Poisson Distribution (P.D.)

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{\lfloor r}$$

$\Downarrow$

Number of successes in a given interval of time

λ = average number of successes in a given interval

Note: (1) Poission case is a $\boxed{\text{limiting case}}$ of Binomial Distribution.
i.e. in Binomial distribution n is finite while in Poisson Distribution $n \to \infty$ (very large)

(2) Mean of Poisson Distribution $= \lambda =$ Variance of P.D.

(3) In the following cases P.D. can be employed

(a) The numbers of telephone calls per hour received by an office.

(b) The number of bacteria in a given culture.

(c) The number of deaths in a district in a given period of time etc.

Ques (1) If X follows P.D. such that $P(X=0) = P(X=1)$ then find mean of X.

Solution: We have $P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{\lfloor r}$

$$P(X=0) = \frac{e^{-\lambda}\lambda^{0}}{\lfloor 0} = e^{-\lambda}$$

$$P(X=1) = \frac{e^{-\lambda}(\lambda)^{1}}{\lfloor 1} = e^{-\lambda}\lambda$$

$$P(X=0) = P(X=1)$$

$$e^{-\lambda} = \lambda e^{-\lambda}$$

$$e^{-\lambda}(1-\lambda) = 0$$

$$1-\lambda = 0 \quad (\because e^{-\lambda} \neq 0)$$

$$\lambda = 1 \Rightarrow \text{Mean of } X = 1 \quad \underline{\text{Ans}}$$

Ques(2) If X follows P.D. such that $P(X=1) = 2P(X=2)$, find the mean and variance of X.

Solution: We have $P(X=r) = \frac{e^{-\lambda}\lambda^r}{\lfloor r}$

$$P(X=1) = \frac{e^{-\lambda}\lambda^1}{\lfloor 1} = \lambda e^{-\lambda}$$

$$P(X=2) = \frac{e^{-\lambda}\lambda^2}{\lfloor 2} = \frac{e^{-\lambda}\lambda^2}{2}$$

$$P(X=1) = 2\,P(X=2)$$

$$\lambda e^{-\lambda} = 2\cdot\frac{e^{-\lambda}\lambda^2}{2}$$

$$\lambda e^{-\lambda} = e^{-\lambda}\lambda^2$$

$$e^{-\lambda}(\lambda-\lambda^2) = 0$$

$$\lambda-\lambda^2 = 0 \quad (e^{-\lambda} \neq 0)$$

$$\lambda(\lambda-1) = 0 \Rightarrow \lambda = 1 \quad (\because \lambda \neq 0)$$

Mean = Variance = $\lambda = 1$ Ans

Ques(3) If a random variable X follows P.D. such that $P(X=2) = \frac{2}{3}P(X=1)$. Find mean of X hence find $P(X=0)$.

Ans: $\lambda = \frac{4}{3}$, $P(X=0) = e^{-\frac{4}{3}}$ (Do by Yourself).

Ques(4) The average number of customers who appear at a counter of a certain bank per minute is 2. Find the probability that during a given minute

(I) No customer appears (II) three or more customers appear (given $e^{-2} = 0.135$).

Solution: Here, we have $\lambda = 2$

Let X = average no of customers appears in one minute

(I) no customer appears

$$P(X=0) = \frac{e^{-2}(\lambda)^0}{0!} \quad (\because \lambda = 2,\ r = 0)$$

$$= e^{-2} = 0\cdot135 \quad \underline{\text{Ans}}$$

(II) Three or more customers appear

$$P(X \geq 3) = 1 - P(X<3) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - \frac{e^{-2}(2)^0}{0!} - \frac{e^{-2}(2)^1}{1!} - \frac{e^{-2}(2)^2}{2!}$$

$$= 1 - e^{-2} - 2e^{-2} - 2e^{-2}$$

$$= 1 - 5e^{-2} = 1 - 5 \times 0\cdot135$$

$$P(X \geq 3) = 0\cdot325 \quad \underline{\text{Ans}}$$