

Series	Solution	aj	II nd	Osidus	osidinary
differen	tial Egy	ation			7

Power Revier - An infinite Review of the form from + ann + ann + ann + -

is called power series.

 $= \underbrace{\sum_{n=0}^{\infty} \frac{n^n}{n!}}_{n=0} \quad a_n = \underbrace{\frac{1}{n!}}_{n!}$

Analytic Functions -

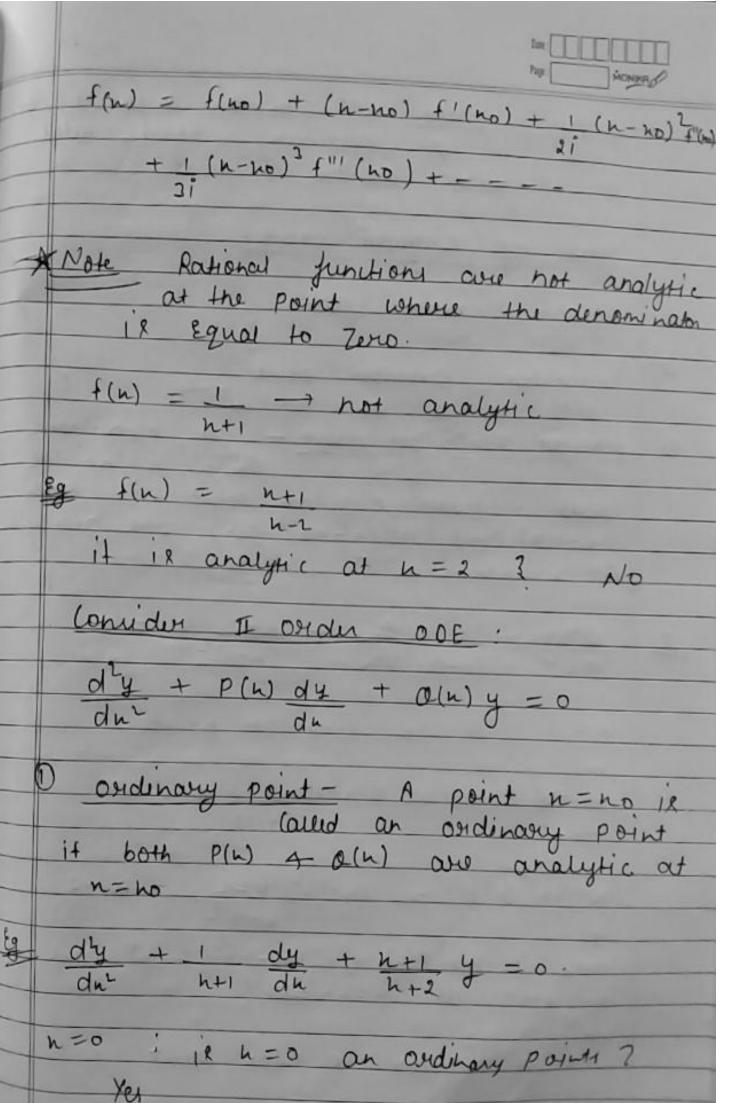
let f: I → R: not I I → intermal

then f is analytic at n=no, if

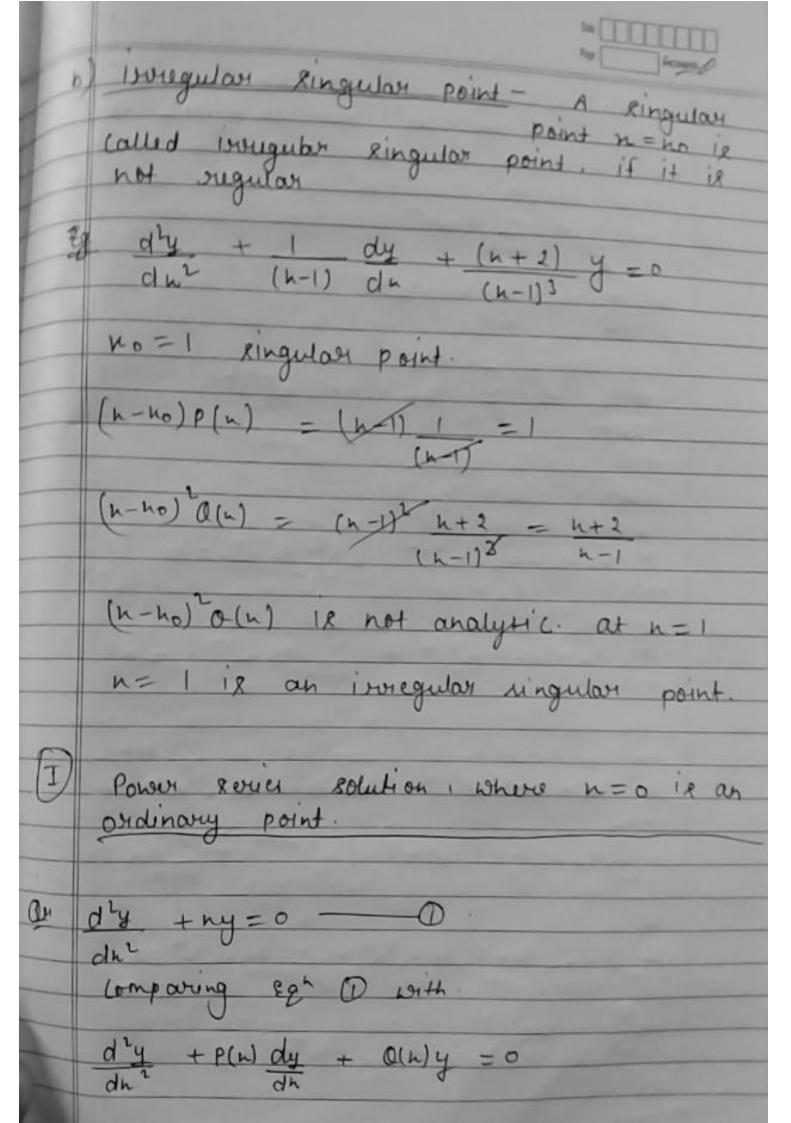
(i) Taylor Enpansion of f(n) at n = no Exists

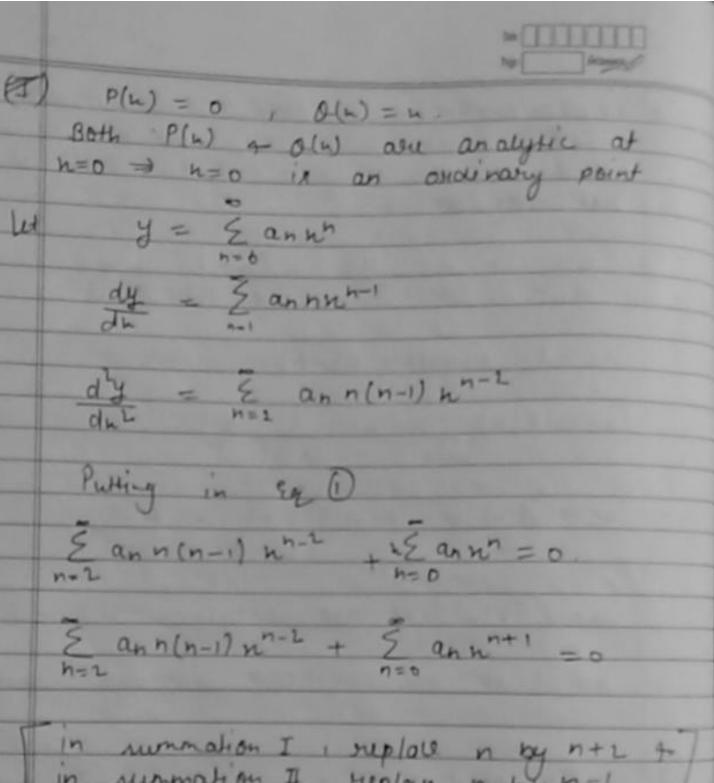
(ii) it converges to f(n) for all n in the internal of convergence.

in one variable Taylor's review Enpansion



Bingular point - The point or ordinary as Ringular point. which 12 not point Called Types of singular pointa Regular Singular point insugular eingular point. 1) Regular Ringular point - A Ringular point n=no in (n-no) p(h) of (n-no)2 o(h) are analytic at h= ho $\frac{d^2y}{dn^2} + \frac{1}{(n-1)} \frac{dy}{dn} + \frac{n+2}{(n-1)^2} \frac{y}{y} = 0$ 13 n=1 it is singular point. (n-hd)P(n) = (n-1)1 = 1 $(n-no)^{2}O(n) = (n+1)^{2}n+2 = n+2$ Both (n-ho) P(h) g (n-ho) 20 (h) are analytic at n=no=1 h=1 i's a regular singular point.





in summation I replace n by n-1

Ean+2(n+2)(n+2-1)nn+2-2 + Ean-12-1-1

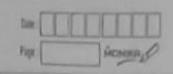
E an+2 (n+2) (n+1)nn+ & an-1 hn=0

n=0 a2(2)(1)=0 [a1=0]

y = Eann

dy = Zannumi

Putting in Eq ()



Jon n > 1 an+2(n+2)(n+1) + an-1 = 0

$$a_{n+2} = -a_{n-1}$$
 $(n+2)(n+1)$

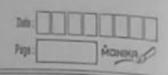
$$n=1$$
: $a_3 = -a_0 = -a_0$
 $3x_2 = 6$

$$n = 2 : qy = -q_1$$

$$n=3$$
 : $q_5 = -q_2 = 0$

$$n=4: 96 = -91 = 90 \cdot 1 = 6x5 = 6$$

$$y = \frac{5}{5} \frac{a_n n^n}{n} = \frac{a_0 + q_1 n + a_2 n^2 + a_3 n^3 + q_4 n^4}{1 + a_5 n^3 + q_4 n^4}$$



$$P(n) = n$$
, $O(n) = -1$
 $1+n^2$

Both P(h) 4 O(h) are analytic at n=0 h=0 is an ordinary point

let y = & ann'

dy = = 20 annn-1

dry = \(\frac{2}{\text{dn}^2} \) \(\frac{1}{\text{dn}^2} \) \(\frac{1}{\text{n}} \) \(\frac{

putting in Egh O

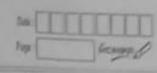
 $(1+n^2)$ $\sum_{n=1}^{\infty}$ $a_n n (n-1) n^{n-2} + n \sum_{n=1}^{\infty} a_n n n^{n-1}$

- \(\sum_{n=0}^{\infty} ann^h = 0.

=) $\sum_{n=2}^{\infty} a_n n(n-1) n^{n-2} + \sum_{n=2}^{\infty} a_n n(n-1) n^n + \sum_{n=2}$

 $\frac{\tilde{z}}{\sum_{n=1}^{n} a_n h_n^n} - \frac{\tilde{z}}{\sum_{n=0}^{n} a_n h_n^n} = 0.$

=) $\frac{1}{2}$ \frac



= = an+2(n+2)(n+1) n + = ann(n-1) 2 +

2 an nun - € an nun = 0.

 $\frac{1}{100} \frac{1}{100} \frac{1}$

n=1 as. 3.2+9/1-9/1=0 [as=0]

n22 an+2 (n+2)(n+1) + an n(n-1) + ann-9n

 $a_{n+2} = -a_n [n^L - 1] = -(n-1) a_n$ (n+2)(n+1) = (n+2)

n=1 $q_4 = -\frac{1}{4}a_1 = -\frac{1}{4}\frac{q_0}{q_0} = -\frac{q_0}{8}$

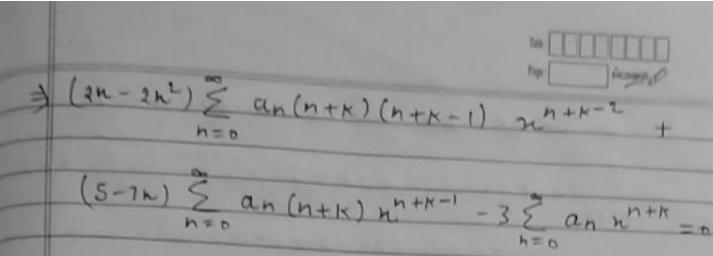
n=3 $q_{5}=-\frac{2}{5}a_{3}=0$

n = 4 $a_6 = -3 = 4390$

y= 90 +91+ azh + azh + ----

 $= 90 + 91n + 90n^{2} - 20n^{4} + 90n^{6} + = 90 + 91n + 20n^{2} - 20n^{4} + 90n^{6} + -$

Frobenius method for levier Robetion when n=0 is a sugular singular point dru + p(u) dy + o(u) y = 0 Can 1: - when roots of indicial Equation are distinct of do not differ by an integer. y = ciy, + czyz Plu an(1-n) d2y + [s-7n] dy - 3y = 0 $\frac{SM}{2n(1-n)} = \frac{-3}{2n(1-n)}$ Both n p(n) + n2 O(n) are analytic of n=0 n=0 il a sugular singular point, Wy = Eann+K no =0 (n-no) P(n) $\frac{dy}{dx} = \frac{5}{n=0} \frac{q_n(n+x)n^{n+x-1}}{n=0}$ n(p(h) 2(1-n) (n-no) 2(h) d24 = 5 an (n+k) (n+k-1)nh+k-2 n2 O(W) = - 3n 2(1-n)



$$=\frac{1}{2}\sum_{n=0}^{\infty} 2a_n (n+k) (n+k-1) n^{n+k-1} - \sum_{n=0}^{\infty} 2a_n (n+k) n^{n+k-1}$$

$$\sum_{n=0}^{\infty} \frac{a_n(n+\kappa)\{2(n+\kappa-1)+s\}n^{n+\kappa-1}}{\sum_{n=0}^{\infty} \frac{a_n\{2(n+\kappa)^2-2(n+\kappa)-7(n+\kappa)+3\}n^n}{n=0}}$$

$$= 2(n+k)^{2} + 2(n+k) + 3(n+k) + 3$$

I an record &, suplace n by n-13

£ an (n+k) (2n+2k+3) nn+k-1 - £ an-,

(n-y+x+1) (2n-2+2x+3) un-1 =0

n=0 : 90 K (3K+3) = 0 90 # 0

K (2K+3) = 0 Indicial Egh

K = 0, -3

 $n\geq 1$: $a_n(n+k)(2n+2k+3)-a_{n-1}(n+k)$ (2n+2k+1)=0

 $an = \frac{(2n+2k+1)a_{n-1}}{(2n+2k+3)}$

n=1 $q_1 = \frac{(2k+3)}{(2k+5)}$ q_0

 $a_2 = \frac{(2K+5)}{(2K+7)} \frac{a_2}{(2K+7)} = \frac{(2K+7)}{(2K+7)} \frac{(2K+7)}{(2K+5)}$

= (3x+3) 90 (2x+3)

 $a_3 = \frac{(2k+3)}{(2k+9)} q_0$

to icson

$$y_1 = y |_{\alpha \in K} = x_1 = y |_{K=0}$$

$$y_1 = \sum_{h=0}^{\infty} a_h n^h + x_1 = \sum_{h=0}^{\infty} a_h n^h$$

$$= 90 + \frac{3}{5} + \frac{3}{5} + \frac{9}{7} + \frac{3}{9} + \frac{3}{7} + \frac{3}{7}$$

$$y_2 = \sum_{n=0}^{\infty} a_n n^{n+kx} = \sum_{n=0}^{\infty} a_n n^{n-3/2}$$

Qu 2n2y" + (2n2-n) y + y =0

 $\int \frac{1}{y'} + \frac{3n^2 - n}{3n^2} + \frac{y'}{2n^2} = 0$

 $y'' + (1-\frac{1}{2n})y' + \frac{y}{2n^2} = 0$

P(n) = 1 - 1, $Q(n) = \frac{1}{2n^2}$

n=0 is a singular point belows both p(n) 4 O(n) are not analytic

 $A = (n-n0) p(n) = n (1-1) = \kappa(2n-1)$

 $B = (n-no)^{2} O(n) = n^{2} \left(\frac{1}{2k^{2}}\right) = \frac{1}{2}$

n=0 is a sugment singular point

W y = = anh++

y'= & an (n+1x) nn+x-1

 $y'' = \sum_{n=0}^{\infty} \alpha_n (n+k)(n+k-1) n^{n+k-2}$

$$\frac{1}{2} = 2 an (n+k) (n+k-1) n^{n+k} + \frac{5}{2} = 2 an (n+k)$$
 $\frac{1}{2} = \frac{1}{2} an (n+k) (n+k-1) n^{n+k} + \frac{5}{2} an n^{n+k} = 0$

from D

Put n=0 in the coeff of least power of n t Equate to zero to find indicial Equation

$$(K-1)(3K-1)=0$$
 $K=1,\frac{1}{2}$



[Replace n by n-1 in $\leq I^{+}$] $= 2 a_{n-1} (n+k-1) n^{n-1+k+1} + \sum_{n=0}^{\infty} a_{n} (n+k-1) n^{n+k} = 0.$

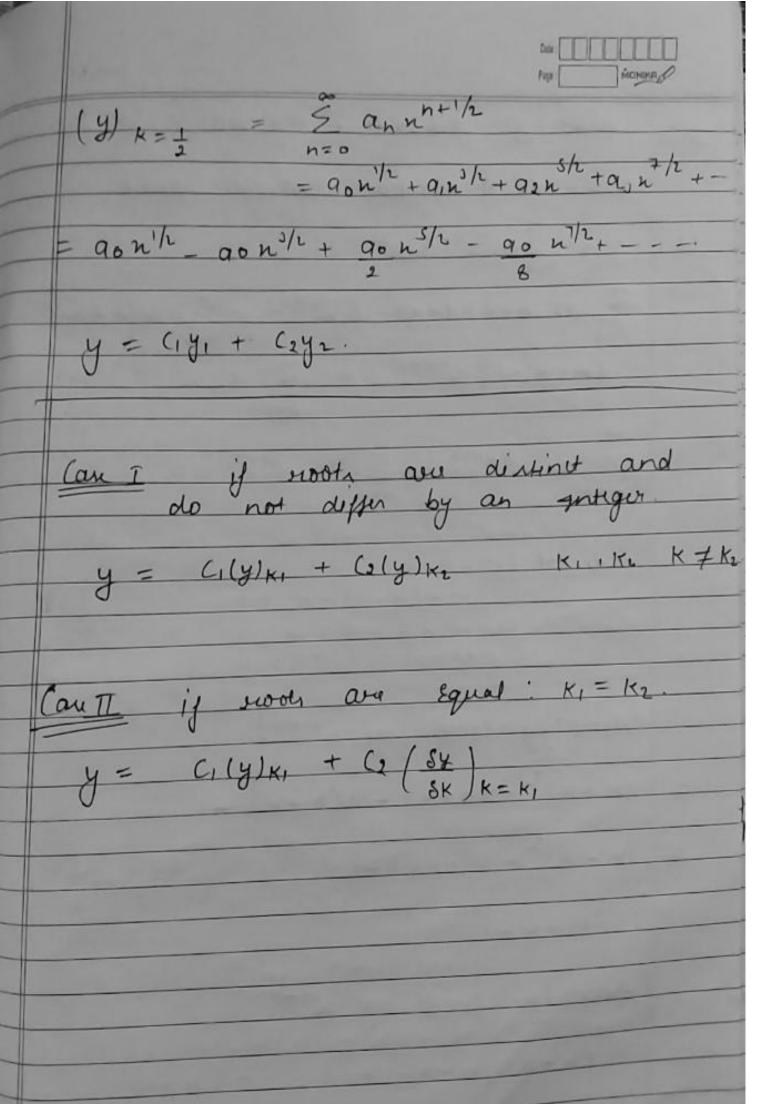
 $n \ge 1$: $2a_{n-1}(n+k-1) + a_n(n+k-1)(2n+2k-1)=0$

 $a_n = -\frac{2a_{n-1}(n+k-1)}{(n+k-1)(2n+2k-1)}$

 $an = -2a_{n-1}$ (2n+2k-1)

 $(y)_{k=1} = \sum_{n=0}^{\infty} a_n n^{n+1} = q_0 n + q_1 n^2 + a_2 n^4 + a_3 n^4$

 $= aon - 290n^{2} + 490n^{3} - 290n^{4} + - - -$



Que n(n-1) y" + (3n-1) y' + y = 0 n=0 is a sugular singular point let, y = Eannhx = an (n+k) (n+k-1) nn+k - = an (n+k) (n+K-1) nn+K-1 + = ann+K - $\sum_{n=0}^{\infty} a_n n^{n+k-1} + \sum_{n=0}^{\infty} a_n n^{n+k} = 0$ indicial 8gh - - K(K-1)-K = 0. - KL+K-K=0 K=0,0 (simplify well of nn+k) $= (n+\kappa)^2 - (n+\kappa) + 3(n+\kappa) + 1$ (n+k) + 2(n+k)+1 $(n+k+1)^2$ (dimplify lock of nn+k-1) - (n+k) (n+K-1+1) = - (n+k)2

$$\sum_{n=0}^{\infty} a_n (n+k+1)^{\frac{1}{2}} n^{n+k} - \sum_{n=0}^{\infty} a_n (n+k)^{\frac{1}{2}} n^{n+k-1} = 0$$

$$= \sum_{n=1}^{\infty} a_{n-1} (n-1+k+1)^{2} n^{n+k-1} - \sum_{n=0}^{\infty} a_{n} (n+k)^{2} n^{n+k-1} = 0$$

$$a_n = \frac{a_{n-1}(n+\kappa)^2}{(n+\kappa)^2} = \frac{a_n = a_{n-1}(n+\kappa)^2}{(n+\kappa)^2}$$

$$n=1$$
 $a_1 = q_0$

$$n = 2$$
 $a_1 = q_1 = q_0$

$$\frac{\delta y}{\delta K} = n^{K} \log_{e} n \, q_{0} \, (1 + n + n^{2} + - - - -)$$

$$\frac{2}{m_{20}}$$
 an $(n+k)(n+k-1+1)$ $n^{n+k-1} - \frac{2}{m_{21}}$ $\alpha_{n-1}n^{n+k+1}$

$$a_n = \underbrace{a_{n-1}}_{(n+k)^2} \qquad n \ge 1$$

$$n=1 \qquad a_1 = \underline{a_0}$$

$$(1+K)^2$$

$$n=2$$
 $\alpha_2 = \frac{\alpha_1}{(2+\kappa)^2} = \frac{\alpha_0}{(1+\kappa)^2(2+\kappa)^2}$

$$q_3 = q_0$$
 $(1+k)^{2}(2+k)^{2}(3+k)^{2}$

$$= a_0 n^{\kappa} \int_{1}^{1} + n + n^2 + \cdots$$

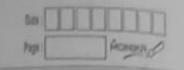
$$= (1+\kappa)^2 (2+\kappa)^2 + \cdots$$

$$\left(\frac{\delta y}{\delta k}\right) = a_0 n^k \log_e n \left[1 + n + n^2 + n^2 + --\right]$$

$$+ q_{0}n^{K} \left[-2n + n^{2} \left\{ -2 - 2 \right\} \right]$$
 $-2 \left[(1+K)^{2} (2+K)^{2} (1+K)^{2} (2+K)^{2} \right]$

$$\left(\frac{\delta y}{\delta k}\right)_{k=0} = a_0 \log_e \left[1 + k + n^2 + --\right] -$$

$$290 \left[\frac{1}{13} + n^2 \left\{ \frac{1}{3} + \frac{1}{1^2 2^3} \right\} \right]$$



Courte when stooks are distinct differ by an integer and making one of more coefficients interminate

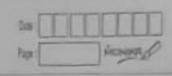
let K = K1, K2

if looff of y becomes infinite at K=K,

=) y= (y)k,

Tan G when roots on distinct differ by an antique and making coeff of infinite $y = C_1(y)_{K_1} + C_2(\frac{5y}{5K})_{K_2}$

* if some coeff. of y becomes injuste at K = K2 first suplate as by bo (K - K2) then differentiate it.



Our n(n-1) dy + (3n-1) dy + y = 0

n=0 is a sugular singular tooint

Let, $y = \sum_{n=0}^{\infty} a_n n^{n+\kappa}$, $y' = \sum_{n=0}^{\infty} a_n (n+\kappa) n^{n+\kappa-1}$

 $y'' = \sum_{n=0}^{\infty} q_n (n+k) (n+k-1) n^{n+k-2}$

=) $(n^2-n)\frac{8}{5}$ an $(n+k)(n+k-1)n^{n+k-2}$ +

(3n-1) $\leq a_n (n+1x) n^{n+K-1} + \leq a_n n^{n+K} = 0$

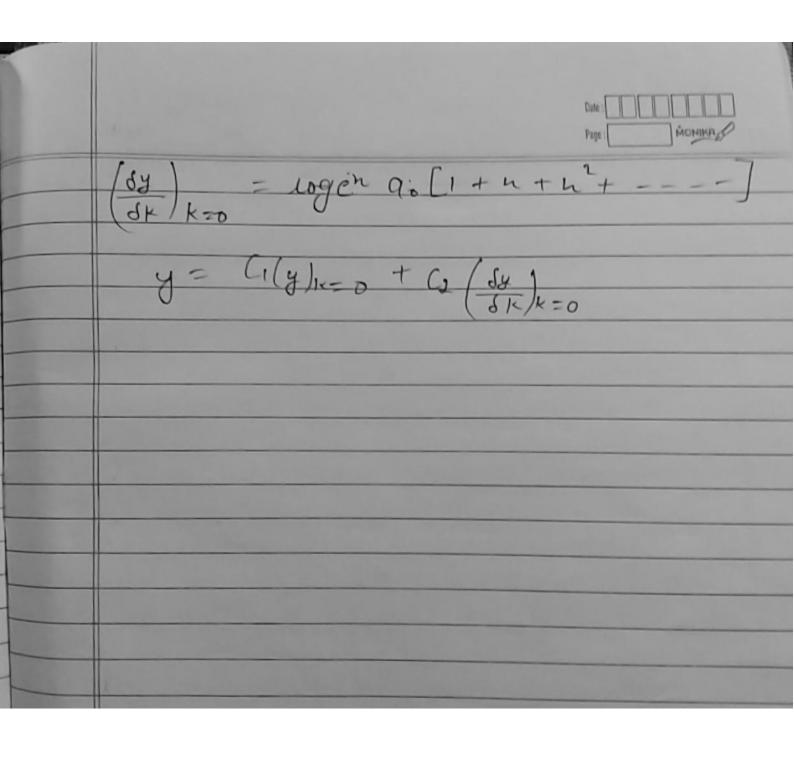
= = an (n+k) (n+k-1) + + = = an (n+k) (n+k-1)

nn+k-1 + \$ 39n (n+k) nn+k - \$ an (n+k) nn+k-1

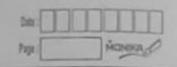
+ \(\frac{2}{9} \an k^n + \times = 0

- K(K-1) - K = 0

Page Michigan D
$\frac{(n+k)^{2} - (n+k) + 3(n+k) + 1}{(n+k)^{2} + 2(n+k) + 1}$ $\frac{(n+k+1)^{2}}{(n+k+1)^{2}}$ $\frac{2}{n=0}$
$n \ge 1 : q_{n-1} (n+k)^2 - q_n (n+k)^2 = 0$ $q_n = q_{n-1}$ $n = 1 : q_1 = q_0$
$n=2$: $q_3 = q_1 = q_0$. $n=3$: $q_3 = q_0$ $y = \sum_{n=0}^{\infty} q_n n^{n+\kappa} = n' \left[q_0 + q_1 n + q_2 n'_{+} - J - 0 \right]$
$y_{k} = 0 \cdot \sum_{n=0}^{\infty} a_{n}n^{n} = q_{0} + q_{1}n + q_{1}h^{2} + q_{2}h^{3} +$ $= q_{0}(1 + n + n^{2} + n^{3} + J$ forom O .
54 - nt logen [90 + 9, n + 9, n +]



Mathe



Can III: when roots are distinct differ by an integer 4 making one or more coefficient of y interminate.

W K = K1 , K2

if coeff. of y becomes interminate for

y = (y) K,

91/0

Quy ny" + 2y' + ny = 0

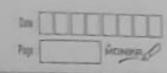
3d n=0 il a sugular lingular point

 $y = \sum_{n=0}^{\infty} q_n x^{n+k}$

€ an (n+k)(n+k-1)nn+k-1 + € 29n (n+k) nn+k-1

+ & ann+k+1 = 0

JE K(K-1) + 2K =0 K2+K=0 K = 0, -1



 $\sum_{n=0}^{\infty} a_n(n+k) (n+k-1+2) + \sum_{n=2}^{\infty} a_{n-2} n^{n+k-1} = 0$

n=1: $a_1(1+k)(k+2)=0$.

91.0 = 0 * (are III (identify)

value.

n > 2

 $\begin{array}{c}
\left(n + k\right)\left(n + k + 1\right)
\end{array}$

n = 2

 $q_2 = -q_0$ $q_2 = -q_0 = -q_0$ $q_1 = -q_0 = -q_0$ $q_1 = -q_0 = -q_0$

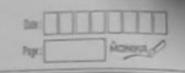
n=3 $\alpha_3 = -\alpha_1$ $\alpha_3 = -\alpha_1 = -\alpha_1$ $\alpha_3 = -\alpha_1$ $\alpha_4 = -\alpha_1$ $\alpha_5 =$

h = 4 $a_{4} = \frac{-a_{2}}{(k+4)(k+5)}$ $a_{4} = \frac{a_{0}}{4!}$

 $= \frac{-90}{(K+2)(K+3)(K+4)(K+5)}$

 $n=5 \quad q_{S} = \frac{q_{I}}{(K+3)(K+7)(K+5)(K+6)} \qquad q_{I} = \frac{q_{I}}{5I}$

y = (y) k, = (y) k = -1



$$\frac{1}{3!} \frac{1}{5!} \frac{1}{7!} = \frac{1}{7!} \frac{1}{7!}$$

$$y = \sum_{n=0}^{\infty} a_n n^{n+K}$$

$$=) \sum_{n=0}^{\infty} q_n(n+k) (n+k-1) n^{n+k} + \sum_{n=0}^{\infty} q_n(n+k) n^{n+k}$$

$$K^2 + 3K + 2 = 0$$

$$= \frac{(n+k)^2 - (n+k) + 4(n+k) + 2}{(n+k)^2 + 3(n+k) + 2}$$

$$= (n+k)^{2} + 3(n+k) + 2$$

= $(n+k+1)(n+k+2)$

$$\frac{2}{8} a_{n} (n+k+1) (n+k+2) n^{n+k} + \frac{2}{6} a_{n-2} n^{n+k}$$

$$n=1$$
 $a_1(K+2)(K+3)=0$ $\int (a_1 II) a_2 a_3 a_4 b_4$
 $for K=-2$ $a_1 \cdot a=0$ $for Y$ (or intended)

$$n > 2$$

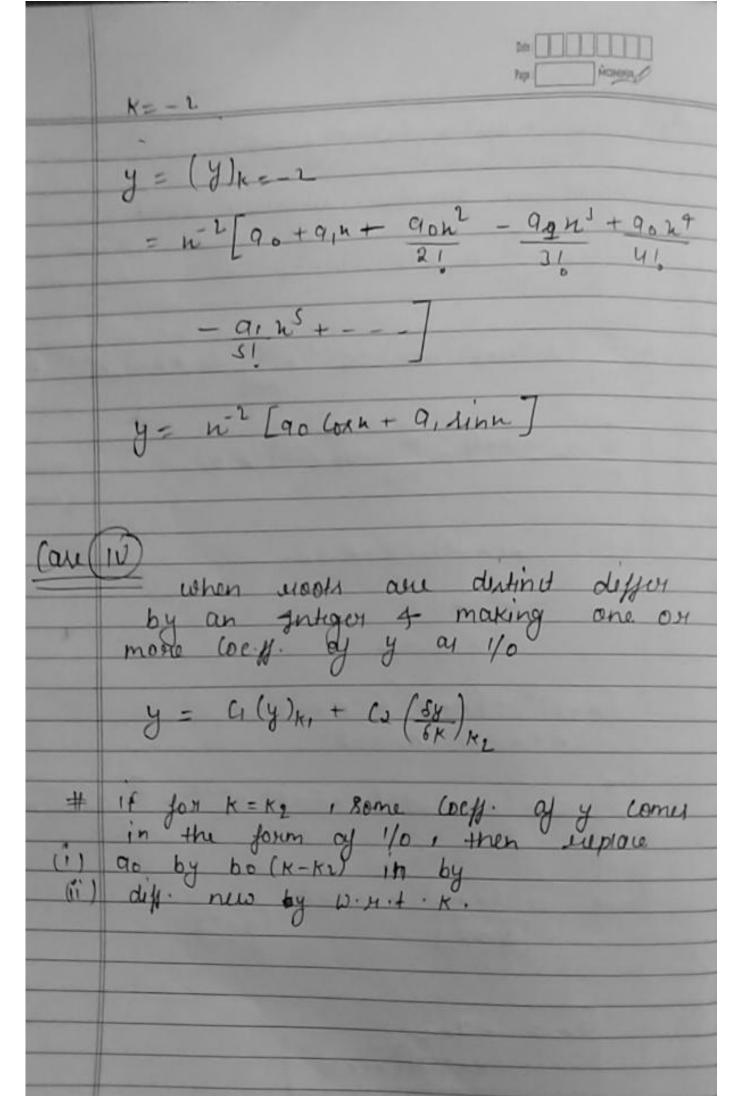
$$Q_2 = \frac{-q_0}{(K+3)(K+4)}$$
 $Q_2 = \frac{-q_0}{1\cdot 2} = \frac{-q_0}{2!}$

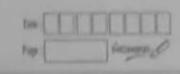
$$q_3 = -q_1$$
 $q_3 = -q_1$
 $(K+4)(K+5)$
 $q_3 = -q_1$
 $q_3 = 3!$

$$q_4 = \frac{-q_1}{(K+5)(K+6)}$$
 $q_4 = \frac{q_0}{13.34}$ q_1

$$=$$
 90 $(K+3)(K+4)(K+5)(K+6)$

$$QS = \frac{QI}{(k+4)(k+5)(k+6)(k+7)}$$





$$y = \sum_{n=0}^{\infty} a_n n^{n+k}$$

$$\frac{1}{8} \sum_{n=0}^{\infty} q_n (n+k) (n+k-1) n^{n+k} + \sum_{n=0}^{\infty} q_n (n+k) n^{n+k} + \sum_{n=0}^{\infty}$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}y_{n}q_{n}n+k=0$$
Combine
$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}y_{n}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}y_{n}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}y_{n}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}y_{n}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}y_{n}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}q_{n}n+k=0$$

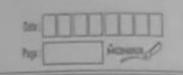
$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k+2-\frac{5}{2}q_{n}n+k=0$$

$$\frac{5}{n=0}q_{n}n+k=0$$

$$n=1$$
 $q_1(K+3)(K-1)=0$ $q_1=0$ (an IV



$$n=2$$
 $q_2 = -q_0$ $q_2 = -q_0$ $q_2 = -q_0$ $q_3 = -q_0$ $q_4 = -q_0$

$$n=3$$
 $q_{j}=-q_{i}=0$ $q_{j}=0$ $(k+1)(k+5)$

$$n=4$$
 $94=-92$ $94=90$ $2.4.6.8$

$$=\frac{90}{K(K+2)(K+9)(K+6)}$$
 $=\frac{90}{45}$
 $=0$

$$(y)_{k=2} = n^2 [q_0 - q_0 n^2 + q_0 n^4 - q_0 n^6]$$

but
$$a_0 = b_0 (k+2)$$
 $v = -2$

$$y = \kappa^{-2} \left[bo (K+2) - bo (K+2) n^{2} + bo n^{3} + K(K+4) (K+6) \right]$$

