# Probability

**Random Experiment:** An Experiment which when performed does not given unique result or outcome but may result one of the several possible outcomes.

Examples : (I) Tossing a coin (II) Rolling of a die

(III) Drawing a card from a 52 playing card.

**Sample Space:** The ~~spal~~ sample space S, is the Collection of all possible outcomes of a random Experiment.

Examples: (I) The sample space corresponds to the tossing of a coin is $S_1 = \{H, T\}$

(II) For tossing two coins

$$S_2 = \{HH, HT, TH, TT\}$$

(III) For tossing three coins

$$S_3 = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

**Event:** Any subset of the sample space is called an Event.

Example: For Rolling a die the sample space $S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1\}$, $E_2 = \{2, 4, 6\}$ $E_3 = \{1, 3, 5\}$

$E_1$, $E_2$ and $E_3$ are the events.

**Sure or certain Event:** The sample space S itself is called Sure event.

**Impossible Event:** An event which does not contain any outcome of the Experiment is called impossible event. It is denoted by $\phi$ (phi).

Example: For rolling of a die the sample space $S = \{1, 2, 3, 4, 5, 6\}$

Define E = Turing up 7 on the face of die

Then E is an impossible event.

**Equally-Likely Events:** Two or more events are said to be equally likely if each has the same chance of occurrence.

**Example:** (I) In tossing of a coin the sample space $S = \{H, T\}$

$E_1 = \{H\}$ $E_2 = \{T\}$.

$E_1$ and $E_2$ are equally likely event.

**Definiton of Probability:** The probability of happening of an event A is denoted by $P(A)$ and is defined as

$$P(A) = \frac{\text{The number of outcomes in favour of A}}{\text{Total number of outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$n(A)$ = number of outcomes in favour of A

$n(S)$ = Total number of outcomes i.e. number of elements in the Sample Space.

**Note:** (I) For any Event A, we have $\boxed{0 \le P(A) \le 1}$

(II) For impossible event A, $P(A) = 0$.

(III) For sure Event A, $P(A) = 1$.

**Ques(1)** For tossing a coin, what is the probability of getting

(I) head (II) tail

**Solution:** Here, the sample space $S = \{H, T\} \Rightarrow n(S) = 2$

(I) $A$ = getting head

$A = \{H\} \Rightarrow n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$ Ans

(II) $B$ = getting tail

$B = \{T\} \Rightarrow n(B) = 1$

$P(B) = \frac{1}{2}$ Ans

Ques(2) Two coins are tossed simultaneously. what is the prob. of getting (I) Two heads (II) At least One tail

Solution: Here, the sample space $S = \{HH, HT, TH, TT\}$

$n(S) = 4$

(I) $A$ = getting two heads

$A = \{HH\} \Rightarrow n(A) = 1$

$P(A) = \frac{1}{4}$

(II) $B$ = At least One tail

$B = \{HT, TH, TT\} \Rightarrow n(B) = 3$

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$.

Ques(3) Three coins are tossed simultaneously. what is the prob. of getting (I) no heads (II) Exactly one head (III) Exactly two heads (IV) at least one head (V) at least two heads (VI) all heads.

Solution: Here, the sample space $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

(I) $A$ = no heads

$A = \{TTT\} \Rightarrow n(A) = 1$

$P(A) = \frac{1}{8}$

(II) $B$ = Exactly one head

$B = \{HTT, THT, TTH\} \Rightarrow n(B) = 3$

$P(B) = \frac{3}{8}$

(III) $C$ = Exactly two head

$C = \{HHT, HTH, THH\} \Rightarrow n(C) = 3$

$P(C) = \frac{3}{8}$

(IV) $D$ = at least one head

$D = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$

$n(D) = 7$

$P(D) = \frac{7}{8}$

(V) $E$ = at least two heads

$E = \{HHH, HHT, HTH, THH\}$

$n(E) = 4$

$P(E) = \frac{4}{8} = \frac{1}{2}$

(VI) $F$ = All heads

$F = \{HHH\} \Rightarrow n(F) = 1$

$P(F) = \frac{1}{8}$

Ques(4): One die is rolled. what is the prob. of getting

(I) a prime number (II) an odd number (III) an even number

Solution: Here, the sample space $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

(I) A = a prime number

$A = \{2, 3, 5\} \Rightarrow n(A) = 3$

$P(A) = \frac{3}{6} = \frac{1}{2}$

(II) B = an odd number

$B = \{1, 3, 5\} \Rightarrow n(B) = 3$

$P(B) = \frac{3}{6} = \frac{1}{2}$

(III) C = an even number

$C = \{2, 4, 6\} \Rightarrow n(C) = 3$

$P(C) = \frac{3}{6} = \frac{1}{2}$ Ans

Ques(5) Two dice are rolled simultaneously. what is the probability of getting (I) an even number as a sum (II) an odd number as a sum (III) Six as a product (IV) the sum as a prime number (V) a doublet i.e. same number on both dice.?

Solution: Here, the sample space S is given by

S =

| | | | | | |
|---|---|---|---|---|---|
| (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

$n(S) = 36$

(I) A = An even number as a sum
Exacly half combination of the above table have even no. as a sum

$n(A) = 18$

$P(A) = \frac{18}{36} = \frac{1}{2}$

(II) B = getting an odd no. as a sum

$n(B) = 18$

$P(B) = \frac{18}{36} = \frac{1}{2}$

(III) C = getting six as a product

$C = \{(1,6), (6,1), (2,3), (3,2)\}$

$n(C) = 4$

$P(C) = \frac{4}{36} = \frac{1}{9}$

(IV) D = sum as a prime number i.e. 2, 3, 5, 7, 11

$D = \{(1,1), (1,2), (2,1), (1,4), (4,1), (2,3), (3,2), (1,6), (6,1), (2,5), (5,2)$
$(3,4), (4,3), (5,6), (6,5)\}$

$n(D) = 15$

$P(D) = \frac{15}{36} = \frac{5}{12}$

(V) E = getting a doublet

$E = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(E) = 6$

$P(E) = \frac{6}{36} = \frac{1}{6}$

Ques (6) what is the probability that a leap year, selected at random, will contain 53 sundays

Solution: A leap year = 366 days = 52 weeks + 2 days ⇒ 52 sundays

For these 2 extra days, we have the following possible ways

(I) Su & Mo (II) Mo & Tu (III) Tu & Wd (IV) Wd & Th (V) Th & Fri

(VI) Fri & Sat (VII) Sat & Su

Total cases = 7 and only 2 are in favour of 53 Sundays

Required Prob. = $\frac{2}{7}$ Ans

Note :- (I) The number of ways selecting r Objects. from n distinct objects is $nC_r = \dfrac{\lfloor n}{\lfloor r \lfloor n-r}$

(II) $3C_2 = \dfrac{\lfloor 3}{\lfloor 2 \lfloor 1} = \dfrac{3\times 2}{2\times 1} = 3$

(III) $15C_2 = \dfrac{\lfloor 15}{\lfloor 2 \lfloor 13} = \dfrac{15\times 14\times \cancel{\lfloor 13}}{2\times \cancel{\lfloor 13}} = 15\times 7 = 105$

Ques (7) A bag contain 6 red , 4 white and 8 blue balls. If 3 balls are drawn at radom find the prob. that each ball is of different colour ?

Solution: Total number of balls $= 6+4+8 = 18$

The number of ways selecting 3 balls out of 18 balls $= 18C_3$

$$= \frac{\lfloor 18}{\lfloor 3 \lfloor 15} = \frac{18\times 17\times 16\times \cancel{\lfloor 15}}{3\times 2\times 1\times \cancel{\lfloor 15}}$$

$$= 3\times 17\times 16 = 51\times 16 = 816$$

E = each ball is of different colour i.e. One is red, one is white and one is blue.

The number of ways by selecting each ball of different colour $= 6C_1 \times 4C_1 \times 8C_1 = 6\times 4\times 8 = 24\times 8 = 192$.

$$P(E) = \frac{192}{816} = \frac{4}{17} \quad \text{Ans}$$

Ques (8) An urn contain 7 white, 5 black and 3 red balls. The balls are drawn at random. Find the probability that

(I) both the balls are red

(II) One ball is red and other is black

(III) one ball is white

Solution: Total number of balls $= 7+5+3 = 15$

$\therefore$ the no of ways in which 2 balls can be drawn from 15 balls

$$= {}^{15}C_2 = \frac{\lfloor 15}{\lfloor 2 \lfloor 13} = \frac{15 \times 14 \times \lfloor 13}{2 \times \lfloor 13} = 15 \times 7 = 105$$

(I) $E_1 =$ both the balls are red.

no of ways selecting two red balls out of 3 $= {}^3C_2 = \frac{\lfloor 3}{\lfloor 2 \lfloor 1} = \frac{3 \times 2 \times 1}{2 \times 1}$

$= 3$

$$P(E_1) = \frac{3}{105} = \frac{1}{35}$$

(II) $E_2 =$ one ball is red and other is black

no of ways selecting one red and one black ball $= {}^3C_1 \times {}^5C_1$

$= 3 \times 5 = 15$

$$P(E_2) = \frac{15}{105} = \frac{1}{7}$$

(III) one ball is white $= E_3$

no of ways selecting one white ball out of 7 $\Rightarrow {}^7C_1 = 7$

We have to draw two balls so one ball either can be black or red. No of ways selecting the second ball out of $= 15-7=8$ balls $= {}^8C_1 = 8$

Total number of ways selecting one white ball $= 7 \times 8 = 56$

$$P(E_3) = \frac{56}{105} = \frac{8}{15}$$

Ques (9) A box contain 3 white, 5 red and 8 blue balls. If 3 balls are drawn at random find the probability that

(I) all at them are white (II) two of them are white

(III) Exactly one is white (IV) at least one is white

(V) each of ball is of different colour.

Ans (I) $\frac{1}{560}$ (II) $\frac{39}{560}$ (III) $\frac{234}{560}$ (IV) $\frac{274}{560}$ (V) $\frac{3}{14}$

Ques (10) If a card is selected from a well-shuffled deck, what is probability of drawing (I) a spade (II) a king

(III) a king of spade?

Solution: Number of ways selecting a card from 52 cards $= {}^{52}C_1 = 52$

(I) $E_1$ = a spade

There are 13 cards of spade, therefore number of ways selecting one spade card $= {}^{13}C_1 = 13$

$$P(E_1) = \frac{13}{52} = \frac{1}{4}$$

(II) $E_2$ = a king

There are 4 cards of king so no of ways selecting one king card

$= {}^{4}C_1 = 4$

$$P(E_2) = \frac{4}{52} = \frac{1}{13}$$

(III) $E_3$ = a king of spade

only one card

$$P(E_3) = \frac{1}{52}$$