(*i*) Hyperbolic (*c*) $B^2 - 4AC > 0$

(*ii*) Parabolic (*a*) $B^2 - 4AC = 0$

(*iii*) Elliptic (*b*) $B^2 - 4AC < 0$

(*iv*) Not classifies (*d*) $A = B = C = 0$

EXERCISE 14.9

Classify the following partial differential equations:

1. $9\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} = 0$

   Ans. Parabolic

2. $3\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$

   Ans. Elliptic

3. $2\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = 0$

   Ans. Hyperbolic

4. $t\frac{\partial^2 u}{\partial t^2} + 3\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial x^2} + 17\frac{\partial u}{\partial x} = 0$

   Ans. Hyperbolic if $xt < \frac{9}{4}$, parabolic if $xt = \frac{9}{4}$, elliptic if $xt > \frac{9}{4}$

5. $\frac{\partial^2 z}{\partial x^2} = \frac{\partial x}{\partial y}$

   (U.P., II Semester, Summer 2003)

   Ans. Parabolic

6. $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$

   Ans. Hyperbolic

7. $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ *(U.P., II Semester, 2010)*

   Ans. Elliptic

OBJECTIVE TYPE QUESTIONS

Choose the correct alternative:

1. The complementary function of $(D^2 - 6DD' + 9D'^2)\, z = 0$ is
   (*i*) $f_1(y + 3x) + x f_2(y + 3x)$
   (*ii*) $f_1(y - 3x) + x f_2(y - 3x)$
   (*iii*) $f_1(y + 3x) + x f_2(y - 3x)$
   (*iv*) $f_1(y + x) + x f_2(y + x)$

   Ans. (*i*)

2. The complementary function of $(D^2 - DD' - 6D'^2)z = 0$ is
   (*i*) $f_1(y - 3x) + f_2(y - 2x)$ (*ii*) $f_1(y + 3x) + f_2(y - 2x)$
   (*iii*) $f_1(y + 3x) - f_2(y - 2x)$ (*iv*) $f_1(y - 3x) - f_2(y - 2x)$

   Ans. (*ii*)

3. The C.F. of $r = c^2 t$ is
   (*i*) $f_1(y - cx) + f_2(y - cx)$
   (*ii*) $f_1(y - cx) + f_2(y + 2x)$
   (*iii*) $f_1(y + cx) + f_2(y - cx)$
   (*iv*) $f_1(y - cx) - f_2(y - cx)$

   Ans. (*iii*)

11. The rule for finding the *P.I.* of $F(D, D')\, z = e^{ax+by}$ is

(*i*) $\dfrac{e^{ax-by}}{F(a,b)}$ (*ii*) $\dfrac{e^{ax+by}}{F(a,b)}$

(*iii*) $\dfrac{F(a,b)}{e^{ax+by}}$ (*iv*) $\dfrac{e^{ax+by}}{F(a^2,b^2)}$

Ans. (*ii*)

12. The rule for finding the *P.I.* of $(D^2 + DD' + D'^2)\, z = \sin(ax+by)$ is

(*i*) $\dfrac{1}{(a^2, ab, b^2)} \sin(ax+by)$

(*ii*) $\dfrac{1}{(-a^2, -ab, b^2)} \sin(ax+by)$

(*iii*) $\dfrac{1}{(a, ab, b)} \sin(ax+by)$

(*iv*) $\dfrac{1}{(-a^2, -ab, -b^2)} \sin(ax+by)$

Ans. (*iv*)

13. P.I. of $(D^2 + DD' - 6D'^2)\, z = e^{2x+3y}$ is

(*i*) $\dfrac{1}{-3} e^{2x+3y}$ (*ii*) $\dfrac{1}{-44} e^{2x+3y}$

(*iii*) $+\dfrac{1}{44} e^{2x+3y}$ (*iv*) $\dfrac{1}{-8} e^{2x+3y}$

Ans. (*ii*)

14. The P.I. of $(D^2 + 6DD' + D'^2)\, z = e^x$ is

(*i*) e^x (*ii*) e^y

(*iii*) e^{x+y} (*iv*) e^{x-y}

Ans. (*i*)

15. The P.I. of $(2D^2 - DD' + 4D'^2)\, z = \cos(2x+3y)$ is

(*i*) $\dfrac{1}{38} \cos(2x-3y)$ (*ii*) $\dfrac{1}{-38} \cos(2x-3y)$

(*iii*) $\dfrac{1}{-38} \cos(2x+3y)$ (*iv*) $\dfrac{1}{38} \cos(2x+3y)$

Ans. (*iii*)

16. The *P.I.* of $(D^2 - DD' - 6D'^2)\, z = x + y$ is

(*i*) $\dfrac{x^2y^2}{2}$ (*ii*) $\dfrac{x^2y}{2}$ (*iii*) $\dfrac{xy^2}{2}$ (*iv*) x^2y

Ans. (*ii*)

17. The *P.I.* of $(D^2 - D'^2)\, z = x - y$ is

(*i*) $\dfrac{x^3}{6} - \dfrac{x^2y}{2}$ (*ii*) $\dfrac{x^3}{6} + \dfrac{x^2y}{2}$

(*iii*) $\dfrac{x^3}{2} - \dfrac{x^2y}{6}$ (*iv*) $x^3 - x^2y$

Ans. (*i*)

18. The *P.I.* of $(D^2 - DD' - 6D'^2)\, z = xy$ is

(*i*) $\dfrac{xy}{6} + \dfrac{x^4}{24}$ (*ii*) $\dfrac{xy^3}{6} + \dfrac{x^4}{24}$

(*iii*) $\dfrac{x^3y}{6} - \dfrac{x^4}{24}$ (*iv*) $\dfrac{x^3y}{6} + \dfrac{x^4}{24}$

Ans. (*iv*)

19. The solution of $\dfrac{\partial^3 z}{\partial x^3} = 0$ is

(*i*) $z = f_1(y) + x f_2(y) + x^2 f_3(y)$ (*ii*) $z = (1 + x + x^2) f(y)$

(iii) $z = f_1(x) + yf_2(x) + y^2 f_3(x)$ (iv) $z = (1 + y + y^2) f(x)$ **Ans. (i)**

20. The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is

(i) $z = f_1(y + x) + f_1(y - x)$ (ii) $z = f_1(y + x) + f_2(y - x)$

(iii) $z = f_2(y + x) + f_2(y - x)$ (iv) $z = f(x^2 - y^2)$ **Ans. (ii)**

21. Particular integral of $(2D^2 - 3DD' + D'^2)z = e^{x+2y}$ is

(i) xe^{x+2y} (ii) $\frac{1}{2}e^{x+2y}$ (iii) $-\frac{x}{2}e^{x+2y}$ (iv) $\frac{x^2}{2}e^{x+2y}$ **Ans. (iii)**

22. Particular integral of $(D^2 - D'^2)z = \cos(x + y)$ is

(i) $\frac{x}{2}\cos(x + y)$ (ii) $x\sin(x + y)$ (iii) $x\cos(x + y)$ (iv) $\frac{x}{2}\sin(x + y)$ **Ans. (iv)**

23. The complementary function of $r - 7s + 6t = e^{x+y}$ is :

(i) $f_1(y - x) + f_2(y - 6x)$ (ii) $f_1(y + x) + f_2(y + 6x)$

(iii) $f_1(y + 2x) + f_2(y - 2x)$ (iv) $f_1(y + 3x) + f_2(y - 4x)$ **Ans. (ii)**

(R.G.P.V Bhopal, II Semester Feb. 2006)

24. $\frac{1}{f(D, D')} x^m y^n =$

(i) $[f(D, D')]^{-1} x^m y^n$ (ii) $[f(D, D')] x^m y^n$

(iii) $[f(D, D')] x^m$ (iv) $[f(D)] x^m y^n$ **Ans. (i)**

25. $\frac{1}{(D - mD)} F(x, y) =$

(i) $\int F(x, c - mx)dx$ (ii) $\int F(c, c + mx)dx$

(iii) $F(x, mx)\,dx$ (iv) $\frac{d}{dx} F(x, mx - c)dx$ **Ans. (i)**

Indicate True and false for the following equation

26. The solution of the partial differential equation $(D^2 + DD' + 6D'^2)\, z = 0$ is

$z = f_1(x) + f_2(x + y)$ **Ans. False**

27. The C.F. of $(D^2 - 3DD' - 4D'^2)\, z = 0$ is

$z = f_1(y + x) + f_2(y - x)$ **Ans. False**

28. The solution of $(D^2 - 6DD' + 8D'^2)\, z = 0$ is

$z = f_1(y + 2x) + f_2(y + 4x)$ **Ans. True**

29. The solution of the partial differential equation $(D^2 - 5DD' - 6D'^2)\, z = 0$ is

$z = f_1(y) + f_2(y + 2x) + f_3(y + 3x)$ **Ans. False**

30. The solution of the partial differential equation $(D^4 + 2D^2D'^2 + 4D'^4)\, z = 0$ is

$z = f_1(y + x) + x f_2(y + 2x)$ **Ans. False**

31. The solution of the partial differential equation $(D^3 - 3D^2D' + 4D'^3)\, z = 0$ is

$z = f_1(y - x) + f_2(y + 2x) + x f_3(y + 2x)$ **Ans. True**

32. The solution of the partial differential equation $\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) z = 0$ is

$z = f_1(y + x) + f_2(y - x)$ **Ans.** False

33. The solution of the partial differential equation $(D^3 + 4D^2 + 4D)\, z = 0$ is

$z = f_1(y) + f_2(y + 2x) + x f_3(y + 2x)$ **Ans.** True

34. The *C.F.* of the partial differential equation

$2r - s - 3t = 5\frac{e^x}{e^y}$ *is* $f_1\left(y - \frac{1}{2}x\right) + x f_2\left(y - \frac{1}{2}x\right)$ **Ans.** False

35. The solution of the linear partial differential equation $\frac{\partial^4 z}{\partial x^4} - 2\frac{\partial^4 z}{\partial x^3 \partial y} + 2\frac{\partial^4 z}{\partial x \partial y^3} - \frac{\partial z^4}{\partial y^4} = 0$ is

$z = f_1(y - x) + f_2(y + x) + x f_3(y + x) + x^2 f_4(y + x)$ **Ans.** True

Fill in the blanks

36. The solution of the partial differential equation $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$ is ....

Ans. $f_1(y - 2x) + f_2\left(y - \frac{x}{2}\right)$

37. The solution of $25r - 40s + 16t = 0$ is ..... **Ans.** $z = f_1\left(y + \frac{4}{5}x\right) + x f_2\left(y + \frac{4}{5}x\right)$

38. The solution of $(D^3 - 6D^2D' + 12DD'^2 - 8D'^3)\, z = 0$ is .....

Ans. $z = f_1(y + 2x) + x f_2(y + 2x) + x^2 f_3(y + 2x)$

39. The solution of $4r - 4s + t = 0$ is ..... **Ans.** $z = f\left(y + \frac{1}{2}x\right) + x f\left(y + \frac{1}{2}x\right)$

40. The solution of $[D^2 + (\alpha + \beta)\, DD' + \alpha\beta\, D'^2]\, z = 0$ is.....

Ans. $z = f_1(y - \alpha x) + f_2(y - \beta x) + \frac{1}{6}x^3 y$