Karl Pearson's coefficient of correlation

Defnition: The Karl Person's coefficient of correlation between two variables X and Y is denoted by $\rho(x,y)$ or γ is defined as

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var\,X}\sqrt{Var\,Y}} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y}$$

where VarX and Var Y denotes the variances of X and Y, while σ_X and σ_Y are S.D.

Note: Correlation coefficient γ is a measure of the linear relationship between two variables X and Y.

Note: The value of γ ranges from $-1 \le \gamma \le 1$.

Note: If $\gamma = \pm 1$, then the variables have perfect correlation.

For $\gamma = +1$ perfect positive correlation.

$\gamma = -1$ perfect negative correlation.

Note: If $\gamma = 0$, then there is no correlation between the variables X and Y.

Ques (1) The covariance between the length and weight of five items is 6 and their standard deviations are 2.45 and 2.61 respectively. Find correlation coefficient between length and weight.

Solution: Let $X \rightarrow$ length $Y \rightarrow$ weight

$$Cov(X,Y) = 6 \quad \sigma_x = 2.45 \quad \sigma_y = 2.61$$

$$r = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y} = \frac{6}{2.45 \times 2.61}$$

$$= \frac{6}{6.3945} = 0.9383$$

$$\boxed{r = 0.9383}$$ Ans

Ques (2) If covariance of 10 pairs of items is 7, variance of X is 36, $\Sigma (Y-\bar{Y})^2 = 90$. Find out r.

Solution: Here, we have $n = 10$, $Cov(X,Y) = 7$

$$Var(X) = 36 \Rightarrow \sigma_x = \sqrt{varx} = 6$$

$$\Sigma (Y-\bar{Y})^2 = 90$$

$$\sigma_y = \sqrt{\frac{(Y-\bar{Y})^2}{n}} = \sqrt{\frac{90}{10}} = \sqrt{9} = 3$$

$$r = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y} = \frac{7}{6 \times 3} = \frac{7}{18} = 0.39 \text{ (Approx)}$$

Ans

Ques (3) The coefficient of correlation between two variables X and Y is 0.3 and covariance is 9. If $Var(X) = 16$ find σ_y.