Sub: Engineering Mathematics – II (BMAS - 1102)

[1] Find rank of following matrices by reducing into Echelon Form

(1.1) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ (1.2) $\begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$ (1.3) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

[2] For which value of ‘**b**’ the rank of the following matrix is **2 ?**

(2.1) $\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$ (2.2) $\begin{bmatrix} 2 & -1 & 4 \\ 9 & 7 & 3 \\ 5 & b & -5 \end{bmatrix}$ (2.3) $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & b \end{bmatrix}$

[3] Test the consistency and hence solve the following systems of linear equations

(3.1) $x - y + 2z = 3,\ x + 2y + 3z = 5,\ 3x - 4y - 5z = -13$

(3.2) $x_1 + 2x_2 - x_3 = 1,\ 3x_1 - 2x_2 + 2x_3 = 2,\ 7x_1 - 2x_2 + 3x_3 = 5$

(3.3) $x + y + z + t = 2,\ x - y + z + t = 0,\ 2x - 3y + 2z + t = -2,\ x + z + t = 1$

[4] Find the values of ‘a’ and ‘b’ for which the system

$2x + 3y + 5z = 9; 7x + 3y - 2z = 8; 2x + 3y + az = b$

has *(i)* no solution *(ii)* unique solution *(iii)* infinitely many solutions

[5] Show that the system of linear equations

$-2x + y + z = a,\ x - 2y + z = b,\ x + y - 2z = c$

has no solution unless $a + b + c = 0$. Find a solution for $a = 1,\ b = 1,\ c = -2.$

[6] Solve the following homogeneous system of linear equations

(6.1) $x - y + z = 0,\ 4x - 3y + 2z = 0,\ 2x - 3y + 4z = 0$

(6.2) $4x + 2y + z + 3w = 0; 6x + 3y + 4z + 7w = 0; 2x + y + w = 0$

[7] Determine ‘b’ such that the system of homogeneous equations

$2x + by + 3z = 0; x + 3y + bz = 0;\ 2x + y + 2z = 0$

has *(i)* Trivial solution *(ii)*Non-trivial solution

[8] Find Eigen values and Eigen vectors for following matrix

(8.1) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ (8.2) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ (8.3) $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ (8.4) $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

[9] Identify if the following matrices are Hermitian/Skew-Hermitian/Unitary.

(9.1) $\begin{bmatrix} -1 & 2i & 4+i \\ -2i & 2 & -2 \\ 4-i & -2 & 5 \end{bmatrix}$ (9.2) $\begin{bmatrix} i & 2+3i & 4i \\ -2+3i & 0 & 5 \\ 4i & -5 & -3i \end{bmatrix}$ (9.3) $\begin{bmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{bmatrix}$

[10] If A and B are Hermitian matrices, show that $AB - BA$ is skew-Hermitian.

[11] Find the general solution of $\frac{d^4x}{dt^4} + 4x = \sin x$.

[12] Find the complete solution of the differential equation $x^2y''+xy'-y=x^2e^x$

[13] Find the basis solutions for differential equation $x^2\frac{d^3y}{dx^3}+3x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

[14] Find the general solution of $\frac{d^2y}{dx^2}-9y=x$

[15] Find the general solution of $(2D^2 + 5D + 3)y = \cos x$.

[16] Find the complete solution of the differential equation

$(D^2+1)^2 y=e^x, when\, D=\frac{d}{dx}$

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[17] Find the value of μ which satisfies the equation $A^{2018}X = \mu X$, where $A=\begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$

[18] If a square matrix A has an Eigen vector $[1 \;\; 4 \;\; 5 \;\; 9]^T$ corresponding to its Eigen value λ, Find an Eigen vector of A^{10} corresponding to the Eigen value λ^{10}.

[19] For what value of λ for which matrix A will be of rank (i) 1, (ii) 2 , (ii) 3.

$$A=\begin{bmatrix} 3 & \lambda & \lambda \\ \lambda & 3 & \lambda \\ \lambda & \lambda & 3 \end{bmatrix}$$

[20] If H is Hermitian matrix and $U = I - HH^{\Theta}$, show that U is Hermitian.