Design and Analysis of Algorithms

Backtracking

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Backtracking

Backtracking algorithms determine problem solutions by systematically searching the solution space for the given problem instance..

This search is facilitated by using a tree organization for the solution space.

Depth first node generation with bounding functions is called backtracking.

Graph Coloring

Let G be a graph and m is an integer number. Nodes of G can be colored in such a way that no two adjacent nodes have the same color yet only m colors are used.

Graph Coloring

```
procedure MCOLORING(k)
//The graph is represented by its boolean adjacency matrix GRAPH(l:n, 1 :n ).
/ /k is the index of the next vertex to color
global integer m, n, X(l :n) boolean GRAPH(l :n, 1 :n)
integer k
loop //generate all legal assignments for X(k)
        call NEXTVALUE(k) //assign to X(k) a legal color//
        if X(k) = 0 then exit endif //no new color possible//
        if k = n
                then print(X) // at most m colors are assigned to n vertices//
                else call MCOLORING(k + 1)
        endif
repeat
end MCOLORING
```

Graph Coloring

```
procedure NEXTVALUE(k)
global integer m, n, X(l:n) boolean GRAPH(l:n, l:n)
integerj, k
loop
        X(k) = (X(k) + 1) mod (m + 1)
        if X(k) = 0 then return endif // all colors have been exhausted
        for j= 1 to n do // check if this color is distinct from adjacent colors
          if GRAPH(k,j) and  //if (k,j) is an edge//
              X(k) = X(j) //and if adjacent vertices have identical colors//
                then exit endif
        repeat
        if j = n + 1 then return endif    //new color found//
repeat //otherwise try to find another color/ I
end NEXTVALUE
```

Graph Coloring

```
graphcolor(int k)
{
for(c=1;c<=m;c++)
     if(issafe(k,c))
       { x[k]=c
          if(k+1<n)
          graphcolor(k+1)
          else
          print x[];
return;
}
```

```
issafe(int k, int c)
{
for(i=0;i<n;i++)
 {   if(G[k][i]==1&&c==x[i]
       return false;
 }
return true;
}
```

Graph Coloring

```
graphcolor(int k)
{
for(c=1;c<=m;c++)
    if(issafe(k,c))
      { x[k]=c
        if(k+1<n)
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        else
        print x[];
return;
}
```

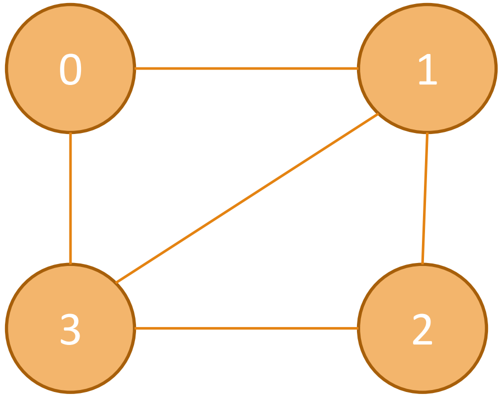

n=4
m=3

Adjacency Graph G

n	0	1	2	3
0	1	1	0	1
1	1	1	1	1
2	0	1	1	1
3	1	1	1	1

X[k]=

0	0	0	0

```
issafe(int k, int c)
{
for(i=0;i<n;i++)
 {   if(G[k][i]==1&&c==x[i])
        return false;
 }
return true;
}
```

Sum of Subset Problem

Suppose we are given *n* distinct positive numbers (usually called weights) and we desire to find all combinations of these numbers whose sum is *M*. This is called the *sum of subsets* problem.

In this case the element X(i) of the solution vector is either one or zero depending upon whether the weight W(i) is included or not.

For a node at level i the left child corresponds to X(i) = 1 and the right to X(i) = 0.

Sum of Subset Problem

The state space tree generated by procedure SUMOFSUB while working on the instance n = 6, M = 30 and W(l :6) = (5, 10, 12, 13, 15, 18).

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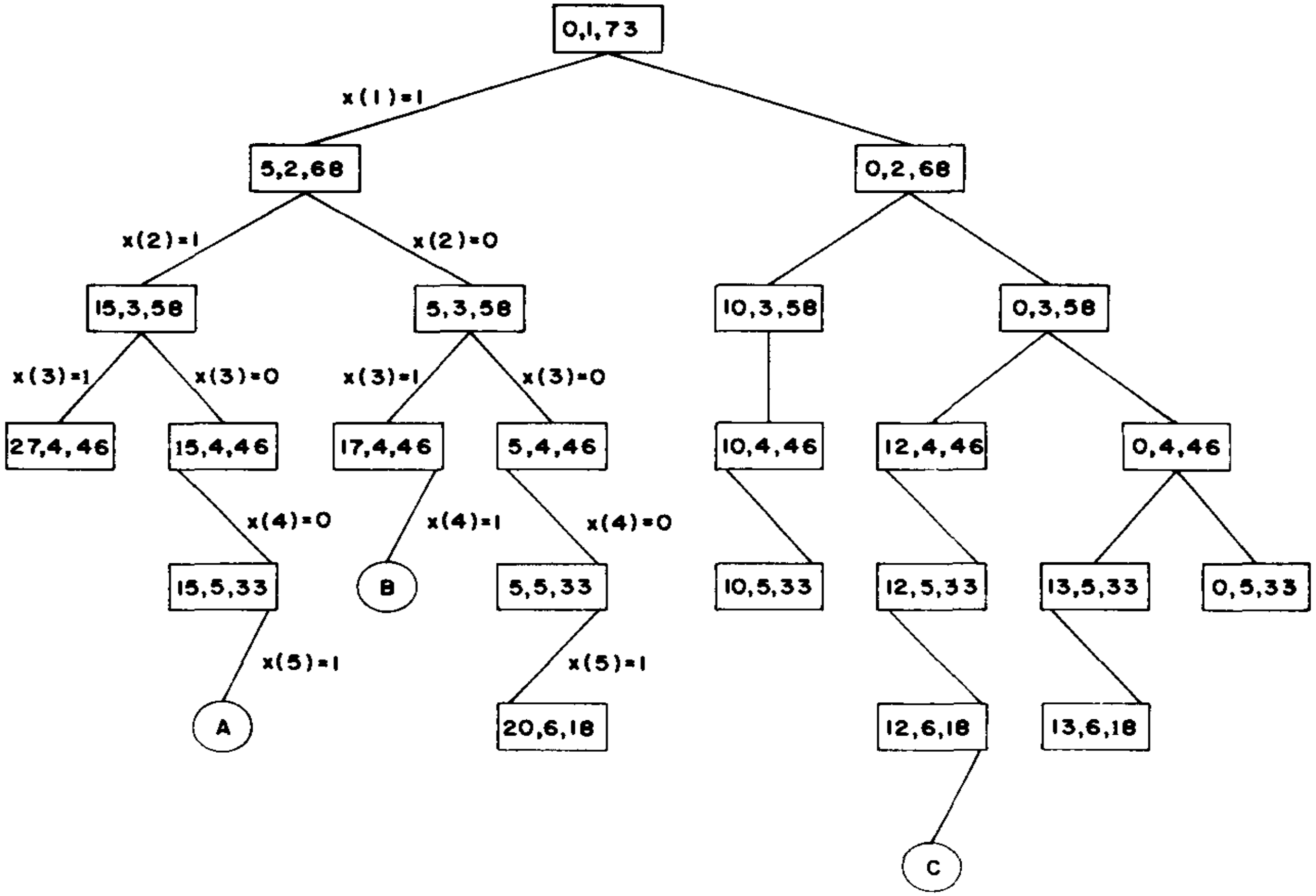
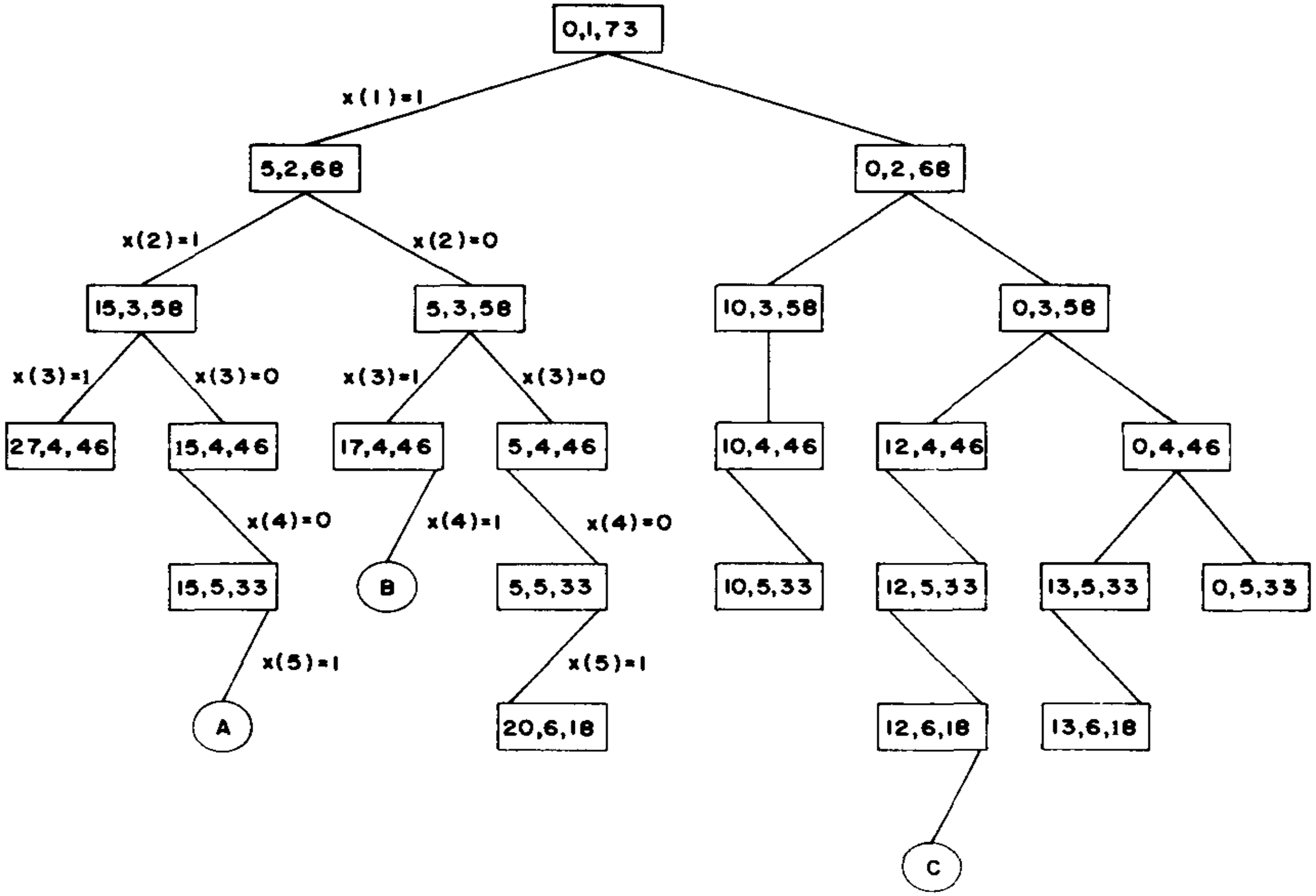


Sum of Subset Problem

The state space tree generated by procedure SUMOFSUB while working on the instance
n = 6, M = 30 and
W(l :6) = (5, 10, 12, 13, 15, 18).
At nodes A, B and C
the output is respectively (1, 1, 0, 0, 1), (1, 0, 1, 1) and (0, 0, 1, 0, 0, 1).



Sum of Subset Problem

```
procedure SUMOFSUB(s, k, r)
  //find all subsets of W(1:n) that sum to M. The values of//
  //X(j), 1 ≤ j < k have already been determined. s = Σ_{j=1}^{k-1} W(j)X(j)//
  //and r = Σ_{j=k}^{n} W(j) The W(j)s are in nondecreasing order.//
  //It is assumed that W(1) ≤ M and Σ_{i=1}^{n} W(i) ≥ M.//
```

```
global integer M, n; global real W(1:n); global boolean X(1:n)
real r, s; integer k, j
//generate left child. Note that s + W(k) ≤ M because B_{k-1} = true//
X(k) ← 1
if s + W(k) = M   //subset found//
     then print (X(j), j ← 1 to k)
     //there is no recursive call here as W(j) > 0, 1 ≤ j ≤ n//
     else
        if s + W(k) + W(k + 1) ≤ M then   //B_k = true//
             call SUMOFSUB(s + W(k), k + 1, r − W(k))
        endif
endif
//generate right child and evaluate B_k//
if s + r − W(k) ≥ M and s + W(k + 1) ≤ M   //B_k = true//
     then X(k) ← 0
          call SUMOFSUB(s, k + 1, r − W(k))
endif
end SUMOFSUB
```

Thank You

State space tree

- All paths from the root to other nodes define the state space of the problem.
- Solution states are those problem states S for which the path from the root to S defines a tuple in the solution space.
- Answer states are those solution states S for which the path from the root to S defines a tuple which is a member of the set of solutions.
- The tree organization of the solution space will be referred to as the state space tree.

N-Queen Problem

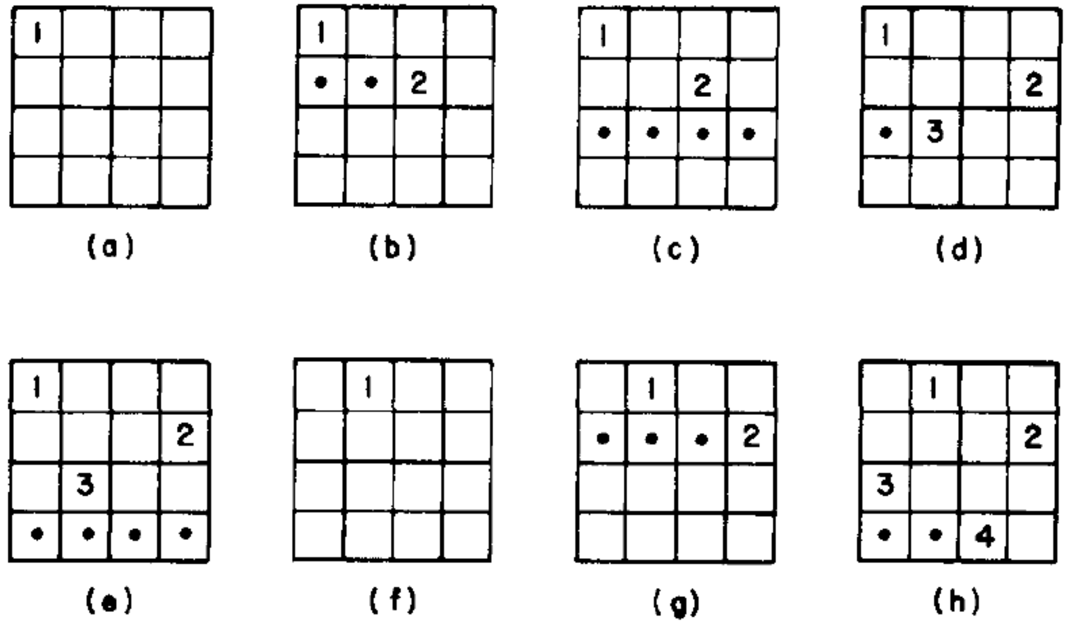
- The n-queens problem is a generalization of the 8-queens problem of n queens are to be placed on a n x n chessboard so that no two attack, i.e., no two queens are on the same row, column or diagonal.

```
Nqueens(k,n){
for i=1 to n{
    if(place(k,i)){
       x[k]=i
       if (k==n){
          for j=1 to n
             print x[i]}
       else
         Nqueens(k+1,n)}}}
```

```
place(k,i){
for j=1 to k
    if(x[j]==i || abs(x[j]-i ==abs(j-k))
         return false
    return true
         }
```

4-Queen Problem

- The 4-queens problem is a generalization of 4 queens are to be placed on a 4 x 4 chessboard so that no two attack, i.e., no two queens are on the same row, column or diagonal.



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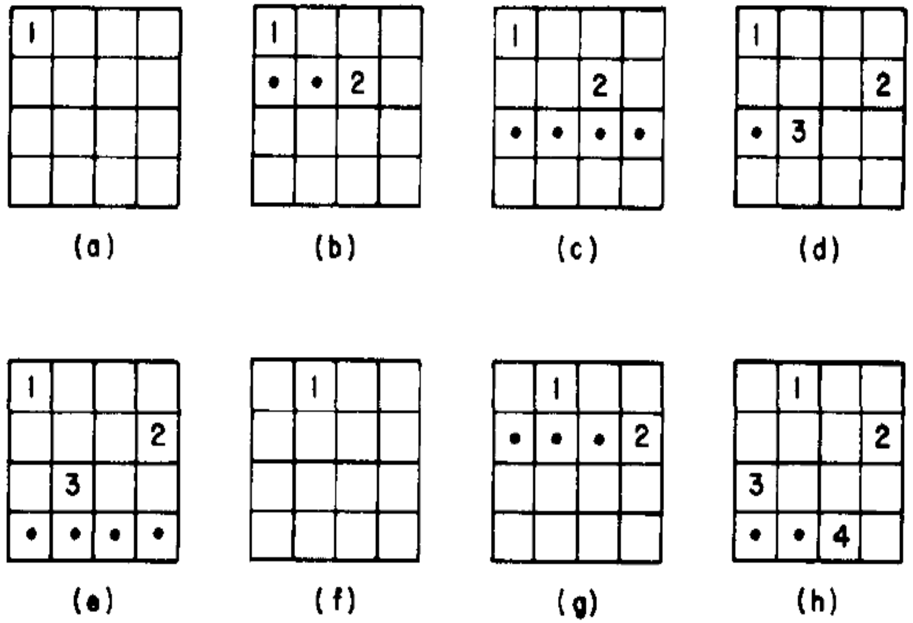

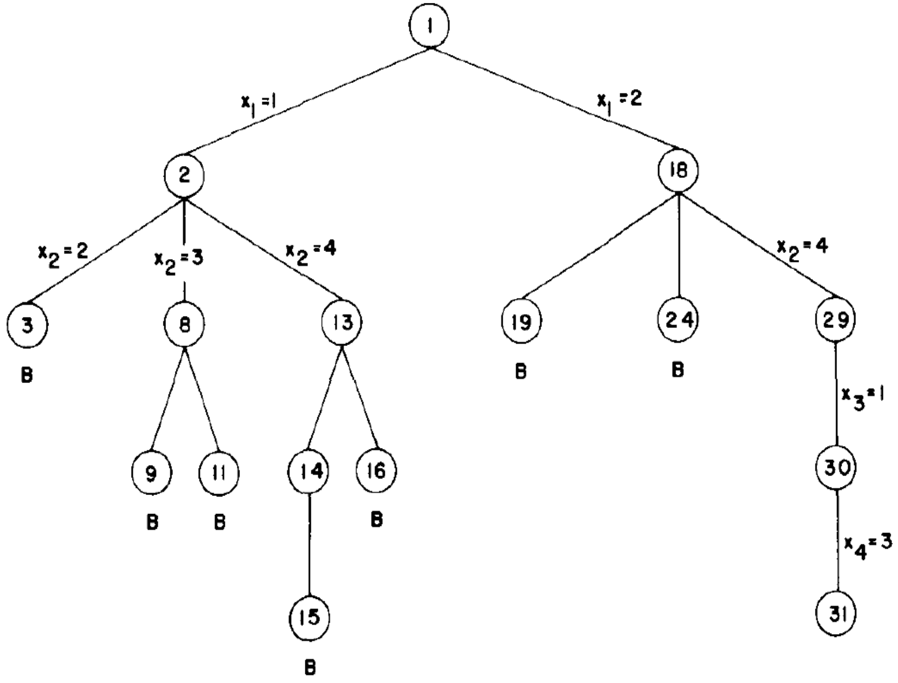

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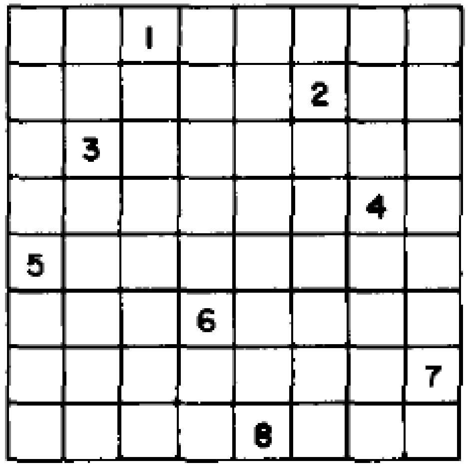
	1		
			2
3			
		4	

		1	
2			
			3
	4		

8-Queen Problem

- The 8-queens problem is a generalization of 8 queens are to be placed on a 8 x 8 chessboard so that no two attack, i.e., no two queens are on the same row, column or diagonal.



(8,5,3,2,2,1,1,1) = 2329

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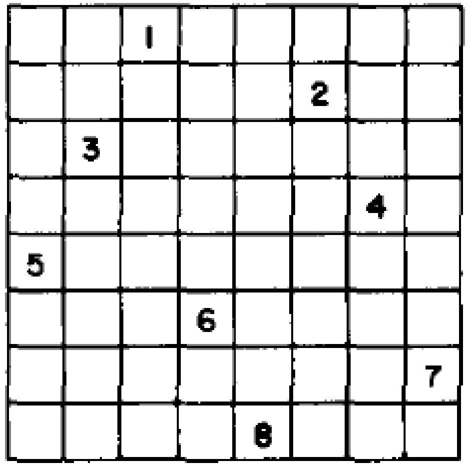

(8,5,3,2,2,1,1,1) = 2329