

Partial differential Equation (PD.E)

An Egn involving partial derivation

An egn involving partial derivatives of a function of two on more to independent variables is (alled pro-E

z = f(n, y)

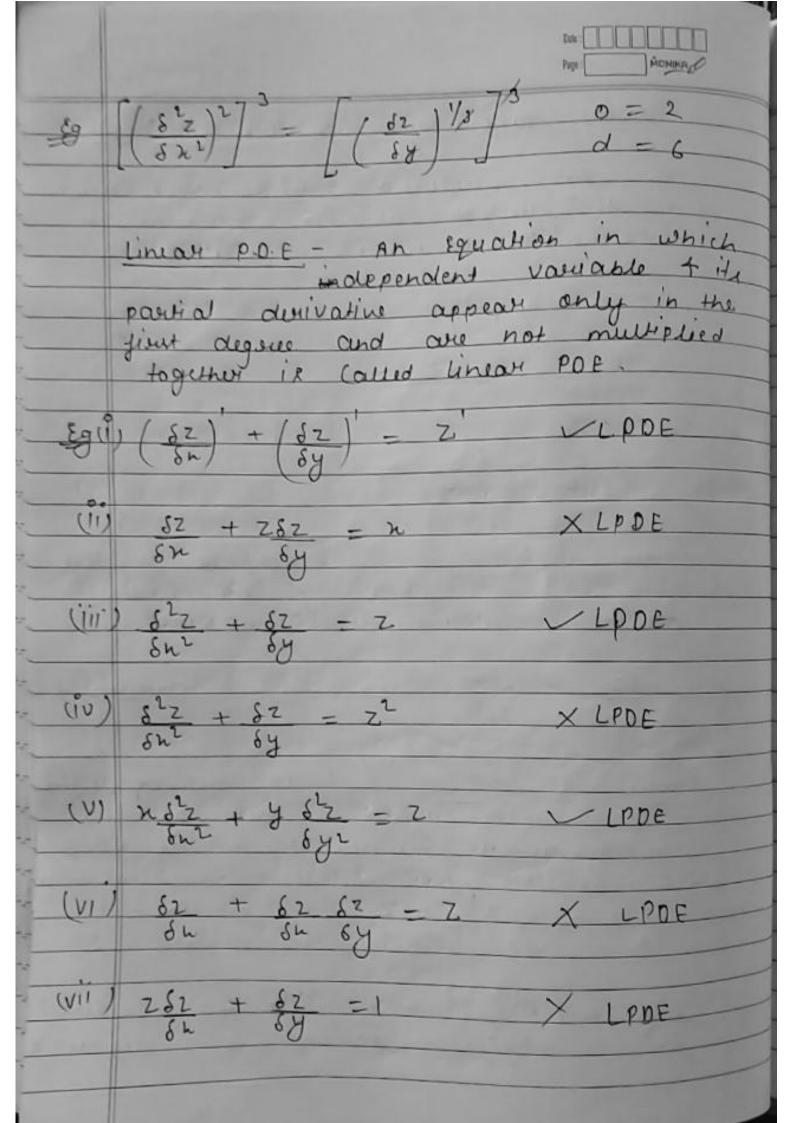
n 82 + y 82 = 1

8²z + 8²z - n

Order of P.D.E. - it is defined as the order of the highest order of the highest

 $\frac{\xi_{0}}{\delta n} = \frac{\delta^{2}z}{\delta n^{2}} + \frac{\delta^{2}z}{\delta y} - \frac{1}{\delta y} = \frac{1}{2}$

Degence of P.D.E - it is defined as the highest power of the highest power of the highest order derivative in the P.O.E when it is modified from radical 4 fractional power of the power of the following powe





Linear Homogeneous PDE -

+ an 8^hz - g(n,y)

where ao, a, a, -- an are control

* order of Each derivative term is some to if order of Each derivative term is not some then it is called linear. non-homogenous pot

898182 + 82 = 1 Homogeneous

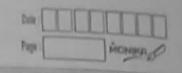
(11) 52 + 82 = 2 non-nonogeneous

Notation

$$0 = \frac{s}{\delta n} , \quad 0' = \frac{s}{\delta y} , \quad 0^{\frac{1}{2}} = \frac{s^{2}}{\delta n^{2}} .$$

 $y = \frac{5^2}{6n^2}, \quad s = \frac{5^{1/2}}{6n6y}, \quad J = \frac{5^{1/2}}{6y^2}$

P = 82 , 9 = 82 84



burneral Solution

if g(n,y) = 0 P.I = 0

Z= C.F

Rules of for find C.F. of Linear Homogene

ON PDE

I) if F(D, O') is homogeneous 4 power of D > power of D'

(9) Replace D by m + D' by I then Equate to zoro. This will give A.E.

(b) Solve A.E find m

(C) if values of m are distinct

m = m, m, m,

CF = f, (y+ m, n) + f2 (y+ m2n) +

foly + mon)

(ii) if values of m are repeated

m = m1, m1, m1

$$CF = f_1(y + m_1 n) + nf_2(y + m_1 n) + n^2 f_3(y + m_2)$$

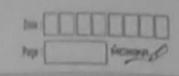
$$\frac{g^{2}}{s^{2}} \left(D^{2} + 00' \right) z = 0 \qquad \frac{5^{2}z}{s^{2}} + \frac{5^{2}z}{s^{2}} = 0$$

$$A \cdot E = m^2 + m = 0$$

$$(m+1)^2 = 0$$

$$A \cdot E = m^3 + 1 = 0$$
 $(m+1)(m^2 - m+1)$
 $m = -1$, $1 + \sqrt{3}i$

$$C \cdot F = f_1 \left(y + (-1)n \right) + f_2 \left(y + \left(\frac{1+\sqrt{3}i^2}{2} \right) n \right) + f_3 \left(y + \left(\frac{1-\sqrt{3}i^2}{2} \right) n \right)$$



Bu (0'+ 200' + 0'2) Z = 0

 $AP m^{2} + 2m + 1 = 0.$ $(m + 1)^{2} = 0$ m = -1, -1

Fi(y-n) + nfo(y-n)

D) when F(D,D') is homogeneous +
power of D' > power of D

- 9) Replace D' by m 4-0 by I in f (0.0')
 4 Equate to Zoro. This will give R.E
- b) solve A.E & find value of m
- Dis it values of m are distinct m=m1, m1, m3.

CF = fi (n+m,y) + fo (n+moy)+ fo (n+moy)

(ii) if value of m ove supeated m = m, , m,

CF = fi(n+miy) + yfo(n+miy)+

y2fo(n+miy)

 $(D^{L}D' + D^{13})z = 0$

 $A \cdot E \quad 0 \rightarrow m \quad , \quad 0 \rightarrow 2$

 $m + m^2 = 0$ $m(m+m^2) = 0$ m = 0, + i

 $CF = f_1(n) + f_2(n+iy) + f_3(n-iy)$

Que (020'3 + 0'203) z = 0

of Kimit sof = 3A

D312 (0'+0) z = 0

for 02 - 0 -m m2 = 0 = m = 0.0

 $CF = f_1(y) + nf_2(y)$

 60×10^{12} $0' \rightarrow m$ $m^2 = 0$ m = 0,0

C.F = f3(h) + y f4(h)

for (0+0') 0-m, 0'=1

(F = fs(y-h) (F = fily) + nfs(y) + fs(n) + yfq(n) + fs(y-h)

Date:	m
Figs:	MONIKA

> $m^3 - 2m^2 + m = 0$. m = 0, 1, 1

CF = fily) + fo (y+ n) + nfo (y+n)

if F(0,0') is non-homogeneous

o) when F (0,0') (an be factorized into linear factors of the form (0-mo'-a)

(i) F(0,0') => (0-m, 0'-a) (0-m,0'-a)

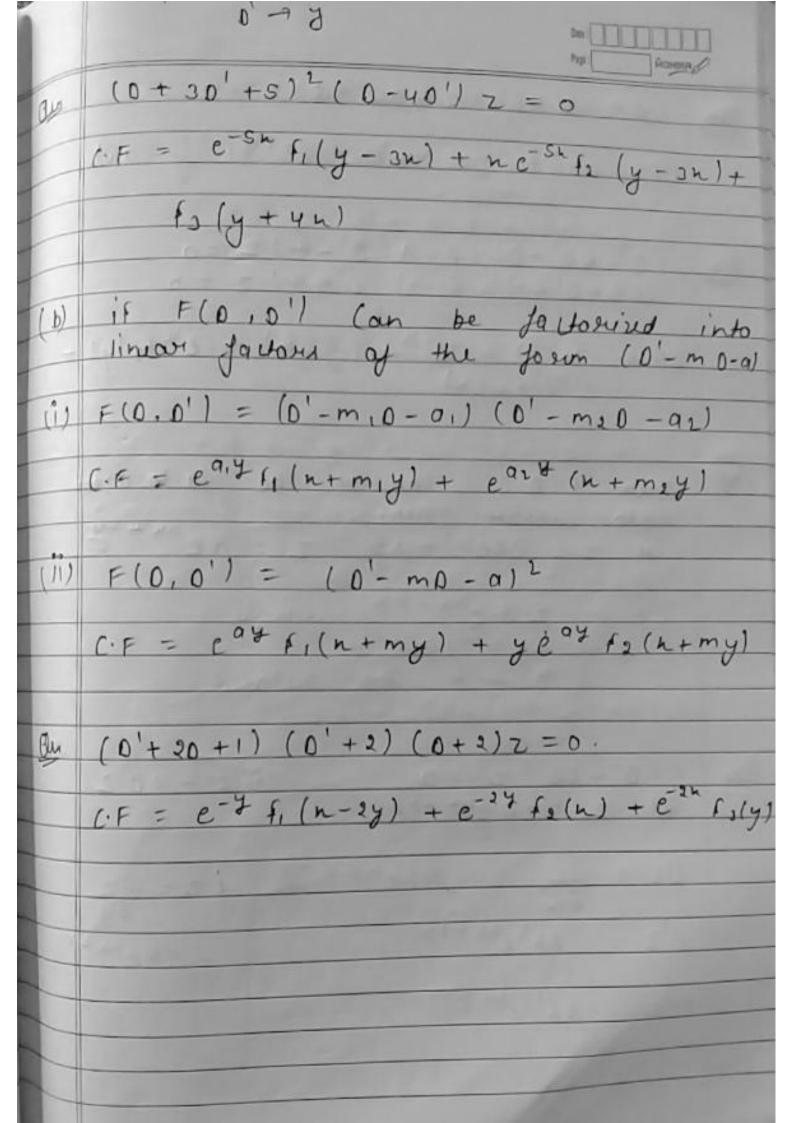
C.F = eqin fi (y+min) + eqinfo (y+mon)

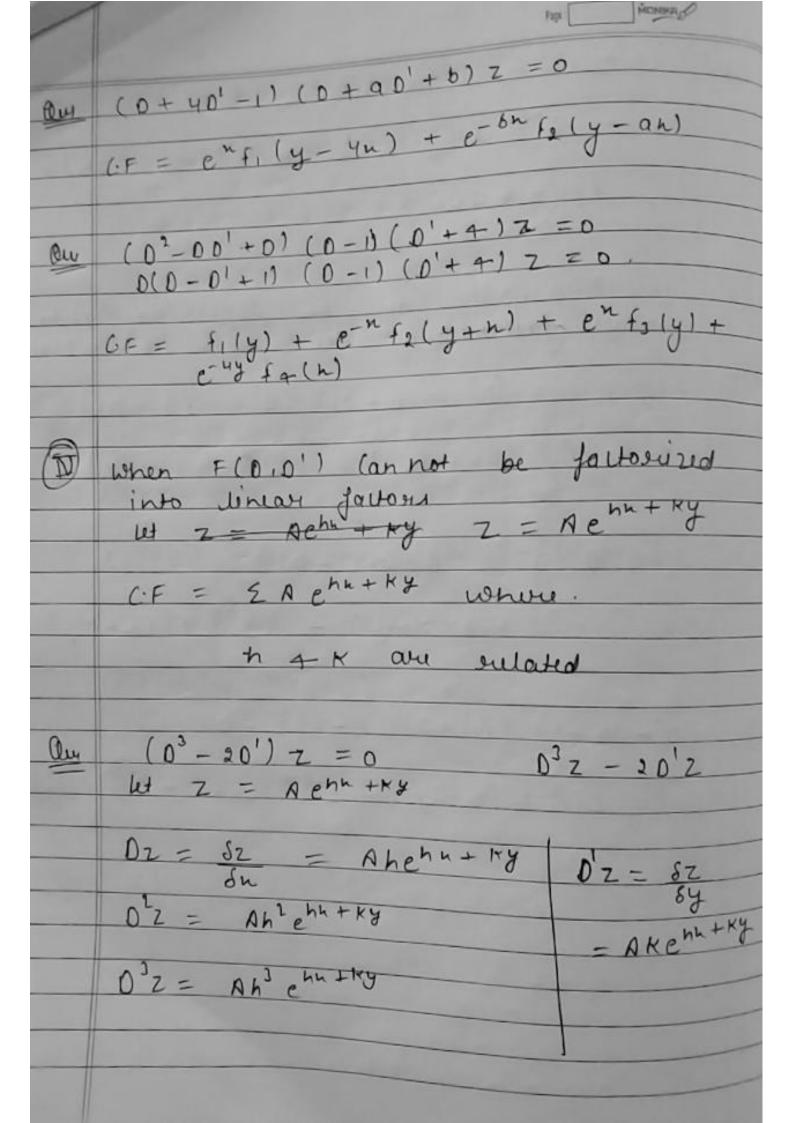
(11) F(0,0') = (0-m, 0'-a)2

CF = earf, (y+m, n) + nearfoly+ mon)

 $\frac{2g}{0-20'+4)(0+30'-1)z=0}$

(·F = e-+n fi(y+2n) + enf2(y-3n)





$$(0^4 - 50^{12}) (0 - 0^1)z = 0$$

$$\frac{\partial u}{\partial x} = \frac{(0^3 + 0^20' + 0^2)}{(0^2 - 2)} = 0$$

$$m^3 + m^2 = 0$$
 $m = 0 \cdot 0 \cdot -1$

$$CF = f_1(y-h) + f_2(y) + nf_3(y) +$$

Our s + ap + bg + abz = 0

(DD' + a D + b D' + a b) Z = 0

(D(p'+q) + b(0'+q))z = 0.

[(0+6) (0+0)]= 0

CF = e-ay film + e-bh foly)

au 31 + 21 + 1 + 20 + 29 + 7 = 0

[02 + 200' + 0'2 + 20 + 20' + 1/2=0

 $[(0+0')^{2}+2(0+0')+1]z=0$

 $[(0+0'+1)^{2}]z=0$

C.F = e-hfi(y-n) + ne-hfe(y-n)



Rules to find AI

F(0,0') Z = ean + by

 $P.J = \frac{1}{F(0.0!)} e^{qn + by}$

= 1 eqn + by F (9,6)

F(9,6) # 0

if F(a,b) = 0

8 F (0,0')

= n ean+by

(01+200'+0'2) z = e24+34

P-1 = 1 C24+34

22+2×2+32

35 62 + 37

to Company

Ou (02-20'+1) 2 = ex+4

 $PT = \frac{1}{(0^2 + 20^1 + 1)}e^{n+y}$

= n enty

= neh+7

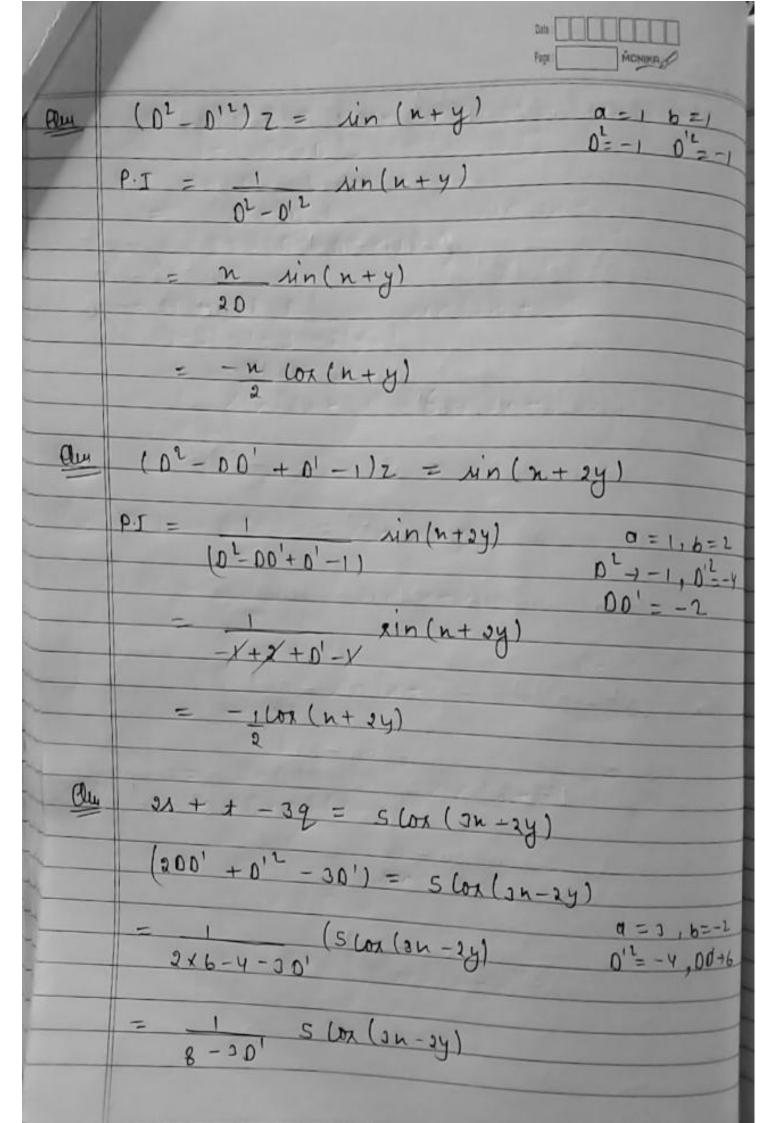
Quy (D3-3013+2012) Z = en+y

P.I = 1 enty

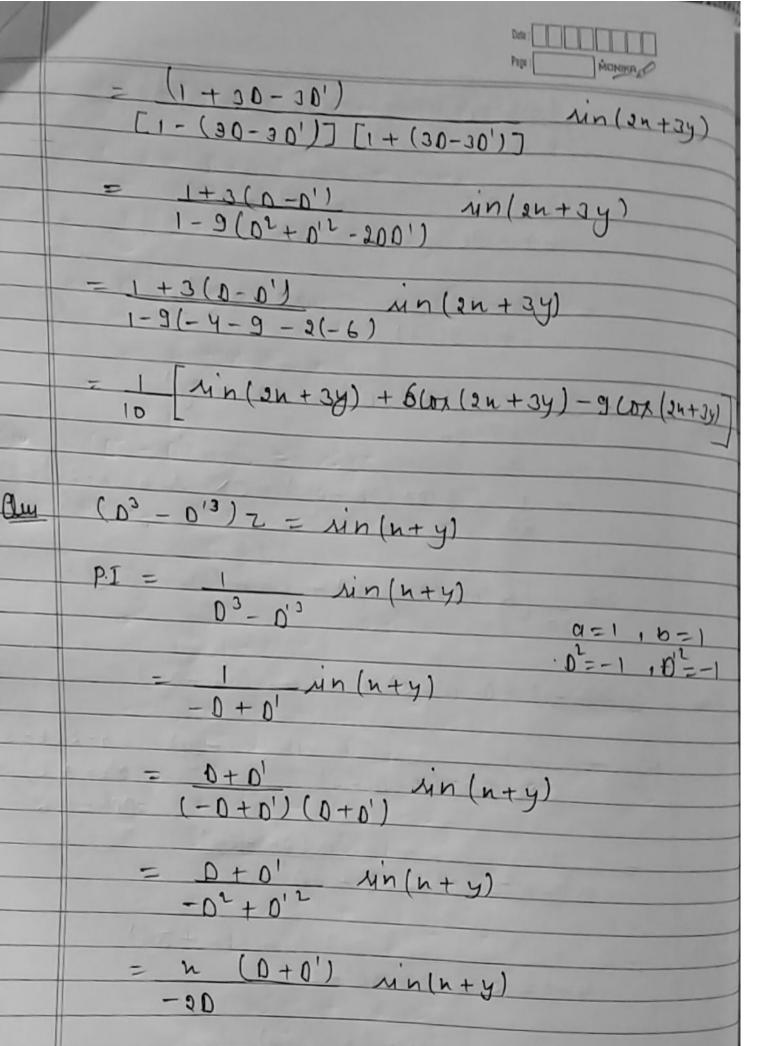
= n en+y

= n en+y

= F(0'.00'.0')z = nin(an+by) (0x (an + by) F(D¹, DD', D¹) Nin(an +by) $D^2 \rightarrow -a^2$ $O^2 \rightarrow -b^2$ DD' -> -ab Promided F(-a2, -ab, -b2) 70 if F(-a2, -ab, -b2) = 0 then proceed as in rule (1) Pu (02+012) 2 = in (2n+3y) P.J = 1 sin(2n+3y) a = 2, b = 3 $0^{2} = -4 \quad 0_{1}^{2} = -3$ -4-9 sin(2n+3y) = -1 in (2n + 2y)



```
= (8+30') S (ox (3n-2y)
  = (8+30') S (0x (3n-2y)
    = (8+30') S cox(3n-2y)
         64-9(-4)
     = 8 (8+30') (ox (3n-2y)
      - 1 [8 cox (3n-2y) + 3x-2in (3x-2y) (-2)]
    = 1 [8 (ox (3n-2y) + 6 sin (3n-2y)]
(0-0'-1)(0-0'-2)z = sin(2n+3y)
PI = \frac{1}{(0-0'-1)(0-0'-2)} \text{ and } (2n+3y) = \frac{1}{0^{\frac{1}{2}}-4,0^{\frac{1}{2}}-9}
    \frac{1}{p^2 + 0'^2 - 20p' - 30 + 30' + 2}
  -4-9+12-30+30'+2
     - 30+30' vin (2+3y)
```



$$= \frac{-n}{2} (0+0') (-\log(n+y))$$

$$= \frac{n}{3} (-\sin(n+y)) + (-\sin(n+y))$$

$$= \frac{1}{3} \sin(n+y)$$

$$=$$

$$=\frac{1}{3}\left[1-(0+0')\right]^{-1}\left[4+3n+6y+\frac{1}{3}(3+2\times6)\right]$$

$$= \frac{1}{3} \left[1 + (0 + 0') \right] \left[9 + 3n + 6y \right]$$

$$= 6 + n + 2y$$

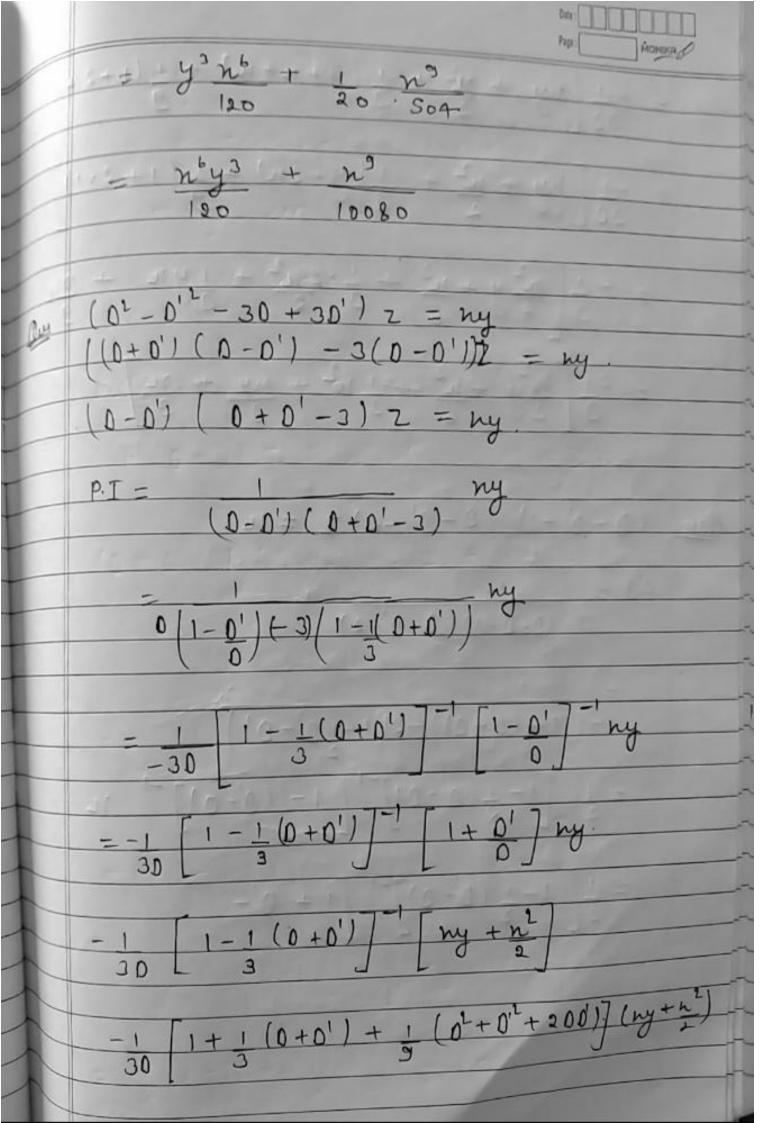
$$P.T = \frac{1}{0^3 - 0^{13}} n^3 y^3$$

$$= \frac{1}{D^3 \left[1 - \left(\frac{D'}{0}\right)^3\right]} n^3 y^3$$

$$= \frac{1}{0^3} \left[1 - \frac{0^{13}}{0^3} \right]^{-1} h^3 y^3$$

$$= \frac{1}{D^{3}} \left[1 + \frac{0^{13}}{D^{3}} \right] \lambda^{3} y^{3}$$

$$=\frac{1}{0^3}\left[n^3y^3+\frac{1}{0^3}\left[6n^3\right]\right]$$



$$= \frac{-1}{30} \frac{ny + y^{2} + \frac{1}{3} (y + n + n) + \frac{1}{3} (1 + 2 + 1)}{9}$$

$$= \frac{-1}{30} \frac{ny + n^{2} + \frac{1}{3} (y + 2n) + \frac{1}{3} (1 + 2 + 1)}{9}$$

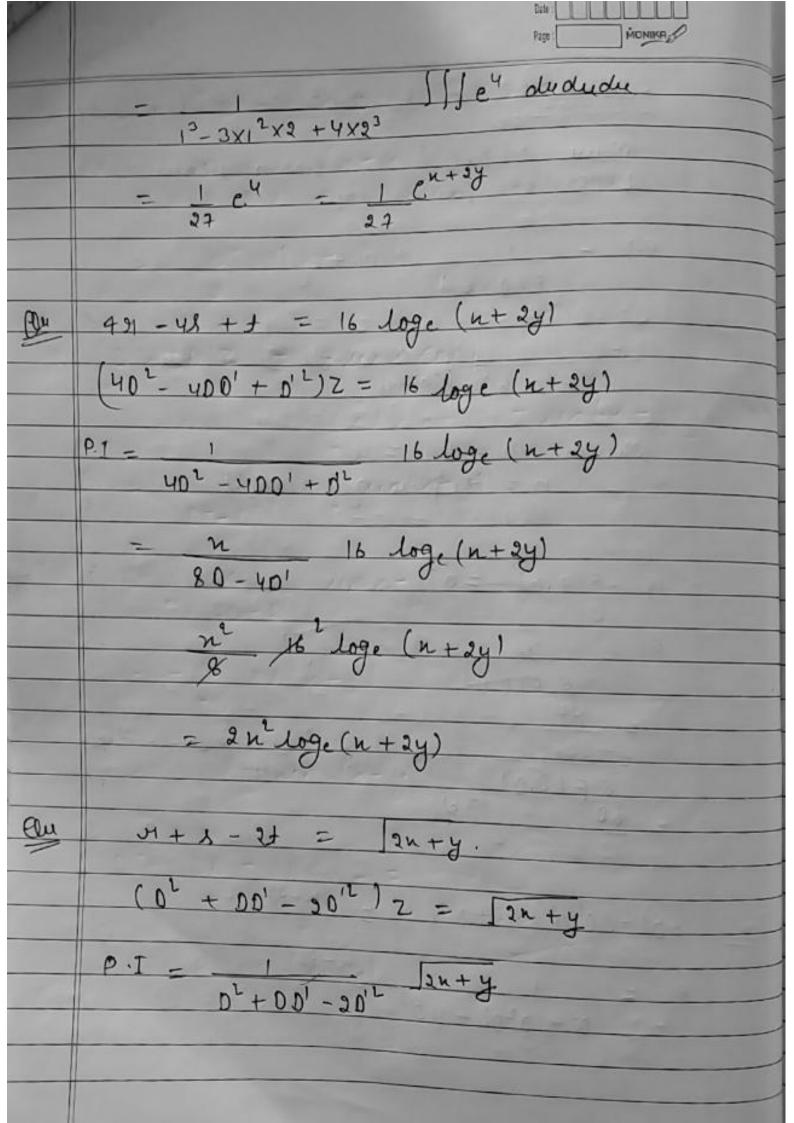
$$= \frac{-1}{30} \frac{n^{2} + n^{3} + \frac{1}{3} (y + 2n) + \frac{1}{3} (1 + 2 + 1)}{9}$$

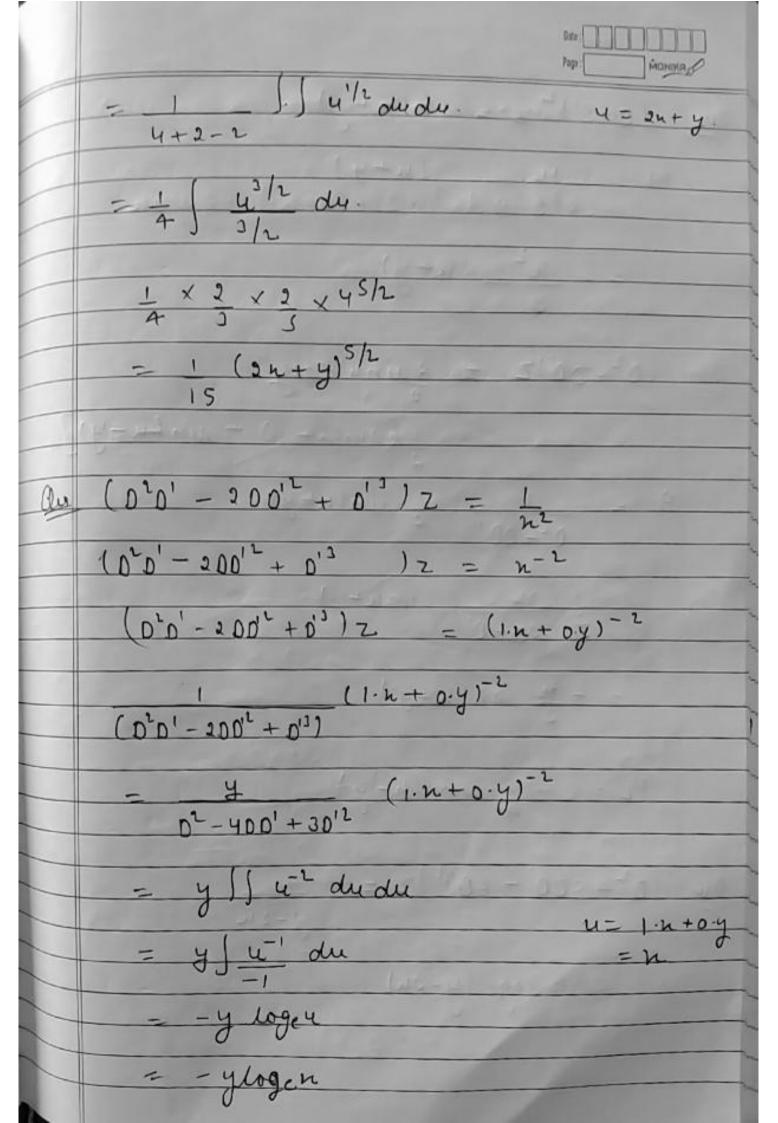
$$= \frac{-1}{3} \frac{n^{2} + n^{3} + \frac{1}{3} (y + 2n) + \frac{1}{3} (y + 2n)}{9}$$

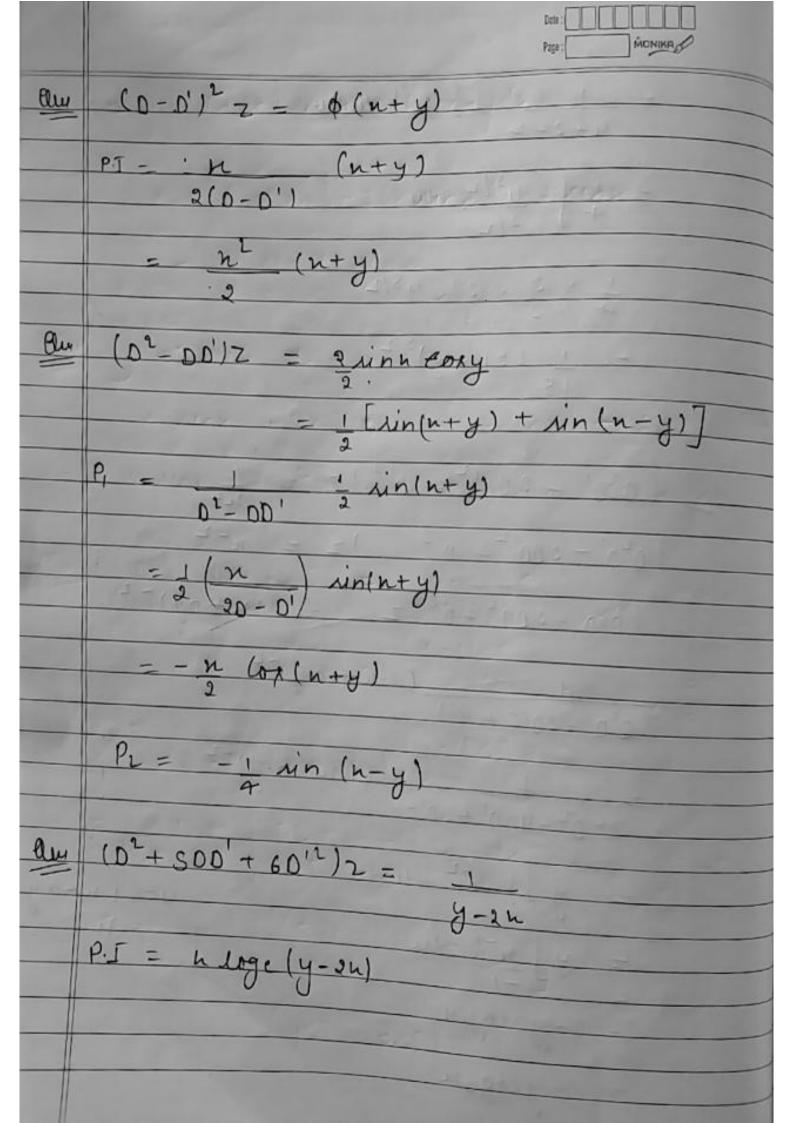
$$= \frac{-1}{3} \frac{n^{2} + n^{3} + \frac{1}{3} n^{2} + \frac{1}{3} n^$$

	Page: Moneya C
	Rule (5) F(D,D')z = P(an+by)
	LPDE is homogeneous or
	$P.J = \frac{1}{F(0,0')} \phi(an + by)$
	F(0,0')
	= 1)) \$\phi(u) du du du F(a,b) htimes
	Where an + by = 4
	where $an + by = u$ n = digene of F(D,D') $F(a_1b) \neq 0$
	F(91b) 7 0
	if F(q,b) = 0
Н	
	$\frac{\rho_{\text{I}}}{\delta \rho} = \frac{h}{\delta \rho} \phi (\alpha n + b y)$
	= n \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
	5 F (0,0') (n-1) times
	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
lu	$(0^3 - 30^20^1 + 40^{13}) = e^{n+24}$
	P.I = 1 en+y
	$\frac{P.I}{D^{3}-3D^{2}D'+4D'^{3}} = \frac{1}{e^{n+\frac{3}{4}}}$

Dete:







	Pape: MONIKA
	Rule 6 General method jos linear nomogeneous PDE
	F(0.0')z = p(n,y)
	$\frac{1}{D-mD'} \phi(n,y) = \int \phi(n,c-mn) dn$ where $y = c-mn$
	where y = C-mn
Щ	$(0^2 + 00' - 60'^2)z = y coxn$
	$P.T = 1 y (02h) m^{2} + m - 6 = 0$ $(D^{2} + DD' - 6D'^{2}) m = -3.2$ $(m + 3) (m - 2)$
	P.J = 1 y (oxh (0+30')(0-20')
	= 1 [1 y Loxh] (0+30') [(0-20') y Loxh] y=624
	- 1 ((-321) Cox n dn)
	= $\frac{1}{0+30!}$ [(-2h) sinh - (-2) (-(oxh)]
	$\frac{1}{0+30'}$ [(1-2h) sinh - 2(0xh)]
\	

