Example 3. *Find the co-efficient of performance and heat transfer rate in the condenser of a refrigerator in kJ/h which has a refrigeration capacity of 12000 kJ/h when power input is 0.75 kW.*

Solution. Refer to Fig. 11.

Refrigeration capacity, $Q_2 = 12000$ kJ/h

Power input, $W = 0.75$ kW $(= 0.75 \times 60 \times 60$ kJ/h$)$

Co-efficient of performance, C.O.P. :

Heat transfer rate :

$$(\text{C.O.P.})_{refrigerator} = \frac{\text{Heat absorbed at lower temperature}}{\text{Work input}}$$

$$\therefore \quad \text{C.O.P.} = \frac{Q_2}{W} = \frac{12000}{0.75 \times 60 \times 60} = 4.44$$

Hence **C.O.P. = 4.44. (Ans.)**

Hence transfer rate in condenser = Q_1

According to the first law

$Q_1 = Q_2 + W = 12000 + 0.75 \times 60 \times 60 = 14700$ kJ/h

Hence, **heat transfer rate = 14700 kJ/h. (Ans.)**

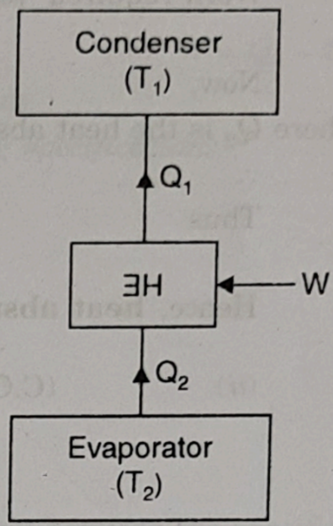


Fig. 11

Example 4. *A domestic food refrigerator maintains a temperature of – 12°C. The ambient air temperature is 35°C. If heat leaks into the freezer at the continuous rate of 2 kJ/s determine the least power necessary to pump this heat out continuously.*

Solution. Freezer temperature,

$$T_2 = -12 + 273 = 261 \text{ K}$$

Ambient air temperature,

$$T_1 = 35 + 273 = 308 \text{ K}$$

Rate of heat leakage into the freezer = 2 kJ/s

Least power required to pump the heat :

The refrigerator cycle removes heat from the freezer at the same rate at which heat leaks into it (Fig. 12).

For minimum power requirement

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}$$

$$\therefore \quad Q_1 = \frac{Q_2}{T_2} \times T_1 = \frac{2}{261} \times 308 = 2.36 \text{ kJ/s}$$

$$\therefore \quad W = Q_1 - Q_2$$

$$= 2.36 - 2 = 0.36 \text{ kJ/s} = 0.36 \text{ kW}$$

Hence, *least power required to pump the heat continuously*

= **0.36 kW. (Ans.)**

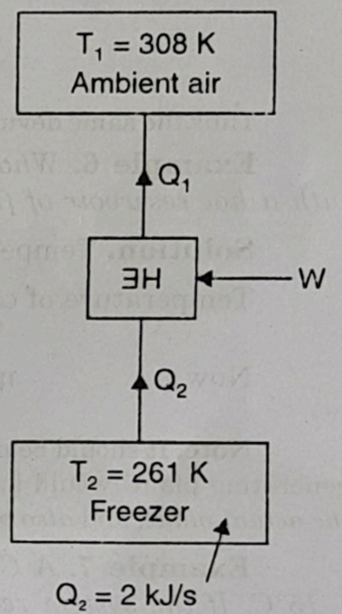


Fig. 12

Example 5. *A house requires 2×10^5 kJ/h for heating in winter. Heat pump is used to absorb heat from cold air outside in winter and send heat to the house. Work required to operate the heat pump is 3×10^4 kJ/h. Determine :*

(i) Heat abstracted from outside ;

(ii) Co-efficient of performance.

Solution. (*i*) Heat requirement of the house, Q_1 (or heat rejected)

$$= 2 \times 10^5 \text{ kJ/h}$$

Work required to operate the heat pump,

$$W = 3 \times 10^4 \text{ kJ/h}$$

Now, $$Q_1 = W + Q_2$$

where Q_2 is the heat abstracted from outside.

$\therefore$ $$2 \times 10^5 = 3 \times 10^4 + Q_2$$

Thus $$Q_2 = 2 \times 10^5 - 3 \times 10^4$$
$$= 200000 - 30000 = 170000 \text{ kJ/h}$$

Hence, **heat abstracted from outside = 170000 kJ/h. (Ans.)**

(*ii*) $$(\text{C.O.P.})_{heat\ pump} = \frac{Q_1}{Q_1 - Q_2}$$

$$= \frac{2 \times 10^5}{2 \times 10^5 - 170000} = 6.66$$

Hence, **co-efficient of performance = 6.66. (Ans.)**

Note. If the heat requirements of the house were the same but this amount of heat had to be abstracted from the house and rejected out, *i.e., cooling of the house in summer*, we have

$$(\text{C.O.P.})_{refrigerator} = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W}$$

$$= \frac{170000}{3 \times 10^4} = \mathbf{5.66}$$

Thus the same device has two values of C.O.P. depending upon the objective.

Example 6. *What is the highest possible theoretical efficiency of a heat engine operating with a hot reservoir of furnace gases at 2100°C when the cooling water available is at 15°C ?*

Solution. Temperature of furnace gases, $T_1 = 2100 + 273 = 2373$ K

Temperature of cooling water, $T_2 = 15 + 273 = 288$ K

Now, $$\eta_{max} \, (= \eta_{carnot}) = 1 - \frac{T_2}{T_1} = 1 - \frac{288}{2373} = \mathbf{0.878 \text{ or } 87.8\%.} \quad \textbf{(Ans.)}$$

Note. It should be noted that a system in practice operating between similar temperatures (*e.g.*, a steam generating plant) would have a thermal efficiency of about 30%. The discrepency is *due to irreversibility in the actual plant, and also because of deviations from the ideal Carnot cycle made for various practical reasons.*

Example 7. *A Carnot cycle operates between source and sink temperatures of 250°C and – 15°C. If the system receives 90 kJ from the source, find :*

(*i*) *Efficiency of the system ;* (*ii*) *The net work transfer ;*

(*iii*) *Heat rejected to sink.*

Solution. Temperature of source, $T_1 = 250 + 273 = 523$ K

Temperature of sink, $T_2 = -15 + 273 = 258$ K

Heat received by the system, $Q_1 = 90$ kJ

(*i*) $$\eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{258}{523} = 0.506 = \mathbf{50.6\%.} \quad \textbf{(Ans.)}$$

(*ii*) **The net work transfer,** $W = \eta_{carnot} \times Q_1$ $\left[\because \eta_{carnot} = \frac{W}{Q_1}\right]$

$= 0.506 \times 90 = $ **45.54 kJ. (Ans.)**

(*iii*) **Heat rejected to the sink,** $Q_2 = Q_1 - W$ $[\because W = Q_1 - Q_2]$

$= 90 - 45.54 = $ **44.46 kJ. (Ans.)**

Example 8. *An inventor claims that his engine has the following specifications :*

Temperature limits ...... *750°C and 25°C*

Power developed ...... *75 kW*

Fuel burned per hour ...... *3.9 kg*

Heating value of the fuel ...... *74500 kJ/kg*

State whether his claim is valid or not.

Solution. Temperature of source, $T_1 = 750 + 273 = 1023$ K

Temperature of sink, $T_2 = 25 + 273 = 298$ K

We know that the thermal efficiency of Carnot cycle is the maximum between the specified temperature limits.

Now, $$\eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{298}{1023} = 0.7086 \text{ or } 70.86\%$$

The actual thermal efficiency claimed,

$$\eta_{thermal} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{75 \times 1000 \times 60 \times 60}{3.9 \times 74500 \times 1000} = 0.9292 \text{ or } 92.92\%.$$

Since $\eta_{thermal} > \eta_{carnot}$, therefore claim of the inventor is **not valid** (or possible). **(Ans.)**

Example 9. *A cyclic heat engine operates between a source temperature of 1000°C and a sink temperature of 40°C. Find the least rate of heat rejection per kW net output of the engine ?*

Solution. Temperature of source,

$T_1 = 1000 + 273 = 1273$ K

Temperature of sink,

$T_2 = 40 + 273 = 313$ K

Least rate of heat rejection per kW net output :

For a reversible heat engine, the rate of heat rejection will be minimum (Fig. 13).

$$\eta_{max} = \eta_{rev.} = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{313}{1273} = 0.754$$

Now $$\frac{W_{net}}{Q_1} = \eta_{max} = 0.754$$

$\therefore$ $$Q_1 = \frac{W_{net}}{0.754} = \frac{1}{0.754} = 1.326 \text{ kW}$$

Now $$Q_2 = Q_1 - W_{net} = 1.326 - 1 = 0.326 \text{ kW}$$

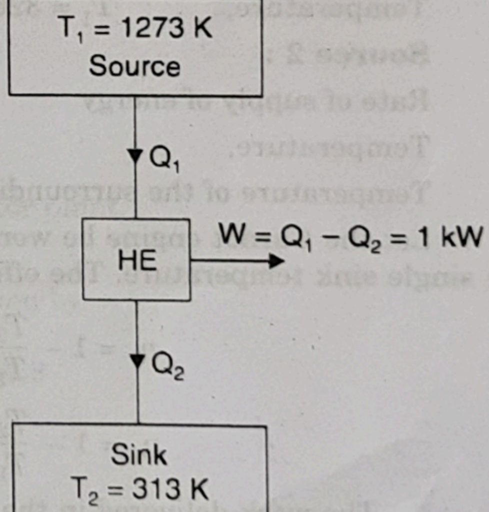


Fig. 13

Hence, **the least rate of heat rejection = 0.326 kW. (Ans.)**

Example 10. *A fish freezing plant requires 40 tons of refrigeration. The freezing temperature is – 35°C while the ambient temperature is 30°C. If the performance of the plant is 20% of the theoretical reversed Carnot cycle working within the same temperature limits, calculate the power required.*

Given : 1 ton of refrigeration = 210 kJ/min.

Solution. Cooling required = 40 tons = 40 × 210

= 8400 kJ/min

Ambient temperature, $T_1 = 30 + 273 = 303$ K

Freezing temperature, $T_2 = -35 + 273 = 238$ K

Performance of plant = 20% of the theoretical reversed Carnot cycle

$$(\text{C.O.P.})_{refrigerator} = \frac{T_2}{T_1 - T_2} = \frac{238}{303 - 238} = 3.66$$

$\therefore$ Actual C.O.P = 0.20 × 3.66 = 0.732

Now work needed to produce cooling of 40 tons is calculated as follows :

$$(\text{C.O.P.})_{actual} = \frac{\text{Cooling reqd.}}{\text{Work needed}}$$

$$0.732 = \frac{8400}{W} \quad \text{or} \quad W = \frac{8400}{0.732} \text{ kJ/min} = 191.25 \text{ kJ/s} = 191.25 \text{ kW}$$

Hence, **power required = 191.25 kW. (Ans.)**

Example 11. *Source 1 can supply energy at the rate of 12000 kJ/min at 320°C. A second source 2 can supply energy at the rate of 120000 kJ/min at 70°C. Which source (1 or 2) would you choose to supply energy to an ideal reversible heat engine that is to produce large amount of power if the temperature of the surroundings is 35°C ?*

Solution. Source 1 :

Rate of supply of energy = 12000 kJ/min

Temperature, $T_1 = 320 + 273 = 593$ K.

Source 2 :

Rate of supply of energy = 120000 kJ/min

Temperature, $T_1 = 70 + 273 = 343$ K

Temperature of the surroundings, $T_2 = 35 + 273 = 308$ K

Let the Carnot engine be working in the two cases with the two source temperatures and the single sink temperature. The efficiency of the cycle will be given by :

$$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{308}{593} = 0.4806 \quad \text{or} \quad 48.06\%$$

$$\eta_2 = 1 - \frac{T_2}{T_1} = 1 - \frac{308}{343} = 0.102 \quad \text{or} \quad 10.2\%$$

$\therefore$ The work delivered in the two cases is given by

$$W_1 = 12000 \times 0.4806 = 5767.2 \text{ kJ/min}$$

and $$W_2 = 120000 \times 0.102 = 12240 \text{ kJ/min.}$$

Thus, choose **source 2. (Ans.)**

Note. The source 2 is selected even though *efficiency* in this case is *lower*, because the *criterion for selection is the larger output.*

Example 12. *A reversible heat engine operates between two reservoirs at temperatures 700°C and 50°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 50°C and – 25°C. The heat transfer to the engine is 2500 kJ and the net work output of the combined engine refrigerator plant is 400 kJ.*

(i) Determine the heat transfer to the refrigerant and the net heat transfer to the reservoir at 50°C ;

(ii) Reconsider (i) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 45 per cent of their maximum possible values.

Solution. Refer Fig. 14.

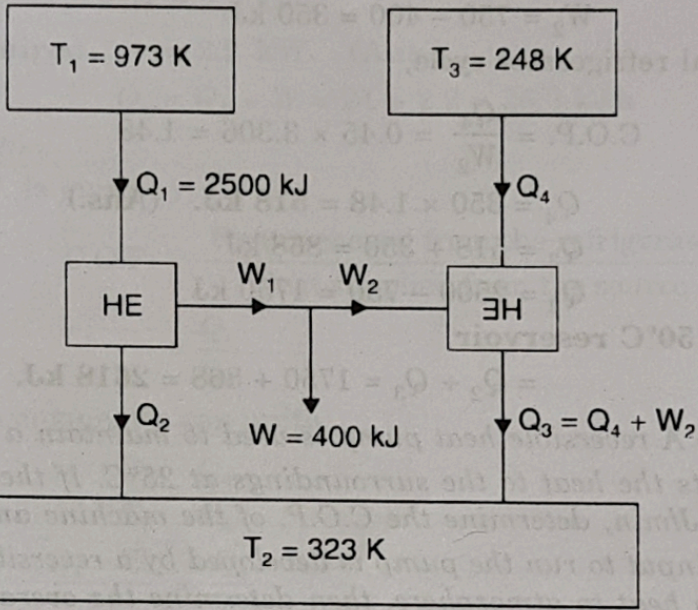


Fig. 14

Temperature, $T_1 = 700 + 273 = 973$ K

Temperature, $T_2 = 50 + 273 = 323$ K

Temperature, $T_3 = -25 + 273 = 248$ K

The heat transfer to the heat engine, $Q_1 = 2500$ kJ

The network output of the combined engine refrigerator plant,

$$W = W_1 - W_2 = 400 \text{ kJ}.$$

(*i*) Maximum efficiency of the heat engine cycle is given by

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{323}{973} = 0.668$$

Again,
$$\frac{W_1}{Q_1} = 0.668$$

$\therefore$
$$W_1 = 0.668 \times 2500 = 1670 \text{ kJ}$$

$$(\text{C.O.P.})_{max} = \frac{T_3}{T_2 - T_3} = \frac{248}{323 - 248} = 3.306$$

Also,
$$\text{C.O.P.} = \frac{Q_4}{W_2} = 3.306$$

Since, $W_1 - W_2 = W = 400$ kJ

$$W_2 = W_1 - W = 1670 - 400 = 1270 \text{ kJ}$$

$\therefore$ $$Q_4 = 3.306 \times 1270 = 4198.6 \text{ kJ}$$

$$Q_3 = Q_4 + W_2 = 4198.6 + 1270 = 5468.6 \text{ kJ}$$

$$Q_2 = Q_1 - W_1 = 2500 - 1670 = 830 \text{ kJ.}$$

Heat rejection to the 50°C reservoir

$$= Q_2 + Q_3 = 830 + 5468.6 = \mathbf{6298.6\ kJ.} \quad \textbf{(Ans.)}$$

(*ii*) Efficiency of actual heat engine cycle,

$$\eta = 0.45\ \eta_{max} = 0.45 \times 0.668 = 0.3$$

$\therefore$ $$W_1 = \eta \times Q_1 = 0.3 \times 2500 = 750 \text{ kJ}$$

$\therefore$ $$W_2 = 750 - 400 = 350 \text{ kJ}$$

C.O.P. of the actual refrigerator cycle,

$$\text{C.O.P.} = \frac{Q_4}{W_2} = 0.45 \times 3.306 = 1.48$$

$\therefore$ $$Q_4 = 350 \times 1.48 = \mathbf{518\ kJ.} \quad \textbf{(Ans.)}$$

$$Q_3 = 518 + 350 = 868 \text{ kJ}$$

$$Q_2 = 2500 - 750 = 1750 \text{ kJ}$$

Heat rejected to 50°C reservoir

$$= Q_2 + Q_3 = 1750 + 868 = \mathbf{2618\ kJ.} \quad \textbf{(Ans.)}$$

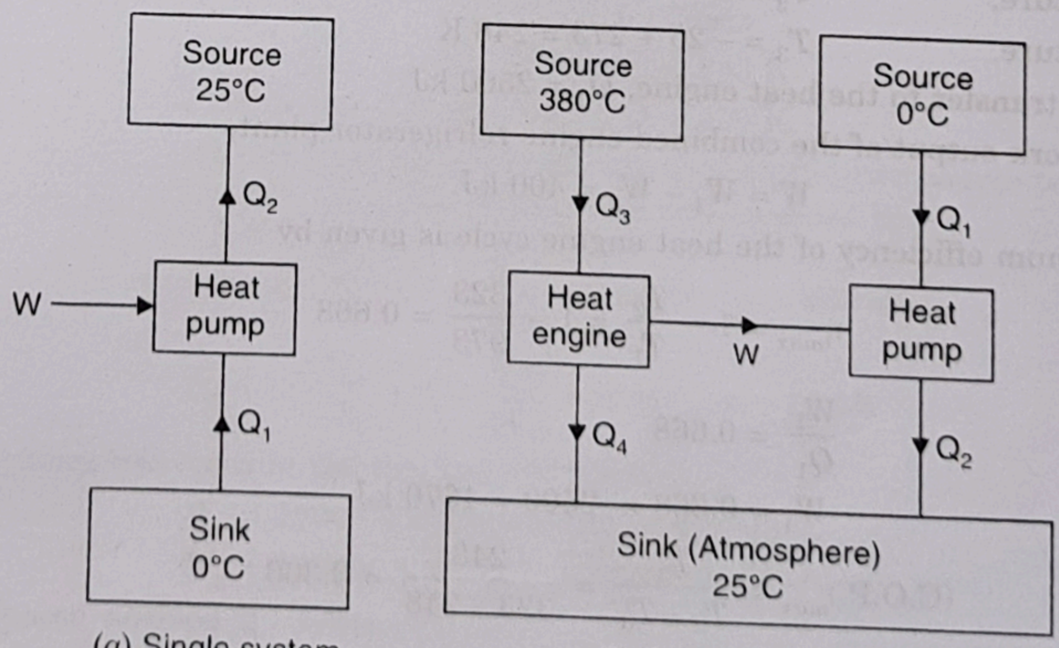
☞**Example 13.** (*i*) *A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 25°C. If the heat removal rate from the refrigerator is 1440 kJ/min, determine the C.O.P. of the machine and work input required.*

(*ii*) *If the required input to run the pump is developed by a reversible engine which receives heat at 380°C and rejects heat to atmosphere, then determine the overall C.O.P. of the system.*

Solution. Refer Fig. 15 (*a*).

(*i*) Temperature, $T_1 = 25 + 273 = 298$ K

Temperature, $T_2 = 0 + 273 = 273$ K



(*a*) Single system (*b*) Combined system

Fig. 15

Heat removal rate from the refrigerator,

$$Q_1 = 1440 \text{ kJ/min} = 24 \text{ kJ/s}$$

Now, co-efficient of performance, for reversible heat pump,

$$\textbf{C.O.P.} = \frac{T_1}{T_1 - T_2} = \frac{298}{(298-273)} = \textbf{11.92.} \quad \textbf{(Ans.)}$$

$$\therefore \quad (\text{C.O.P.})_{ref.} = \frac{T_2}{T_1 - T_2} = \frac{273}{298-273} = 10.92$$

Now,

$$10.92 = \frac{Q_1}{W} = \frac{24}{W}$$

$$\therefore \quad W = 2.2 \text{ kW}$$

i.e., **Work input required** $= \textbf{2.2 kW.}$ **(Ans.)**

$$Q_2 = Q_1 + W = 24 + 2.2 = 26.2 \text{ kJ/s}$$

(*ii*) Refer Fig. 15 (*b*).

The overall C.O.P. is given by,

$$\text{C.O.P.} = \frac{\text{Heat removed from the refrigerator}}{\text{Heat supplied from the source}}$$

$$= \frac{Q_1}{Q_3} \quad ...(i)$$

For the reversible engine, we can write

$$\frac{Q_3}{T_3} = \frac{Q_4}{T_4}$$

or $$\frac{Q_4 + W}{T_3} = \frac{Q_4}{T_4}$$

or $$\frac{Q_4 + 2.2}{(380+273)} = \frac{Q_4}{(25+273)}$$

or $$\frac{Q_4 + 2.2}{653} = \frac{Q_4}{298}$$

or $$298(Q_4 + 2.2) = 653\, Q_4$$

or $$Q_4(653 - 298) = 298 \times 2.2$$

or $$Q_4 = \frac{298 \times 2.2}{(653-298)} = 1.847 \text{ kJ/s}$$

$$\therefore \quad Q_3 = Q_4 + W = 1.847 + 2.2 = 4.047 \text{ kJ/s}$$

Substituting this value in eqn. (*i*), we get

$$\textbf{C.O.P.} = \frac{24}{4.047} = \textbf{5.93.} \quad \textbf{(Ans.)}$$

If the purpose of the system is to supply the heat to the sink at 25°C, then

$$\textbf{Overall C.O.P.} = \frac{Q_2 + Q_4}{Q_3} = \frac{26.2 + 1.847}{4.047} = \textbf{6.93.} \quad \textbf{(Ans.)}$$

Example 14. *An ice plant working on a reversed Carnot cycle heat pump produces 15 tonnes of ice per day. The ice is formed from water at 0°C and the formed ice is maintained at 0°C. The heat is rejected to the atmosphere at 25°C. The heat pump used to run the ice plant is* *coupled to a Carnot engine which absorbs heat from a source which is maintained at 220°C by burning liquid fuel of 44500 kJ/kg calorific value and rejects the heat to the atmosphere. Determine :*

(*i*) *Power developed by the engine ;*

(*ii*) *Fuel consumed per hour.*

Take enthalpy of fusion of ice = 334.5 kJ/kg.

Solution. (*i*) Fig. 16 shows the arrangement of the system.

Amount of ice produced per day = 15 tonnes.

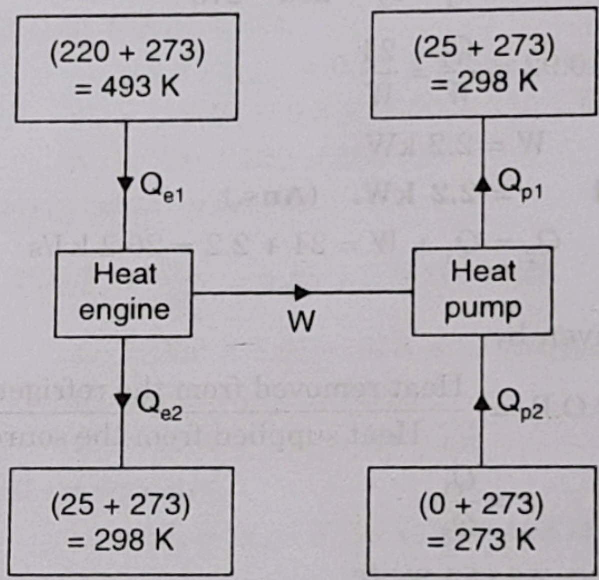


Fig. 16

∴ The amount of heat removed by the heat pump,

$$Q_{p_2} = \frac{15 \times 1000 \times 334.5}{24 \times 60} = 3484.4 \text{ kJ/min}$$

$$\text{C.O.P. of the heat pump} = \frac{Q_{p_2}}{W} = \frac{273}{298 - 273}$$

$$\therefore \quad W = Q_{p_2} \times \frac{298 - 273}{273} = 3484.4 \times \frac{25}{273} = 319.08 \text{ kJ/min}$$

This work must be developed by the Carnot engine,

$$W = \frac{319.08}{60} = 5.3 \text{ kJ/s} = 5.3 \text{ kW}$$

Thus **power developed by the engine = 5.3 kW. (Ans.)**

(*ii*) The efficiency of Carnot engine is given by

$$\eta_{carnot} = \frac{W}{Q_{e_1}} = 1 - \frac{298}{493} = 0.396$$

$$\therefore \quad Q_{e_1} = \frac{W}{0.396} = \frac{5.3}{0.396} = 13.38 \text{ kJ/s}$$

$$\therefore \quad Q_{e_1 (per\ hour)} = 13.38 \times 60 \times 60 = 48168 \text{ kJ}$$

∴ **Quantity of fuel consumed/hour**

$$= \frac{48168}{44500} = \mathbf{1.082 \text{ kg/h}}. \quad \textbf{(Ans.)}$$

Example 15. *Two Carnot engines work in series between the source and sink temperatures of 550 K and 350 K. If both engines develop equal power determine the intermediate temperature.*

Solution. Fig. 17 shows the arrangement of the system.

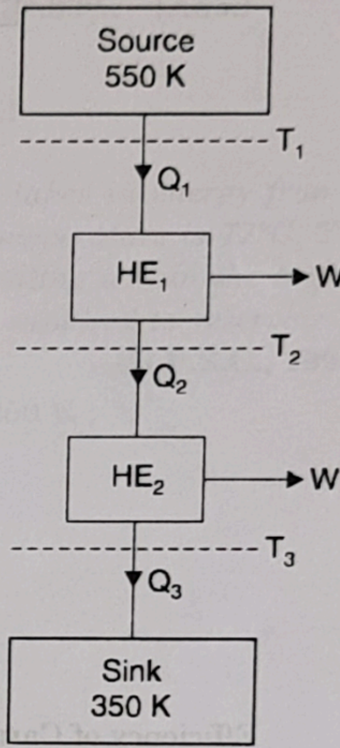


Fig. 17

Temperature of the source, $T_1 = 550$ K

Temperature of the sink, $T_3 = 350$ K

Intermediate temperature, T_2 :

The efficiencies of the engines HE_1 and HE_2 are given by

$$\eta_1 = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{W}{Q_2 + W} \quad \text{...}(i)$$

$$\eta_2 = \frac{W}{Q_2} = \frac{T_2 - T_3}{T_2} = \frac{W}{Q_3 + W} \quad \text{...}(ii)$$

From eqn. (*i*), we get

$$W = (Q_2 + W)\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \quad W\left[1 - \left(\frac{T_1 - T_2}{T_1}\right)\right] = Q_2\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \quad W\left(\frac{T_2}{T_1}\right) = Q_2\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \quad W = Q_2\left(\frac{T_1 - T_2}{T_2}\right) \quad \text{...}(iii)$$

From eqn. (*ii*), we get

$$W = Q_2\left(\frac{T_2 - T_3}{T_2}\right) \quad \text{...}(iv)$$

Now from eqns. (*iii*) and (*iv*), we get

$$T_1 - T_2 = T_2 - T_3$$

$$2T_2 = T_1 + T_3 = 550 + 350$$

$$\therefore \quad T_2 = 450 \text{ K}$$

Hence **intermediate temperature = 450 K. (Ans.)**

Example 16. *A Carnot heat engine draws heat from a reservoir at temperature T_1 and rejects heat to another reservoir at temperature T_3. The Carnot forward cycle engine drives a Carnot reversed cycle engine or Carnot refrigerator which absorbs heat from reservoir at temperature T_2 and rejects heat to a reservoir at temperature T_3. If the high temperature T_1 = 600 K and low temperature T_2 = 300 K, determine :*

(*i*) *The temperature T_3 such that heat supplied to engine Q_1 is equal to the heat absorbed by refrigerator Q_2.*

(*ii*) *The efficiency of Carnot engine and C.O.P. of Carnot refrigerator.*

Solution. Refer Fig. 18.

Temperature, $T_1 = 600$ K

Temperature, $T_2 = 300$ K

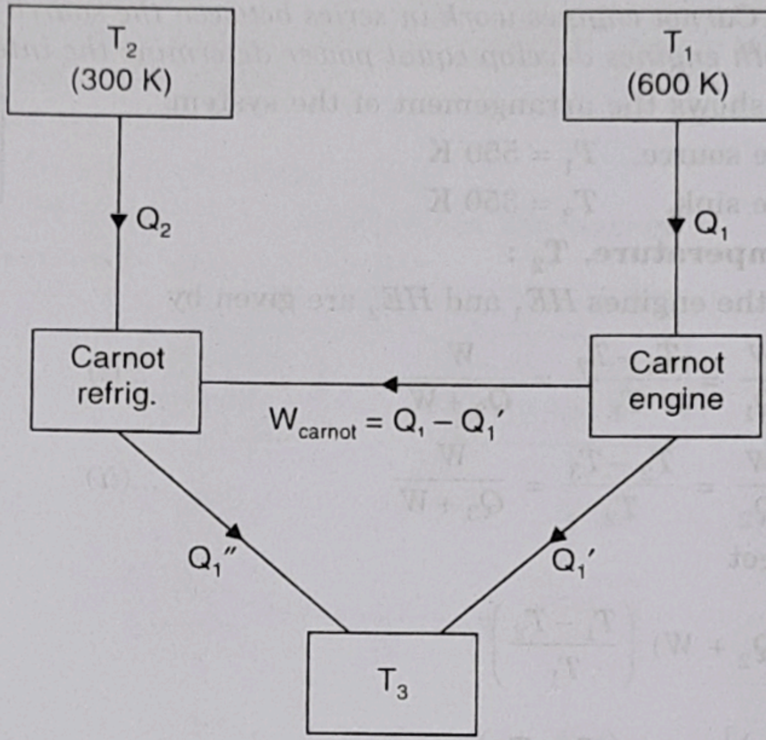


Fig. 18

Efficiency of Carnot engine,

$$\eta_{carnot\ engine} = \frac{Q_1 - Q_1'}{Q_1} = \frac{T_1 - T_3}{T_1}$$

$$= \frac{\text{Work of Carnot engine}}{\text{Heat supplied to the Carnot engine}} = \frac{W_{carnot}}{Q_1}$$

or
$$W_{carnot} = Q_1\left(\frac{T_1 - T_3}{T_1}\right)$$

Also
$$\text{C.O.P.}_{(carnot\ refrigerator)} = \frac{Q_2}{Q_1'' - Q_2} = \frac{T_2}{T_3 - T_2}$$

$$= \frac{\text{Heat absorbed}}{W_{carnot}} = \frac{Q_2}{W_{carnot}}$$

or
$$W_{carnot} = Q_2\left(\frac{T_3 - T_2}{T_2}\right)$$

(*i*) **Temperature, T_3 :**

From eqns. (*i*) and (*ii*), we get

$$Q_1\left(\frac{T_1 - T_3}{T_1}\right) = Q_2\left(\frac{T_3 - T_2}{T_2}\right)$$

$\therefore$
$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}\left(\frac{T_1 - T_3}{T_3 - T_2}\right)$$

or
$$\frac{Q_2}{Q_1} = 1 = \frac{300}{600}\left(\frac{600 - T_3}{T_3 - 300}\right)$$

or
$$600 - T_3 = 2(T_3 - 300)$$
$$600 - T_3 = 2T_3 - 600 \quad \text{or} \quad T_3 = 400\text{ K}$$

Hence, **temperature, T_3 = 400 K. (Ans.)**

☞**Example 20.** *In a power plant cycle, the temperature range is 164°C to 51°C, the upper temperature being maintained in the boiler where heat is received and the lower temperature being maintained in the condenser where heat is rejected. All other processes in the steady flow cycle are adiabatic. The specific enthalpies at various points are given in Fig. 20.*

Verify the Clausius Inequality.

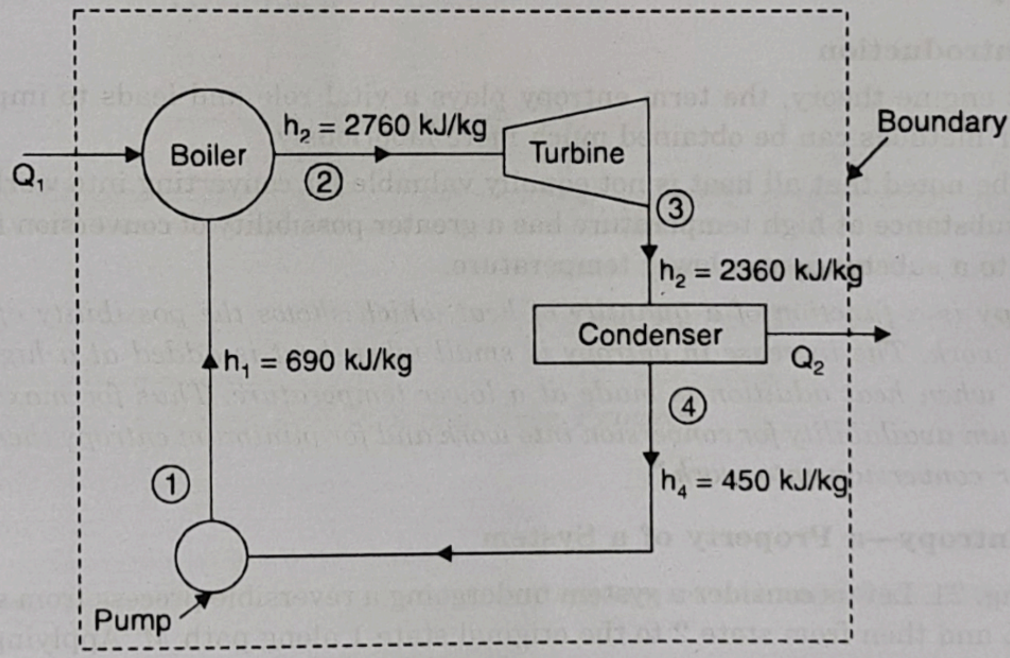


Fig. 20

Solution. Temperature maintained in boiler, $T_1 = 164 + 273 = 437$ K

Temperature maintained in condenser, $T_2 = 51 + 273 = 324$ K

Heat transferred in the boiler per kg of fluid,

$$Q_1 = h_2 - h_1 = 2760 - 690 = 2070 \text{ kJ/kg}$$

Heat transferred out at the condenser per kg of fluid,

$$Q_2 = h_4 - h_3 = 450 - 2360 = -1910 \text{ kJ/kg}$$

Since there is no transfer of heat at any other point, we have per kg

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{2070}{437} + \left(\frac{-1910}{324}\right)$$

$$= 4.737 - 5.895$$

$$= -1.158 \text{ kJ/kg K} < 0.$$

The Clausius Inequality is proved. The steady flow cycle is obviously irreversible.

If the cycle is reversible between the same temperature limits and the heat supplied at higher temperature is same, the heat rejected can be calculated as follows :

$$\eta_{reversible} = 1 - \frac{T_2}{T_1} = 1 - \frac{324}{437} = 0.2586 \text{ or } 25.86\%$$

$\therefore$ Heat rejected per kg is given by

$$Q_2 = (1 - 0.2586) \times Q_1 = (1 - 0.2586) \times 2070 = 1534.7 \text{ kJ/kg}$$

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{2070}{437} - \frac{1534.7}{324} = 4.73 - 4.73 = 0$$

i.e.,

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_{added}}{T_{source}} - \frac{Q_{rejected}}{T_{sink}} = 0$$

Thus **Clausius Equality sign for a reversible engine is verified.**

12. ENTROPY

12.1. Introduction

In heat engine theory, the term entropy plays a vital role and leads to important results which by other methods can be obtained much more laboriously.

It may be noted that all heat is not equally valuable for converting into work. Heat that is supplied to a substance at high temperature has a greater possibility of conversion into work than heat supplied to a substance at a lower temperature.

"***Entropy*** *is a function of a quantity of heat which shows the possibility of conversion of that heat into work. The increase in entropy is small when heat is added at a high temperature and is greater when heat addition is made at a lower temperature. Thus for maximum entropy, there is minimum availability for conversion into work and for minimum entropy there is maximum availability for conversion into work.*"

12.2. Entropy—a Property of a System

Refer Fig. 21. Let us consider a system undergoing a reversible process from state 1 to state 2 along path L and then from state 2 to the original state 1 along path M. Applying the Clausius theorem to this reversible cyclic process, we have

$$\oint_R \frac{\delta Q}{T} = 0$$

(where the subscript designates a reversible cycle)

Hence when the system passes through the cycle 1-L-2-M-1, we have

$$\int_{1(L)}^{2} \frac{\delta Q}{T} + \int_{2(M)}^{1} \frac{\delta Q}{T} = 0 \qquad ...(16)$$

Now consider another reversible cycle in which the system changes from state 1 to state 2 along path L, but returns from state 2 to the original state 1 along a different path N. For this reversible cyclic process, we have

$$\int_{1(L)}^{2} \frac{\delta Q}{T} + \int_{2(N)}^{1} \frac{\delta Q}{T} = 0 \qquad ...(17)$$

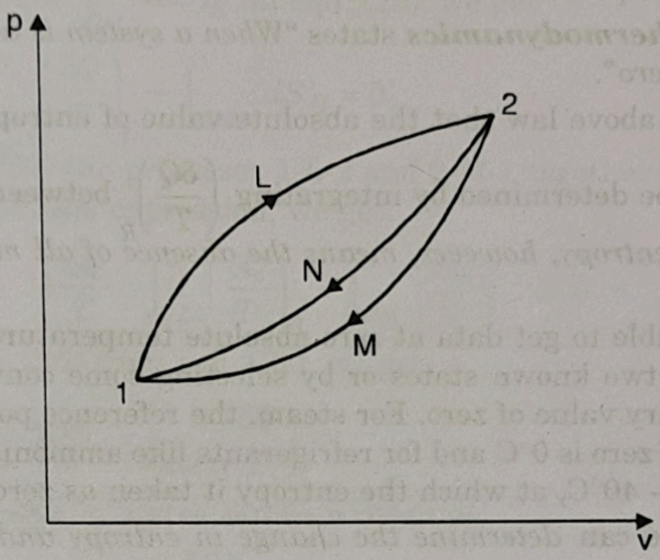


Fig. 21. Reversible cyclic process between two fixed end states.

Subtracting equation (17) from equation (16), we have

$$\int_{2(M)}^{1} \frac{\delta Q}{T} - \int_{2(N)}^{1} \frac{\delta Q}{T} = 0$$

or
$$\int_{1}^{2(M)} \frac{\delta Q}{T} = \int_{1}^{2(N)} \frac{\delta Q}{T}$$

As no restriction is imposed on paths L and M, except that they must be reversible, the *quantity* $\frac{\delta Q}{T}$ *is a function of the initial and final states of the system and is independent of the path of the process. Hence it represents a property of the system.* This property is known as the **"entropy".**

12.3. Change of Entropy in a Reversible Process

Refer Fig. 21.

Let S_1 = Entropy at the initial state 1, and

S_2 = Entropy at the final state 2.

Then, the change in entropy of a system, as it undergoes a change from state 1 to 2, becomes

$$S_2 - S_1 = \int_{1}^{2} \left(\frac{\delta Q}{T}\right)_R \qquad ...(18)$$

Lastly, if the two equilibrium states 1 and 2 are infinitesimal near to each other, the integral sign may be omitted and $S_2 - S_1$ becomes equal to dS.

3. It remains unchanged in all adiabatic frictionless processes.

4. It increases if temperature of heat is lowered without work being done as in a throttling process.

17. ENTROPY CHANGES FOR A CLOSED SYSTEM

17.1. General Case for Change of Entropy of a Gas

Let 1 kg of gas at a pressure p_1, volume v_1, absolute temperature T_1 and entropy s_1, be heated such that its final pressure, volume, absolute temperature and entropy are p_2, v_2, T_2 and s_2 respectively. Then by law of conservation of energy,

$$dQ = du + dW$$

where, dQ = Small change of heat,

du = Small internal energy, and

dW = Small change of work done (pdv).

Now $$dQ = c_v dT + pdv$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = \frac{c_v dT}{T} + \frac{pdv}{T}$$

But $$\frac{dQ}{T} = ds$$

and as $$pv = RT$$

$$\therefore \quad \frac{p}{T} = \frac{R}{v}$$

Hence $$ds = \frac{c_v dT}{T} + R\frac{dv}{v}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R\int_{v_1}^{v_2} \frac{dv}{v}$$

or $$(s_2 - s_1) = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1} \quad \text{...(28)}$$

This expression can be reproduced in the following way :

According to the gas equation, we have

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or $$\frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{v_2}{v_1}$$

Substituting the value of $\frac{T_2}{T_1}$ in eqn. (28), we get

$$s_2 - s_1 = c_v \log_e \frac{p_2}{p_1} \times \frac{v_2}{v_1} + R \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + c_v \log_e \frac{v_2}{v_1} + R \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + (c_v + R) \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}$$

$$\therefore \quad s_2 - s_1 = c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1} \quad \text{...(29)}$$

Again, from gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

Putting the value of $\frac{v_2}{v_1}$ in eqn. (28), we get

$$(s_2 - s_1) = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

$$= c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{p_1}{p_2} + R \log_e \frac{T_2}{T_1}$$

$$= (c_v + R) \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

$$= c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

$$\therefore \quad s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}. \quad \text{...(30)}$$

17.2. Heating a Gas at Constant Volume

Refer Fig. 25. Let 1 kg of gas be heated at constant volume and let the change in entropy and absolute temperature be from s_1 to s_2 and T_1 to T_2 respectively.

Then $\quad Q = c_v(T_2 - T_1)$

Differentiating to find small increment of heat dQ corresponding to small rise in temperature dT.

$$dQ = c_v dT$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = c_v \cdot \frac{dT}{T}$$

or
$$ds = c_v \cdot \frac{dT}{T}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_v \int_{T_1}^{T_2} \frac{dT}{T}$$

or
$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} \quad \text{...(31)}$$

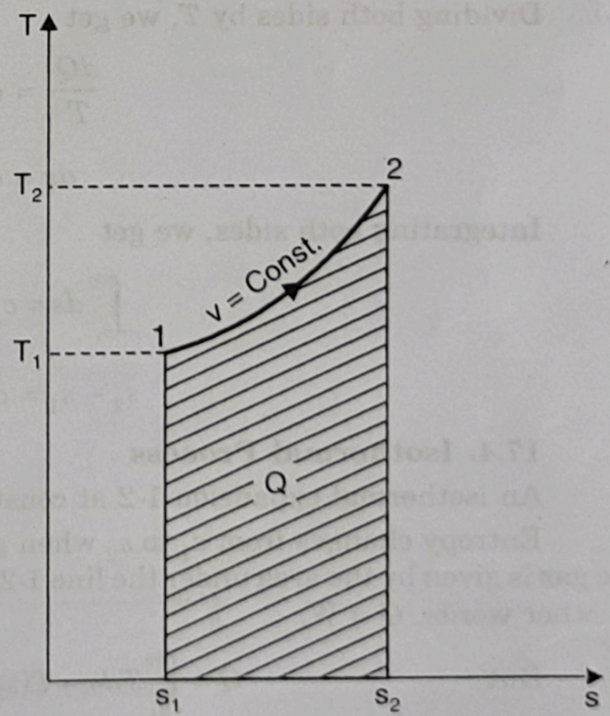


Fig. 25. *T-s* diagram : Constant volume process

17.3. Heating a Gas at Constant Pressure

Refer Fig. 26. Let 1 kg of gas be heated at constant pressure, so that its absolute temperature changes from T_1 to T_2 and entropy s_1 to s_2.

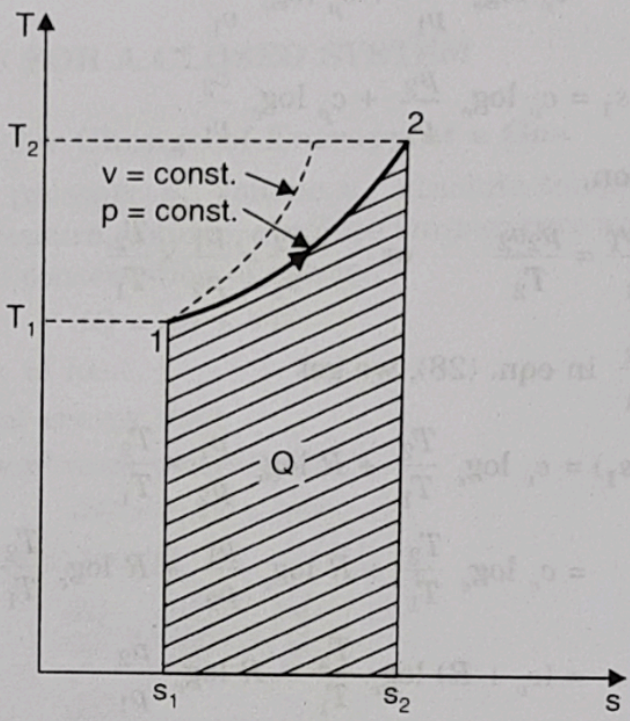


Fig. 26. *T-s* diagram : Constant pressure process.

Then, $$Q = c_p(T_2 - T_1).$$

Differentiating to find small increase in heat, dQ of this gas when the temperature rise is dT.

$$dQ = c_p \cdot dT$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = c_p \cdot \frac{dT}{T}$$

or

$$ds = c_p \cdot \frac{dT}{T}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_p \int_{T_1}^{T_2} \frac{dT}{T}$$

$$s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} \qquad \text{...(32)}$$

17.4. Isothermal Process

An isothermal expansion 1-2 at constant temperature T is shown in Fig. 27.

Entropy changes from s_1 to s_2 when gas absorbs heat during expansion. The heat taken by the gas is given by the area under the line 1-2 which also represents the work done during expansion. In other words, $Q = W$.

But $$Q = \int_{s_1}^{s_2} T ds = T(s_2 - s_1)$$

and $$W = p_1v_1 \log_e \frac{v_2}{v_1} = RT_1 \log_e \frac{v_2}{v_1} \text{ per kg of gas} \qquad [\because \; p_1v_1 = RT_1]$$

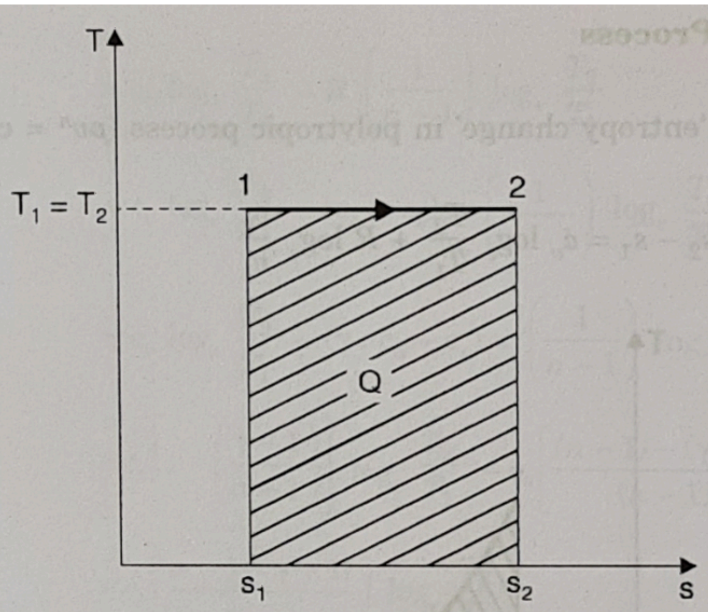


Fig. 27. *T-s* diagram : Isothermal process.

$\therefore \qquad T(s_2 - s_1) = RT_1 \log_e \dfrac{v_2}{v_1}$

or $\qquad s_2 - s_1 = R \log_e \dfrac{v_2}{v_1}$. $\qquad [\because \quad T_1 = T_2 = T]$ ...(33)

17.5. Adiabatic Process (Reversible)

During an adiabatic process as heat is neither supplied nor rejected,

$$dQ = 0$$

or $$\frac{dQ}{dT} = 0$$

or $$ds = 0 \qquad \text{...(34)}$$

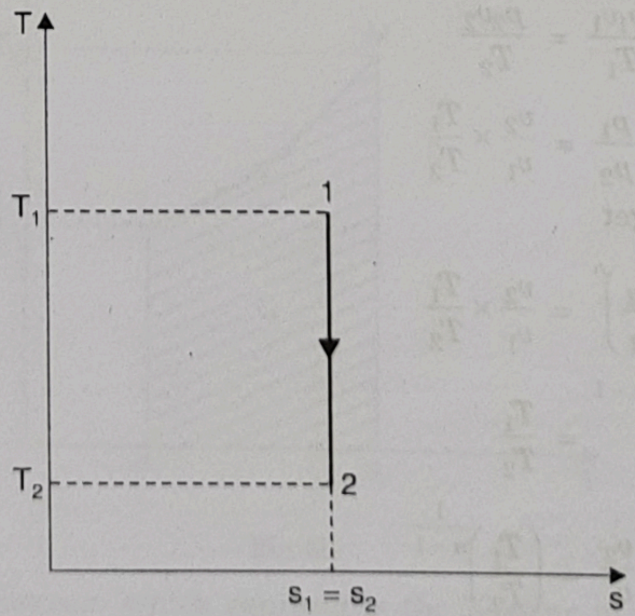


Fig. 28. *T-s* diagram : Adiabatic process.

This shows that there is no change in entropy and hence it is known as *isentropic process*.

Fig. 28 represents an adiabatic process. It is a vertical line (1-2) and therefore area under this line is nil ; hence heat supplied or rejected and entropy change is zero.

17.6. Polytropic Process

Refer Fig. 29.

The expression for 'entropy change' in polytropic process (pv^n = constant) can be obtained from eqn. (28)

i.e.,
$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$$

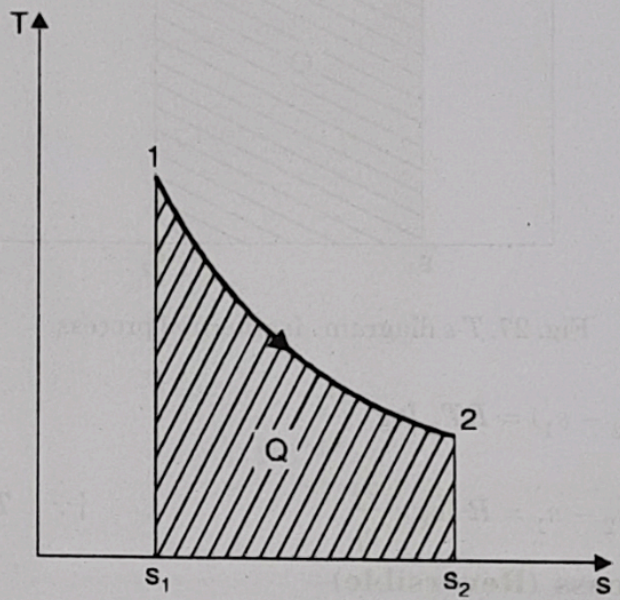


Fig. 29. *T-s* diagram : Polytropic process.

Also
$$p_1 v_1^{\,n} = p_2 v_2^{\,n}$$

or
$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n \quad ...(i)$$

Again, as
$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or
$$\frac{p_1}{p_2} = \frac{v_2}{v_1} \times \frac{T_1}{T_2} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\left(\frac{v_2}{v_1}\right)^n = \frac{v_2}{v_1} \times \frac{T_1}{T_2}$$

or
$$\left(\frac{v_2}{v_1}\right)^{n-1} = \frac{T_1}{T_2}$$

or
$$\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

Substituting the value of $\frac{v_2}{v_1}$ in eqn. (28), we get

$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}} = c_v \log_e \frac{T_2}{T_1} + R\left(\frac{1}{n-1}\right) \log_e \frac{T_1}{T_2}$$

$$= c_v \log_e \frac{T_2}{T_1} - R\left(\frac{1}{n-1}\right) \log_e \frac{T_2}{T_1}$$

$$= c_v \log_e \frac{T_2}{T_1} - (c_p - c_v) \times \left(\frac{1}{n-1}\right) \log_e \frac{T_2}{T_1} \qquad [\because \quad R = c_p - c_v]$$

$$= c_v \log_e \frac{T_2}{T_1} - (\gamma \,.\, c_v - c_v) \times \left(\frac{1}{n-1}\right) \log_e \frac{T_2}{T_1} \qquad [\because \quad c_p = \gamma \,.\, c_v]$$

$$= c_v\left[1 - \left(\frac{\gamma - 1}{n-1}\right)\right] \log_e \frac{T_2}{T_1} = c_v\left[\frac{(n-1)-(\gamma-1)}{(n-1)}\right] \log_e \frac{T_2}{T_1}$$

$$= c_v\left(\frac{n - 1 - \gamma + 1)}{n-1}\right) \log_e \frac{T_2}{T_1}$$

$$= c_v \,.\, \left(\frac{n-\gamma}{n-1}\right) \log_e \frac{T_2}{T_1} \text{ per kg of gas}$$

$$\therefore \qquad s_2 - s_1 = c_v \left(\frac{n-\gamma}{n-1}\right) \log_e \frac{T_2}{T_1} \text{ per kg of gas} \qquad \ldots(35)$$