GLA UNIVERSITY



DIGITAL IMAGE PROCESSING

By:

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Outline



- Morphological Image Processing
 - Introduction, Logical Operations involving Binary Images,
 - Dilation and Erosion, Opening and Closing, The Hit-or-Miss Transformation,
 - Morphological Algorithms Boundary Extraction, Region Filling, Extraction of Connected Components, Convex Hull, Thinning, Thickening
- Image Segmentation
 - Point, Line & Edge detection, Thresholding, Region-based Segmentation,
 - Region Extraction Pixel Based Approach & Region Based Approach,
 - Edge and Line Detection Basic Edge Detection, Canny Edge Detection,
 - Edge Linking Hough Transform.
- Representation & Description
 - Representation Boundary Following, Chain Codes,
 - Boundary Descriptors Shape Numbers





Morphology



- Mathematical tool for processing shapes in image, including boundaries, skeletons, convex hulls, etc
- Morphological operations are typically applied to remove imperfections introduced during segmentation, and so typically operate on binary images



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Set Theory



- Set (Ω) : A collection of objects (elements)
- Membership (\in) : If ω is an element of a set Ω , we can write $\omega \in \Omega$
- Subset (\subset): Let A, and B are two sets., If for every $a \in A$, we also have $a \in B$, then the set A is a subset of B, that is, $A \subset B$
 - If $A \subset B$ and $B \subset A$, then A = B
- Empty set (\emptyset)
- Complement: If $A \subset \Omega$, then its complement set $A^c = \{\omega | \omega \in \Omega$, and $\omega \notin A\}$

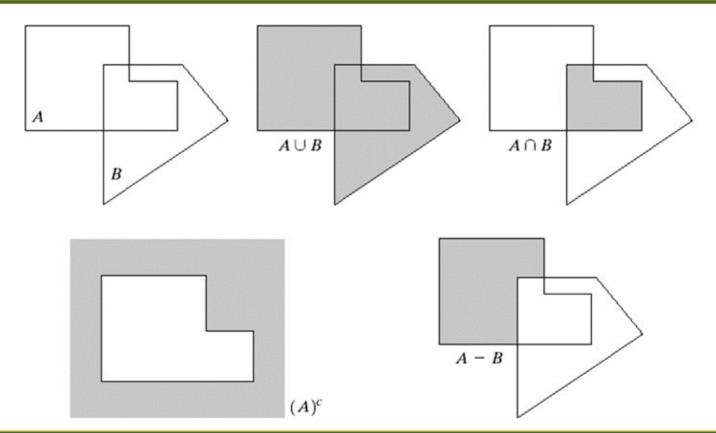
Set Theory



- Union (\cup): $A \cup B = \{\omega | \omega \in A \text{ or } B\}$
- Intersection (\cap): $A \cap B = \{\omega | \omega \in A \text{ and } B\}$
- Set difference (-): $B \setminus A = B \cap A^c$
 - Note that $B-A \neq A-B$
- Disjoint sets: A and B are disjoint (mutually exclusive) if $A \cap B = \emptyset$

Example sets operations





Reflection and Translation



Reflection

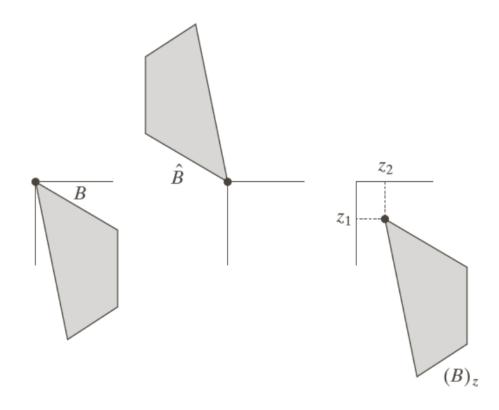
- The reflection of a set B, denoted by \hat{B} , is defined as

$$\hat{B} = \{w | w = -b, for b \in B\}$$

- Translation
 - The translation of a set B by point $z = (z_1, z_2)$, denoted by $(B)_z$ is defined as $(B)_z = \{c | c = b + z, for b \in B\}$

Reflection and Translation

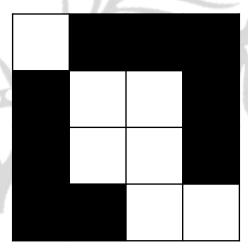




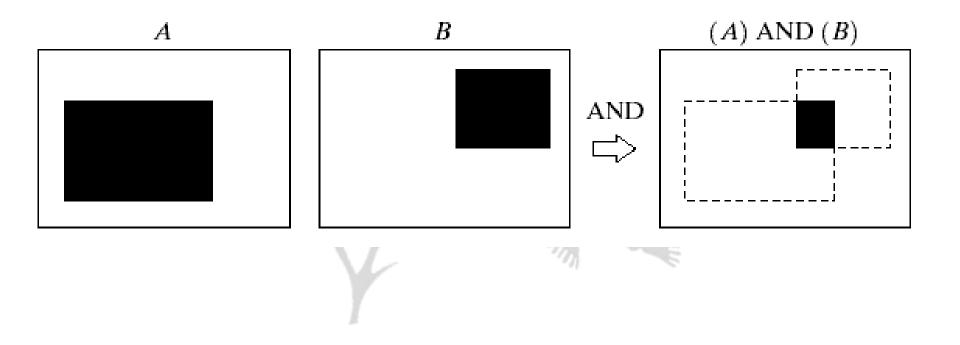
Binary Image



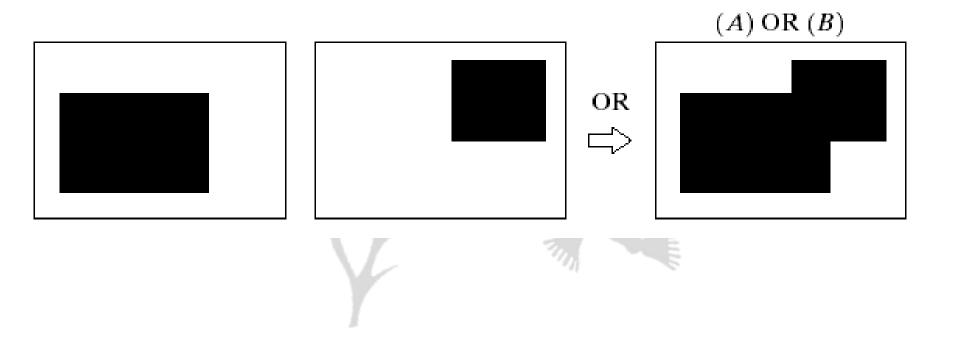
- Binary image
 - bi-valued function of x and y
- Morphological theory views
 - binary image as a set of its foreground (1-valued) pixels



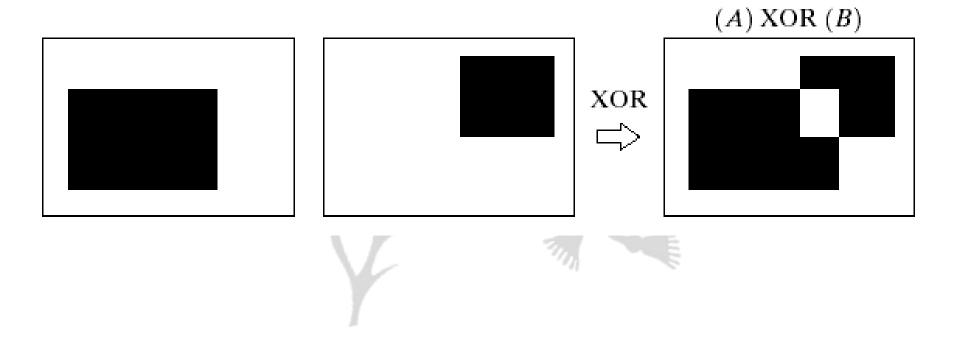




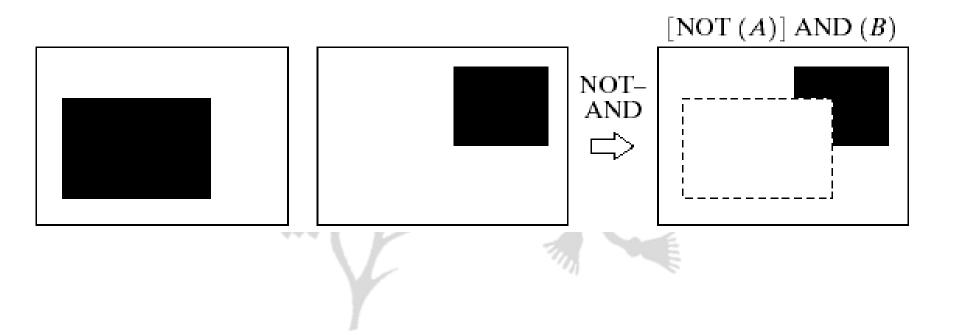








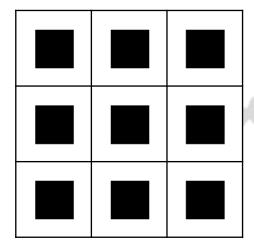


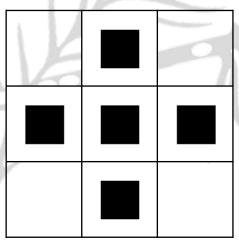


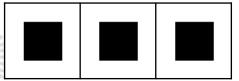
Basic components in Morphology



- Every operation has two elements
 - Input Image
 - Structuring element
- The results of the operation mainly depends upon the structuring element chosen



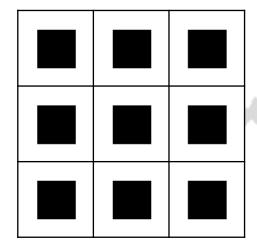


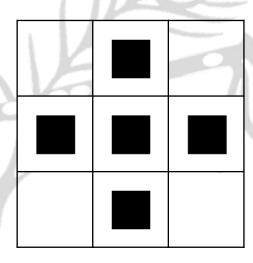


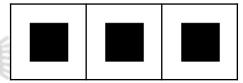
Structuring Elements



- Small sets or sub-images used to analyze an image for properties of interest
- Structuring elements can be any size and any shape

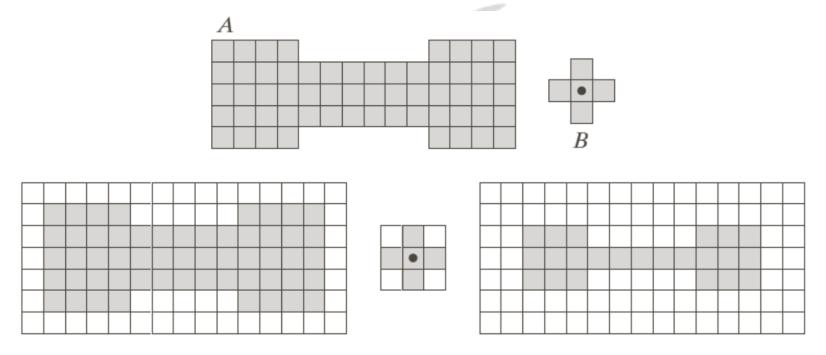






Structuring Elements





Fundamental Operations



- Fundamentally, morphological image processing is like spatial filtering
- The structuring element is moved across every pixel in the original image to give a new value of a pixel in processed image
- The value of this pixel depends on the operation performed
- There are two basic morphological operations
 - Dilation
 - Erosion



DILATION AND EROSION



- Dilation is an operation that grows or thickens objects in a binary image
- The specific manner of this thickening is controlled by a shape referred to as a structuring element
- The structuring element is translated throughout the domain of the image to see where it overlaps with 1-valued pixels
- The output image is 1 at each location of the origin such that the structuring element overlaps at least one 1-valued pixel in the input image

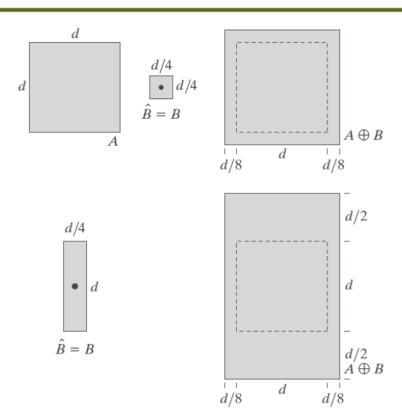


• The dilation of I and S is denoted by I⊕S

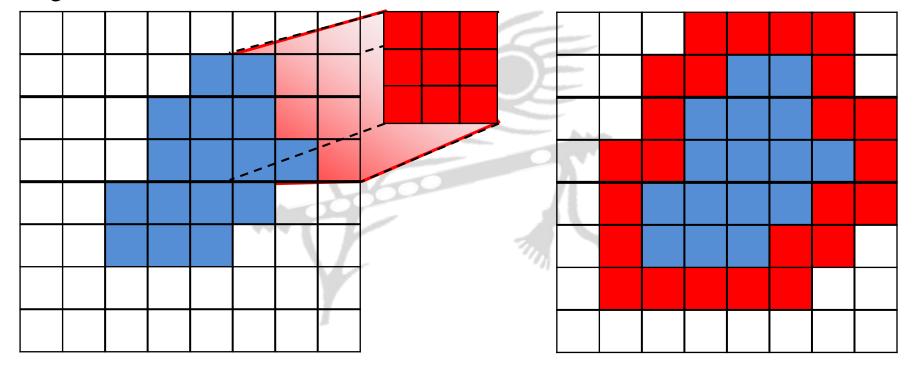
$$I \oplus S = \{ z \, | \, (\hat{S})_z \cap I \neq \emptyset \}$$

- Theoretical way of generation:
 - Obtain the reflection of S about its origin
 - Shift this reflection by z
 - Dilation of I by S is the set of all structuring element origin locations where the reflected and translated S overlaps at least some portion of I

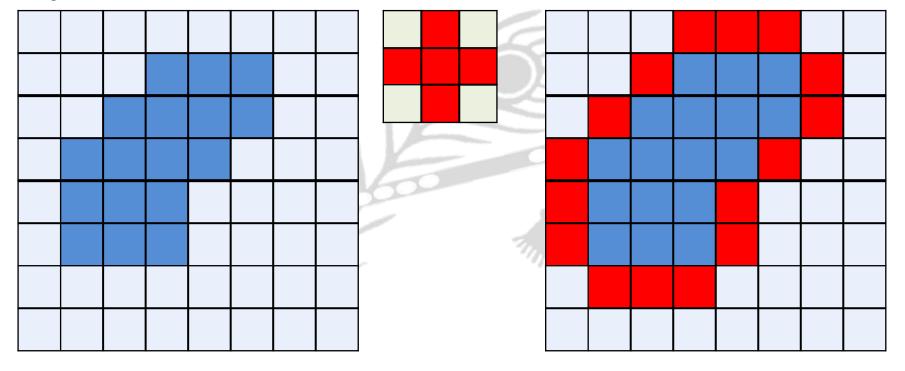








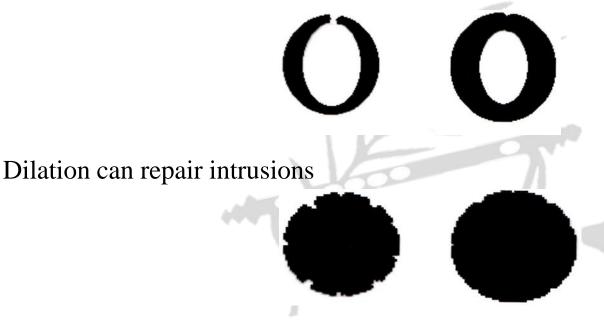




What is Dilation for...?



• Dilation can repair breaks



Properties of Dilation



• Dilation is commutative

$$A \oplus B = B \oplus A$$

Dilation is associative

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

Dilation is invariant to translation

$$A_h \oplus B = (A \oplus B)_h$$

Erosion



• The erosion of I by S, denoted $I \ominus S$

$$I \quad \Theta \quad S = \{z \mid (S)_z \subseteq I\}$$

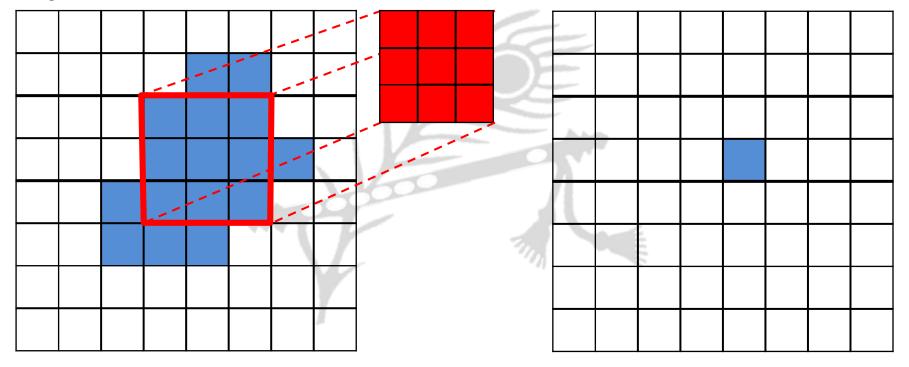
- The set of all points z such that, S translated by z, is contained by I

$$I \ \ominus S = \{z | (S)_Z \cap I^c = \emptyset\}$$

- In other words, erosion of I by S is the set of all structuring element origin locations where the translated S has no overlap with the background of I
- Erosion "shrinks" or "thins" objects in a binary image

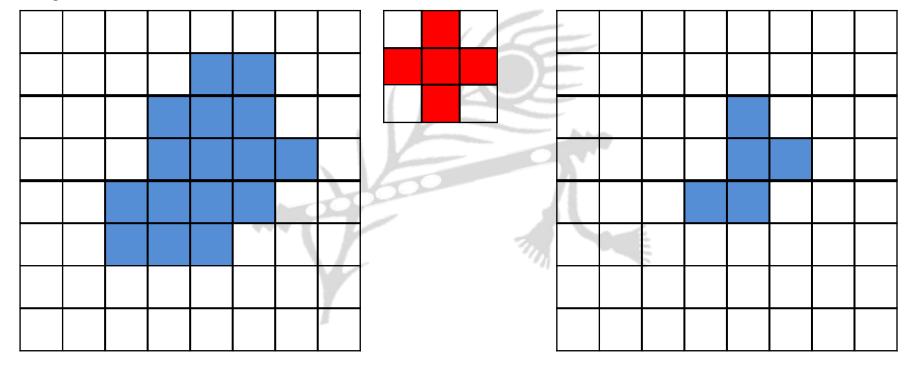
Erosion





Erosion

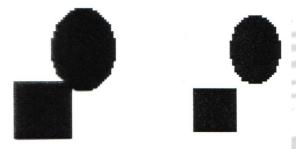




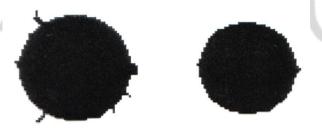
What is Erosion for...?



Erosion can split apart joined objects



• Erosion shrinks objects and removes random outer edges





• Binary image

	0	1	2	3	4	5	6	7
0	X							
1								
2								
3							1	
4						3		
2 3 4 5 6							1	1
6								
7							-	in .

 \leftarrow Image (I) Structure Element (S) →

	-1	0	1
-1			
0		X	
1			

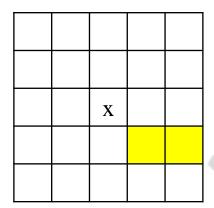
$$I = \{(2,2), (3,2), (3,3), (4,3), (4,4), (5,4)\}$$

$$U = \{(0,0), ..., (7,7)\}$$

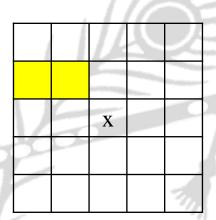
$$S = \{(-1,-1), (0,-1)\}$$



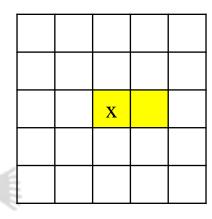
• Reflection and Translation operations



$$I = \{(1,1), (1,2)\}$$



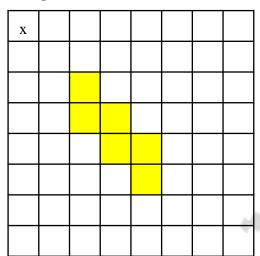
$$\hat{I} = \{(-1,-1), (-1,-2)\}$$

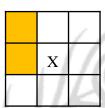


$$I_{(-1,-1)} = \{(0,0),(0,1)\}$$

Dilation (by coordinate system)

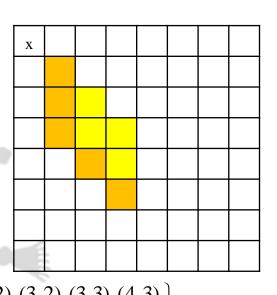






$$S = {\overline{(-1,-1),(0,-1)}}$$

$$I \oplus S = \{p \mid p = i + s, i \in I, s \in S\}$$



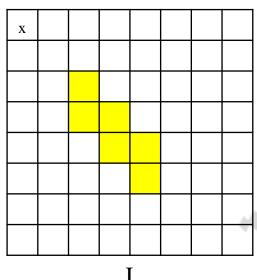
$$I = \{(2,2), (3,2), (3,3), (4,3), (4,4), (5,4)\}$$

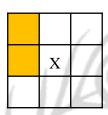
$$I \oplus S = \begin{cases} (1,1), (2,1), (2,2), (3,2), (3,3), (4,3) \\ (2,1), (3,1), (3,2), (4,2), (4,3), (5,3) \end{cases}$$
$$= \{ (1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,2), (4,3), (5,3) \}$$

Dilation (another definition)

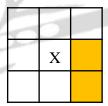


• Eg

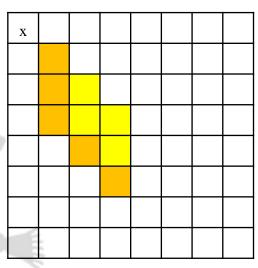




$$S = \{(-1,-1), (0,-1)\}$$



$$S = \{(1,1), (0,1)\}$$

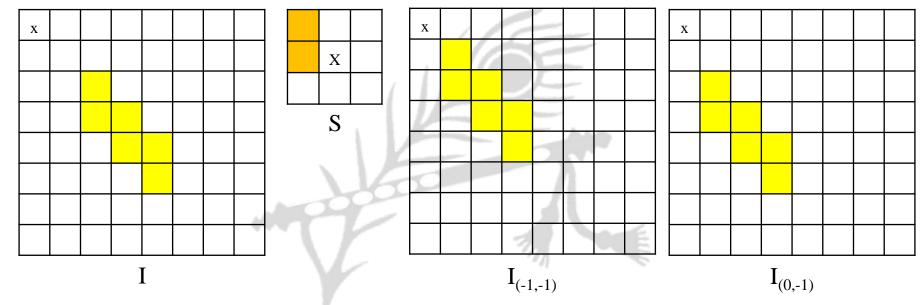


$$I \oplus S = \{ p \mid [(\hat{S})_p \cap I] \neq \emptyset \}$$
$$= \{ p \mid [(\hat{S})_p \cap I] \subseteq I \}$$

Dilation (as Union of object translation)

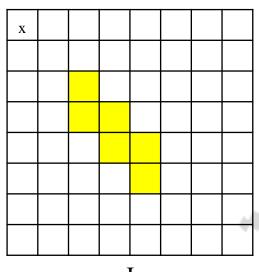


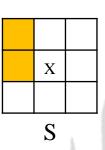
• Eg

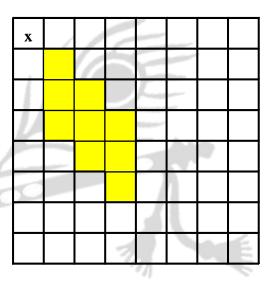


Dilation (as Union of object translation)







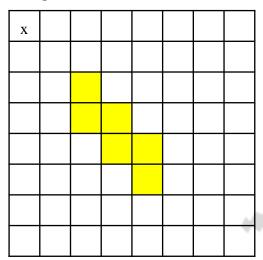


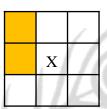
$$I \oplus S = \bigcup_{s \in S} I_s$$

Erosion (by coordinate system)



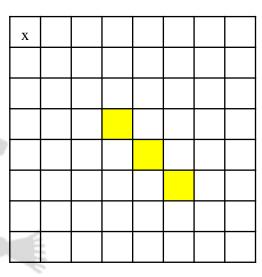
• Eg:





$$S = {\overline{(-1,-1),(0,-1)}}$$

$$I\Theta S = \{ p \mid p+s \in I, s \in S \}$$



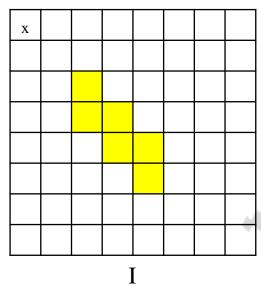
$$I = \{(2,2), (3,2), (3,3), (4,3), (4,4), (5,4)\}$$

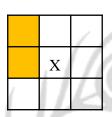
$$(3,3) + \{(-1,-1),(0,-1)\} = \{(2,2),(3,2)\} \in I$$

 $(4,4) + \{(-1,-1),(0,-1)\} = \{(3,3),(4,3)\} \in I$
 $(5,5) + \{(-1,-1),(0,-1)\} = \{(4,4),(5,4)\} \in I$

Erosion (another definition)

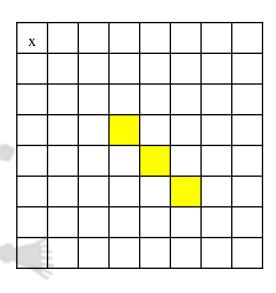






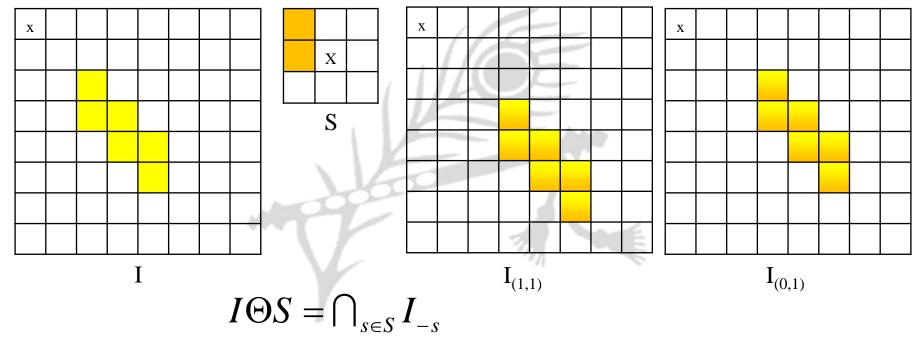
$$S = \{(-1,-1), (0,-1)\}$$

$$I\Theta S = \{ p \mid (S)_p \cap I^c = \phi \}$$
$$= \{ p \mid [(S)_p] \subseteq I \}$$



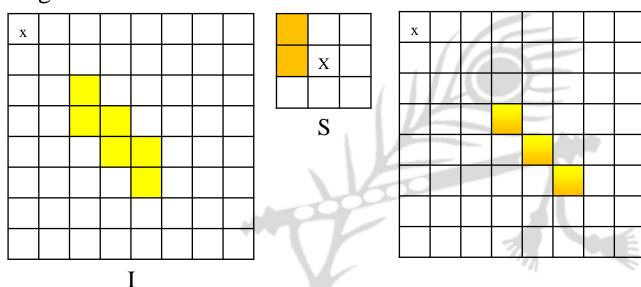
Erosion (as Intersection of object translation)





Erosion (as Intersection of object translation)

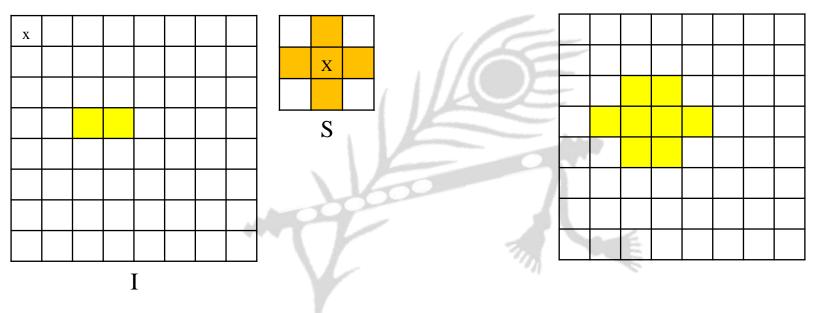




$$I\Theta S = \bigcap_{s \in S} I_{-s}$$

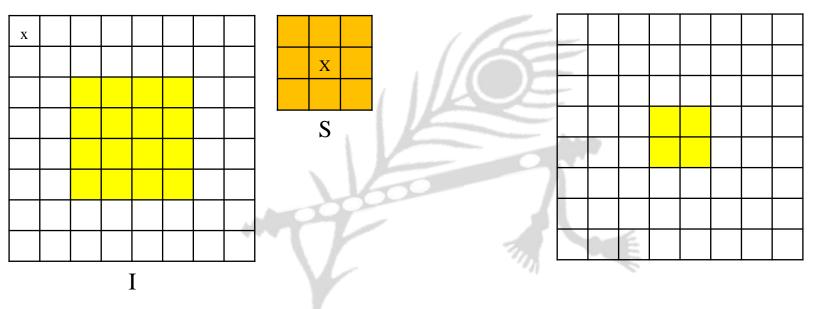


• Find $I \oplus S$



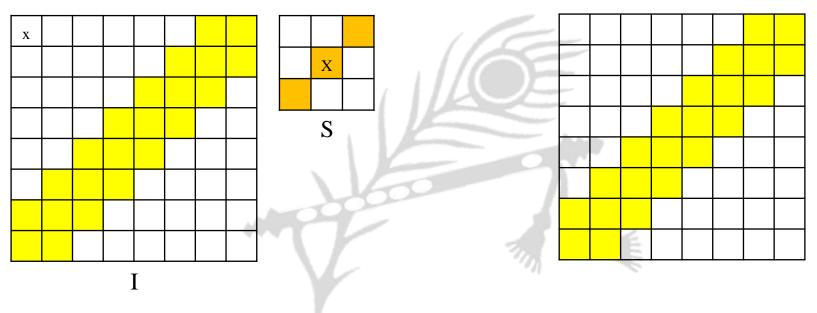


• Find $I\Theta S$



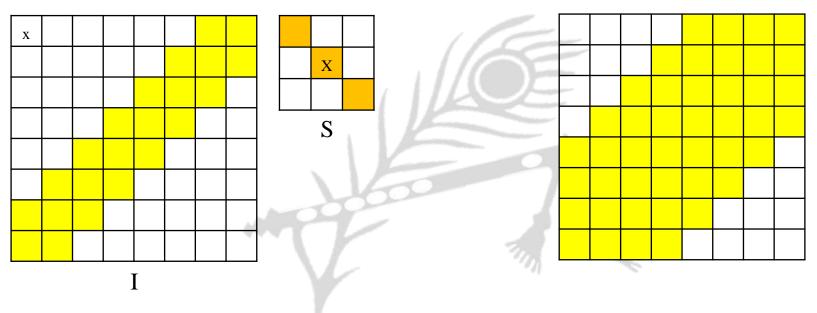


• Find $I \oplus S$



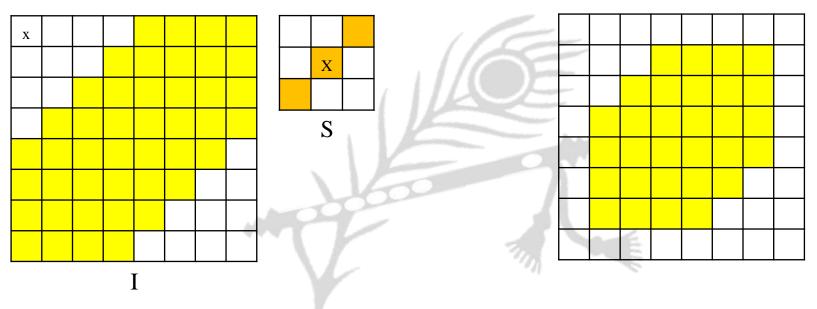


• Find $I \oplus S$



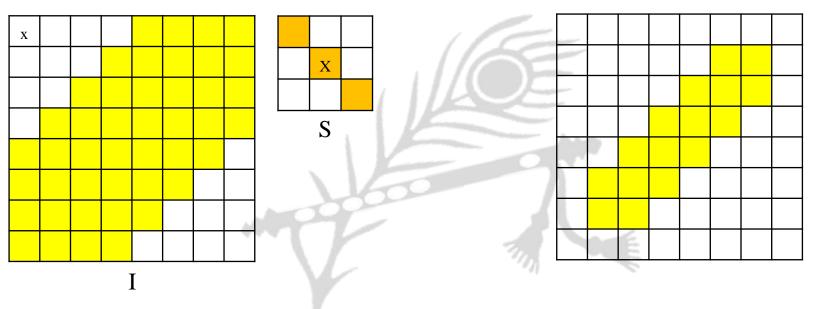


• Find $I\Theta S$





• Find $I\Theta S$



Duality



• Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \ominus B)^{c} = \{z \mid (B)_{z} \subseteq A\}^{c}$$

$$= \{z \mid (B)_{z} \cap A^{c} = \emptyset\}^{c}$$

$$= \{z \mid (B)_{z} \cap A^{c} \neq \emptyset\}$$

$$= A^{c} \oplus B$$

Combining Dilation and Erosion



- Dilation and Erosion are not inverse transformations
- If an image is eroded & then dilated (or vice-versa), the original image can not be obtained
- In practical applications, dilation and erosion are used most often in various combinations
- Three of the most common combinations of dilation and erosion are
 - Opening
 - Closing
 - Hit or miss transformation



OPENING AND CLOSING

Opening and Closing



- Opening is erosion followed by dilation
- The opening is given as

$$A \circ B = (A \Theta B) \oplus B$$

- Closing is dilation followed by erosion
- The closing is given as

$$A \cdot B = (A \oplus B)\Theta B$$

Opening and Closing



Opening

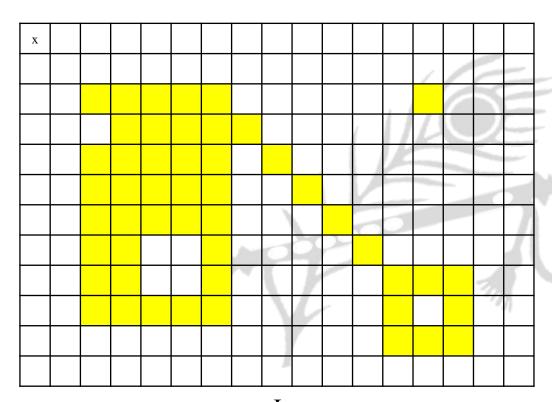
- smooth the contours of an object
- breaks narrow strips
- eliminates thin edges
- it is less destructive than the Erosion

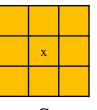
Closing

- smooth sections of the contours
- fuses narrow breaks & long thin gulfs
- eliminates small holes & fills gaps in the contour

Closing

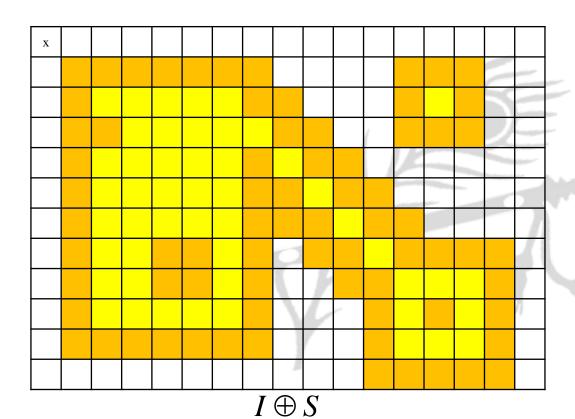


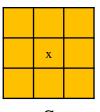




Closing

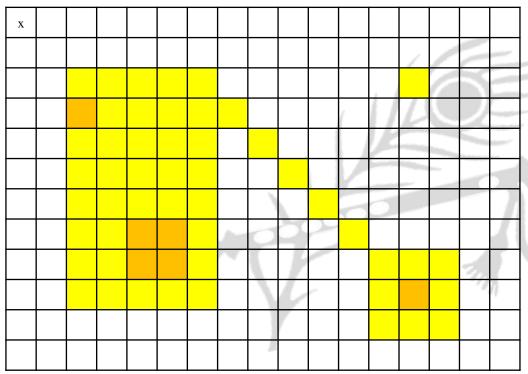


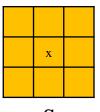




Closing



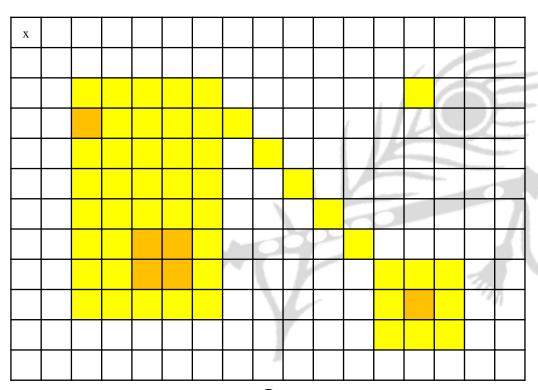


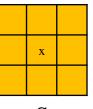


$$I \bullet S = (I \oplus S) \Theta S$$

Opening

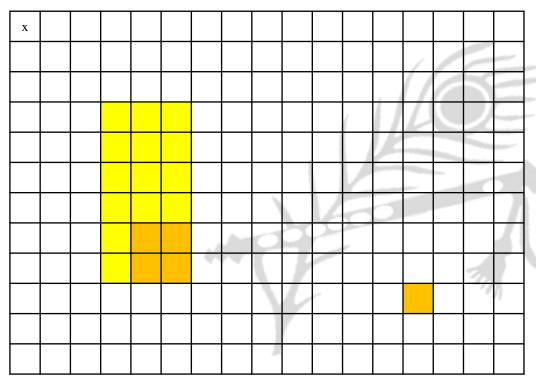


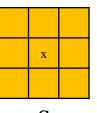




Opening



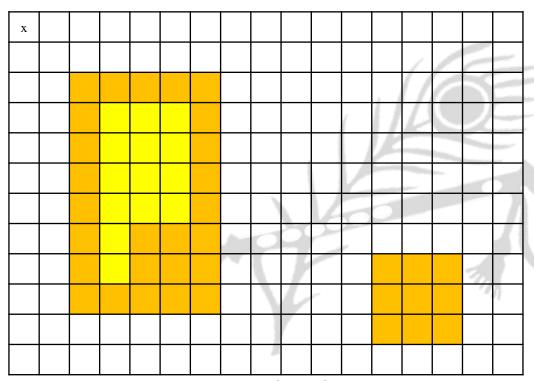


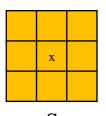


S

Opening







5

$$I \circ S = (I \Theta S) \oplus S$$



HIT OR MISS TRANSFORM



- A basic tool for shape detection
- It is a morphological operator for finding local patterns of pixels
- The hit-or-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image
- Concept:
 - Hit object
 - Miss background



• It is given as

$$I \otimes S = (I \Theta S) \cap (I^c \Theta (W - S))$$

• It can be written as

$$I \circledast S = (I \Theta S_1) \cap (I^c \Theta S_2)$$

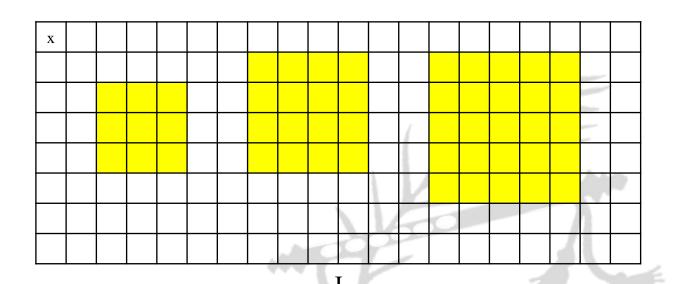
- where,
 - S_1 is the set formed from elements of S associated with an object (S in this case)
 - S_2 is the set of elements of S associated with the corresponding background (W S)
- The set contains all the points at which, S_1 found a match (hit) in I and S_2 found a match in I^c

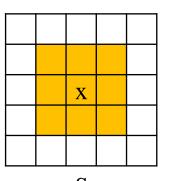


• Using the definition of set difference & the dual relationship between erosion & dilation, the equation can be rewritten as

$$I \circledast S = (I \Theta S_1) - (I \oplus \hat{S}_2)$$

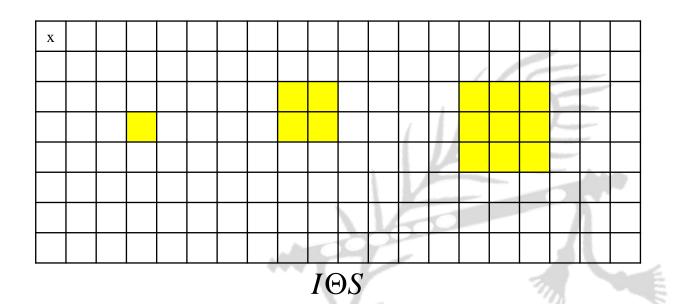


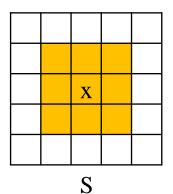




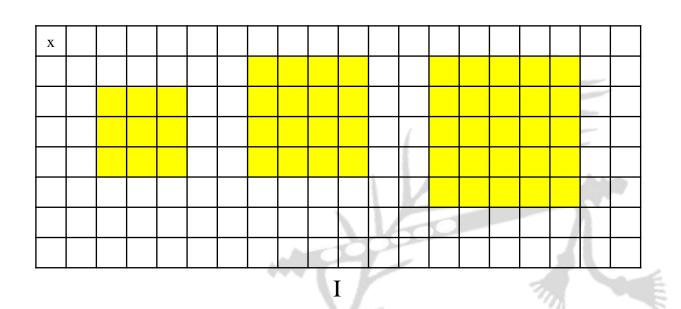
N

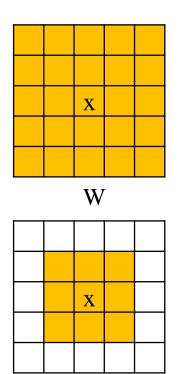




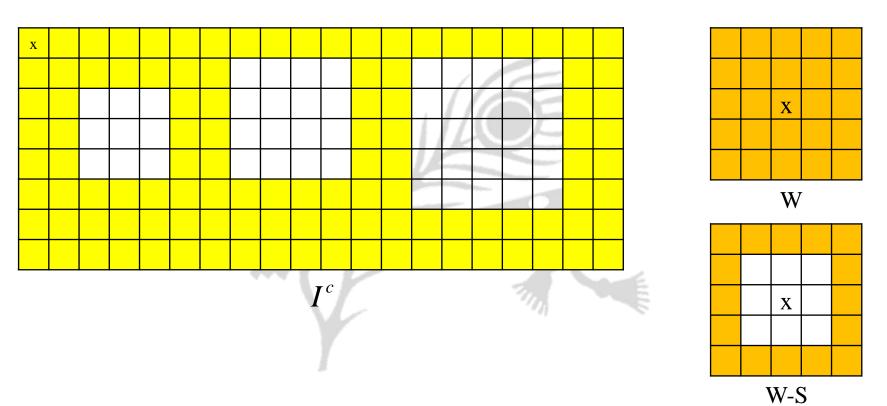




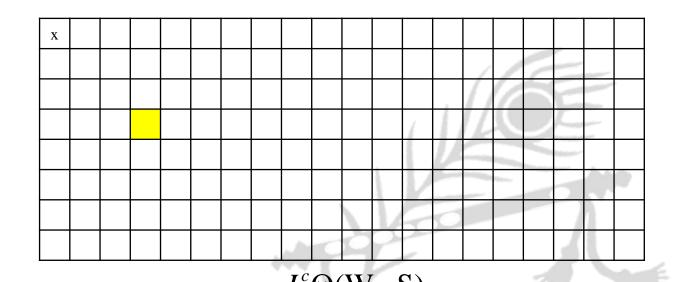


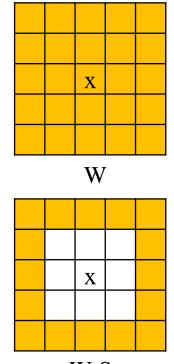




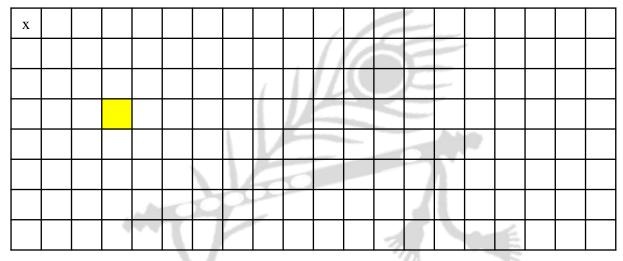










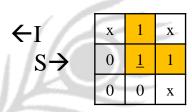


$$I \circledast S = (I \Theta S) \cap (I^c \Theta (W - S))$$



• Find I *S

0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	0	0	0
0	1	1	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0





0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0
0	1	1	1	1	1	1	0	0	0
0	1	1	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

2

 $I\Theta S$



1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	1	1
1	0	0	0	0	0	0	0	1	1
1	0	0	0	0	0	0	0	1	1
1	0	0	0	0	0	0	1	1	1
1	0	0	0	0	0	1	1	1	1
1	0	0	0	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

1



1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	1	1
1	0	0	0	0	0	0	0	1	1
1	0	0	0	0	0	0	0	1	1
1	0	0	0	0	0	0	1	1	1
1	0	0	0	0	0	1	1	1	1
1	0	0	0	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

X	1	X
0	1	1
0	0	X
Teritologic person	S	

x 0 x

1 0 0

1 1 x

W-S

 I^{c}

Eg:



0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	1	1	1	1
0	1	0	0	0	1	1	1	1	1
0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0

X	1	X
0	1	1
0	0	X
Marian de la companione d La companione de la compa	S	

X	0	X
1	0	0
1	1	X

W-S

$$I^c\Theta(W-S)$$

Eg:



					A STATE OF THE PARTY OF THE PAR		e contra	
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0 0 0 0 0 0 1	0 0 0 0 0 0 0 0 0 0 1 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0	0 0



• Find I *S

0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	1	1	1	0
0	1	1	1	0	0	0	0	1	1	1	0
0	1	1	1	0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0



	0	0	0	0	0
	0	1	1	1	0
	0	1	1	1	0
inig	0	1	1	1	0
	0	0	0	0	0

Eg



• Find I *S

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$$I\Theta S$$

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$$I^c\Theta(W-S)$$



BOUNDARY EXTRACTION



- The boundary of a region R is the set of pixels in the region that have one or more neighbours that are not in R
- The boundary of a set I can be obtained by first eroding I by S and then performing the set difference between S and its erosion
- It is given by

$$\beta(I) = I - (I\Theta S)$$



• Eg:

- find $\beta(I)$

1	1	1	0	1	1	1	1	1	0
1	1	1	0	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1



1	1	1
1	1	1
1	1	1



1	1	1	0	1	1	1	1	1	0
1	1	1	0	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	1	1	0	0
0	1	0	0	0	1	1	1	0	0
0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0

 $I\Theta S$



1	1	1	0	1	1	1	1	1	0
1	0	1	0	1	0	0	0	1	0
1	0	1	1	1	0	0	0	1	1
1	0	0	0	0	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1

$$I - (I\Theta S)$$

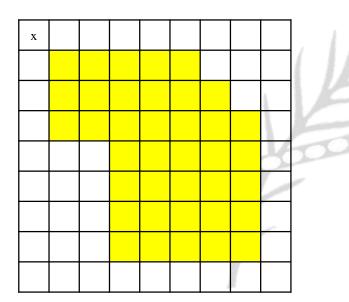
	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	1	1	1	0	0
Nego	0	1	0	0	0	1	1	1	0	0
	0	1	1	1	1	1	1	1	1	0
	0	0	0	0	0	0	0	0	0	0

 $I\Theta S$

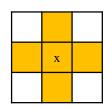


• Eg:

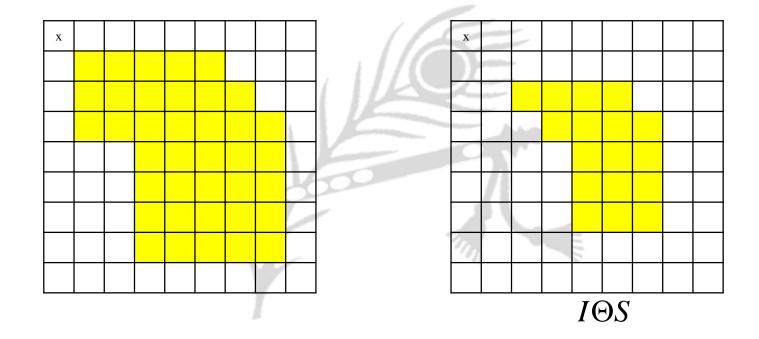
- find $\beta(I)$



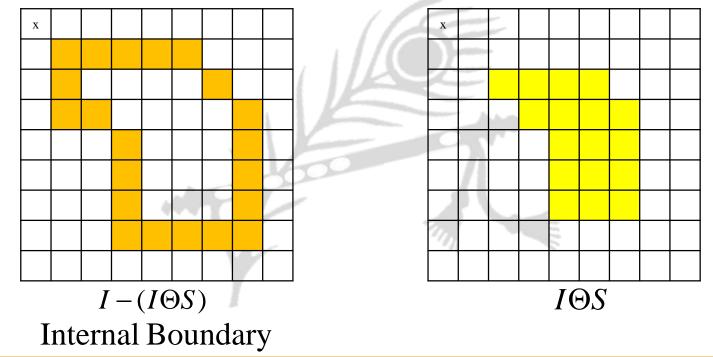




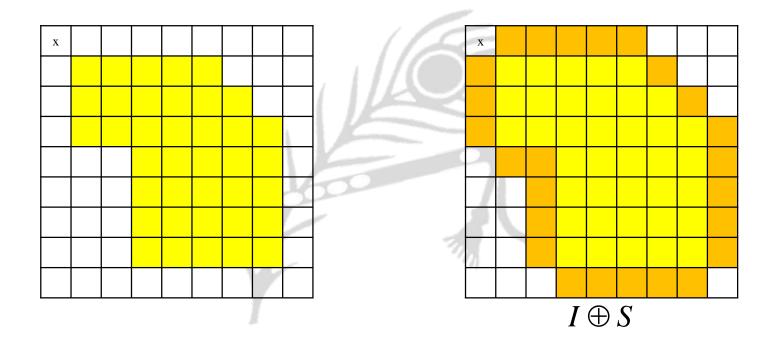




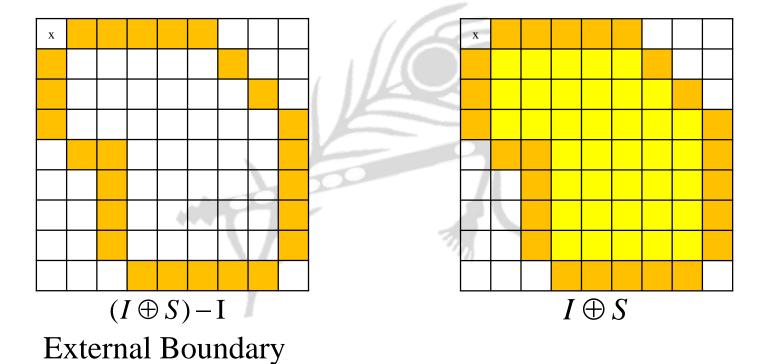












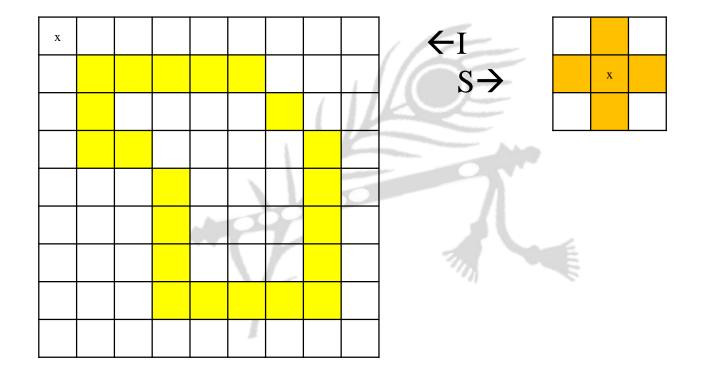


REGION FILLING

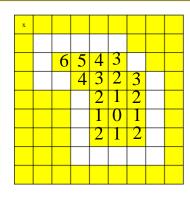


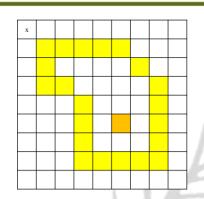
- Region filling is used to fill the selected region of the object
- Steps includes
 - Choose a seed point X₀
 - Iterate $X_k = (X_{k-1} \oplus S) \cap I^c$ until convergence

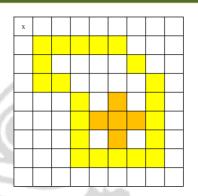


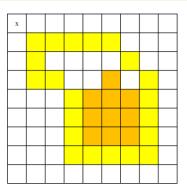


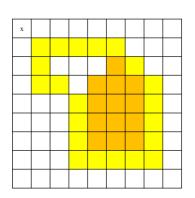


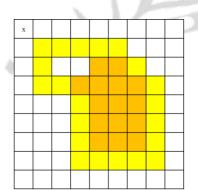


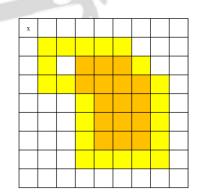


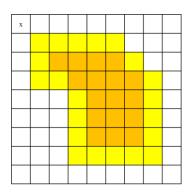






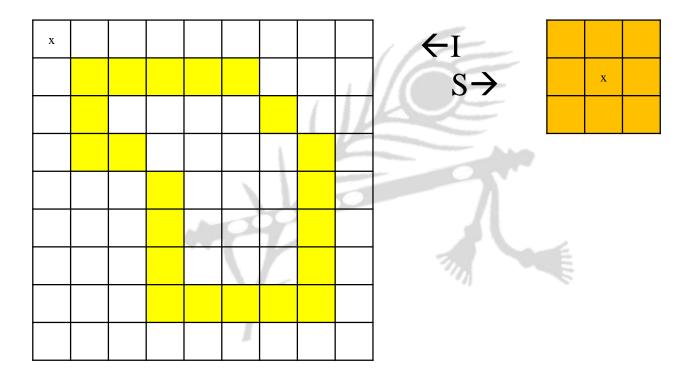






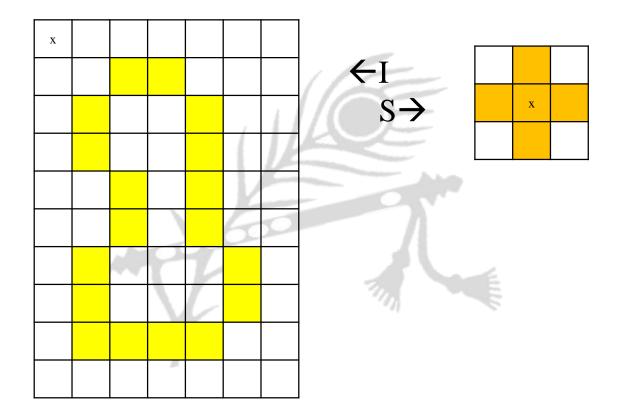


• Eg:





• Eg:





EXTRACTION OF CONNECTED COMPONENTS

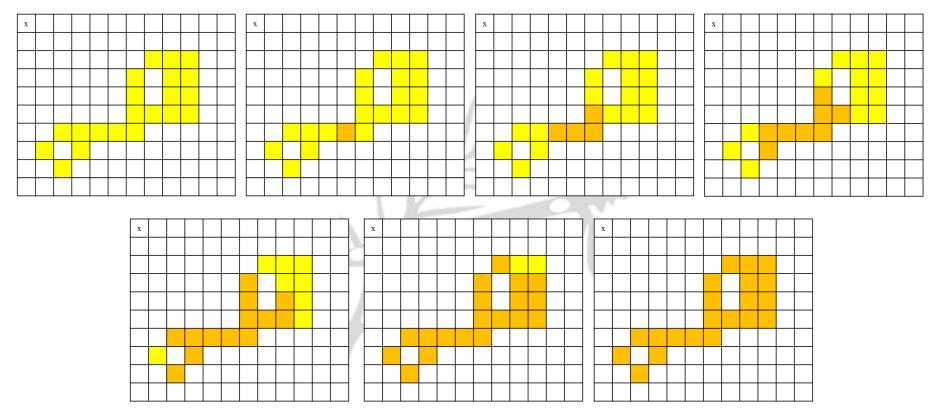


- Connected component labeling is used in computer vision to detect connected regions in the images
- It groups the pixels into components based on the pixel connectivity
- Steps includes
 - Choose a seed point X₀
 - Iterate $X_k = (X_{k-1} \oplus S) \cap I$ until convergence



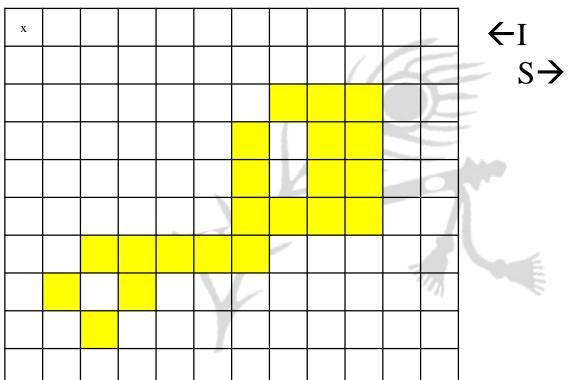
X						\leftarrow I S \rightarrow x
				7		
				Y		
				N.		







• Eg:



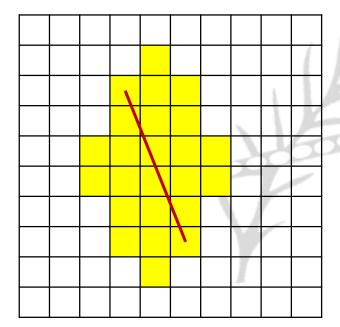


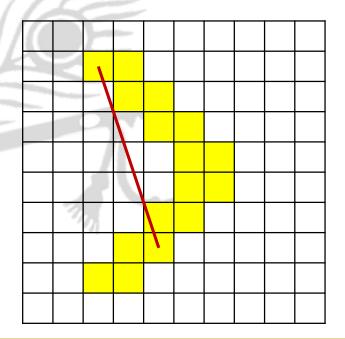






• A set I is said to be convex if the straight line segment joining any two points in I lies entirely within I

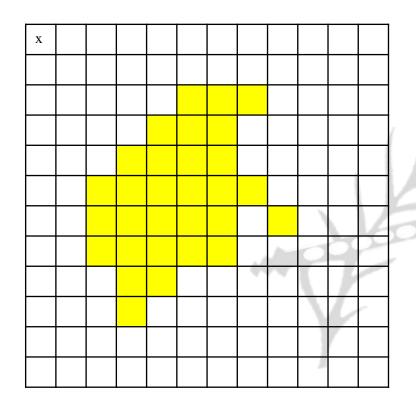




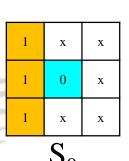


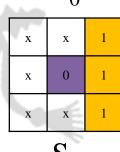
- Convex Hull (H) = Minimum convex set containing set I
- Hull Deficiency (D) = H I
- Steps include
 - Choose a seed point X₀
 - do i = 0 to 3
 - Iterate $X_k = (X_{k-1} \circledast S_i) \bigcup I$ until convergence
 - Minimize convex set using bounding box of I











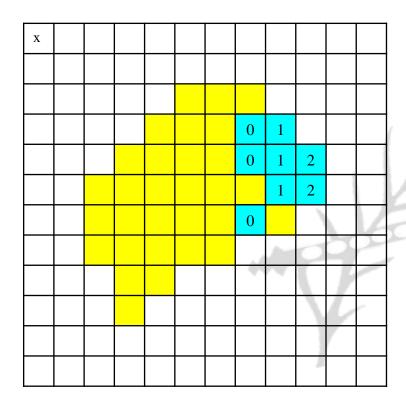
	X	
	х	
•		
		_
	X	
	X	
	1	

X

X

 S_3





1	Х	X
1	0	Х
1	X	X

 S_0



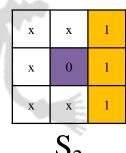
X										
					0	1				/
					0	1	2			
						1	2			L
					0		-		Z	P
					3	A	0		4	2
			3	4	4	4		1		
		4	5				/	L	e	
		6								
							II.			

1	1	1
X	0	х
X	Х	х

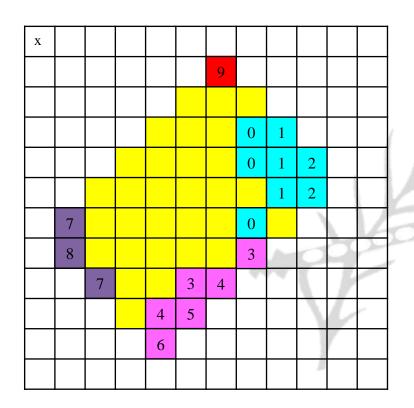
 \mathbf{S}_1

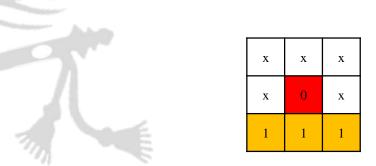


X										
						0	1			
						0	1	2		
							1	2		
	7					0				V
	8					3	Ą	0		1
		7		3	4	6	7		1	
			4	5				1	Ļ	E Common
			6							
								J.		









 S_3



X										
					9					
						0	1			
						0	1	2		
							1	2		
	7					0		1		V
	8					3	A	0	0	7
		7		3	4	4	7		ŀ	
			4	5					Ļ	green .
			6							
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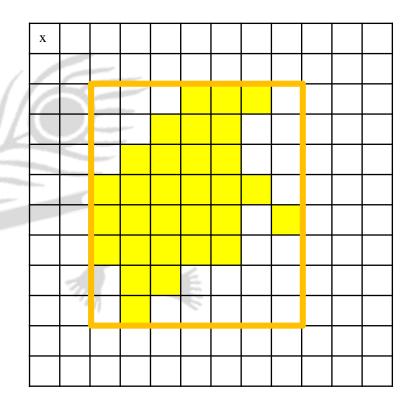


X										
					_					
					9					
						0	1			
						0	1	2		
							1	2		
	7					0				Z
	8					3	Arr		ď	4
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			6							
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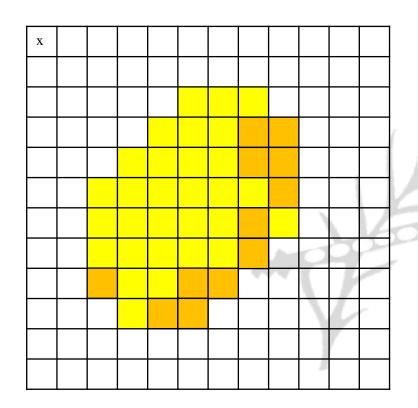


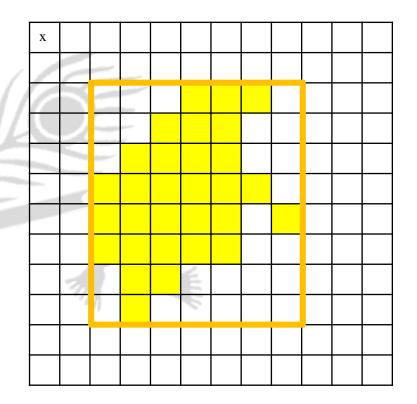
X									
					0	1			
					0	1			
						1			
					0		1		Z
					3	Arr	Ø		-
	7		3	4	-			ŀ	
		4	5				\	L	E ^{colo}
							щ		



Convex Hull

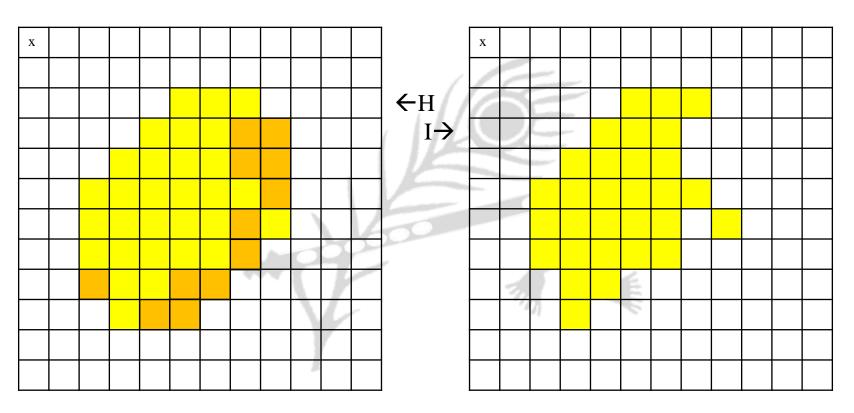






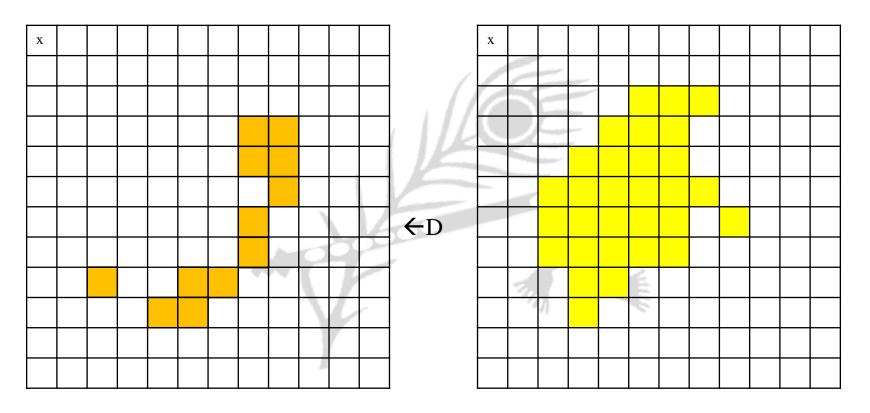
Convex Hull





Convex Hull







MORPHOLOGICAL THINNING AND THICKENING



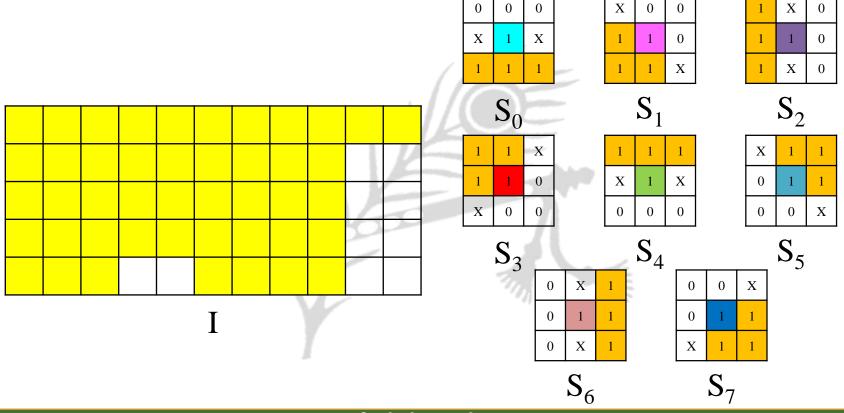
- It is an operation that is used to remove selected foreground pixels from binary images
- It is the process of reducing an object in a digital image to the minimum size
- It is given by

$$\begin{split} I \otimes S &= I - (I \circledast S_i) \\ I \otimes S &= I \cap (I \circledast S_i) \\ I \otimes \{S\} &= ((((I \circledast S_0) \circledast S_1) \circledast S_2) \dots \circledast S_7) \end{split}$$



- Thinning is basically a search & delete process
- It removes only those boundary pixels from the image whose deletion
 - Does not connectivity of their neighbours locally
 - Does not reduce the length of the already thinned curve
- Critical pixel
 - Its deletion changes the connectivity of its neighbourhood locally
- End pixel
 - Its deletion reduces the length of an already thinned curve





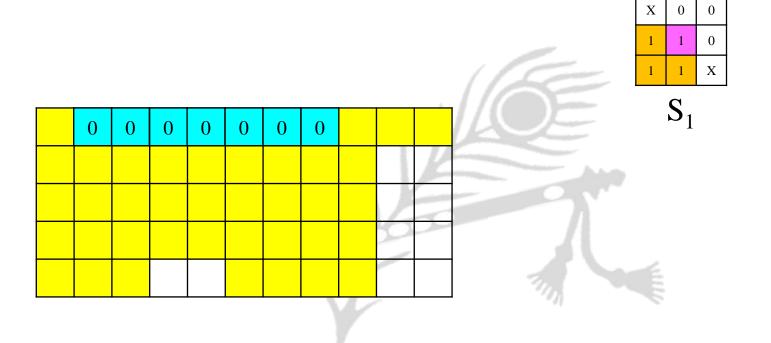


0	0	0	0	0	0	0		
								K
							×	
							9	

0	0	0
X	1	X
1	1	1

 S_0





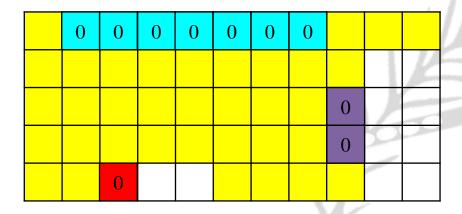


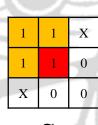
0	0	0	0	0	0	0			
									K
							0	1	
							0	Ŋ)	

1	X	0
1	1	0
1	X	0

 \mathbf{S}_2

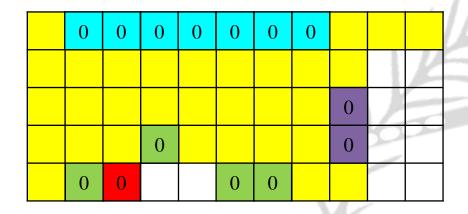


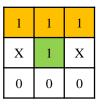




S

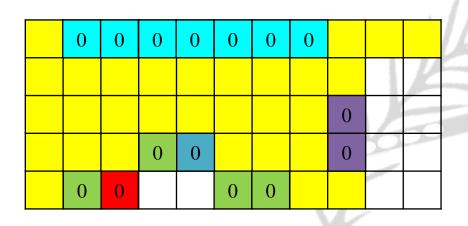






 S_{α}

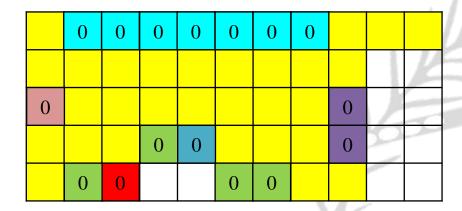




X	1	1
0	1	1
0	0	X

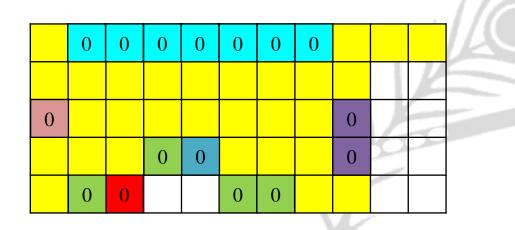
 S_5





	Min. str.		
0	X	1	
0	1	1	
0	X	1	





0	0	X
0	1	1
X	1	1

 S_{7}

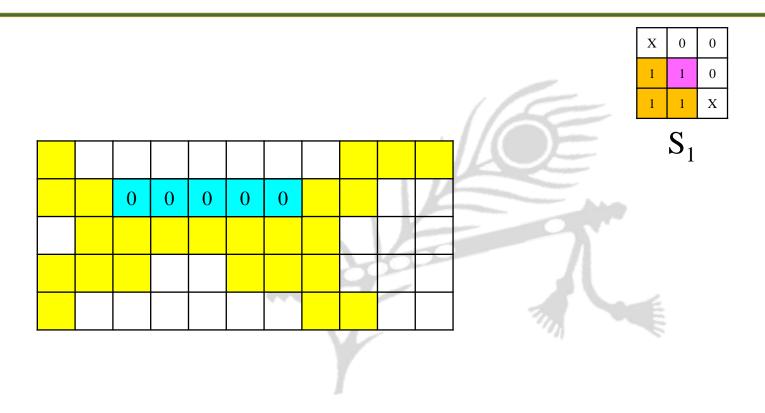


	0	0	0	0	0			L
						7	1	
							9	
					444			

0	0	0
X	1	X
1	1	1

 S_0





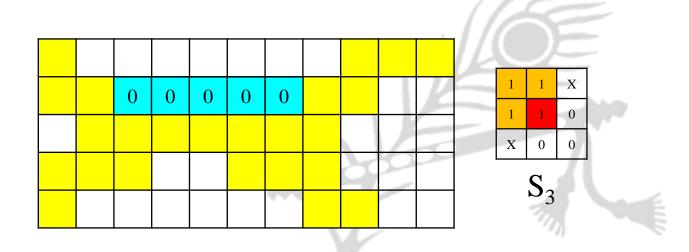


								ě	16
	0	0	0	0	0		1	K	
							Z		
									37
									-//

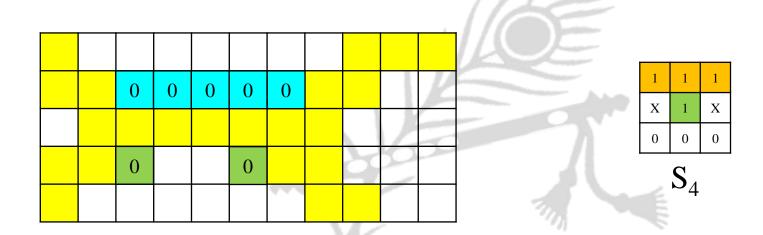
1	X	0
1	1	0
1	X	0

 \mathbf{S}_2

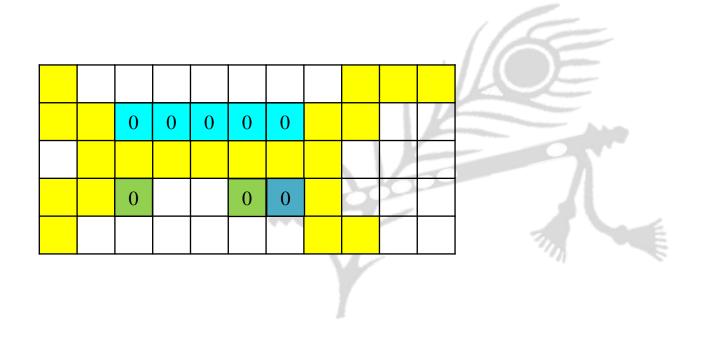








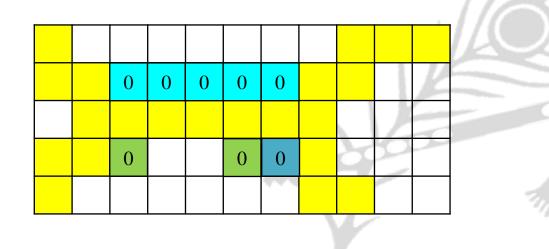




X	1	1
0	1	1
0	0	X

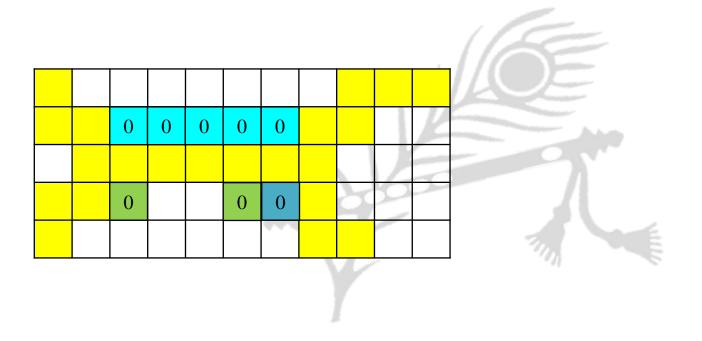
 S_5





70000000		
0	X	1
0	1	1
0	X	1

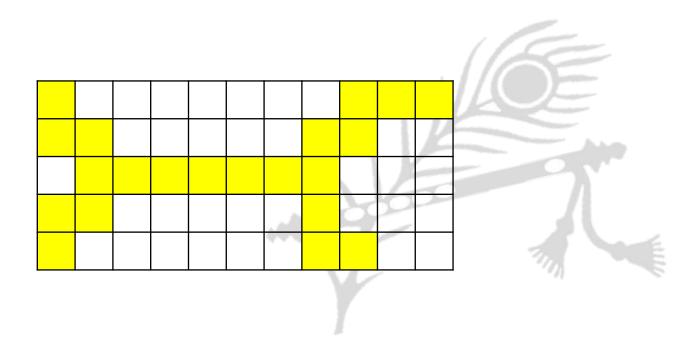




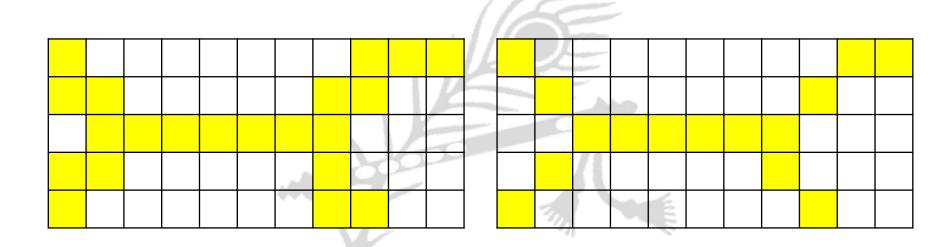
0	0	X
0	1	1
X	1	1

 S_7



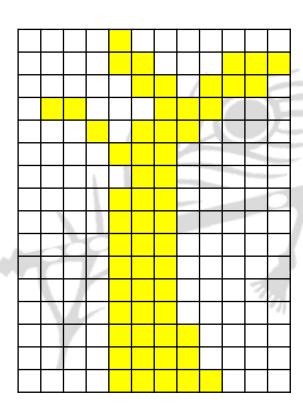








• Eg:



Thickening



- Thickening is the morphological dual of thinning
- It is defined as

$$I \odot S = I \cup (I \circledast S_i)$$

$$I \odot \{S\} = ((((I \circledast S_0) \circledast S_1) \circledast S_2) \dots \circledast S_7)$$

- Approach
 - Take I^c
 - res = Apply thinning on I^c
 - Take res^c

$$I \odot \{S\} = |(I^c \otimes S) \text{ followed by isolated pixel removal }|^c$$

RECAP



$$I \oplus S = \{ z \mid (\hat{S})_z \cap I \neq \emptyset \}$$

$$I\Theta S = \{z \mid (S)_z \cap I^c = \emptyset\}$$

$$I \circ S = (I \Theta S) \oplus S$$

$$I \bullet S = (I \oplus S) \Theta S$$

$$I \circledast S = (I \Theta S) \cap (I^c \Theta (W - S))$$

$$\beta(I) = I - (I\Theta S)$$

$$\beta(I) = (I \oplus S) - I$$

RECAP



• Region Filling

$$X_k = (X_{k-1} \oplus S) \cap I^c$$

• Connected Components

$$X_k = (X_{k-1} \oplus S) \cap I$$

Convex Hull

$$X_k = (X_{k-1} \circledast S_i) \bigcup I$$

• Thinning

$$I \otimes \{S\} = ((((I \otimes S_0) \otimes S_1) \otimes S_2) \dots \otimes S_7)$$

• Thickening

$$I \odot \{S\} = |(I^c \otimes S) \text{ isolated pixel removal }|^c$$



SEGMENTATION



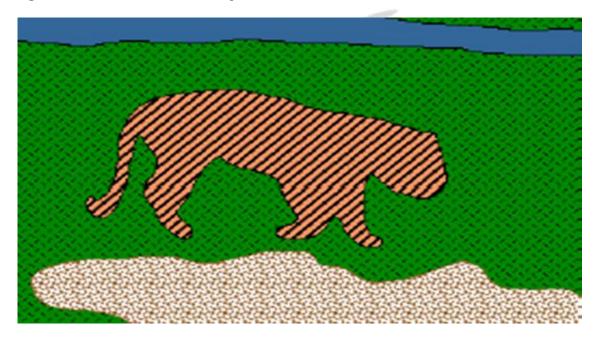
- Image segmentation is a technique to extract the attributes of the image
- Segmentation attempts to partition the pixels of an image into groups that strongly correlate with the objects in an image
- Mostly used in any automated computer vision application



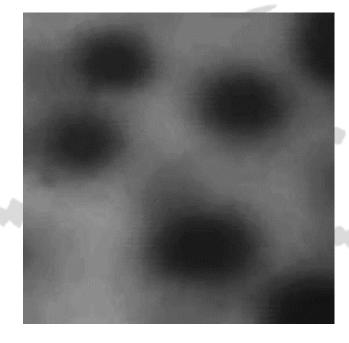




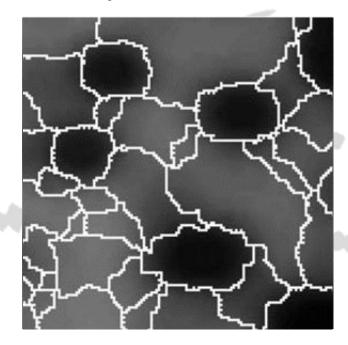












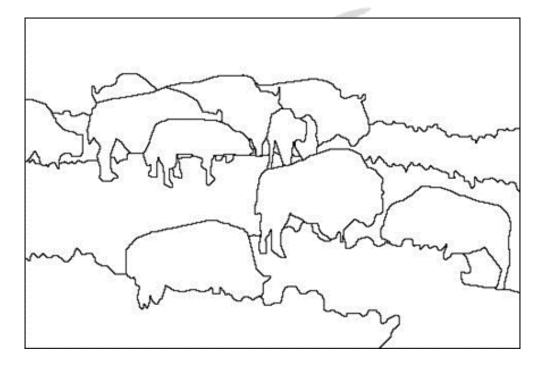




Segmentation



• Separate image into coherent objects



Segmentation



- The purpose of image segmentation is to partition an image into meaningful regions with respect to a particular application
- The segmentation is based on measurements taken from the image which includes intensity, color, texture, depth, etc

Segmentation (Approach)



- Segmentation algorithms generally are based on one of 2 basis properties of intensity values
- Discontinuity (abrupt changes)
 - to partition an image based on abrupt changes in intensity (such as edges)
- Similarity (homogeneity)
 - to partition an image into regions that are similar according to a set of predefined criteria
 - Two types: pixel based and region based

Segmentation (Fundamental)



- Let v be the spatial domain on which the image is defined
- Image segmentation divides v into n regions R_i (i = 1 to n), such that

$$\bigcup_{i=1}^n R_i = v$$

$$R_{i} \cap R_{j} = \phi$$

Segmentation (Basic Idea)



- All the image segmentation methods assume that
 - the intensity values are different in different regions
 - within each region of the corresponding object, the intensity values are similar
- For this, we need to apply threshold

Thresholding



- Gray level thresholding is the simplest segmentation process
- Correct thresholding leads to better segmentation
- Thresholding is computationally inexpensive and fast which can easily be applied in real time
- Thresholding as a transformation function

$$T = T[x, y, p(x, y), f(x, y)]$$

- where
- f(x, y) is the gray level of the point (x, y)
- p(x, y) denotes some local property of the point (e.g. average gray level of a pixels centred at (x, y))

Thresholding (Types)



- Global Thresholding
 - If T is only a function of f(x, y), then the threshold is called global threshold
 - Apply the same threshold to the whole image
- Local Thresholding
 - If T is a function of both f(x, y) & p(x, y), then the threshold is called local
 - The threshold depends on local property
- Dynamic (Adaptive) Thresholding
 - The threshold depends on spatial coordinates (x, y)

Thresholding (Types)



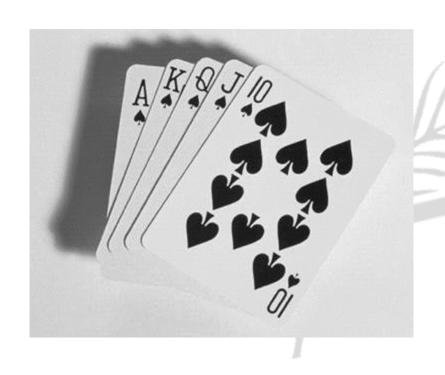
• Thresholding is the transformation of an input image f to an output (segmented) binary image g

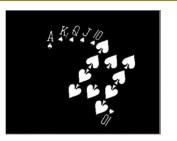
$$g(x, y) = \begin{cases} 1 & if \ f(x, y) > T \\ 0 & otherwise \end{cases}$$

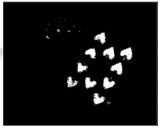
- where, T is threshold
- From a grayscale image, thresholding can be used to create binary images

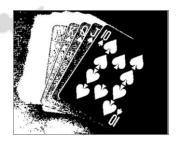
Thresholding











Thresholding (Eg)



- Disk = 255 and background = 127
- Segment the image into two regions disk and background
- What will be the threshold values?



Global Thresholding



- Based on the histogram of an image
- Partition the image histogram using a single global threshold
- The success of this technique strongly depends on how well the histogram can be partitioned

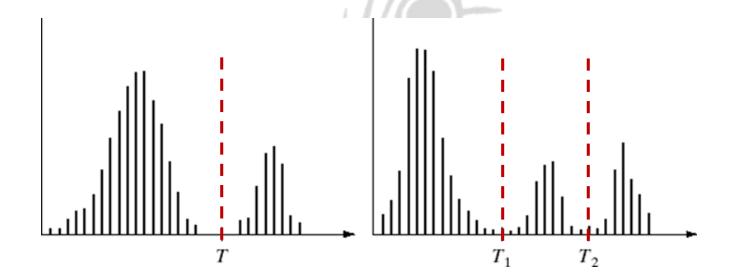
Global Thresholding (Algorithm)



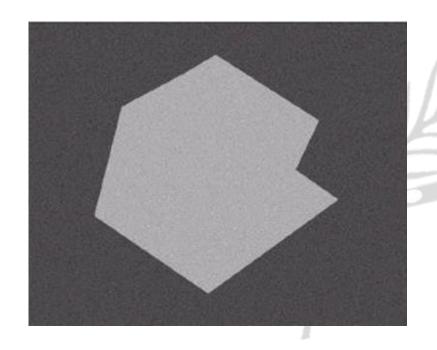
- Select an initial estimate for T (typically the average grey level in the image)
- Segment the image using T to produce two groups of pixels:
 - G1 consisting of pixels with grey levels >T
 - G2 consisting pixels with grey levels \leq T
- Compute the average grey levels of pixels in G1 to give μ1 and G2 to give μ2
- Compute a new threshold value $T = (\mu 1 + \mu 2)/2$
- Repeat steps 2 4 until the difference in T in successive iterations is less than a predefined limit

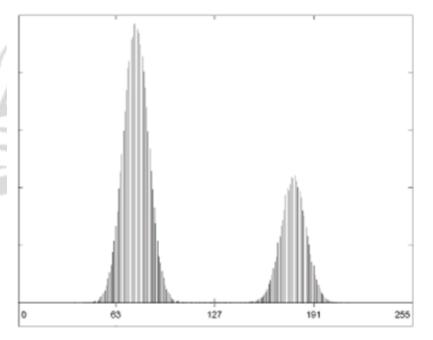


- Single value thresholding only works for bimodal histograms (two peaks)
- Images with other kinds of histograms need more than a one threshold



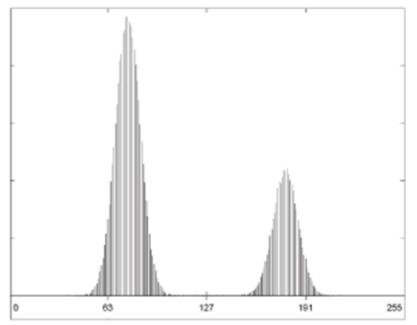




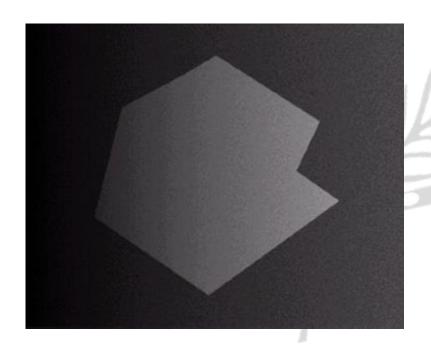


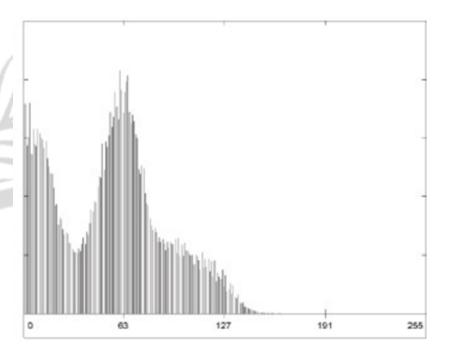




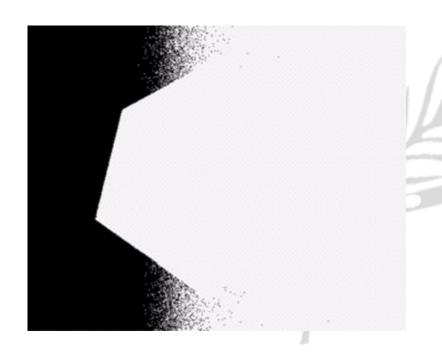


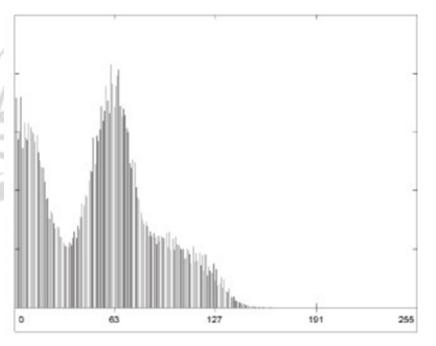








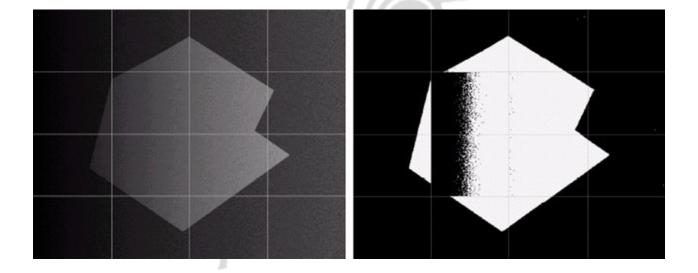




Adaptive Thresholding



• The threshold for each pixel depends on its location within an image, this technique is said to adaptive



Region



- A group of connected pixels with similar properties
- For correct interpretation, image must be partitioned into regions that correspond to objects or parts of an object

Region based approach



- Let R represent the entire image region
- Segmentation R into n sub regions, R_1 , R_2 , ..., R_n , such that

$$\bigcup_{i=1}^n R_i = R$$

 R_i is a connected region, i = 1, 2, ..., n

$$R_i \cap R_j = \phi$$
, for all i and $j, i \neq j$

Region based approach



- The fundamental drawback of histogram based region detection is that histograms provide no spatial information
- Region growing approaches exploit the important fact that pixels which are close together have similar gray values

Region growing



- Choose the seed pixels (1 for every segment)
- Check the neighboring pixels and add them to the region if they are similar to the seed
- Repeat previous step for each of the newly added pixels, stop if no more pixels can be added

	-				
1	K	1	9	9	9
2	1	1	9	9	9
4	5	1	9	9	9
4	5	5	5	3	9
3	3	3	3	3	3

Region growing



- Choose the seed pixels (1 for every segment)
- Check the neighboring pixels and add them to the region if they are similar to the seed
- Repeat previous step for each of the newly added pixels, stop if no more pixels can be added

1	1	9	9	9
1	1	9	9	9
5	1	9	9	9
5	5	5	3	9
3	3	3	3	3

Region growing



- Choose the seed pixels (1 for every segment)
- Check the neighboring pixels and add them to the region if they are similar to the seed
- Repeat previous step for each of the newly added pixels, stop if no more pixels can be added

			Section .	
1	1	9	9	9
1	1	9	9	9
5	1	9	9	9
5	5	5	3	9
3	3	3	3	3

Region splitting



- Split starts from the assumption that the entire image is homogeneous
- If this is not true (by the homogeneity criterion), then split image into four sub images
- This splitting procedure is repeated recursively until we split the image into homogeneous regions

I

I ₁	I_2
I_3	I_4

I_1	I	2
T	I_{41}	I_{42}
I_3	I ₄₃	I_{44}

Region splitting

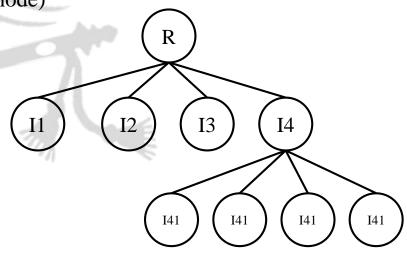


- If an image of dimensions N x N, that is in the powers of 2 ($N = 2^n$):
 - All regions produced by the splitting algorithm are squares having dimensions M x M, where M is a power of 2 as well (M = 2m, $M \le n$)

- Since the procedure is recursive, it produces an image that can be described by a tree, in which each nodes have four child (except leaf node)

- Such a tree is called a Quadtree

I_1	, ob I	2
T	I_{41}	I_{42}
I_3	I ₄₃	I ₄₄



Region splitting



- Advantage
 - Created regions are adjacent and homogenous
- Disadvantage
 - Over splitting, since no merge is performed

- Improvement
 - Split and Merge

Region splitting and Merging



- After splitting
- Merging phase
 - If 2 adjacent regions are homogenous, they are merged
- Repeat merging step, until no further merging is possible

Segmentation (Discontinuity)



- There are basic three types of grey level discontinuities:
 - Points
 - Lines
 - Edges
- We typically find discontinuities using masks and correlation
- Discrete form of derivative is used

Segmentation (Discontinuity)



- The filter is expected to be isotropic
 - response of the filter is independent of the direction of discontinuities in an image
- The digital implementation of the 2-Dimensional Laplacian is obtained by summing 2 components

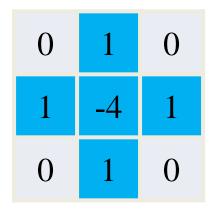
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

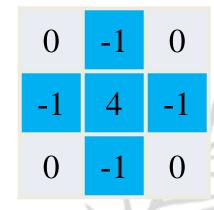
0	1	0
1	-4	1
0	1	0

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Segmentation (Discontinuity)







← 90° isotropic

Segmentation (Discontinuity→Point)



• A point has been detected at the location on which the mask center if

$$|R| >= T$$

- where
- T is a nonnegative threshold
- R is the sum of products of the coefficients with the gray levels contained in the region encompassed by the mark

Segmentation (Discontinuity → Point)



7	7	7	7	7	7	7
7	10	7	7	7	7	7
7	7	7	7	7	7	7
7	7	7	7	7	7	7
7	7	7	7	4	7	7
7	7	7	7	7	7	7
7	7	7	7	7	7	7

-1	-1	-1
-1	8	-1
-1	-1	-1

P

	24	-3	0	0	0	
	-3	-3	0	0	0	
-	0	0	3	3	3	
	0	0	3	-24	3	
	0	0	3	3	3	
	WHIP/					

I ° P

Segmentation (Discontinuity → Point)



7	7	7	7	7	7	7
7	10	7	7	7	7	7
7	7	7	7	7	7	7
7	7	7	7	7	7	7
7	7	7	7	4	7	7
7	7	7	7	7	7	7
7	7	7	7	7	7	7

-1	-1	-1
-1	8	-1
-1	-1	-1

P

	24	3	0	0	0	
	3	3	0	0	0	
9	0	0	3	3	3	
	0	0	3	24	3	
	0	0	3	3	3	
	Ann					

$$R_p = |I \circ P|$$

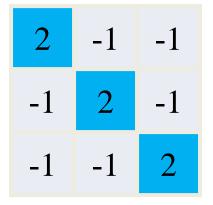
Segmentation (Discontinuity → Line)



- The next level of complexity is to try to detect lines
- The masks below will extract lines that are one pixel thick and running in a particular direction

-1	-1	-1
2	2	2
-1	-1	-1

-1	-1	2	-1	2	-1
-1	2	-1	-1	2	-1
2	-1	-1	-1	2	-1



Segmentation (Discontinuity → Line)



1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	3
1	1	1	1	1	1	3	1
1	1	1	1	1	3	1	1
3	3	3	3	3	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

-1	-1	-1
2	2	2
-1	-1	-1
-	Н	A CONTRACTOR OF THE PARTY OF TH

	0	0	0	0	2	2	
(Section and Section)	0	0	0	2	2	0	
	6	6	6	0	0	2	
	12	12	12	6	2	2	
	6	6	6	4	2	0	
	0	0	0	0	0	0	
		77.13					

I

 $|I \circ H|$



1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	3
1	1	1	1	1	1	3	1
1	1	1	1	1	3	1	1
3	3	3	3	3	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

-1	-1	2
-1	2	<i>-</i> 1
2	-1	-1
-1	2 5	ALC:

 $D_{45^{\circ}}$

	1	0	0	0	0	2	4	
		0	0	0	2	4	12	
Sil Sinch		0	0	0	0	12	4	
R	4	0	0	0	6	4	2	
		0	0	0	4	2	0	
		0	0	0	0	0	0	
			11/2					

I

$$|I \circ D_{45^{\circ}}|$$



1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	3
1	1	1	1	1	1	3	1
1	1	1	1	1	3	1	1
3	3	3	3	3	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

-1	2	-1
-1	2	-1
-1	2	-1
	V	A STREET

	0	0	0	0	2	2	
	0	0	0	2	2	0	
est o	0	0	0	0	0	2	
	0	0	0	0	2	2	
	0	0	0	2	2	0	
).	0	0	0	0	0	0	
		1814					

I

 $|I \circ V|$



1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	3
1	1	1	1	1	1	3	1
1	1	1	1	1	3	1	1
3	3	3	3	3	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

2	-1	-1
-1	2	-1
-1	-1	2

D_{-45°}

	0	0	0	0	4	4	
il Disser-	0	0	0	4	4	0	
Si Senior	0	0	0	6	0	4	
	0	0	0	0	4	4	
	0	0	0	2	4	0	
	0	0	0	0	0	0	
		200					

I



0	0	0	0	4	4	
0	0	0	4	4	12	
6	6	6	6	12	4	
12	12	12	6	4	4	
6	6	6	4	4	0	
0	0	0	0	0	0	

$L=\max\{ I^{\circ}H , I^{\circ}V , I^{\circ}I \}$) _{45°} , I°D_45° }
--	-------------------------------

	0	0	0	0	2	2	
4	0	0	0	2	2	0	
1	6	6	6	0	0	2	
	12	12	12	6	2	2	
	6	6	6	4	2	0	
Nation of the last	0	0	0	0	0	0	
Marie .							
	and Con-		7	(A			
,			1				
	0	0	0	0	2	2	

				V 4			
	0	0	0	0	2	2	
	0	0	0	2	2	0	0.11
	0	0	0	0	0	2	en Francisco
	0	0	0	0	2	2	Tips.
	0	0	0	2	2	0	
	0	0	0	0	0	0	

0	0	0	0	2	4	
0	0	0	2	4	12	
0	0	0	0	12	4	
0	0	0	6	4	2	
0	0	0	4	2	0	
0	0	0	0	0	0	

0	0	0	0	4	4	
0	0	0	4	4	0	
0	0	0	6	0	4	
0	0	0	0	4	4	
0	0	0	2	4	0	
0	0	0	0	0	0	



- Edges characterize boundaries
- Edges in images are areas with strong intensity contrasts a jump in intensity from one pixel to the next
- Edge detecting in an image significantly reduces the amount of data and filters out useless information, while preserving the important structural properties in

an image





- Difference between an edge and a line
 - An edge, for instance be the border between a block of blue color and a block of yellow





- In contrast, a line can be a small number of pixels of different color
- For a line, there may usually be one edge on each side of the line



- Edges are the most common approach for detecting meaningful discontinuities in gray level
- The process of edge detection can be broadly classified into two categories
 - Derivative approach
 - Edge pixels (or edgels) are detected by taking derivative followed by thresholding
 - Incorporate noise cleaning scheme
 - Pattern fitting approach
 - A series of edge approximating functions in the form of edge templates over neighbourhood are analysed
 - Parameters along with their properties corresponding to the best fitting function are determined
 - Based on this information, it is decided whether an edge is present or not
 - These are edge filters



Edge linking

 process takes an unordered set of edge pixels produced by an edge detector as i/p to form an ordered list of edgels

Edge following

- process takes the entire edge strength or gradient image as i/p & produces geometric primitives such as lines or curves

Segmentation (Discontinuity \rightarrow Edge \rightarrow Derivative)



- First Order Derivative/Gradient Methods
 - Robert operator
 - Sobel operator
 - Prewitt operator
- Second Order Derivative
 - Laplacian
 - Laplacian of Gaussian
 - Difference of Gaussian
- Optimal Edge Detection
 - Canny Edge Detection

Segmentation (Discontinuity \rightarrow Edge \rightarrow Derivative)



1	1	1	1	2	2	2
1	1	1	1	2	2	2
1	1	1	1	2	2	2
2	2	2	2	1	1	1
2	2	2	2	1	1	1
2	2	2	2	1	1	1
2	2	2	2	1	1	1

 $R = |I^{\circ}G_{45^{\circ}}| + |I^{\circ}G_{-45^{\circ}}|$ Robert $G_{45^{\circ}}$

4	<i>ar</i>	100
	0	1
pil	-1	0
,	G ₋	45°

-1	-2	-1
0	0	0
1	2	1

 S_{H}

	N.Y		
The second	-1	0	1
ſ	-2	0	2
	-1	0	<i>u</i> 21

$$\begin{array}{ccc} S_{\text{V}} & P_{\text{V}} \\ \text{Sobel} & \text{Prewitt} \\ S = |I^{\circ}S_{H}| + |I^{\circ}S_{V}| & P = |I^{\circ}P_{H}| + |I^{\circ}P_{V}| \end{array}$$

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

 P_{H}

Prewitt

Segmentation (Discontinuity → Edge → Derivative)



- Motivation
 - Detect sudden changes in image intensity
 - Gradient: sensitive to intensity changes
- Gradient

$$\nabla f = \left[\frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \right]^{\frac{1}{2}}$$

Segmentation (Discontinuity \rightarrow Edge \rightarrow Derivative)



The gradient of the image I at location (x, y) is the vector

$$\nabla I = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial I(x, y)}{\partial x} \\ \frac{\partial I(x, y)}{\partial y} \end{bmatrix}$$

- Magnitude
- Gradient

$$|\nabla I| = \sqrt{G_x^2 + G_y^2}$$

$$|\nabla I| = \sqrt{G_x^2 + G_y^2}$$

$$\theta(x, y) = \tan^{-1} \left(\frac{G_y}{G_x}\right)$$

Segmentation (Discontinuity → Edge → Derivative)



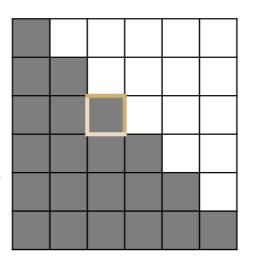
- Eg:
 - Find the strength & the direction of the edge at the highlighted pixel
 - Pixels in gray are 0 and white are 1

Solution

- Derivative is computed by using a 3x3 neighbourhood
 - subtract the pixels in the top row from bottom row (x direction)
 - similarly obtain the derivative in the y direction

-1	-1	-1
0	0	0
1	1	1

		200
-1	0	1
-1	0	1
-1	0	1

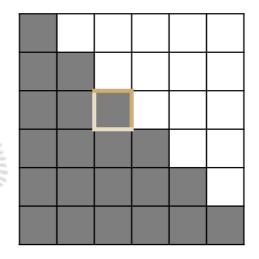


Segmentation (Discontinuity → Edge → Derivative)



- Eg:
 - Find the strength & the direction of the edge at the highlighted pixel
 - Pixels in gray are 0 and white are 1

$$\nabla I = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial I(x, y)}{\partial x} \\ \frac{\partial I(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$
$$|\nabla I| = \sqrt{G_x^2 + G_y^2} = 2\sqrt{2}$$
$$\theta(x, y) = \tan^{-1} \left(\frac{G_y}{G_x}\right) = -1 = -45^{\circ}$$



Simple edge detection using gradient



- Compute gradient vector at each pixel by convolving image with horizontal and vertical derivative filters
- Compute gradient magnitude at each pixel
- If magnitude at a pixel exceeds a threshold, report a possible edge point



