Case - V

$$\boxed{e^{iax} = \cos ax + i \sin ax}$$

$\cos ax$ = Real part of e^{iax}

$\sin ax$ = Img part of e^{iax}

$$\frac{1}{f(D)} x^m \sin ax = \text{Img part of } e^{iax} \frac{1}{f(D+ia)} x^m$$

$$\frac{1}{f(D)} x^m \cos ax = \text{Real part of } e^{iax} \frac{1}{f(D+ia)} x^m$$

Solve $\quad (D^2+2D+1)y = x \cos x$

A.E $\quad m^2+2m+1=0$

$m = -1, -1$

$$C.F = (c_1 + c_2 x)e^{-x}$$

$$P.I = \frac{1}{(D+1)^2} x \cos x$$

$$= \frac{1}{(D+1)^2} x \text{ Real part of } e^{ix}$$

$$= \text{Real part of } e^{ix} \frac{1}{(D+1)^2} x e^{ix}$$

$$= R.P \text{ of } \frac{1}{(D^2+2D+1)} x e^{ix}$$

$$= R.P \text{ of } e^{ix} \frac{1}{(D+i)^2 + 2(D+i) + 1} x$$

$$= R.P \text{ of } e^{ix} \frac{1}{D^2 \cancel{-1} + 2Di + 2D + 2i \cancel{+1}} x$$ (2)

$$= R.P \text{ of } e^{ix} \frac{1}{D^2 + 2D(1+i) + 2i} x$$

$$= R.P \text{ of } e^{ix} \frac{1}{2i} \left\{ 1 + \left(\frac{1+i}{i}\right) D + \frac{D^2}{2i} \right\}^{-1} x$$

$$= R.P \text{ of } e^{ix} \frac{1}{2i} \left[1 - \left\{ \left(\frac{1+i}{i}\right) D + \frac{D^2}{2i} \right\} \right] x$$

$$= R.P \text{ of } e^{ix} \frac{1}{2i} \left[x - \frac{1+i}{i} \right]$$

$$= R.P \ \frac{1}{2} (\cos x + i \sin x) \left[\frac{x}{i} + (1+i) \right]$$

$$= R.P \text{ of } \frac{1}{2} (\cos x + i \sin x) (-xi + i + 1)$$

$$= \frac{1}{2} R.P \text{ of } (\cos x + i \sin x) \{ i(1-x) + 1 \}$$

$$= \frac{1}{2} \left[\cos x - (1-x) \sin x \right]$$

Note:- $\frac{1}{2}\left[i \ldots \cos x + \cos x - (1-x) \sin x + \cancel{i \sin x} \right]$

Q $(D^2-1) = x\sin x + x^2e^x$ ③

$m^2-1=0$

$m=\pm 1$

$C.F = c_1e^x + c_2e^{-x}$

$$P.I = \frac{1}{(D^2-1)}x\sin x + \frac{1}{(D^2-1)}x^2e^x$$

$$P_1 \quad + \quad P_2$$

$$P_1 = \frac{1}{(D^2-1)}x\sin x = \frac{1}{(D^2-1)}x \text{ Imp of } e^{ix}$$

$$= \text{Img P} \frac{1}{(D^2-1)} x e^{ix}$$

$$= \text{Img Part of } e^{ix} \frac{1}{(D+i)^2-1}x$$

$$= \text{Img Pt of } e^{ix} \frac{1}{D^2-1+2Di-1}x$$

$$= \text{Img Pt of } e^{ix} \frac{1}{D^2+2Di-2}x$$

$$= \text{Img Pt of } e^{ix} \frac{1}{(-2)\left\{1-\frac{(D^2+2Di)}{2}\right\}}x$$

$$= \text{Img Pt of } e^{ix} \frac{1}{(-2)}\left\{1-\frac{(D^2+2Di)}{2}\right\}^{-1}x$$

Img Pt of $\frac{e^{ix}}{(-2)}\left\{1+\frac{(D^2+2Di)}{2}\right\}x$

$= \text{Img Pt of } \frac{e^{ix}}{(-2)}\left\{x+i\right\}$

$\text{Img Pf}\left\{-\frac{1}{2}(\cos x+i\sin x)(x+i)\right]$

$= -\frac{1}{2}\left[x\cos x + i\cos x + ix\sin x - 1\sin x\right]$

$\cancel{= -\frac{1}{2}\left[x\cos x - \frac{1}{2}\sin x\right]}$

$= -\frac{1}{2}\left[x\sin x + \cos x\right]$

$P_2 = \frac{1}{(D^2-1)}x^2e^x = e^x\frac{1}{(D+1)^2-1}x^2$

$= e^x\frac{1}{D^2+\cancel{1}+2D-\cancel{1}}x^2$

$= e^x\frac{1}{D^2+2D}x^2 = \frac{e^x}{2D\left(1+\frac{D}{2}\right)}x^2$

$= \frac{e^x}{2D}\left(1+\frac{D}{2}\right)^{-1}x^2 = \frac{e^x}{2D}\left\{1-\frac{D}{2}+\frac{(-1)(\cancel{-1}-1)}{\cancel{2}}\frac{D^2}{4}\right\}x^2$

$= \frac{e^x}{2D}\left\{1-\frac{D}{2}+\frac{D^2}{4}\right\}x^2$

$$= \frac{e^x}{2D}\left\{x^2 - \frac{2x}{2} + \frac{2}{4}\right\}$$ (5)

$$= \frac{e^x}{2D}\left\{x^2 - x + \frac{1}{2}\right\}$$

$$= \frac{e^x}{2}\left\{\frac{x^3}{3} - \frac{x^2}{2} - \frac{1}{2}x\right\}$$

$$y = C.F + P_1 + P_2$$

Case - VI When Q is the any function of x

(i) $$\frac{1}{(D-\alpha)}Q = e^{\alpha x}\int e^{-\alpha x} Q\, dx$$

(ii) $$\frac{1}{(D+\alpha)}Q = e^{-\alpha x}\int e^{\alpha x} Q\, dx$$

Q Solve $(D^2+1)y = \operatorname{cosec} x$

$$m^2+1=0$$

$$m = \pm i$$

$$C.F = C_1 \cos x + C_2 \sin x$$

$$P.I = \frac{1}{(D^2+1)}\operatorname{cosec} x = \frac{1}{(D+i)(D-i)}\operatorname{cosec} x.$$

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omogeneous ...

$$\frac{1}{(D+i)(D-i)} = \frac{A}{(D+i)} + \frac{B}{(D-i)}$$

$$1 = A(D-i) + B(D+i)$$ $D+i=0$, $D=-i$

$$1 = -2iA$$ $D-i=0$, $D=+i$

$$A = \frac{-1}{2i}$$

$$B = \frac{1}{2i} \qquad = \frac{-1}{2i(D+i)} + \frac{1}{2i(D-i)}$$

$$P.I = \frac{1}{(D+i)(D-i)} \operatorname{cosec} x$$

$$= \frac{1}{2i}\left[\frac{1}{(D-i)} - \frac{1}{(D+i)}\right]\operatorname{cosec} x$$

$$= \frac{1}{2i}\left[\frac{1}{(D-i)}\operatorname{cosec} x - \frac{1}{(D+i)}\operatorname{cosec} x\right]$$

$$P_1 \qquad - \qquad P_2$$

$$P_1 = \frac{1}{(D-i)}\operatorname{cosec} x = e^{ix}\int e^{-ix}\operatorname{cosec} x \, dx$$

$$\frac{1}{(D-\alpha)} Q = e^{\alpha x}\int e^{-\alpha x} Q \, dx$$

$$= e^{ix}\int (\cos x - i\sin x)\operatorname{cosec} x \, dx$$

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$$= e^{ix}\int(\cot x - i)\,dx$$ (7)

$$= e^{ix}(\log \sin x - ix)$$

Similarly

$$P_2 = \frac{1}{(D+i)}\operatorname{cosec} x = e^{-ix}\int e^{ix}\operatorname{cosec} x\,dx$$

$$= e^{-ix}\int(\cos x + i\sin x)\operatorname{cosec} x\,dx$$

$$= e^{-ix}\int(\cot x + i)\,dx$$

$$= e^{-ix}(\log \sin x + ix)$$

$$= \frac{1}{2i}\left[e^{ix}(\log \sin x - ix) - e^{-ix}(\log \sin x + ix)\right]$$

$$\log \sin x \left(\frac{e^{ix} - e^{-ix}}{2i}\right) - x\frac{(e^{ix} + e^{-ix})}{2}$$

$$= \log \sin x\,(\sin x) - x\cos x$$

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