$$+a_1\left\{x-\frac{x^3}{(m+3)(m+4)}+\frac{x^5}{(m+3)(m+4)(m+5)(m+\ldots}\right.$$

Now, $$(y)_{m=-1}=x^{-1}\left[a_0\left(1-\frac{x^2}{1.2}+\frac{x^4}{1.2.3.4}-\ldots\right)+a_1\left(x-\frac{x^3}{2.3}+\frac{x^5}{2.3.4.5}-\ldots\right)\right]$$

$$=x^{-1}\,[a_0\cos x+a_1\sin x]$$

Hence complete solution is given by

$$y=(y)_{m=-1}$$

$$\Rightarrow \qquad y=\frac{1}{x}(a_0\cos x+a_1\sin x).$$

Note. All those problems, in which $x=0$, was an ordinary point of $y''+P(x)y'+Q\ldots$ can also be solved by Frobenius method as given in Art. 7.9 and explained in above illustrative...

EXERCISE 7.5

Solve the following differential equations:

1. $(1-x^2)\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}+y=0$ **Ans.** $y=a_0\left(1-\dfrac{1}{2}x^2+\dfrac{1}{8}x^4+\ldots\right)+b\left(x-\dfrac{1}{2}x^3+\dfrac{1}{4}\ldots\right.$

2. $(2+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+(1+x)\,y=0$

Ans. $y=a_0\left(1-\dfrac{1}{4}x^2-\dfrac{1}{12}x^3+\dfrac{5}{56}x^4+\ldots\right)+b\left(x-\dfrac{1}{6}x^3-\dfrac{1}{24}\ldots\right.$

3. $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+4y=0$ **Ans.** $y=a_0(1-2x^2)+a_1\cdot\left(x-\dfrac{x^3}{2}-\dfrac{x^5}{8}\ldots\right.$

OBJECTIVE TYPE QUESTIONS

Choose the correct alternative:

1. The singular point of $x(x-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+x^2y=0$ is

(*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) 3

2. The singular point of $x^2(x-4)\dfrac{d^2y}{dx^2}+5(x-4)\dfrac{dy}{dx}+6y=0$ is

(*i*) 2 (*ii*) 3 (*iii*) 4 (*iv*) 0

3. The singular point of $x(x-1)(x-2)\dfrac{d^2y}{dx^2}+x(x-1)\dfrac{dy}{dx}+2x(x-1)y=0$ is

(*i*) 1 (*ii*) 2 (*iii*) 3 (*iv*) 4 **Ans.** (*ii*)

4. The irregular singular point of $(x-1)(x-2)^3\dfrac{d^2y}{dx^2}+(x-1)^2\dfrac{dy}{dx}+3(x-1)y=0$ is

(*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) 3 **Ans.** (*iii*)

5. The regular singular point of $\dfrac{d^2y}{dx^2}+\dfrac{1}{x}\dfrac{dy}{dx}+\dfrac{1}{x^2}y=0$ is

(*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) 3 **Ans.** (*i*)

6. At $x=0$ the differential equation $\dfrac{d^2y}{dx^2}+\dfrac{1}{x^2}\dfrac{dy}{dx}+\dfrac{1}{x^3}y=0$ has

(*i*) Ordinary point (*ii*) Regular singular point

(*iii*) irregular singular point (*iv*) None of these point **Ans.** (*iii*)

7. If $x=0$ is an ordinary point of a differential equation, then for its solution we take $y=$

(*i*) $\sum a^{m+k}x^k$ (*ii*) $\sum a_2 x^1$

(*iii*) $\sum a_k x^{m+k}$ (*iv*) $\sum_{k=0}^{\infty} a_k x^k$ **Ans.** (*iv*)

8. If $x=0$ is a regular singular point of a differential equation, then for its solution we take $y=$

(*i*) $\sum a_k x^{m+k}$ (*ii*) $\sum a_k x^m$

(*iii*) $\sum a_k x^k$ (*iv*) None of these **Ans.** (*i*)

9. If $x=0$ is an irregular singular point of a differential equation, then for its solution we take $y=$

(*i*) $\sum a_k x^{m+k}$ (*ii*) $\sum a_k x^k$

(*iii*) $\sum a^k x^m$ (*iv*) None of these **Ans.** (*iv*)

Indicate True or False for the following :

10. $x=0$ is an ordinary point of $(x-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+5=0.$ **Ans.** True

11. $x=0$ is a singular point of $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+4y=0.$ **Ans.** True.

12. $x=1$ is a singular point of $(x-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+2y=0.$ **Ans.** True.

13. $x=1$ is a not an ordinary point of $(x-1)^2\dfrac{d^2y}{dx^2}+4(x-1)\dfrac{dy}{dx}+4y=0.$ **Ans.** True.

14. $x=0$ and $x=3$ are regular singular points of $x(x-3)^2\dfrac{d^2y}{dx^2}+2(x-3)\dfrac{dy}{dx}+(x+3)y=0.$ **Ans.** True.

Fill up the blanks :

15. For the solution of differential equation $x\dfrac{d^2y}{dx^2}+3x^2\dfrac{dy}{dx}+4x^3y=0$ at $x=0$.

We take $y=$ .... **Ans.** $y=\sum a_k x^k$

16. For the solution of the differential equation $x^2\dfrac{d^2y}{dx^2}+4x\dfrac{dy}{dx}+6y=0$ at $x=0$.

We take $y=$ .... **Ans.** $y=\sum a_k x^{m+k}$

17. For the solution of the differential equation $x^2(x-4)\dfrac{d^2y}{dx^2}+(x-4)\dfrac{dy}{dx}+y=0$ at $x=0$.

We take $y=$ .... **Ans.** $y=\sum a_k x^{m+k}$

18. For the solution of the differential equation $\dfrac{d^2y}{dx^2}+x^3\dfrac{dy}{dx}+4y=0$ at $x=0$.

We take $y=$ .... **Ans.** $y=\sum a_k x^k$

19. The name of the method by which we solve a differential equation

$x^2(x-1)^2\dfrac{d^2y}{dx^2}+2x(x-5)\dfrac{dy}{dx}+6y=0$ at $x=0$ is .... **Ans.** Frobenius