Covariance: $X \to x_1, x_2 \dots x_n$ (n observations)
$Y \to y_1, y_2 \dots y_n$

Covariance between X and Y, denoted by Cov(x,y) is defined as

$$Cov(X,Y) = \frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{n} = \frac{\Sigma xy}{n}$$

where $x = X - \bar{X}$ and $y = Y - \bar{Y}$

Ques (1) Find Cov(X,Y) between X and Y if

X : 1   2   3   4   5

Y : 2   4   6   8   10

Solution:

X	Y	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})(Y-\bar{Y})$
1	2	-2	-4	8
2	4	-1	-2	2
3	6	0	0	0
4	8	1	2	2
5	10	2	4	8
$\Sigma X = 15$	$\Sigma Y = 30$			20

Here, we have $n = 5$, $\bar{X} = \frac{\Sigma X}{n} = \frac{15}{5} = 3$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{30}{5} = 6$$

$$Cov(X,Y) = \frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{n} = \frac{20}{5} = 4$$

Ques (2) Find Cov(X,Y) between X and Y if

X : 3   4   5   6   7

Y : 8   7   6   5   4

Note: If $\bar{x}$ or $\bar{y}$ is not an integer. Then

$$Cov(x,y) = \frac{\sum xy}{n} - \left(\frac{\sum x}{n}\right)\left(\frac{\sum y}{n}\right)$$

Ques (3) Calculate the $Cov(x,y)$ for the following data

x : 1   2   3   4   5   6   7   8   9   10

y : 6   9   6   7   8   5   12   3   17   1

Solution:

X	Y	XY
1	6	6
2	9	18
3	6	18
4	7	28
5	8	40
6	5	30
7	12	84
8	3	24
9	17	153
10	1	10
$\sum x = 55$	$\sum y = 74$	$\sum xy = 411$

Here, we have $n = 10$, $\bar{x} = \sum \frac{x}{n} = \frac{55}{10} = 5.5$ (not an integer)

$$\bar{y} = \frac{74}{10} = 7.4$$

$$Cov(x,y) = \frac{\sum xy}{n} - \left(\frac{\sum x}{n}\right)\left(\frac{\sum y}{n}\right)$$

$$= \frac{411}{10} - \left(\frac{55}{10}\right)\left(\frac{74}{10}\right)$$

$$= 41.1 - 5.5 \times 7.4$$

$$= 41.1 - 40.7$$

$$= 0.4 \quad \text{Ans}$$

Ques: (4) Find the covariance between X and Y for the following data

X :	1	2	3	4	5	6	7	8	9	10
Y :	10	9	8	8	6	12	4	3	18	1

Soln: Step deviation Method: Values of X or/and Y are large

Let $u = X - A$, $v = Y - B$

where A and B are arbitrary constants. Then

$$Cov(X, Y) = \sum \frac{uv}{n} - \left(\frac{\sum u}{n}\right)\left(\frac{\sum v}{n}\right)$$

Ques: (5) Find the Covariance between X and Y for the following data

X :	66	67	68	69	70	71	72
Y :	68	65	70	70	69	70	69

Solution:

X	Y	$u = X - A$ $= X - 69$	$v = Y - B$ $= Y - 70$	
66	68	−3	−2	6
67	65	−2	−5	10
68	70	−1	0	0
69 = A	70 = B	0	0	0
70	69	1	−1	−1
71	70	2	0	0
72	69	3	−1	−3
		$\sum u = 0$	$\sum v = -9$	$\sum uv = 12$

n = 7

$$Cov(X, Y) = \sum \frac{uv}{n} - \left(\frac{\sum u}{n}\right)\left(\frac{\sum v}{n}\right)$$

$$= \frac{12}{7} - \left(\frac{0}{7}\right)\left(\frac{-9}{7}\right) = 1.7 \text{ Ans}$$