^2 z}{\delta x^2} + \frac{\delta z}{\delta y} = z^2$ × LPDE

(v) $x\frac{\delta^2 z}{\delta x^2} + y\frac{\delta^2 z}{\delta y^2} = z$ ✓ LPDE

(vi) $\frac{\delta z}{\delta x} + \frac{\delta z}{\delta x}\frac{\delta z}{\delta y} = z$ × LPDE

(vii) $z\frac{\delta z}{\delta x} + \frac{\delta z}{\delta y} = 1$ × LPDE

Linear Homogeneous PDE -

$$a_0 \frac{\delta^n z}{\delta x^n} + a_1 \frac{\delta^n z}{\delta x^{n-1} \delta y} + a_2 \frac{\delta^n z}{\delta x^{n-2} \delta y^2} +$$

$$\cdots\cdots + a_n \frac{\delta^n z}{\delta y^n} = g(x, y)$$

where $a_0, a_1, a_2 - - - a_n$ are constant

* order of Each derivative term is same
* if order of Each derivative term is not same then it is called Linear non-homogeneous PDE

Eg (i) $\frac{\delta z}{\delta x} + \frac{\delta z}{\delta y} = 1$ Homogeneous

(ii) $\frac{\delta^2 z}{\delta x^2} + \frac{\delta z}{\delta y} = 2$ non-homogeneous

Notation

$$D \equiv \frac{\delta}{\delta x}, \quad D' = \frac{\delta}{\delta y}, \quad D^2 \equiv \frac{\delta^2}{\delta x^2} \ldots$$

$$r = \frac{\delta^2 z}{\delta x^2}, \quad s = \frac{\delta^2 z}{\delta x \delta y}, \quad t = \frac{\delta^2 z}{\delta y^2}$$

$$p = \frac{\delta z}{\delta x}, \quad q = \frac{\delta z}{\delta y}$$

General Solution

$$Z = C.F + P.I$$

if $g(x, y) = 0$ $\quad\quad P.I = 0$

$$Z = C.F$$

Rules ~~of~~ to find C.F of Linear Homogeneous PDE –

(I) if $F(D, D')$ is homogeneous & power of $D \geq$ power of D'

(a) Replace D by m & D' by 1 then Equate to zero. This will give A.E

(b) Solve A.E find m

(c) (i) if values of m are distinct

$$m = m_1, m_2, m_3$$

$$C.F = f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x)$$

(ii) if values of m are repeated

$$m = m_1, m_1, m_1$$

$C.F = f_1(y + m_1 x) + x f_2(y + m_1 x) + x^2 f_3(y + m_1 x)$

Eg $(D^2 + DD')z = 0$ $\quad \frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta x \delta y} = 0$

$A.E = m^2 + m = 0$

$m = 0, -1$

$C.F = f(y + 0x) + f_2(y - 1x)$

Eg $(D + D')^2 z = 0$

$(m + 1)^2 = 0$

$m = -1, -1$

$C.F = f_1(y - x) + x f_2(y - x)$

Qu $(D^3 + D'^3)z = 0$

$A.E = m^3 + 1 = 0 \qquad (m+1)(m^2 - m + 1)$

$m = -1, \frac{1 \pm \sqrt{3}i}{2}$

$C.F = f_1(y + (-1)x) + f_2\left(y + \left(\frac{1 + \sqrt{3}i}{2}\right)x\right) +$

$f_3\left(y + \left(\frac{1 - \sqrt{3}i}{2}\right)x\right)$

Qu. $(D^2 + 2DD' + D'^2) z = 0$

A E $m^2 + 2m + 1 = 0$.
$(m+1)^2 = 0$
$m = -1, -1$

$f_1(y-x) + x f_2(y-x)$

(II) When $F(D, D')$ is homogeneous & power of $D' \geq$ power of D

a) Replace D' by m & D by 1 in $f(D, D')$ & Equate to zero. This will give A.E

b) Solve A.E & find value of m

c) (i) if values of m are distinct
$m = m_1, m_2, m_3$.

$C.F = f_1(x + m_1 y) + f_2(x + m_2 y) + f_3(x + m_3 y)$

(ii) if value of m are repeated
$m = m_1, m_1, m_1$

$C.F = f_1(x + m_1 y) + y f_2(x + m_1 y) + y^2 f_3(x + m_1 y)$

Que $(D^2D' + D'^3)z = 0$

A.E $\quad D' \to m \quad , D \to 1$

$$m + m^3 = 0$$
$$m(\cancel{m} + m^2) = 0$$
$$m = 0, \pm i$$

$C.F = f_1(x) + f_2(x+iy) + f_3(x-iy)$

Que $(D^2D'^3 + D'^2D^3)z = 0$

A.E $= \cancel{(m^3 + m^2) = 0}$

$$D^2D'^2(D' + D)z = 0$$

for $D^2 \rightarrow \quad D \to m$

$$m^2 = 0 \Rightarrow m = 0, 0$$

$C.F = f_1(y) + xf_2(y)$

for $D'^2 \quad D' \to m$

$$m^2 = 0 \Rightarrow m = 0, 0$$

$C.F = f_3(x) + yf_4(x)$

for $(D + D') \quad D \to m \, , \, D' = 1$

$$m + 1 = 0 \qquad m = -1$$

$C.F = f_5(y - x)$

$CF = f_1(y) + xf_2(y) + f_3(x) + yf_4(x) + f_5(y-x)$

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Qu $D(D^2 - 2DD' + D'^2)z = 0$

$(D^3 - 2D^2D' + DD'^2)z = 0$

$D \to m,\ D' \to 1$

$$m^3 - 2m^2 + m = 0$$

$$m = 0, 1, 1$$

$$C.F = f_1(y) + f_2(y+x) + xf_3(y+x)$$

(III) if $F(D, D')$ is non-homogeneous

a) when $F(D, D')$ can be factorized into linear factors of the form $(D - mD' - a)$

(i) $F(D, D') \Rightarrow (D - m_1D' - a_1)(D - m_2D' - a_2)$

$$C.F = e^{a_1x} f_1(y + m_1x) + e^{a_2x} f_2(y + m_2x)$$

(ii) $F(D, D') = (D - m_1D' - a)^2$

$$C.F = e^{ax} f_1(y + m_1x) + xe^{ax} f_2(y + m_2x)$$

Eg $(D - 2D' + 4)(D + 3D' - 1)z = 0$

$$C.F = e^{-4x} f_1(y + 2x) + e^{x} f_2(y - 3x)$$

$D' \to y$

Qn $(D + 3D' + 5)^2 (D - 4D')z = 0$

$C.F = e^{-5x} f_1(y - 3x) + x e^{-5x} f_2(y - 3x) + f_3(y + 4x)$

(b) if $F(D, D')$ can be factorised into linear factors of the form $(D' - mD - a)$

(i) $F(D, D') = (D' - m_1 D - a_1)(D' - m_2 D - a_2)$

$C.F = e^{a_1 y} f_1(x + m_1 y) + e^{a_2 y}(x + m_2 y)$

(ii) $F(D, D') = (D' - mD - a)^2$

$C.F = e^{ay} f_1(x + my) + y e^{ay} f_2(x + my)$

Qn $(D' + 2D + 1)(D' + 2)(D + 2)z = 0.$

$C.F = e^{-y} f_1(x - 2y) + e^{-2y} f_2(x) + e^{-2x} f_3(y)$

Qu $(D + 4D' - 1)(D + aD' + b)z = 0$

$C.F = e^{x} f_1(y - 4x) + e^{-bx} f_2(y - ax)$

Qu $(D^2 - DD' + D)(D-1)(D'+4)z = 0$

$D(D - D' + 1)(D-1)(D'+4)z = 0.$

$CF = f_1(y) + e^{-x} f_2(y+x) + e^{x} f_3(y) + e^{-4y} f_4(x)$

(IV) When $F(D, D')$ can not be factorized into linear factors

let ~~$z = Ae^{hx+ky}$~~ $z = Ae^{hx+ky}$

$C.F = \Sigma A e^{hx+ky}$ where.

h & k are related

Qu $(D^3 - 2D')z = 0$ $\qquad D^3 z - 2D'z$

let $z = Ae^{hx+ky}$

$Dz = \frac{\delta z}{\delta x} = Ahe^{hx+ky}$

$D^2 z = Ah^2 e^{hx+ky}$

$D^3 z = Ah^3 e^{hx+ky}$

$D'z = \frac{\delta z}{\delta y} = Ake^{hx+ky}$

$= Ah^3 e^{hx+ky} - 2AKe^{hx+ky}$

$= Ae^{hx+ky}(h^3 - 2k) = 0.$

$C.F = \sum Ae^{hx+ky}$, where $h^3 - 2k = 0$

Qu $(D^4 - 5D'^2)(D - D')z = 0$

$C.F = \sum Ae^{hx+ky} + f_1(y+x)$

where $h^4 - 5k^2 = 0$

Qu $(D^3 + D^2D' \cancel{+ D'^2})(D'^2 - 2)z = 0$

$D \to m, \quad D' \to 1$

$m^3 + m^2 = 0 \qquad m = 0, 0, -1$

$C.F = f_1(y-x) + f_2(y) + xf_3(y) +$

$\sum Ae^{hx+ky}$ where $k^2 - 2 = 0$

Qu $(D^2 - D'^2 - 2D + 2D')z = 0$

$(D-D')(D+D') - 2(D-D')z$

$(D-D')(D+D'-2)z = 0$

$C.F = f_1(y+x) + e^{2x}f_2(y-x)$

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Qu $s + ap + bq + abz = 0$

$(DD' + aD + bD' + ab)z = 0$

$(D(D'+a) + b(D'+a))z = 0$

$[(D+b)(D'+a)]z = 0$

$C.F = e^{-ay} f_1(x) + e^{-bx} f_2(y)$

Qu $r + 2s + t + 2p + 2q + z = 0$

$[D^2 + 2DD' + D'^2 + 2D + 2D' + 1]z = 0$

$[(D+D')^2 + 2(D+D') + 1]z = 0$

$[(D+D'+1)^2]z = 0$

$C.F = e^{-x} f_1(y-x) + xe^{-x} f_2(y-x)$

Rules to find PI

(1) $F(D, D')z = e^{ax+by}$

$$P.I = \frac{1}{F(D, D')} e^{ax+by}$$

$$= \frac{1}{F(a, b)} e^{ax+by} \quad F(a, b) \neq 0.$$

if $F(a, b) = 0$

$$PI = \frac{x}{\frac{\delta}{\delta D} F(D, D')} e^{ax+by}$$

$$= \frac{x}{\frac{\delta}{\delta D} F(D, D') / (a, b)} e^{ax+by}$$

Qu $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$

$$P.I = \frac{1}{D^2 + 2DD' + D'^2} e^{2x+3y}$$

$$\frac{1}{2^2 + 2\times2\times3 + 3^2} e^{2x+3y}$$

$$\frac{1}{35} e^{2x+3y}$$

Que $(D^2 - 2D' + 1)\, z = e^{x+y}$

$$P.I = \frac{1}{(D^2 + 2D' + 1)} e^{x+y}$$

$$= \frac{x}{2D} e^{x+y}$$

$$= \frac{x}{2} e^{x+y}$$

Que $(D^3 - 3D'^3 + 2D'^2)\, z = e^{x+y}$

$$P.I = \frac{1}{D^3 - 3D'^3 + 2D'^2} e^{x+y}$$

$$= \frac{x}{3D^2} e^{x+y}$$

$$= \frac{x}{3} e^{x+y}$$

Rule (2)

$F(D^2, DD', D'^2)z = \sin(ax+by)$

or

$\cos(ax+by)$

$$P.I = \frac{1}{F(D^2, DD', D'^2)} \sin(ax+by)$$

$D^2 \rightarrow -a^2$
$D'^2 \rightarrow -b^2$
$DD' \rightarrow -ab$

$$= \frac{1}{F(-a^2, -ab, -b^2)} \sin(ax+by)$$

Provided $F(-a^2, -ab, -b^2) \neq 0$

if $F(-a^2, -ab, -b^2) = 0$

then proceed as in rule (1)

Qu $(D^2 + D'^2)z = \sin(2x+3y)$

$$P.I = \frac{1}{D^2 + D'^2} \sin(2x+3y)$$

$a = 2, b = 3$
$D^2 = -4 \quad D'^2 = -9$

$$\frac{1}{-4-9} \sin(2x+3y)$$

$$= -\frac{1}{13} \sin(2x+3y)$$

Qu $(D^2 - D'^2)z = \sin(x+y)$

$a=1 \quad b=1$

$D^2=-1 \quad D'^2=-1$

$$P.I = \frac{1}{D^2-D'^2}\sin(x+y)$$

$$= \frac{x}{2D}\sin(x+y)$$

$$= -\frac{x}{2}\cos(x+y)$$

Qu $(D^2 - DD' + D' - 1)z = \sin(x+2y)$

$$P.I = \frac{1}{(D^2-DD'+D'-1)}\sin(x+2y)$$

$a=1, b=2$

$D^2 \to -1, D'^2=-4$

$DD' = -2$

$$= \frac{1}{\cancel{-1}+\cancel{2}+D'\cancel{-1}}\sin(x+2y)$$

$$= -\frac{1}{2}\cos(x+2y)$$

Qu $2s + t - 3q = 5\cos(3x-2y)$

$$(2DD' + D'^2 - 3D') = 5\cos(3x-2y)$$

$$= \frac{1}{2\times 6 - 4 - 3D'}(5\cos(3x-2y)$$

$a=3, b=-2$

$D'^2=-4, DD'=6$

$$= \frac{1}{8-3D'}\, 5\cos(3x-2y)$$

$$= \frac{(8+3D')}{(8-3D')(8+3D')} 5\cos(3x-2y)$$

$$= \frac{(8+3D')}{64-9D'^2} 5\cos(3x-2y)$$

$$= \frac{(8+3D')}{64-9(-4)} 5\cos(3x-2y)$$

$$= \frac{5}{\cancel{100}_{20}}(8+3D')\cos(3x-2y)$$

$$= \frac{1}{20}[8\cos(3x-2y) + 3x - \sin(3x-2y)(-2)]$$

$$= \frac{1}{20}[8\cos(3x-2y) + 6\sin(3x-2y)]$$

Sw $(D-D'-1)(D-D'-2)z = \sin(2x+3y)$

$$P.I = \frac{1}{(D-D'-1)(D-D'-2)}\sin(2x+3y)$$

$a=2, b=3$

$D^2=-4, D'^2=-9$

$DD'=-6$

$$= \frac{1}{D^2+D'^2-2DD'-3D+3D'+2}\sin(2x+3y)$$

$$= \frac{1}{-4-9+12-3D+3D'+2}\sin(2x+3y)$$

$$= \frac{1}{1-3D+3D'}\sin(2x+3y)$$

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$$= \frac{(1 + 3D - 3D')}{[1 - (3D - 3D')][1 + (3D - 3D')]} \sin(2x + 3y)$$

$$= \frac{1 + 3(D - D')}{1 - 9(D^2 + D'^2 - 2DD')} \sin(2x + 3y)$$

$$= \frac{1 + 3(D - D')}{1 - 9(-4 - 9 - 2(-6)} \sin(2x + 3y)$$

$$= \frac{1}{10}\left[\sin(2x + 3y) + 6\cos(2x + 3y) - 9\cos(2x + 3y)\right]$$

Que $(D^3 - D'^3)z = \sin(x + y)$

$$P.I = \frac{1}{D^3 - D'^3} \sin(x + y)$$

$a = 1, \ b = 1$

$D^2 = -1, \ D'^2 = -1$

$$= \frac{1}{-D + D'} \sin(x + y)$$

$$= \frac{D + D'}{(-D + D')(D + D')} \sin(x + y)$$

$$= \frac{D + D'}{-D^2 + D'^2} \sin(x + y)$$

$$= \frac{x}{-2D} (D + D') \sin(x + y)$$

$$= -\frac{x}{2}(D+D')(-\cos(x+y))$$

$$= \frac{x}{2}[-\sin(x+y) + (-\sin(x+y))].$$

$$- \frac{\cancel{2}x}{\cancel{2}} \sin(x+y)$$

$$- x\sin(x+y)$$

(3) Rule

$$F(D, D')z = x^m y^n \quad , \quad m, n \geq 0$$

Qu $(D+D'-1)(D+2D'-3)z = 4+3x+6y$

$$P.I = \frac{1}{(D+D'-1)(D+2D'-3)}(4+3x+6y)$$

$$= \frac{1}{(-1+D+D')(-3+D+2D')}(4+3x+6y)$$

$$= \frac{1}{-[(+1 \cancel{+} (D+D')](-3)[1-\frac{1}{3}(D+2D')]}(4+3x+6y)$$

$$= \frac{1}{3}[1-(D+D')]^{-1}\left[1-\frac{1}{3}(D+2D')\right]^{-1}(4+3x+6y)$$

$$= \frac{1}{3}[1-(D+D')]^{-1}\left[1+\frac{1}{3}(D+2D')\right](4+3x+6y)$$

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$$= \frac{1}{3}\left[1-(D+D')\right]^{-1}\left[4+3x+6y+\frac{1}{3}(3+2\times 6)\right]$$

$$= \frac{1}{3}\left[1+(D+D')\right]\left[9+3x+6y\right]$$

$$= \frac{1}{3}\left[9+3x+6y+3+6\right]$$

$$= \frac{1}{3}\left[18+3x+6y\right]$$

$$= 6+x+2y$$

<u>Qus</u> $(D^3-D'^3)z = x^3y^3$

$$P.I = \frac{1}{D^3-D'^3}x^3y^3$$

$$= \frac{1}{D^3\left[1-\left(\frac{D'}{D}\right)^3\right]}x^3y^3$$

$$= \frac{1}{D^3}\left[1-\frac{D'^3}{D^3}\right]^{-1}x^3y^3$$

$$= \frac{1}{D^3}\left[1+\frac{D'^3}{D^3}\right]x^3y^3$$

$$= \frac{1}{D^3}\left[x^3y^3+\frac{1}{D^3}\left[6x^3\right]\right]$$

$$= \frac{1}{D^3}\left[x^3y^3+\cancel{6}\frac{x^6}{\cancel{120}_{20}}\right]$$

$$= \frac{y^3 x^6}{120} + \frac{1}{20} \cdot \frac{x^9}{504}$$

$$= \frac{x^6 y^3}{120} + \frac{x^9}{10080}$$

Qn $(D^2 - D'^2 - 3D + 3D')z = xy$

$$((D+D')(D-D') - 3(D-D'))z = xy.$$

$$(D-D')(D+D'-3)z = xy.$$

$$P.I = \frac{1}{(D-D')(D+D'-3)} xy$$

$$= \frac{1}{D\left(1-\frac{D'}{D}\right)(-3)\left(1-\frac{1}{3}(D+D')\right)} xy$$

$$= \frac{1}{-3D}\left[1 - \frac{1}{3}(D+D')\right]^{-1}\left[1-\frac{D'}{D}\right]^{-1} xy$$

$$= -\frac{1}{3D}\left[1 - \frac{1}{3}(D+D')\right]^{-1}\left[1+\frac{D'}{D}\right] xy.$$

$$-\frac{1}{3D}\left[1 - \frac{1}{3}(D+D')\right]^{-1}\left[xy + \frac{x^2}{2}\right]$$

$$-\frac{1}{3D}\left[1 + \frac{1}{3}(D+D') + \frac{1}{9}(D^2+D'^2+2DD')\right]\left(xy + \frac{x^2}{2}\right)$$

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$$= \frac{-1}{30}\left[xy + \frac{x^2}{2} + \frac{1}{3}(y+x+x) + \frac{1}{9}(1+2+1)\right]$$

$$= \frac{-1}{30}\left[xy + \frac{x^2}{2} + \frac{1}{3}(y+2x) + \frac{1}{9}(1+2\times1)\right]$$

$$= -\frac{1}{3}\left[\frac{x^2}{2}y + \frac{x^3}{6} + \frac{1}{3}\cdot\frac{\cancel{2}x^2}{\cancel{2}} + \frac{1}{3}yx + \frac{3x}{9}\right]$$

$$= \frac{-1}{3}\left[\frac{x^2}{2}y + \frac{x^3}{6} + \frac{1}{3}x^2 + \frac{xy}{3} + \frac{x}{3}\right].$$

Rule - 5 $\quad F(D, D')z = \phi(ax+by)$

Where $F(D,D')$ is homogeneous or LPDE is homogeneous

$$P.I = \frac{1}{F(D,D')}\phi(ax+by)$$

$$= \frac{1}{F(a,b)}\underbrace{\int\ldots\iint}_{n\,times}\phi(u)\,du\,du\ldots du$$

Where, $ax+by = u$

n = degree of $F(D,D')$

$F(a,b) \neq 0$

if $F(a,b) = 0$

$$P.I = \frac{x}{\frac{\delta}{\delta D}F(D,D')}\phi(ax+by)$$

$$= \frac{x}{\frac{\delta}{\delta D}F(D,D')\Big|_{(a,b)}}\underbrace{\int\ldots\iint}_{(n-1)\,times}\phi(u)\,du\,du\ldots du$$

Que $\quad (D^3 - 3D^2D' + 4D'^3) = e^{x+2y}$

$$P.I = \frac{1}{D^3-3D^2D'+4D'^3}e^{x+2y}$$

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$$= \frac{1}{1^3 - 3\times1^2\times2 + 4\times2^3} \iiint e^u \, du\,du\,du$$

$$= \frac{1}{27} e^u = \frac{1}{27} e^{x+2y}$$

Qu $4r - 4s + t = 16 \log_e (x+2y)$

$$(4D^2 - 4DD' + D'^2)z = 16 \log_e (x+2y)$$

$$P.I = \frac{1}{4D^2 - 4DD' + D'^2} 16 \log_e (x+2y)$$

$$= \frac{x}{8D - 4D'} 16 \log_e (x+2y)$$

$$\frac{x^2}{\cancel{8}} \cancel{16}^{2} \log_e (x+2y)$$

$$= 2x^2 \log_e (x+2y)$$

Qu $r + s - 2t = \sqrt{2x+y}$.

$$(D^2 + DD' - 2D'^2) z = \sqrt{2x+y}$$

$$P.I = \frac{1}{D^2 + DD' - 2D'^2} \sqrt{2x+y}$$

$$= \frac{1}{4+2-2} \iint u^{1/2}\, du\, du \qquad u = 2x+y$$

$$= \frac{1}{4} \int \frac{u^{3/2}}{3/2}\, du$$

$$\frac{1}{4} \times \frac{2}{3} \times \frac{2}{5} \times u^{5/2}$$

$$= \frac{1}{15} (2x+y)^{5/2}$$

<u>Qu</u> $(D^2D' - 2DD'^2 + D'^3)z = \frac{1}{x^2}$

$$(D^2D' - 2DD'^2 + D'^3)z = x^{-2}$$

$$(D^2D' - 2DD'^2 + D'^3)z = (1.x + 0y)^{-2}$$

$$\frac{1}{(D^2D' - 2DD'^2 + D'^3)} (1.x + 0.y)^{-2}$$

$$= \frac{y}{D^2 - 4DD' + 3D'^2} (1.x + 0.y)^{-2}$$

$$= y \iint u^{-2}\, du\, du$$

$$= y \int \frac{u^{-1}}{-1}\, du \qquad u = 1.x + 0.y = x$$

$$= -y \log_e u$$

$$= -y \log_e x$$

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Qu $(D-D')^2 z = \phi(x+y)$

$P.I = \frac{x}{2(D-D')} (x+y)$

$= \frac{x^2}{2}(x+y)$

Qu $(D^2-DD')z = \frac{2}{2}\sin x \cos y$

$= \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$P_1 = \frac{1}{D^2-DD'} \frac{1}{2}\sin(x+y)$

$= \frac{1}{2}\left(\frac{x}{2D-D'}\right)\sin(x+y)$

$= -\frac{x}{2}\cos(x+y)$

$P_2 = -\frac{1}{4}\sin(x-y)$

Qu $(D^2+5DD'+6D'^2)z = \frac{1}{y-2x}$

$P.I = x \log_e(y-2x)$

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Rule ⑥ General method for linear homogeneous PDE

$$F(D,D')z = \phi(x,y)$$

$$\frac{1}{D-mD'}\phi(x,y) = \int \phi(x, c-mx)\,dx$$

where, $y = c-mx$

Que $(D^2+DD'-6D'^2)z = y\cos x$

$$P.I = \frac{1}{(D^2+DD'-6D'^2)} y\cos x$$

$m^2+m-6=0$
$m=-3,2$
$(m+3)(m-2)$

$$P.I = \frac{1}{(D+3D')(D-2D')} y\cos x$$

$$= \frac{1}{(D+3D')}\left[\frac{1}{(D-2D')} y\cos x\right]$$

$y = c-2x$

$$= \frac{1}{(D+3D')}\left[\int (c-2x)\cos x\,dx\right]$$

$$= \frac{1}{D+3D'}\left[(c-2x)\sin x - (-2)(-\cos x)\right]$$

$$= \frac{1}{D+3D'}\left[(c-2x)\sin x - 2\cos x\right]$$

$\int x^2 \sin x$

$x^2(-\cos x) - 2x(-\sin x) + 2\cos x$

diff. int. diff. int.

$y = c_1 + 3x$

$$= \int\left[(c_1+3x)\sin x - 2\cos x\right]dx.$$

$$(c_1+3x)(-\cos x) - 3(-\sin x) - 2\sin x$$

$$= -y\cos x + \sin x$$

Que $\int x^3 \cos x \, dx.$

$$= x^3\sin x + (+3x^2\cos x) + (-6x\sin x) + 6\cos x$$

$$= x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x.$$

Que $(D^2 + DD' - 2D'^2) = (y-1)e^x$ $\quad m = 1,\ = -2$

$$P.I = \frac{1}{D^2 + DD' - 2D'^2}(y-1)e^x$$

$D' = 1$

$D = m$

$$= \frac{1}{(D-D')(D+2D')}(y-1)e^x$$

$$= \frac{1}{(D-D')}\int (c+2x-1)e^x \, dx$$

$y = c + 2x$

$$= \frac{1}{(D-D')}\left[(c+2x-1)e^x - 2e^x\right]$$

$$= \frac{1}{(D-D')}(ye^x - 3e^x)$$

$$\int \{ (c_1 - x) e^x - 3e^x \} dx \qquad y = c_1 - x$$

$$(c_1 - x) \cdot e^x - (-1) e^x - 3e^x$$

$$= ye^x - 2e^x$$

$$= (y-2) e^x$$

Qu $(D^2 + 2DD' + D'^2) z = x \sin y$

$$= \frac{1}{(D+D')(D+D')} x \sin y$$

$$\frac{1}{(D+D')} \left[\frac{1}{(D+D')} x \sin y \right]$$

$$y = c + x$$

$$\frac{1}{(D+D')} \int x \sin (c+x) \, dx$$

$$\frac{1}{D+D'} \left[x - \cos (c+x) - 1 - \sin (c+x) \right]$$

$$\frac{1}{D+D'} \left[-x \cos y + \sin y \right]$$

$$\int \left[-x \cos (c+x) + \sin ((c+x) \right] dx.$$

$$- \left[x \sin (c+x) - 1 - \cos ((c+x) \right] - \cos (c+x)$$

$$- x \sin y - \cos y - \cos y$$

$$- x \sin y - 2 \cos y$$