(Probability)

Addition Theorem: If A, B are any two events of a sample space then $\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$

$P(A\cup B)$ stand for occurrence of at least one of two events A and B.

Note: (I) $P(S) = 1$

(II) $P(\phi) = 0$

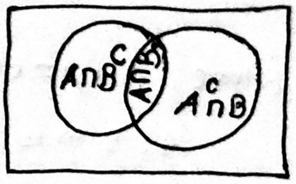
Addition Theorem for Mutually Exclusive Events:

Two events A and B are said to be MEE if $A\cap B = \phi$

$$P(A\cap B) = P(\phi) = 0$$

$$\boxed{P(A\cup B) = P(A) + P(B)}$$

Note (I) $P(A\cap B^c) = P(A) - P(A\cap B)$



(II) $P(A^c\cap B) = P(B) - P(A\cap B)$

(III) $P(A^c\cap B^c) = P(A\cup B)^c$
$= 1 - P(A\cup B)$

Ques (1) Let A and B be two events of a sample space S and let $P(A) = \frac{1}{2}$ and $P(B) = \frac{3}{10}$ and $P(A\cap B) = \frac{1}{10}$. Find the prob. for each of the following events

(I) A or B (II) A but not B (III) B but not A (IV) neither A nor B.

(I) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$= \frac{1}{2} + \frac{3}{10} - \frac{1}{10} = \frac{5+3-1}{10} = \frac{7}{10}$

(II) $P(A\cap B^c) = P(A) - P(A\cap B) = \frac{1}{2} - \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$

(III) $P(A^c\cap B) = P(B) - P(A\cap B) = \frac{3}{10} - \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$

(IV) $P(A^c\cap B^c) = P(A\cup B)^c = 1 - P(A\cup B) = 1 - \frac{7}{10} = \frac{3}{10}$

Ques(2) If E and F are two events associated with a random experiment for which $P(E) = 0.60$ $P(E \text{ or } F) = 0.85$ and $P(E \text{ and } F) = 0.42$ find $P(F)$.

Solution: We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(E) = 0.60$ $P(E \cup F) = 0.85$ $P(E \cap F) = 0.42$

$$\boxed{P(E \cup F) = P(E) + P(F) - P(E \cap F)}$$

$0.85 = 0.60 + P(F) - 0.42$

$P(F) = 0.85 + 0.42 - 0.60$

$P(F) = 0.67$

Ques(3) In a single throw of two dice, find the probability of obtaining either a sum of 9 or a sum of 11.

Solution: Here, $n(S) = 6 \times 6 = 36$

A: sum is 9 : $\{(3,6), (6,3), (4,5), (5,4)\}$

B : sum is 11 : $\{(5,6)\ (6,5)\}$

$n(A) = 4$ $n(B) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$, $P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

$P(A \cap B) = 0$ $\{\because n(A \cap B) = 0\}$

~~PtA~~

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B)$

$= \frac{1}{9} + \frac{1}{18} = \frac{3}{18} = \frac{1}{6}$ Ans

Ques(4) From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability that it is either a heart or a queen.

Solution: Here $n(S) = 52$

A: drawing a card of heart

B: drawing of card of queen

$P(A) = \frac{13}{52} = \frac{1}{4}$, $P(B) = \frac{4}{52} = \frac{1}{13}$

$P(A \cap B) = \frac{1}{52}$ (only one card is there of heart queen)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52}$

$= \frac{13 + 4 - 1}{52} = \frac{16}{52} = \frac{4}{13}$

$P(A \cup B) = \frac{4}{13}$ Ans

Ques (5) Two unbaised dice are thrown. Find the prob. that neither a doublet nor a total of 10 will appear.

Solution: Here, $n(S) = 36$.

$A =$ doublet $= \{(1,1), (2,2), (3,3), (4,4), (5,5) (6,6)\}$

$B =$ total of 10 $= \{(4,6), (6,4), (5,5)\}$

$n(A) = 6$, $n(B) = 3$

$P(A) = \frac{6}{36} = \frac{1}{6}$, $P(B) = \frac{3}{36} = \frac{1}{12}$

$A \cap B = \{(5,5)\} \Rightarrow n(A \cap B) = \{(5,5)\}$

$P(A \cap B) = \frac{1}{36}$

$P(A^c \cap B^c) = P(A \cup B)^C$

$= P(S) - P(A \cup B)$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [\frac{1}{6} + \frac{1}{12} - \frac{1}{36}\}$

$= 1 - \left(\frac{6 + 3 - 1}{36}\right) = 1 - \frac{8}{36} = \frac{28}{36}$

$P(A^c \cap B^c) = \frac{7}{9}$ Ans

Ques (6) Find the probability that a leap year selected at random contains either 53 sundays or 53 mondays. (Ans: $\frac{3}{7}$)

Ques (7) Two dice are thrown together. What is the probability that the sum of the numbers on the faces is divisible by 3 or ~~by~~ 4. (Ans: $\frac{5}{9}$)

Ques (8) A bag contains 5 Red, 6 White and 7 Black balls. Two balls are drawn at random. What is prob. that the both balls are red or both are black.

(Ans: $\frac{31}{153}$)