Correlation Coefficient

Ques: (1) The coefficient of correlation between X and Y is 0.4 and covariance is 10. If Var (X) = 9, find second moment about mean of Y.

Solution: Here, we have $r = 0.4$, $Cov(X, Y) = 10$

$$\sigma_x^2 = 9 \Rightarrow \sigma_x = \sqrt{9} = 3$$

$$r = \frac{Cov(X, Y)}{\sigma_x \cdot \sigma_y} \Rightarrow 0.4 = \frac{10}{3 \times \sigma_y} \Rightarrow 1.2\,\sigma_y = 10$$

$$\sigma_y = \frac{10}{1.2} = 8.33$$

The second moment about mean of Y is

$$\mu_2 = Var(Y) = \sigma_y^2 = (8.33)^2 = 69.39 \text{ Ans}$$

Ques: (2) Given the following information:

$$r = 0.8, \quad \Sigma xy = 60, \quad \sigma_y = 2.5 \text{ and } \Sigma x^2 = 90$$

where x and y are deviations from the respective means, find the number of items (n).

Solution: Here we have $r = 0.8$, $\Sigma xy = 60$

where $x = X - \bar{X}$ and $y = Y - \bar{Y}$

$$\Sigma (X - \bar{X})(Y - \bar{Y}) = 60 \quad \text{—— } (*)$$

$$\Sigma x^2 = 90 \Rightarrow \Sigma (X - \bar{X})^2 = 90$$

$$\sigma_x = \sqrt{\frac{\Sigma (X - \bar{X})^2}{n}} = \sqrt{\frac{90}{n}}$$

We know that $$r = \frac{Cov(X, Y)}{\sigma_x \sigma_y} \quad \text{—— } (1)$$

$$Cov(X, Y) = \frac{1}{n} \Sigma (X - \bar{X})(Y - \bar{Y})$$

$$Cov(X, Y) = \frac{60}{n} \quad (\text{From Eq}^n (*))$$

From Eq^n (1), we have

$$0.8 = \frac{\frac{60}{n}}{\sqrt{\frac{90}{n}} \times 2.5} \Rightarrow 2.5 \times 0.8 = \frac{60}{n} \times \sqrt{\frac{n}{90}}$$

$$2 = \frac{60}{n} \times \frac{\sqrt{n}}{\sqrt{90}}$$

$$2 = \frac{60}{\sqrt{90}} \times \frac{1}{\sqrt{n}}$$

$$2\sqrt{90} \times \sqrt{n} = 60$$

$$\sqrt{90} \times \sqrt{n} = 30$$

Squaring both sides

$$90n = 900 \Rightarrow n = 10.$$

Ques: (3) Calculate coefficient of correlation from the following information and comment on the result

$\sigma_x = 10$, $\sigma_y = 12$, $\bar{x} = 25$, $\bar{y} = 35$

Summation of the product of deviation from actual arithmetic mean of X and Y = 24

No of observations = 20.

Various formulas for computing the correlation coefficient

We know that $\gamma = \frac{\text{cov}(X, Y)}{\sigma_X \cdot \sigma_Y}$ — (1)

$$\gamma = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\sqrt{\Sigma(X-\bar{X})^2} \cdot \sqrt{\Sigma(Y-\bar{Y})^2}} = \frac{\Sigma xy}{\sqrt{\Sigma x^2} \times \sqrt{\Sigma y^2}} \quad \text{— (2)}$$

where $x = X - \bar{X}$ and $y = Y - \bar{Y}$

$$\gamma = \frac{n\Sigma XY - \Sigma X \,\Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2} \cdot \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}} \quad \text{— (3)}$$