

Instead of BTB, we use stack.
This is called Return address Predictor.
By using stack.

Advance ILP

Simple Pipelining.

ICC ICC ICC ICC ICC
IF ID EX MEM WB

$$\underline{E} = A \cos(\omega_0 t) F_0 F_1 F_2 = 500$$

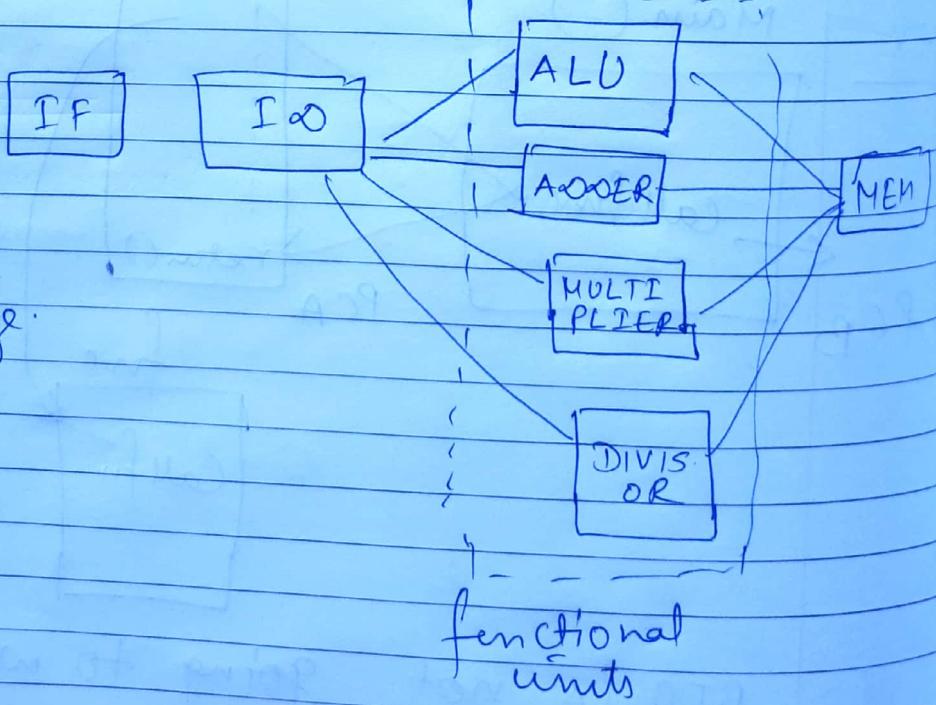
ADROFO F1 F2 = 5CC

Ex. $\text{AODAO} \rightarrow \text{FO F1 F2} \rightarrow \text{4CC}$

MULT. \circ F_0 F_1 $F_2 \rightarrow$ occ

$\text{at} \nu \cdot \omega \text{ for } f_1, f_2 \rightarrow 40\text{cc}$

Pipelining



multicyclic pelining
precipitation stage.

Add: 4cc

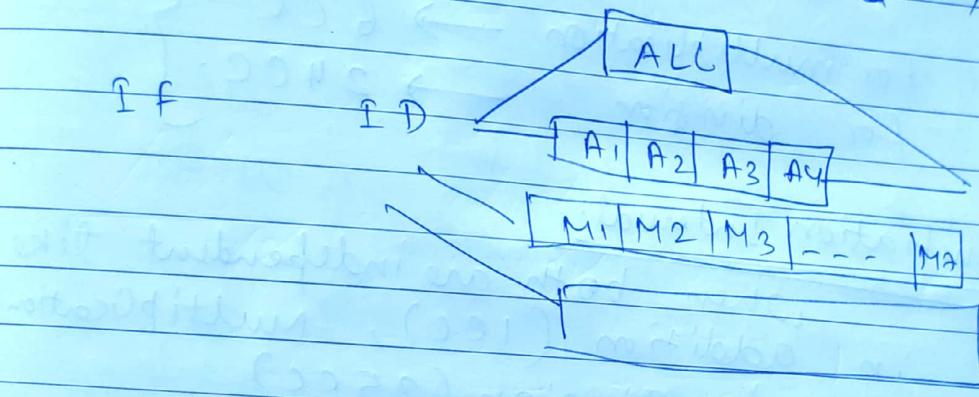
Mult: 10cc

DIV: 40cc

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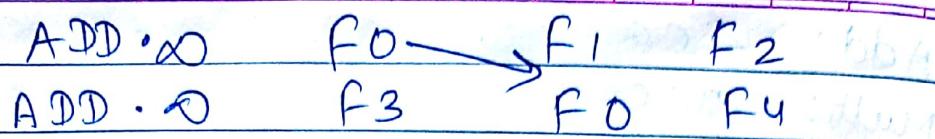
Ex: Div: $F_0 F_1 F_2 = 44\text{ cc}$
 $A \times A \times F_3 F_4 F_5 = 9\text{ cc}$
Mult: $F_6 F_7 F_8 = 16\text{ cc}$

Q. Add: 4cc, Mult \rightarrow 7cc, DIV: 25cc, ALU = 1cc



Latency \rightarrow of two instruction of same type first one is producing the result and second one is consuming the result; the no. of clock cycle waiting by the second instruction is latency.

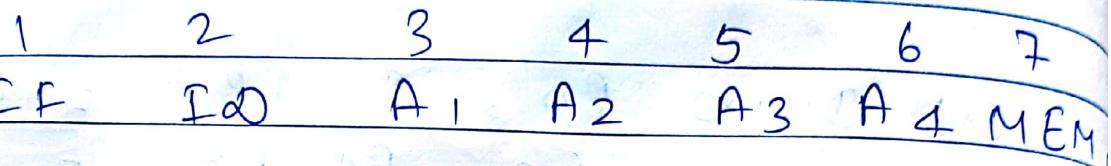
The gap in terms of CC ~~are in terms~~ b/w the two instruction are of same type.



ADD : ∞

F0 F1 F2 F3

F0 F1 F2 F3



IF ID Execution A₁

Latency of 3 CC.

for multiplication \rightarrow 6 CC }
 for division \rightarrow 24 CC }

Initiation Interval:

when both are independent like
 in addition (1 CC), multiplication (1 CC)
 But in division (25 CC)

\hookrightarrow As it is complex.

Hazards

\rightarrow Structural

\rightarrow Data

\rightarrow Control

9 10 11 12

A₃ A₄ MEM WB

MULT. ∞

ADD. ∞

IF I O M₁ M₂ M₃ M₄ M₅ M₆ M₇ M₈
B

I_{II} O A₁ A₂ A₃ M_W

801^h is Extra buffer/Registers

Data → R A_W (Read after write)

→ W A_R

→ W A_W

44CC.

DIV. ∞ F₀ F₁ F₂

A_{000.00} F₃ F₄

9 CC

DIV. D F₀ F₁ F₂

ADD. ∞ F₁ F₃ F₄

ADD. ∞ F₁ F₃ F₄

DIV. D

ADD. ∞

F₀ F₁ F₂

F₀ F₃ F₄

!

!

ADD. ∞

F_x F₀ F₅

Techniques to Optimize and Cache

- Software techniques / compiler techniques
- H/w (Hardware techniques)

Compiler techniques

- Register renaming
- loop unrolling
- software pipelining
- trace scheduling
- predicate instructions
- Multiple Issue processor

{ Data Hazards.

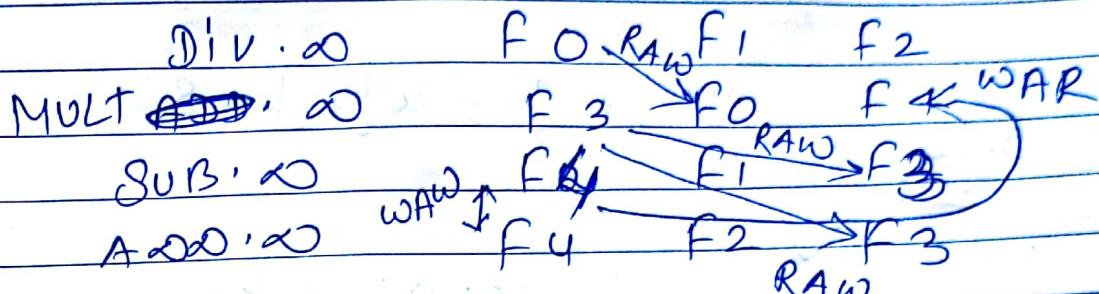
{ Control Hazards

H/w Techniques / Dynamic Scheduling

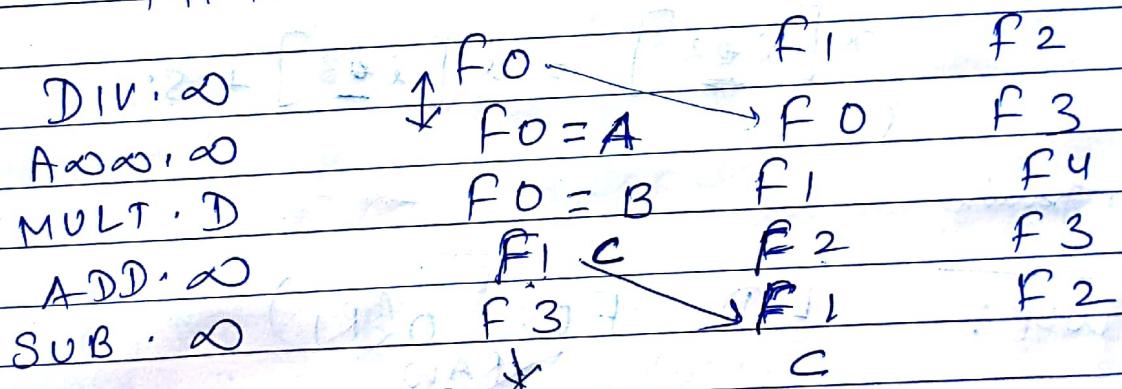
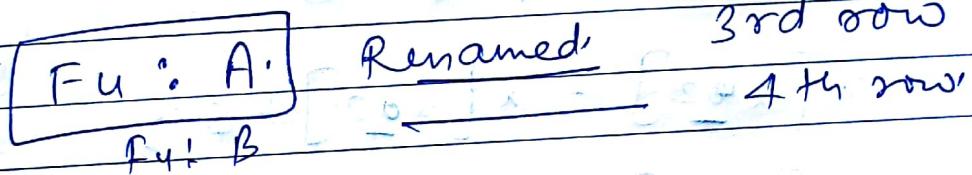
- dynamic score boarding
- Tomasulo's Algo
- Tomasulo's Algo + ROB
- Tomasulo's Algo + ROB + LSG
- ↓
(Load Store) Queue



Register Renaming.



we have a pool of registers {A, B, C, ..., Z}
By using this, RAW, WAR, WAW can
be removed.



LOOP UnRolling.

for($i = 999;$ $i > 0;$ $i--$)

1000 times.

$$x[i] = x[i] + s;$$

for($i = 1000;$ $i > 0;$ $i = i - 4$)

$$\{ x[i] = x[i] + s;$$

$$x[i+1] = x[i] + s$$

250 times.

$$x[i+2] = x[i+2] + s;$$

$$\{ x[i+3] = x[i+3] + s;$$

Assembly Programming

loop:

LD F0, 0(R1)

ADD.0 F4 \rightarrow F0 F2
RAW ↓

Store F4, 0(R1)

DADDUI

R1 R1 +- 8
↓ RAW

B NEZ R1 R2 loop.

double
Addition
Unsigned
immediate

Unrolling 4 times:

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① LD F₀ O(R₁)

ADD.0 F₄ F₀ - F₂

SD F₄ O(R₁)

② LD F₀ - 8(R₁)

ADD.0 F₄ F₀, F₂

SD F₄ - 8(R₁)

③ LD F₀ - 16(R₁)

ADD.0 F₄ F₀, F₂

SD F₄ - 16(R₁)

LD F₀ - 24(R₁)

④ ADD.0 F₄ F₀, F₂

SD F₄ - 24(R₁)

DADDUF R₁ R₁ ≠ -32

BNE 2 R₁ R₂ loop.

The latency b/w LD and ADD is 1cc
and b/w ADD and SD is 2cc.

LD F0, O(R1)
 ADD F4, F0, F2
 SD F4 O(R1)
 LD F6 -8(R1)
 ADD F12 F6 F2
 SD F12 -8(R1)
 LD F8 -16(R1)
 ADD F14 F8 F2
 SD F14 -16(R1)
 LD F10 -24(R1)
 ADD F16 F10, F2
 SD F16, -24(R1)

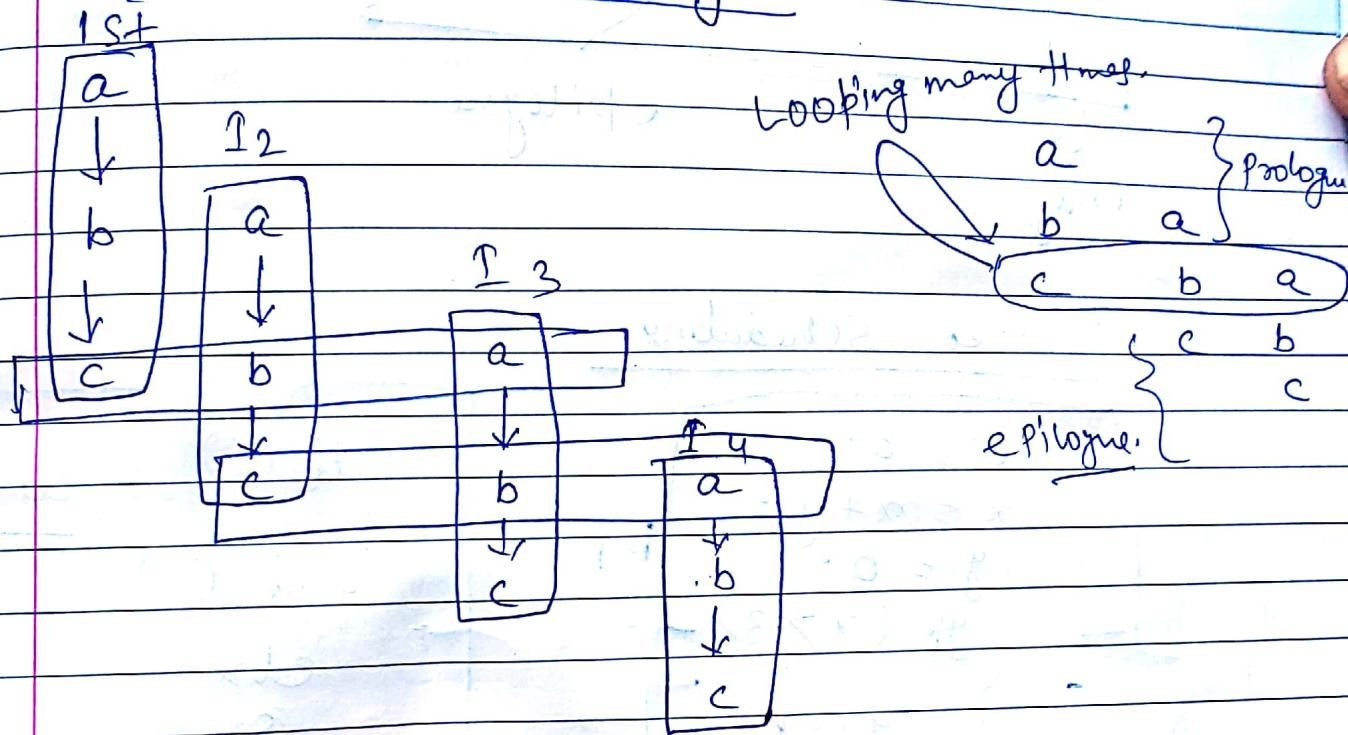
~~ADD R1, R1 ≠ -32~~
 BNE 2 R1, R2 loop

a LD F0, O(R1)
 b LD F6, -8(R1)
 c LD F8, -16(R1)
 a LD F10, -24(R1)
 b ADD F4, F0, F2
 c ADD F12, F6, F2
 a ADD F14, F8, F2
 b ADD F16, F10, F2
 c SD F4 O(R1)
 b SD F12 -8(R1)
 c SD F14 -16(R1)
~~ADD R1, R1 ≠ -32~~
 b SD F16 -8(R1)
 c BNE 2 R1, R2 loop

Drawbacks of loop unrolling!

Page No.:

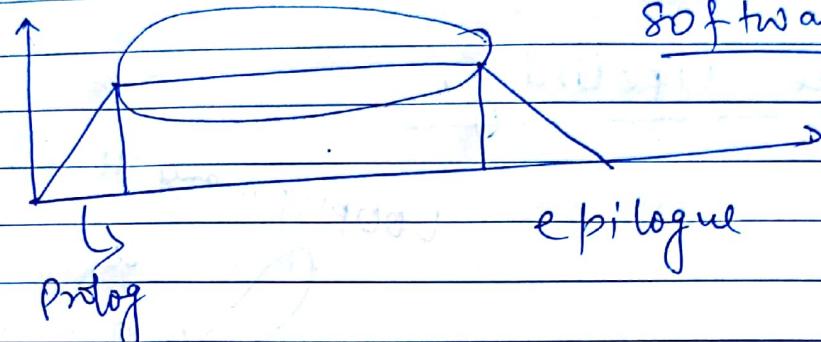
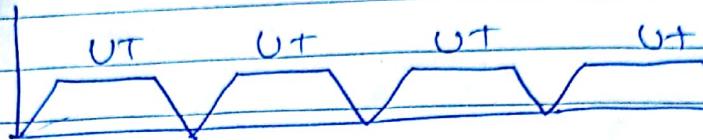
- Software Pipelining.



loop:

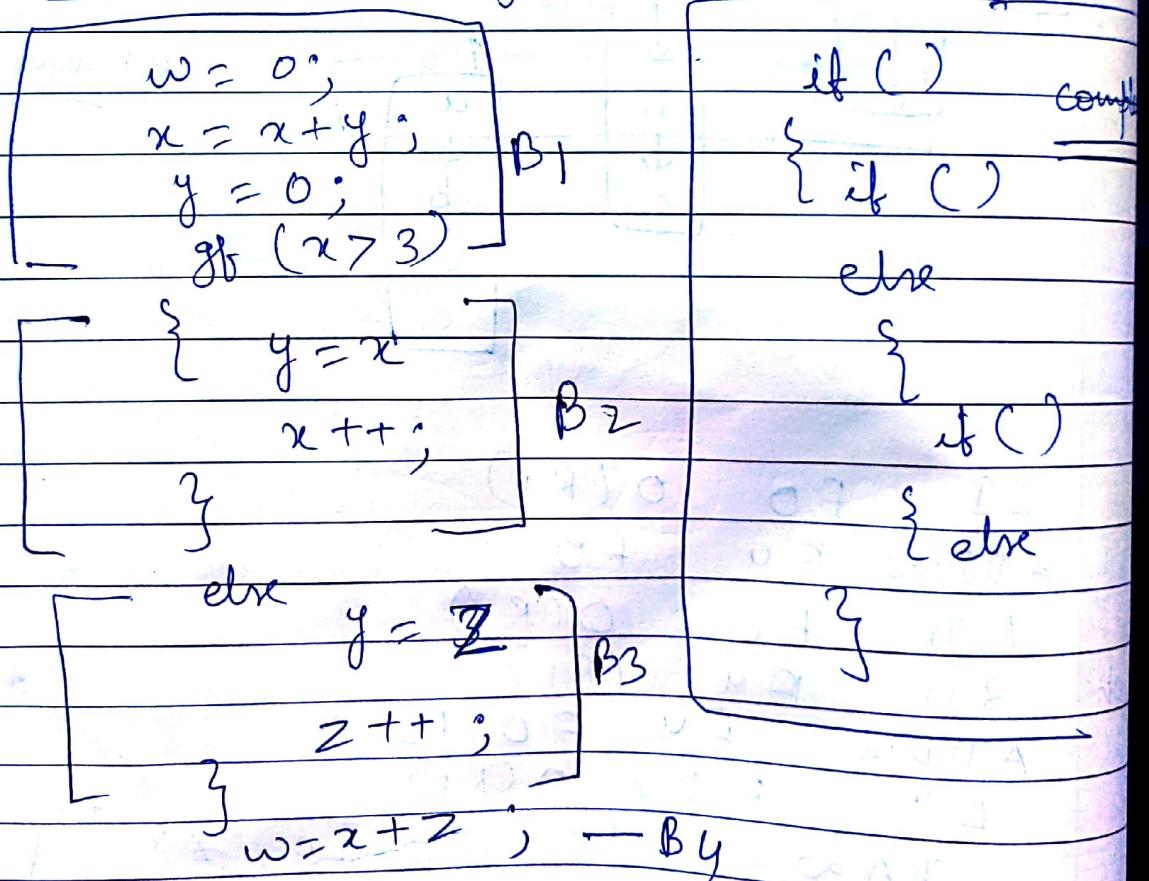
LD	FO	O(R1)
ADD.0	F4	FO F2
LD	FO	O(R1)
SD	F4	O(R1)
ADD.0	F4	FO F2
LD	FO,	O(R1)
DA	0001	(R1, R2 ≠ ?)
BNEZ		R1, R2 loop
SD	F4	O(R1)
ADD.0	F4	00, F2
SD	F4	O(R1)

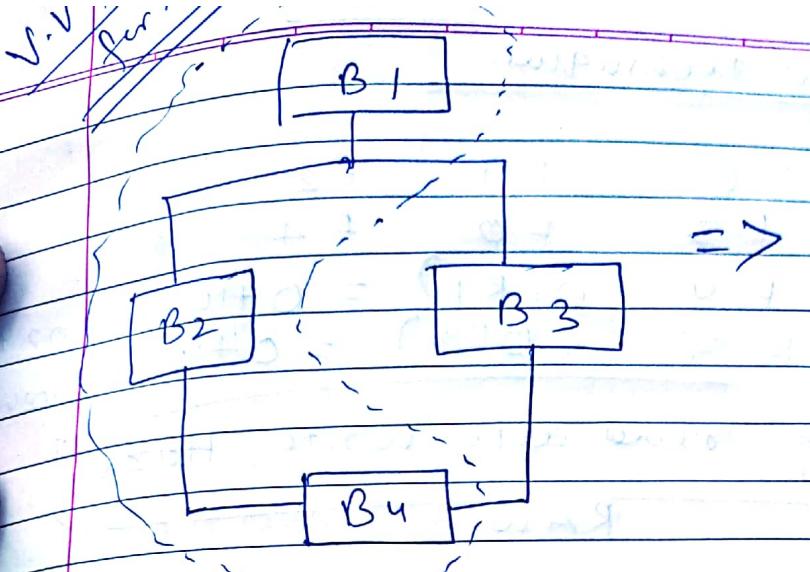
loop unrolling



Software Pipelining

Traces Scheduling.





$w = 0$;
 $x = x + y$;
 $y = 0$;
 $y = x$
 $x++$
 $w = x + z$;
 $if (l < \infty)$

Tail computation

Recovery
code

if 50% of is true and false then this technique
is inefficient in tail computation

Predicate instructions

$gf(R_1 == 0)$

$R_2 = R_2 + R_4 \Rightarrow$

$R_7 = !R_1 \text{ predicated on } R_7$

$R_8 = R_2$

$R_2 = R_2 + R_4 \Rightarrow$

$R_6 = R_3 + R_5$

$R_6 = R_3 + R_5$

$R_4 = R_2 + R_3$

$R_4 = R_8 + R_3$

Predicated on R_4

Predicated on R_1

H/W Techniques.

MULT ∞	F0	F1	F2
ADD $\cdot \infty$	F3	F0	F4
SD	F4	O(R1)	$= 0 + 1024 = 1024$
LD	F3	(OR2)	$= 0 + 1024 = 1024$

Memory read after write Hazard

RAW

Memory WAR

LD F4 O(R1)

SD FO OR(R2)

Memory WAW

SD F4 (OR1)

SD FO O(R2)

#

Div ∞ F0

F1 F2

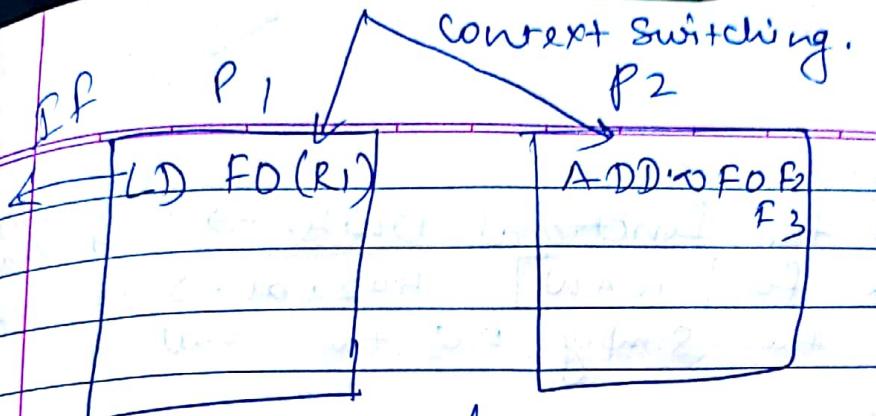
ADD ∞ F1

F3 F4



Exception is detected

When any exception is happened to O_{1,2} after that no instruction should be completed due to error



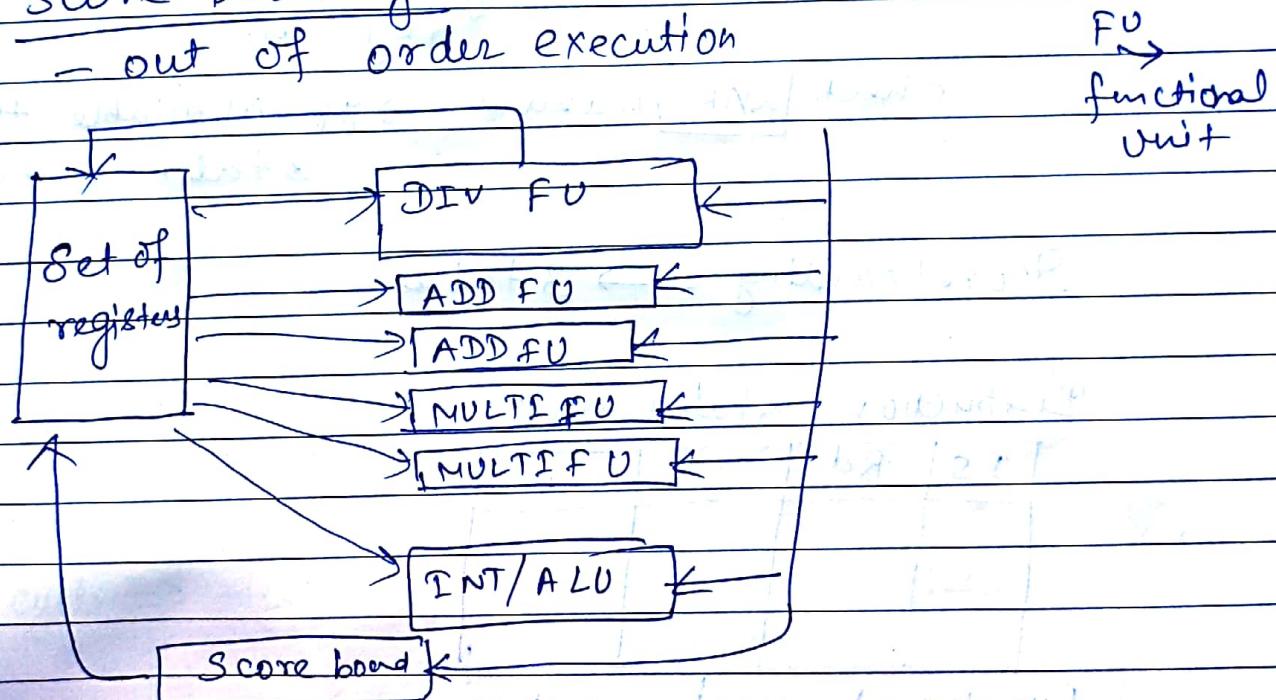
Hyperthreading (Simultaneous Multithreading)

11/03/2018

H/W Techniques:

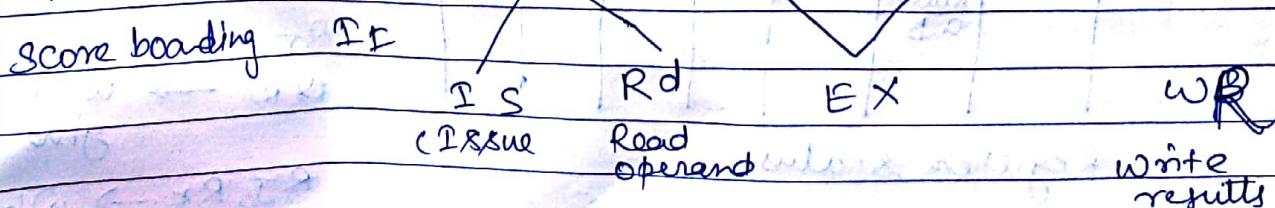
1) Score Boarding

- out of order execution



FU
functional unit

Simplest IF ID, EX, MEM, WB
pipelining



Issue stage:

- check for functional units. → If unavailable do stall
- check for **WAW** Hazards. → If it is then simply put the stall.

Read operands:

→ Both the operands are available.

Execute: ~~IFP~~ perform execution using functional units.

Write results → writing into the destination register.

check **WAR** Hazard → If available then stall.

Scoreboarding → 3 tables.

Instruction status unit

IS	Rd	Ex	WR

Functional unit status

BUSY	OP.	F _i	F _j	F _k	R _j	R _k
	Div Multi Sub					

Register status

F_j, F_k → inputs
 F_i → destination register
 R_j → Who will give f_j?
 R_k → Who will give f_k?
 R_j, R_k → Whether operand available

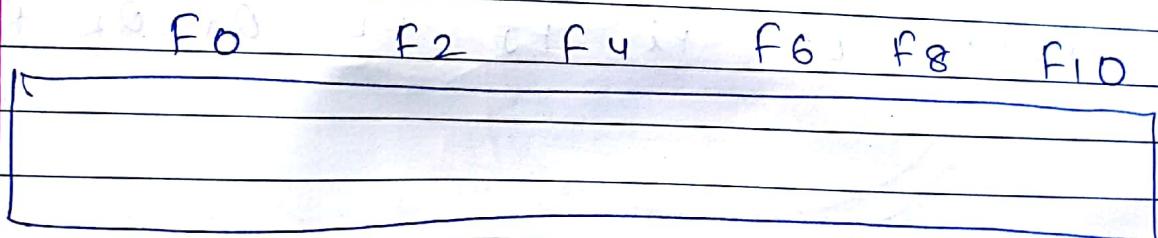
Instruction Status Unit

			I S	Rd	E X	W R	Page No.:
LD	F6	34(R2)					9
LD	F2	45(R3)	1	2	3	4	
MULT D	F0 F2	F4	5	6	7	8	
SUB D	F8 F6	F2	6	9	19	20	
DIV D	F10 F0	F6	7	9	11	12	
ADD D	F6 F8	F2	8	21	61	62	
			13	14	16	22	

functional unit.

	BUSY	OP	F1	F2	F3	F4	QJ	QK	RJ	RK
INT	NO	yes	LDI							
MULT I	NO			F6	34	R2	—	—	—	—
ADD	NO									
DIV D	NO									

Register Unit



Clock cycle \Rightarrow 0

Adder = 2CC

Div D = 40CC

MULT = 10CC

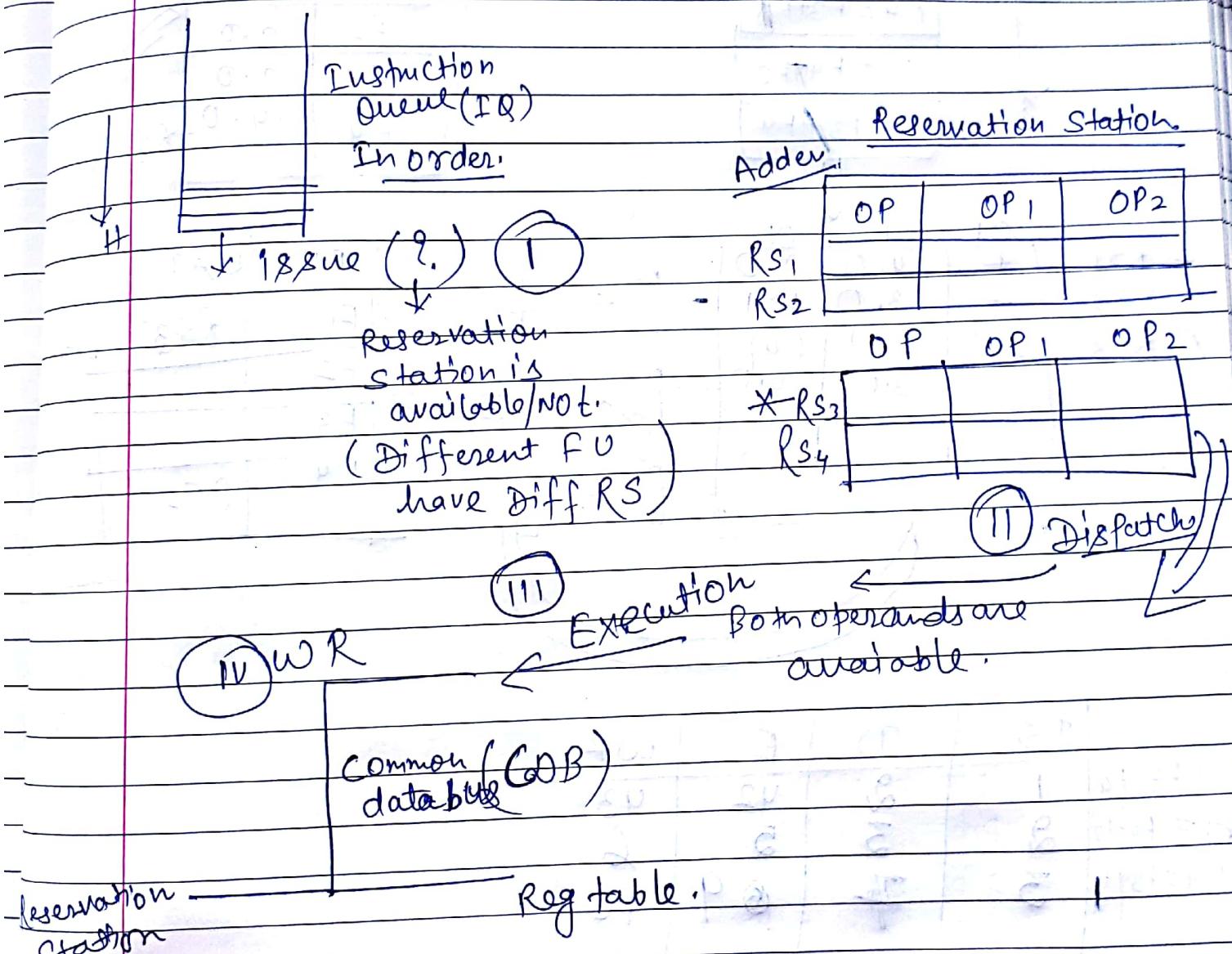
INT = 1CC

Drawbacks of scoreboarding

- There is no forwarding.
- It is going to solve any hazards.
- Only put the stalls.

Tomasulo's Algo.

g+ solves WAR & WAW Hazard.



WAW

Div \rightarrow 40CC
Add \rightarrow 2CC

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~~$F_2 = F_3 + F_4$~~

Register table

F_1	1.0	R_S
F_2	2.0	
F_3	3.0	
F_4	4.0	R_{S2}

$$F_4 = F_3 + F_1$$

$$F_1 = F_4 - F_2$$

$$F_1 = F_3 + F_4$$

	OP	OP ₁	OP ₂
+ RS ₁	+	4.0	2.0
+ RS ₂	+	3.0	RS ₁
/ RS ₃		F_3 3.0	F_4 4.0

RRT/RAT

F_1	RS_3
F_2	
F_3	
F_4	RS_2

	IS	D	E	WR
$F_1 = F_3 / F_4$	1	2	42	43
$F_1 = F_4 + F_2$	2	3	5	6
$F_4 = F_3 + F_1$	3	7	9	10

WAR problem

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$$\begin{aligned} F_4 &\equiv F_1 - F_2 \\ F_1 &\equiv F_3 + F_4 \\ F_1 &\equiv F_4/F_2 \end{aligned}$$

$$F_3 = F_4 + F_3$$

$$F_4 = F_2 + F_3$$

$$F_1 = F_4/F_2$$

$$\begin{array}{lll} OP & OP_1 & OP_2 \\ \hline RS_1 + & F_2 & F_3 \\ RS_2 + & RSI & 3.0 \\ / RS_3 - & F_4 & F_2 \\ \hline RS_1 & 4.0 & 2.0 \end{array}$$

$$\begin{array}{llll} I & S & E & W/R \\ \hline 1 & 2 & 42 & 43 \\ 2 & 3 & 5 & 6 \\ 3 & 7 & 9 & 10 \end{array}$$

Reg table:

$$\begin{array}{ll} F_1 & 1.0/RS_3 \\ F_2 & 2.0 \\ F_3 & 3.0 \\ F_4 & 4.0/RS_1 \end{array}$$

RRT

$$\begin{array}{ll} F_1 & RS_3 \\ F_2 & \\ F_3 & \\ F_4 & \end{array}$$

~~Prob~~

- (I)
- (II)
- (III)
- (IV)

$$F_1 = F_2 + F_3$$

$$F_4 = F_1 - F_2$$

$$F_1 = F_2 / F_3$$

$$F_2 = F_4 + F_1$$

~~$F_1 = 1.0$~~

~~$F_2 = 2.0$~~

~~$F_3 = 3.0$~~

~~$F_4 = 4.0$~~

Add
div.

w/

$$\begin{cases} S + RS_1 \\ + RS_2 \\ \hline RS_3 \end{cases}$$

$$d.0$$

$$OP_1$$

$$OP_2$$

$$RS_1 / 5.0$$

$$2.0$$

$$3.0$$

$$3.0$$

Reg.

$$F_1 = 1.0 / RS$$

$$F_2 = 2.0$$

$$F_3 = 3.0$$

$$F_4 = 4.0 / R$$

$$\begin{cases} F_1 = F_2 + F_3 \\ F_4 = F_1 - F_2 \\ F_1 = F_2 / F_3 \\ F_2 = F_4 + F_1 \end{cases}$$

$$IS$$

$$ID$$

$$Ex$$

$$WR$$

~~7~~

~~6~~

~~7~~

~~7~~

~~11~~

~~12~~

~~9~~

~~11~~

~~12~~

~~13~~

~~18~~

~~19~~

~~RS~~~~F1~~~~F2~~~~F3~~~~F4~~

~~1~~

~~2~~

~~6~~

~~7~~

~~2~~

~~8~~

~~12~~

~~13~~

~~3~~

~~4~~

~~29~~

~~30~~

~~8~~

~~31~~

~~35~~

~~36~~



$$F_1 = F_2 + F_3 \quad 1 \quad 2 \quad 6 \quad 7$$

$$F_4 = F_1 - F_2 \quad 2 \quad 8 \quad 12 \quad 13$$

$$F_1 = F_2 / F_x \quad 20' \quad 24$$

$$F_2 = F_4 + F_0 \quad 8 \quad 14 \quad 18 \quad 19$$

10.8

10.9

10

11

F S	D	EX	WR	C	F ₀	1.0
$F_1 = F_2 + F_3$	1	2	6	7	F_1	$1.0 / ROB_1 /$ ROB_2
$F_4 = F_1 - F_2$	2	8	12	13	F_2	2.0
$F_1 = F_2 / F_3$	3	4	29	30	F_3	0.0
$F_2 = F_4 + F_0$	9	10	14	15	F_4	3.0
			32			

S	T	(Rob entry)	T	X D	X	C/RC
OP ₁	OP ₂	OP ₁ , P ₂	ROB ₀	+ F ₁	- 2.0	H
+ RS ₁	+ RS ₂	2.0	0.0	ROB ₀ ROB ₁	- F ₄	
- RS ₂	-	ROB ₀	2.0	ROB ₁ ROB ₂	/ F ₁	-
/ RS ₃	-	2.0	0.0	ROB ₂ ROB ₃		

R RT

F ₀	R
F ₁	ROB ₀ /
F ₂	ROB ₂
F ₃	
F ₄	ROB ₁

$$F_1 = F_3 + F_4$$

$$F_1 = F_4/F_2$$

$$F_4 = F_3 - F_2$$

$$F_1 = 1.0$$

$$F_2 = 0.7$$

$$F_3 = 2.0$$

$$F_4 = 3.0$$

+RS
1

OP

OP₁

OP₂

ROB
Ruty.

+RS
2

2.0

3.0

ROBO

+RS
3

1

FS	D	EX	WR	C
$F_1 = F_3 + F_4$	1	2	6	7
$F_1 = F_4/F_2$	2	3	28	29
$F_4 = F_3 - F_2$	3	4	8	9

Tomasulo's Alg + ROB

\Rightarrow Branch hazards

\Rightarrow Speculation.

① MULT D F₂ F₀ F₁

BNES Q F₂, 0, Label

SGADD D F₄ F₃ F₁

Label

ADD F₃ F₄ F₂

MULT \rightarrow IDCC

IS D Ex WR C

1	2	12	13	14
---	---	----	----	----

2

3

4

RS ₁	RS ₂	RS ₃	OP	OP ₁	OP ₂	ROB even.	F ₀	O.D
			*	0.0	1.0	ROBO	F ₁	1.0
	+			3.0	1.0	ROB ₂	F ₂	2.0 ROBO
							F ₃	3.0
							F ₄	4.0

RRT

	T D V C/NT
ROBO	* F ₂ - H
ROB ₁	BNES Q F ₂ --
ROB ₂	+ F ₄ --
ROB ₃	

F ₀	
F ₁	
F ₂	ROBO
F ₃	
F ₄	

IS D EX WR C

LD R₁ O(R₁) 1 2 3
 SD R₂ O(R₃) 2 3 4
 LD R₄ O(R₄) 3 4 5
 SD R₅ O(R₀)
 LD R₅ O(R₁)

T	D	V	C/NC
L	R ₁	—	H
S	R ₂	—	

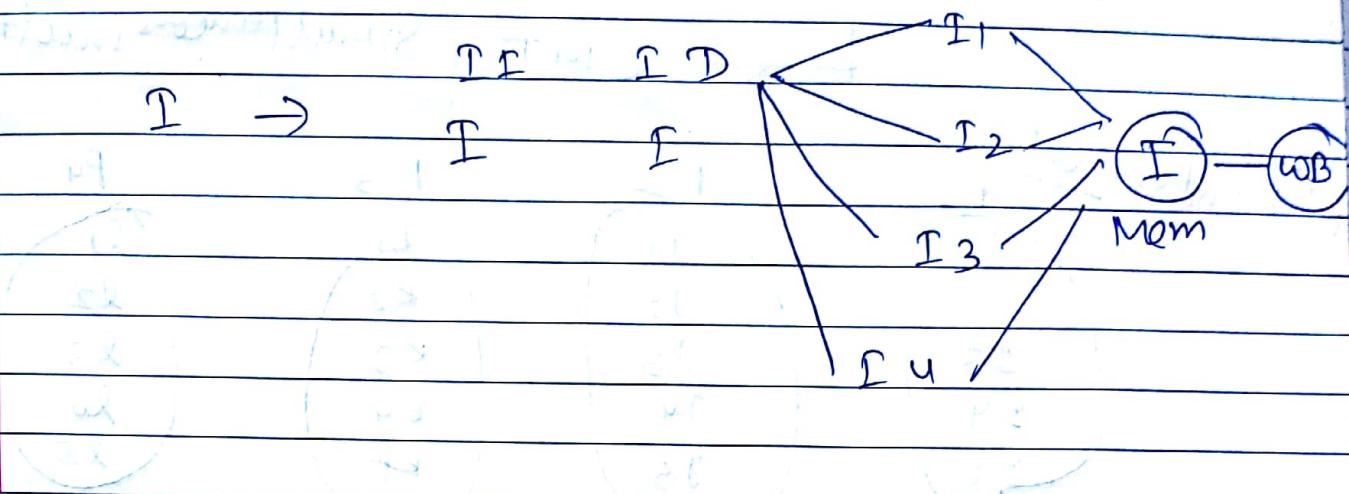
LS	EA	V	C/NC	After Calculation of 1st Instruction is 104,
L	104	—	H	
S	204	—		

Store to load forwarding

~~Multi-issue Processor~~

VLIW

very long instruction word.



	IF	ID	EX	M	WB
I_1	1				
I_2	1				
I_3					
I_4					

A ⁵ VLIW can issue two load/store operations and 4 floating point operations and one decremental branch instruction in one clock cycle.

LD ₁	SD ₁				
LD ₂	SD ₂	1CC	LD ₁	LD ₂	---
---	---	2CC	LD ₃	LD ₄	---
LD ₇	SD ₇	3CC	LD ₅	LD ₆	ADD, ADD ₂ -
ADD ₁	DADD ₁	4CC	LD ₇	-	ADD ₃ ADD ₄ -
ADD ₂	BNE ₂	5CC	-	-	ADD ₅ ADD ₆ -
---	---	6CC	SD ₁	SD ₂	A007 - -
LD ₇	SD ₇	7CC	SD ₃	SD ₄	- ADD ₀₀₀₁ -
---	---	8CC	SD ₅	SD ₆	- - ADD ₀₀₀ -
A007		9CC	SD ₇	-	BNE/2
		INC			

Simultaneous Multithreading.

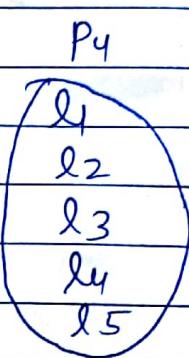
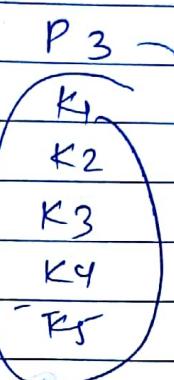
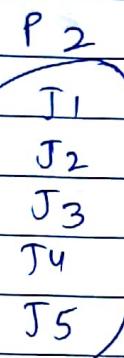
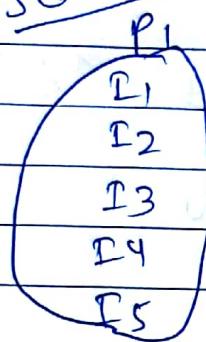
4 Issue processor (4 instructions in 1cc)

Super scalar

fine grained

SMT (Simultaneous multithreading)

① Super Scalar



1cc I₁ I₂

2cc I₃ I₄

3cc

4cc I₅ I₆

when there is a

big delay then move
from one processor
to another for
fetching instruction.

② fine grained

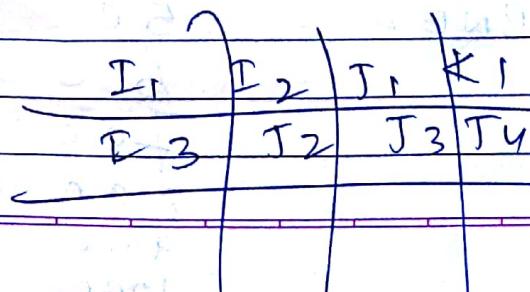
1cc I₁ I₂

2cc J₁ J₂

3cc K₁ K₂

4cc L₁ L₂ L₃

SMT



If there is
a gap then
jump to
and run
processor

Data level Parallelism.

Array Processors

No. of Processors
connected in parallel
manner.Single instruction stream multiple data
streamSIMD computer organization:- (vector architecture)

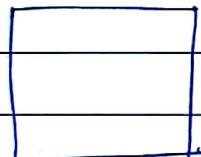
$$C_i = A_i + B_i * C_i$$

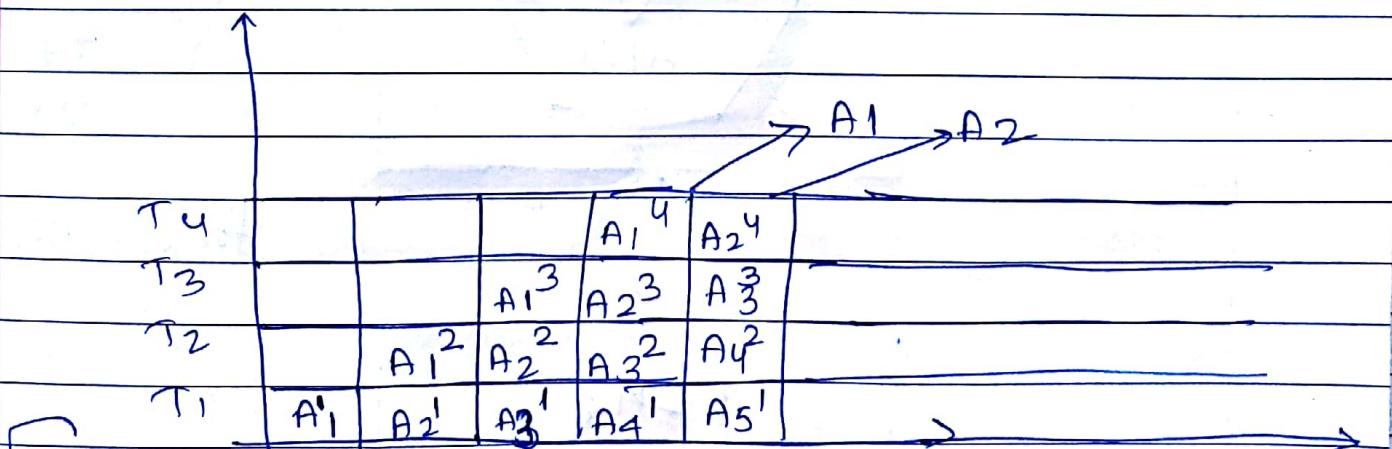
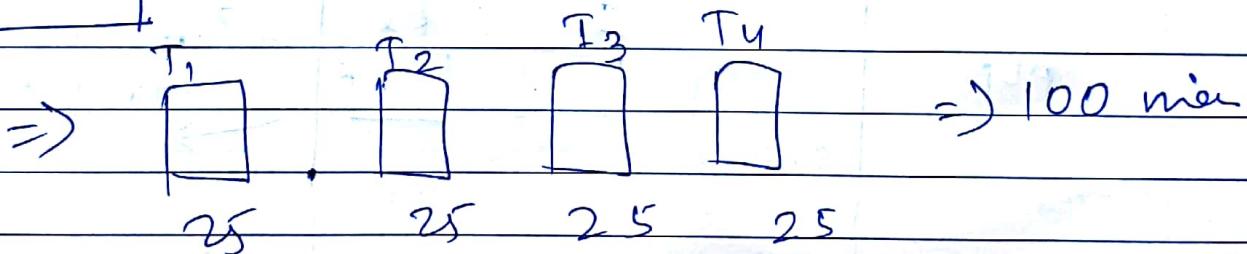
 $i = 0 \text{ to } n-1$

$$A = (A_0, A_1, \dots, A_{n-1})$$

$$B = (B_0, B_1, \dots, B_{n-1})$$

$$C = (C_0, C_1, \dots, C_{n-1})$$

 \Rightarrow Total time = 400 mins.



$$4 + 99 = 103 \approx 100$$

 This is Pipelining.

$$C_i = A_i + B_i * C_i$$

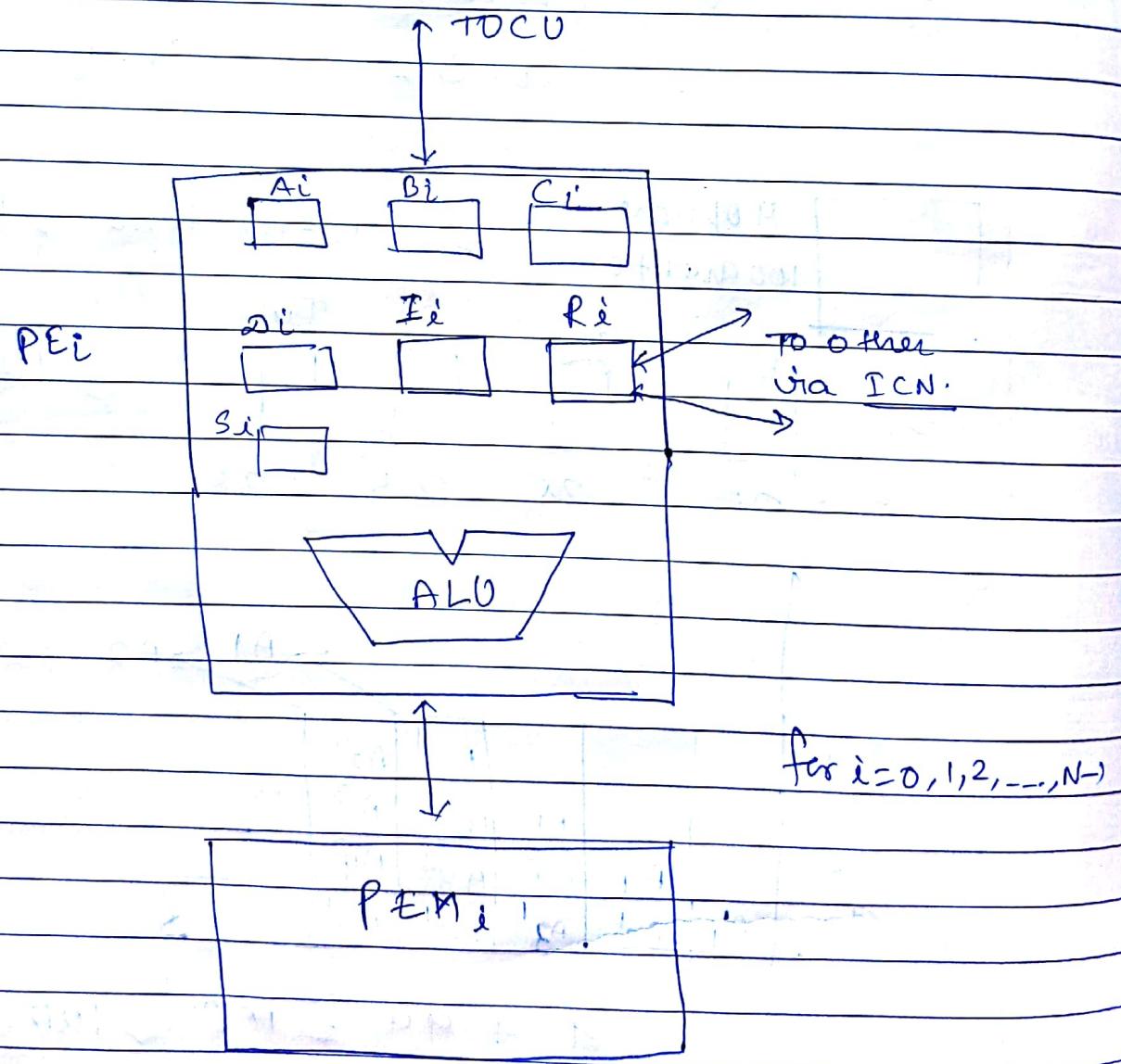
SIMD Computer $C = \langle N; F, I, M \rangle$

$N \rightarrow$ No. of processing elements in the system

$F \rightarrow$ set of data routing function

$I \rightarrow$ the set of machine instruction

$M \rightarrow$ the set of masking schemes.



Components in a
processing element

Illustration of Data Routing and Masking schemes

$$A = A_0, A_1, \dots, A_{n-1}$$

We need to compute $S(k) = \sum_{i=0}^k A_i$ for $k=0, 1, 2, \dots, n-1$

$$S(0) = A_0$$

$$S(1) = A_0 + A_1$$

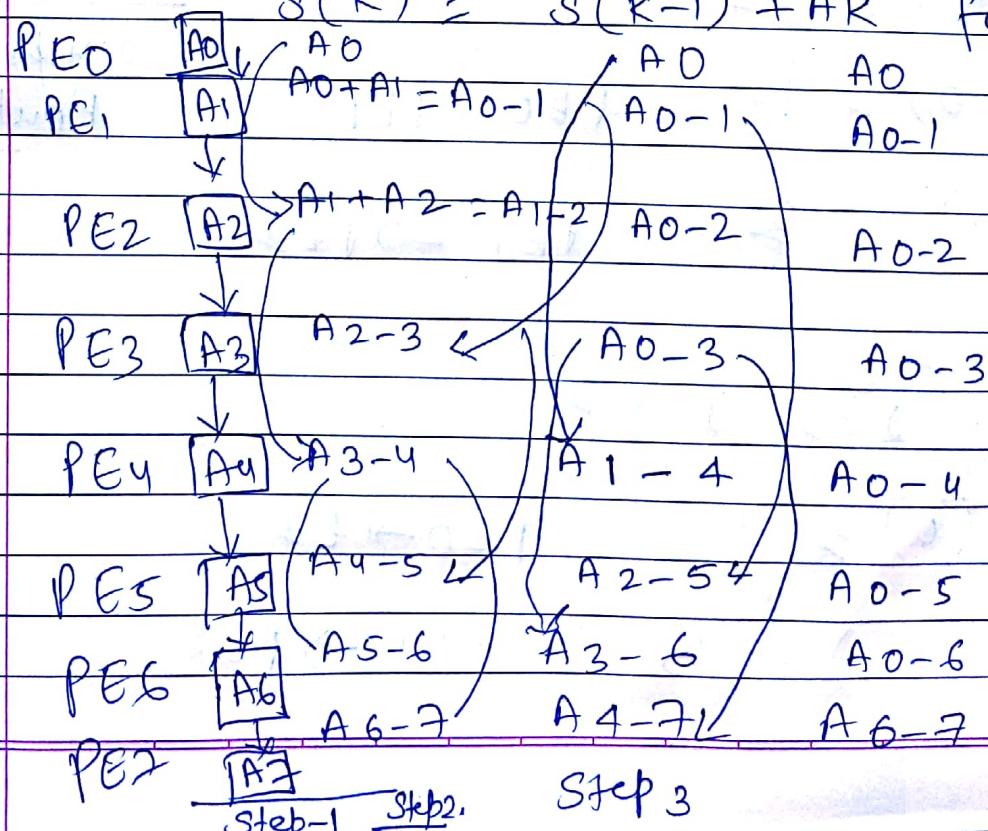
$$S(2) = A_0 + A_1 + A_2$$

$$S(n-1) = A_0 + A_1 + \dots + A_{n-1}$$

Time $\rightarrow O(n)$ with the help of recursion.

$$S(0) = A_0$$

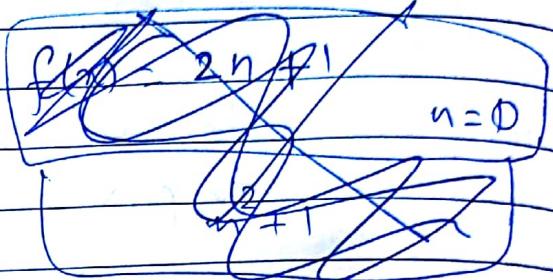
$$S(k) = S(k-1) + A_k \text{ for } k=0, 1, 2, \dots, n-1$$



④ Page No.:
PE0 PE1 PE2 PE4

Time complexity = $O(\log_2 n)$

PE1 PE3 PE5



$$R_i(j) = j + 2^i \quad 0 \leq i \leq \log(n)-1,$$

$$0 \leq j \leq n-1$$

for $n=8$:

$$0 \leq i \leq 2, 0 \leq j \leq 7$$

for $i=0$

$$R_0(j) = j+1 \rightarrow 0, 1, 2$$

$$R_0(0) = 1 \quad PE0 \rightarrow PE1 \quad \text{3 steps of Routing.}$$

$$R_0(1) = 2 \quad PE1 \rightarrow PE2$$

for $i=1$
 ~~$R_1(j) = j+2$~~

$$R_1(0) = 2 \quad PE0 \rightarrow PE2$$

$$R_1(1) = 3 \quad PE1 \rightarrow PE3$$

for i = 2

$$R_2(j) = j + 4$$

$$R_2(0) = 4 \quad P(E_0) \rightarrow PE_4$$

Masking scheme. Step 1 0, 1, 1, 1, 1, 1, _

0 → When not perform addition.

1 → when addition ~~not~~ performs.

Step 2.

Step 3 → 0 0 0 0 1 1 1 1

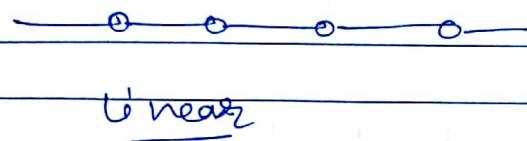


Type of Interconnection network.

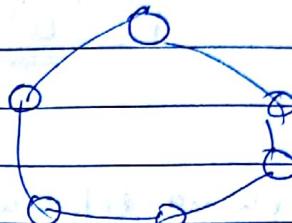
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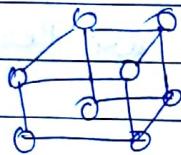
Static vs Dynamic networks.



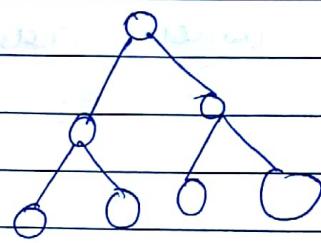
Linear



Ring



cube



tree

If vertices of cube is replaced by ~~3 cut~~ 3 circles
then cube ^{connected} circled.

two classes of dynamic network:-

- single - stage
- multi stage.

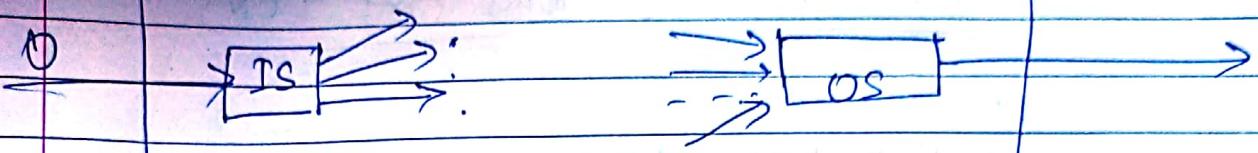
IS \rightarrow Input selector



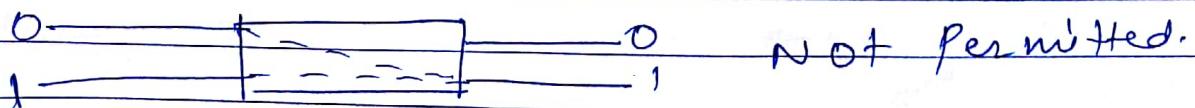
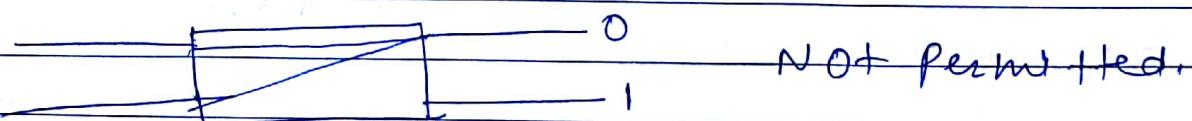
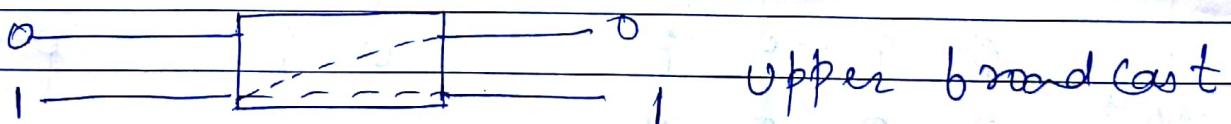
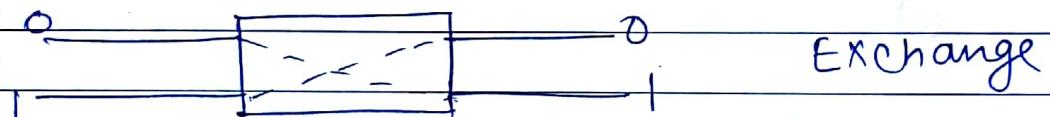
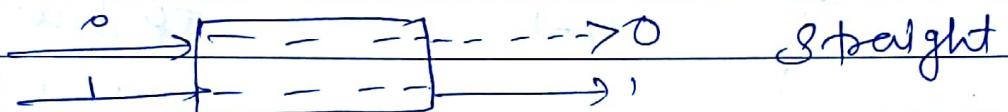
$1 \times D$ deMultiplexer

OS \rightarrow output selector.

$D \times 1$ multiplexer



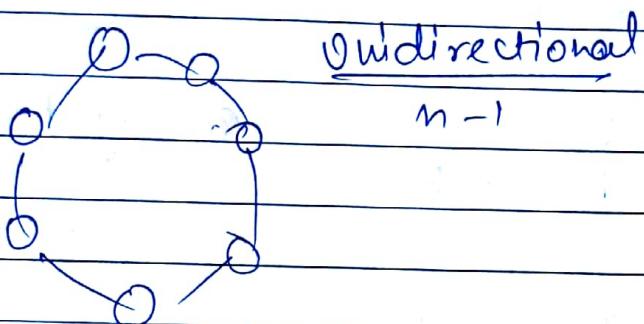
not allowed
and illegal
and not recommended



Evaluation criteria of ICN

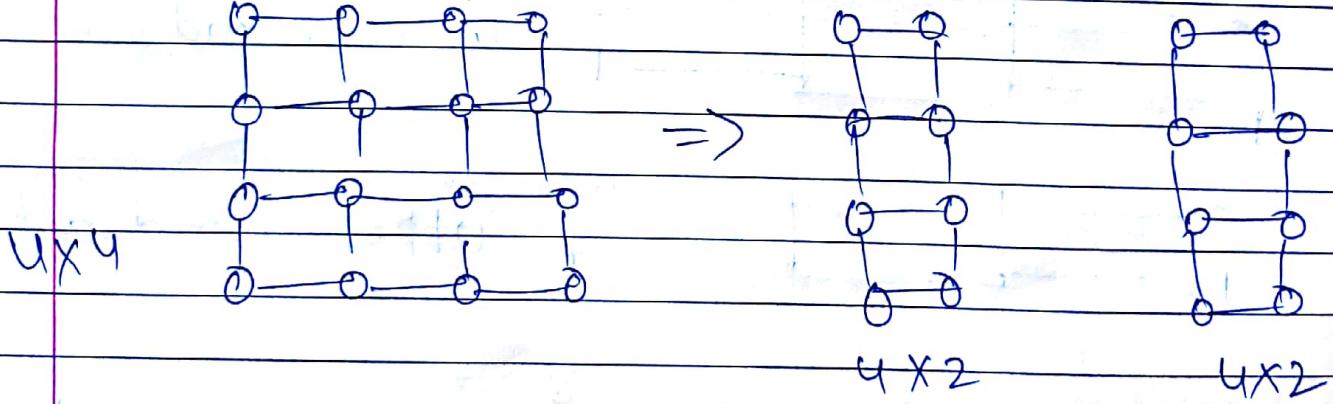
- diameter.
- Bisection width
- no. of edges per Node
- Maximum edge length.

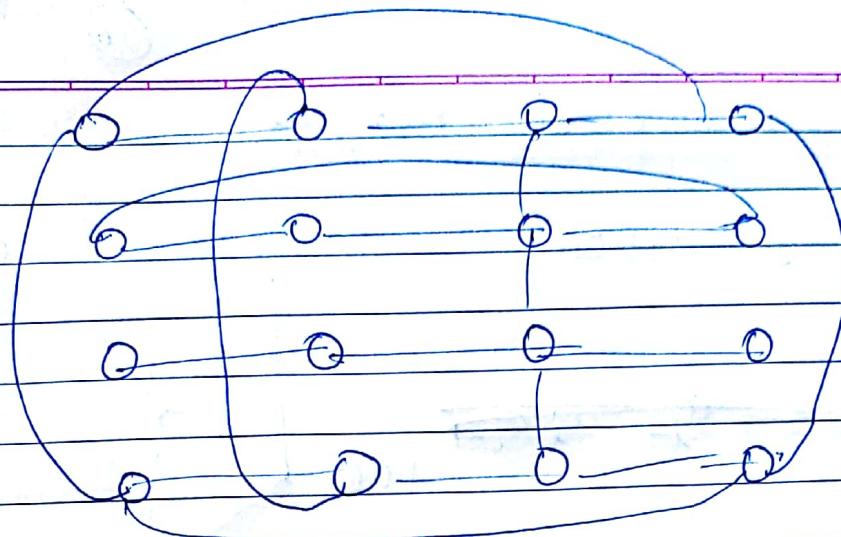
If we increase the no. of connections, diameter will decrease.



if changes
to bidirectional
 $\left(\frac{n-1}{2}\right)$

- Bisection width





4x4 2D - wrap around mesh

of no. of connections is more to reduce diameter, it will create hazards in the form of heat due to interference, overlapping of networks.

Dynamic networks. → Three classes:-

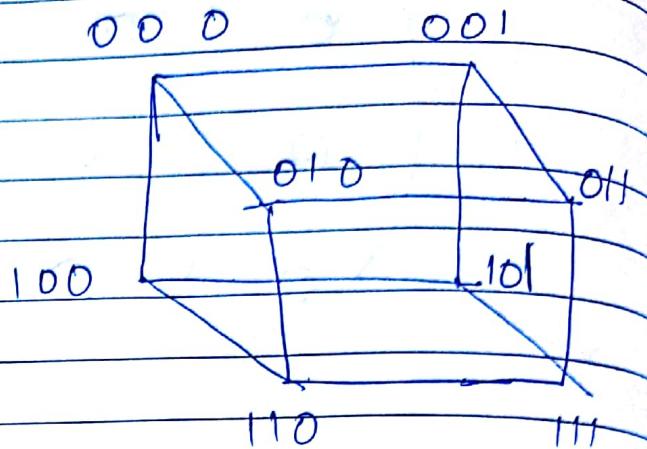
- Blocking
- Rearrangeable
- Non-blocking

Simultaneous connected b/w source and Receiver.

Cube interconnection Network.

Cube routing function

$$c_i(a_{n-2} a_{n-1} \dots a_{i+1} a_i a_{i-1} \dots a_2 a_1 a_0)$$



A 3 cube of 8 nodes

$$c_i(a_{n-2} a_{n-1} \dots a_{i+1} a_i a_{i-1} \dots a_2 a_1 a_0)$$

$$= a_{n-2} a_{n-1} \dots a_{i+1} a_i a_{i-1} \dots a_2 a_1 a_0$$

$$c_0(000) = 001$$

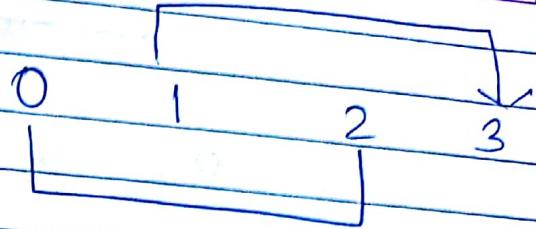
$$c_0(001) = 000$$

$$c_0(101) = 100$$

$$0 \leftrightarrow 1 \quad 2 \leftrightarrow 3 \quad 4 \leftrightarrow 5 \quad 6 \leftrightarrow 7 \quad c_0$$

$$C_1(000) = 010$$

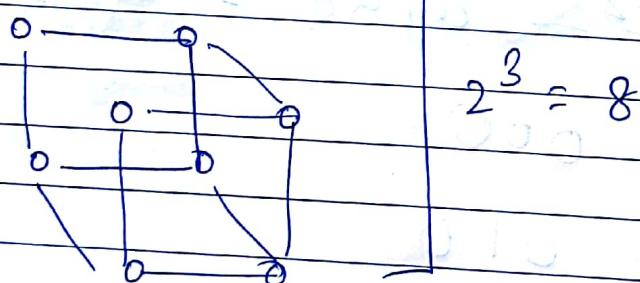
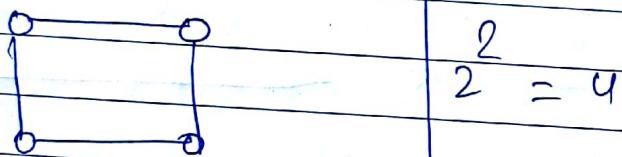
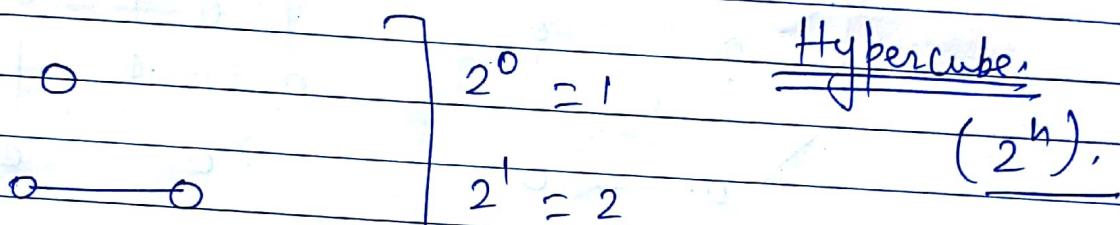
: diagonal connectivity



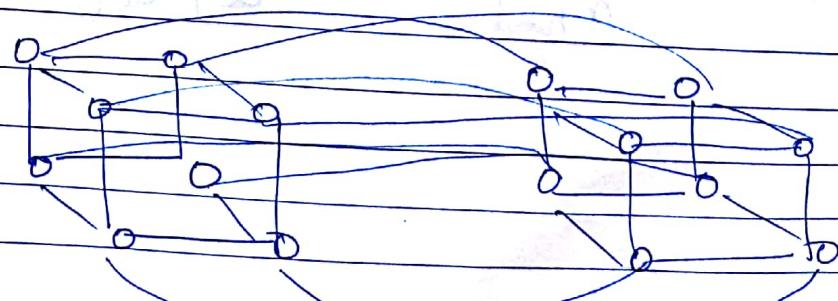
$$C_2(000) = 100$$

$$C_2(111) = 011$$

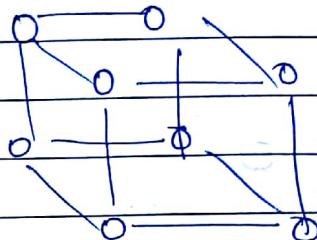
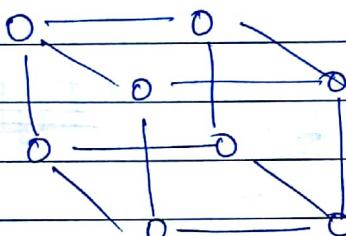
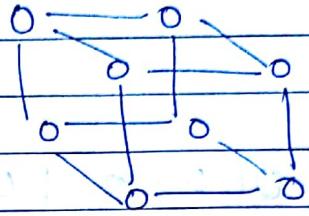
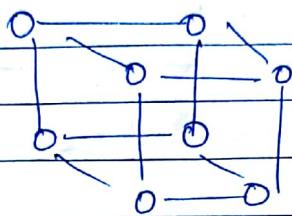
vertical
connectivity.



Horizontal
connectivity



5D Hyper Cube



Shuffle function

$$S(a_{n-1} a_{n-2} \dots a_1 a_0) = a_{n-2} \dots a_0 a_{n-1}$$

$$S(000) = 000$$

$$S(001) = 010$$

Inverse.

$$a_{n-1} \dots a_1 a_0 = a_{n-1} \dots a_1 a_0$$

Exchange routing function

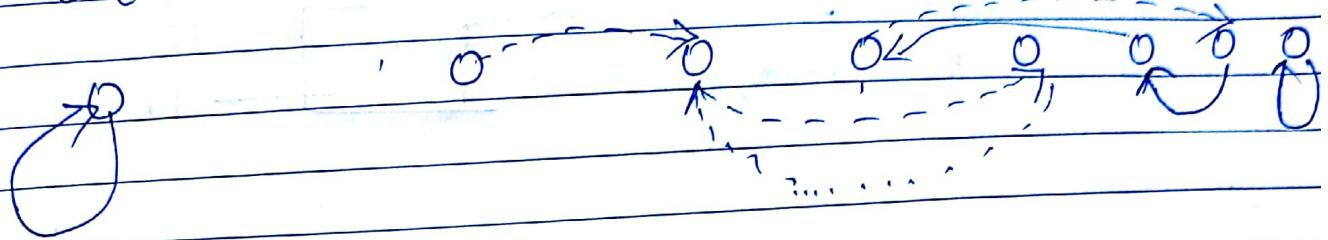
(15)

Page No.:

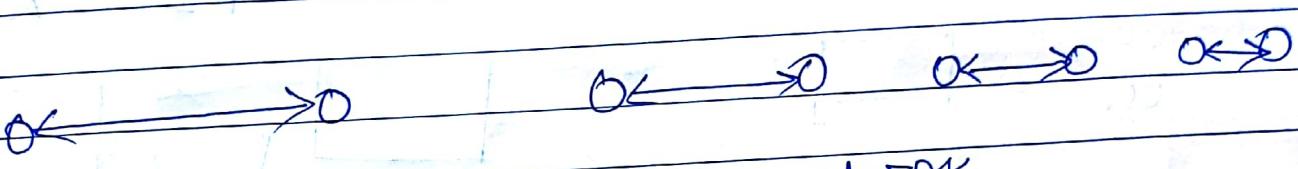
$$E(A) = C \circ A$$

0 0 0 0 0 1 0 1 0 0 1 1 1 0 0 1 0 1 1 1 0 1 1 1

shuffle:



Exchange:



8 node - shuffle exchange network.

Task: Prove that the diameter of an n -processor (n^p) shuffle exchange network is $O(\log n)$.

Butterfly function

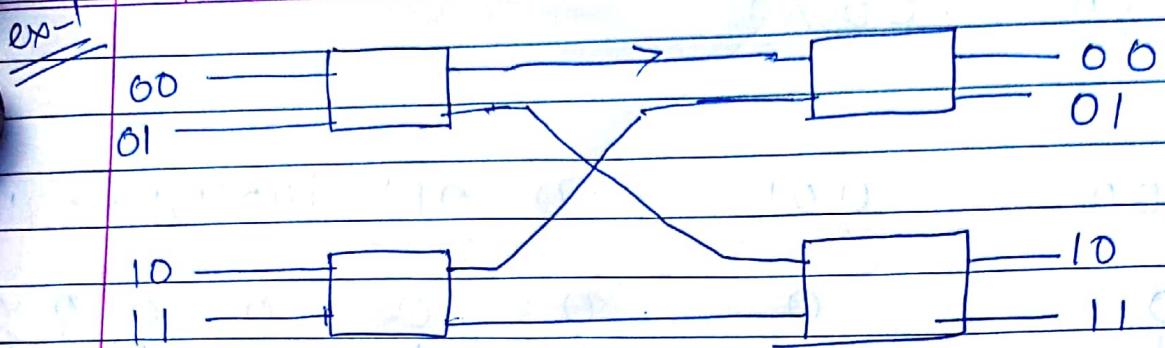
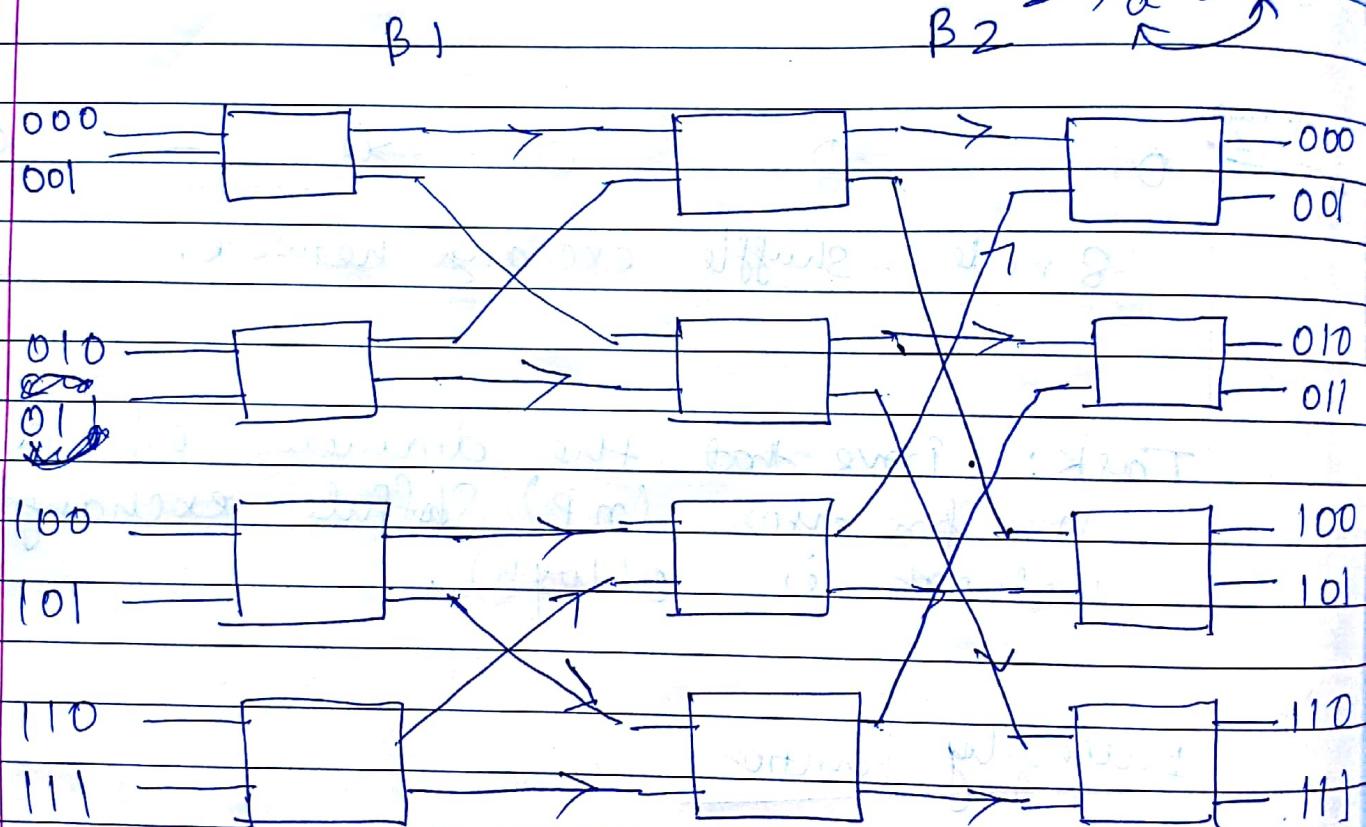
$$\beta_k (b_{n-1} \ b_{n-2} \ \dots \ b_{k+1} \ b_k \ b_{k-1} \ \dots \ b_2 \ b_1 \ b_0)$$

$$= b_{n-1} \ b_{n-2} \ \dots \ b_{k+1} \ b_0 \ b_{k-1} \ \dots \ b_2 \ b_1 \ b_0$$

$k \rightarrow 1 \text{ to } n-1$

$[0, n-1]$

$0 < k \leq n-1$

~~ex-2~~

-1 (single bit right rotation)

0

→ 0 0 0

→ 0 0 1

→ 0 1 0

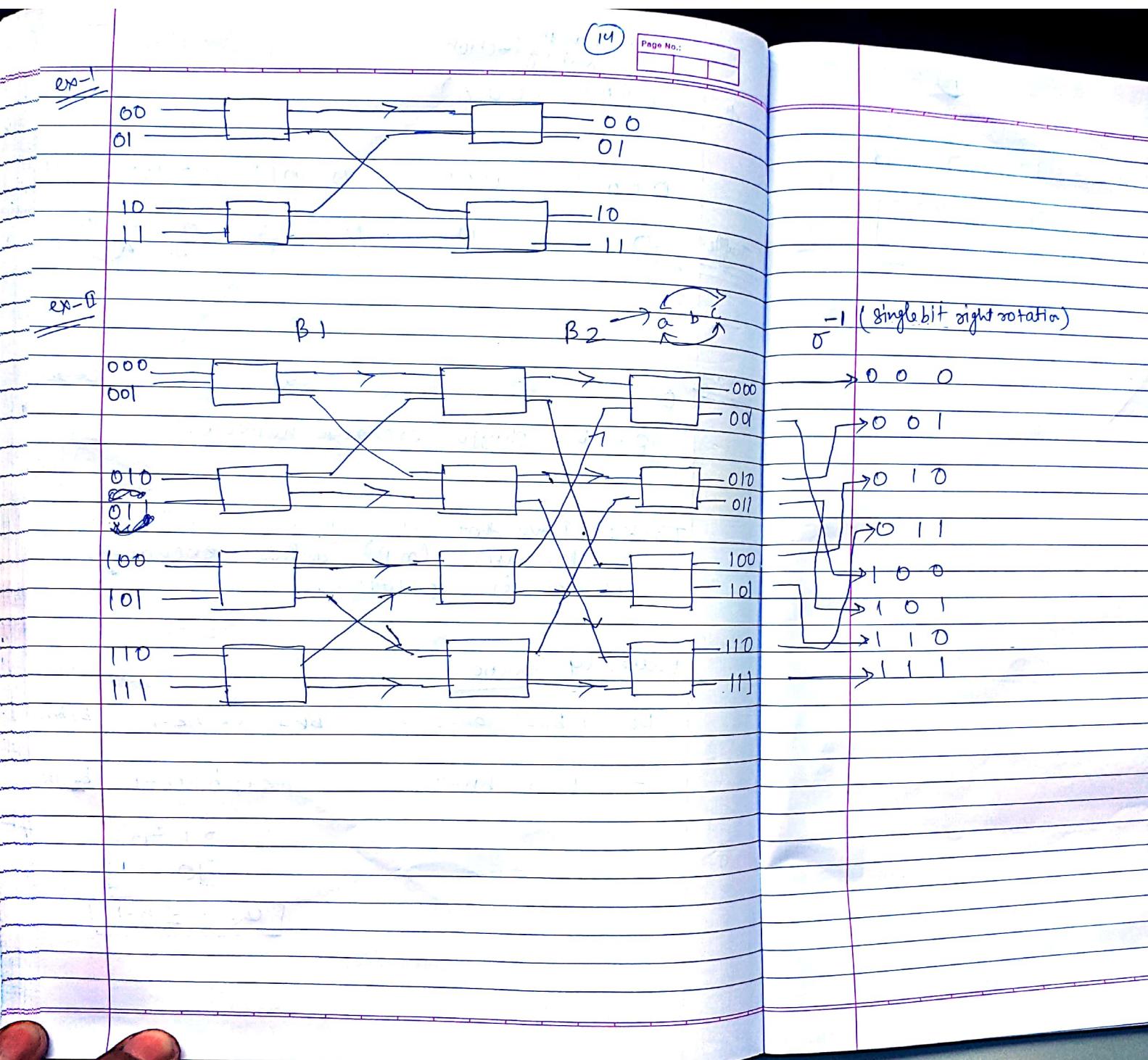
→ 0 1 1

→ 1 0 0

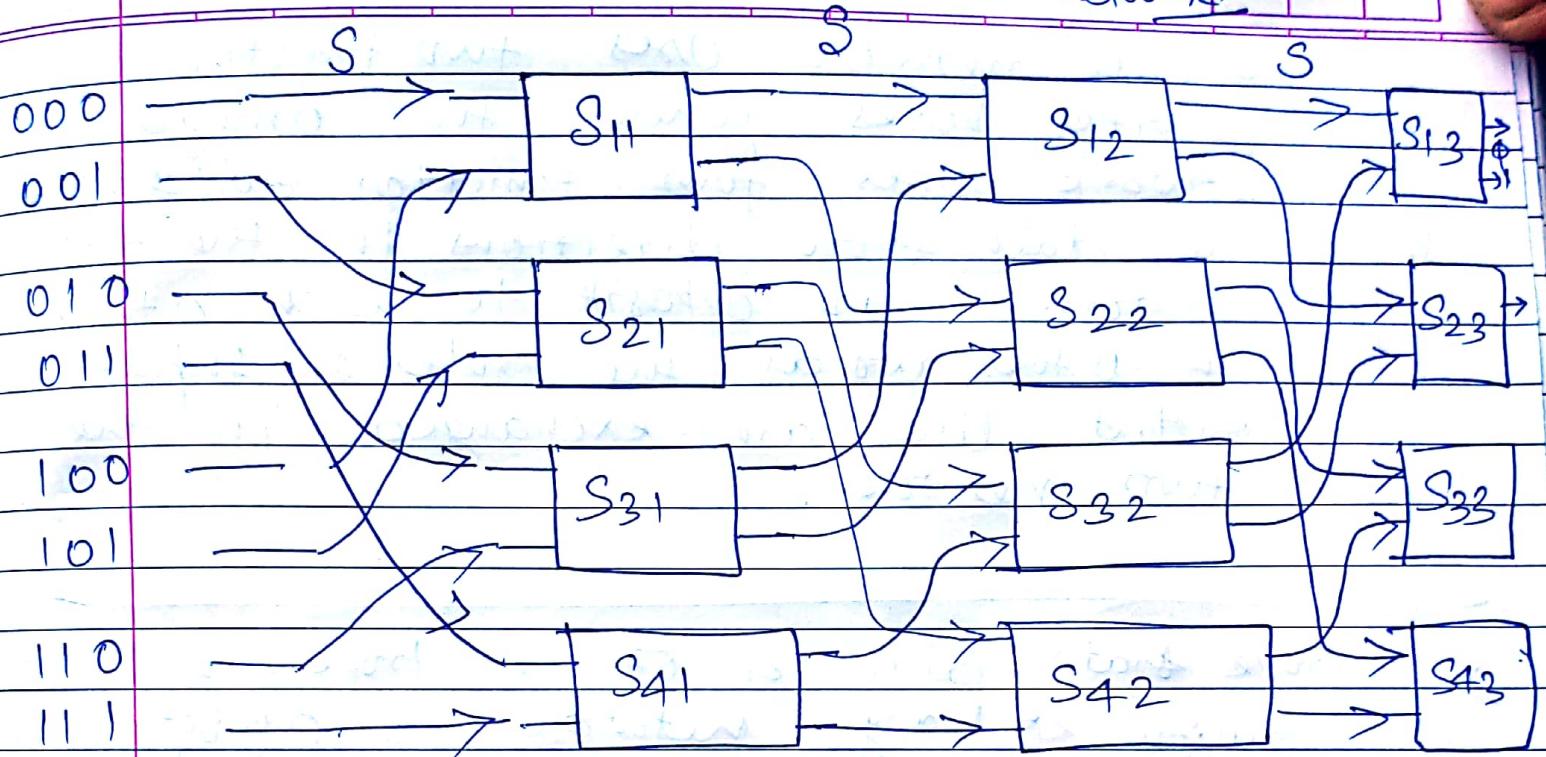
→ 1 0 1

→ 1 1 0

→ 1 1 1



3 stage 8×8 Omega switching network



Reposition Omega network. (changing position

$S_{32} \leftrightarrow S_{22}$

- # The cube network uses two function switch boxes whereas the omega network uses four function switch boxes.
- # The data flow directions in the two networks are opposite to each other. In other words, the roles of input-output lines are exchanged in the two network.

✓ Prove that diameter of n-processor shuffle exchange network is $O(\log n)$

(19)

Page No.:		

S

E

0 0 0 —————→ 0 0 0 —————

0 0 1 0 0 1

~~0 0 0~~

0 1 0

0 1 1

1 0 0

1 0 1

1 1 0

1 1 1

0 0 0

0 0 1

0 1 0

0 1 1

1 0 0

1 0 1

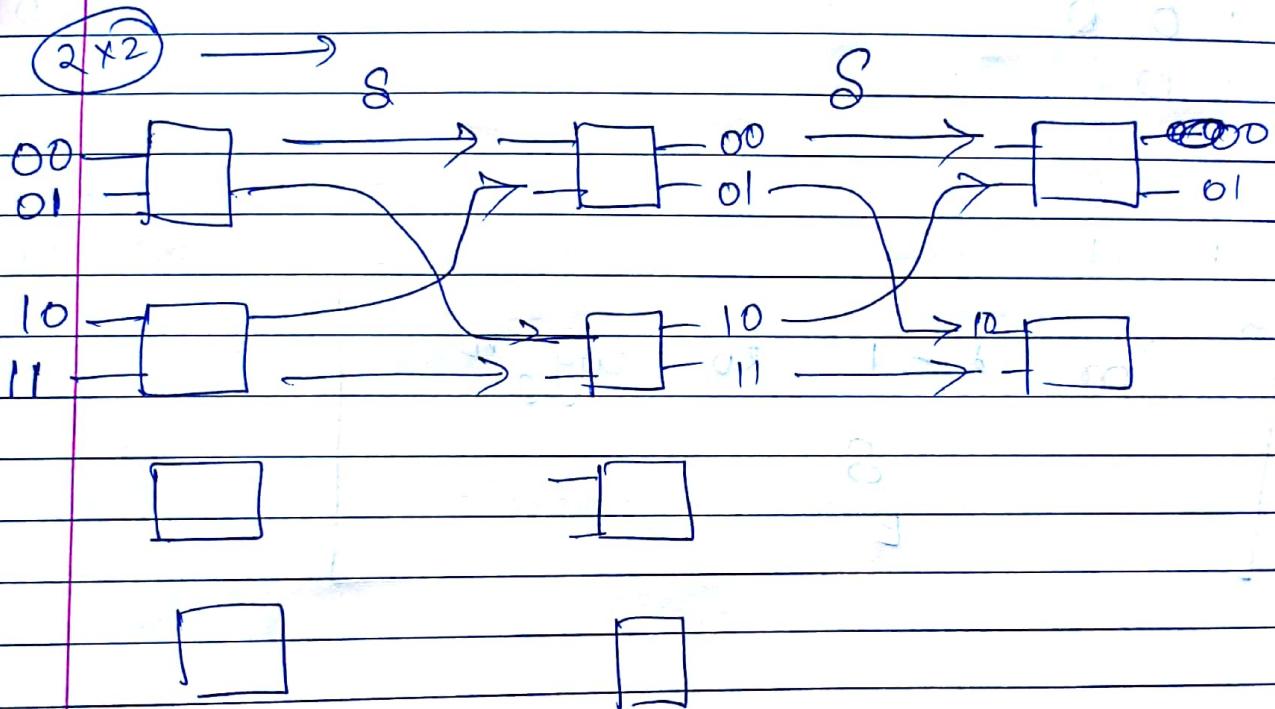
1 1 0

1 1 1

[for $i = 1$ to $\log_2 n$]

S
E

let us consider a matrix of size $m \times m$. To prove the Inverse of Transpose matrix can be obtained by ~~m-perfect shuffle~~ or m -shuffle.



$$A^{-1} = ad$$

(2) Page No.:

$$\begin{bmatrix} \phi & 0 \\ 0 & \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$[2 \times 2]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$m=2 \quad 2^2 \times 2^2 = 16$$

0 0 0 0	0 0 0 1	0 0 1 0	0 0 1 1
0 1 0 0	0 1 0 1	0 1 1 0	0 1 1 1
1 0 0 0	1 0 0 1	1 0 1 0	1 0 1 1
1 1 0 0	1 1 1 0	1 1 1 0	1 1 1 1

\rightarrow
~~1000~~

row col

1 0 0 1

1 1
✓

0 0 1 1

1 1
✓

0 1 1 0

Other row

$$2^m \times 2^m = 2^{2m}$$

bits = 2m

L row | J th Column
 $a_{2n-1} a_{2n-2} \dots a_m | a_{m-1} a_{m-2} \dots a_2 a_1 a_0$

↓
 m shuffle.

$a_{m-1} a_{m-2} \dots a_2 a_1 a_0$ | $a_{2m-1} a_{2m-2} \dots a_m$
 Jth column | Lth row.

00	0100	1000	0000	0000
100	0110	1010	0010	0010
001	0101	1001	0001	0001
000	0111	1011	0011	0011

Ans

Ans

Ans

Ans

Ans

01/04/2018.

Page No.:

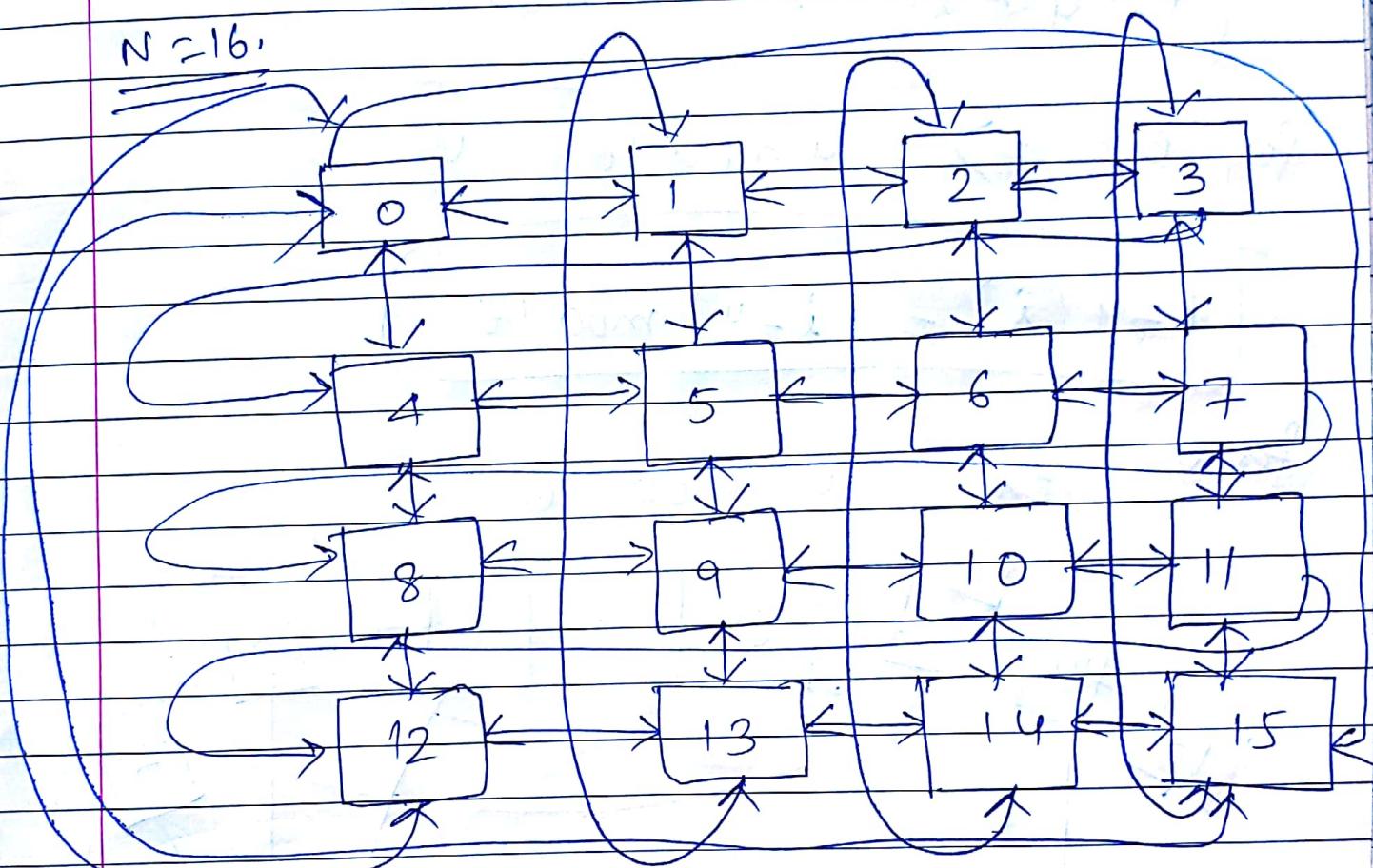
Iliac Routing function

i → address of Processor

$$R_{\pm 1}(i) = (i \pm 1) \bmod N$$

$$R_{\pm r}(i) = (i \pm r) \bmod n$$

where $r = \sqrt{N}$, $0 \leq i \leq N-1$



$$R+1(i) = (i+1) \bmod 16$$

$$r = 4$$

$$0 \leq i \leq 15$$

$$\text{for } 0 \Rightarrow 1 \bmod 16 = 1$$

$$\text{for } 1 \Rightarrow 2 \bmod 16 = 2$$

$$\text{for } 2 \Rightarrow 3 \bmod 16 = 3$$

$$\text{for } 3 \Rightarrow 4 \bmod 16 = 4$$

$$R-1(i) = (i-1) \bmod 16$$

for 0 $\Rightarrow -1 \bmod 16 = 15$

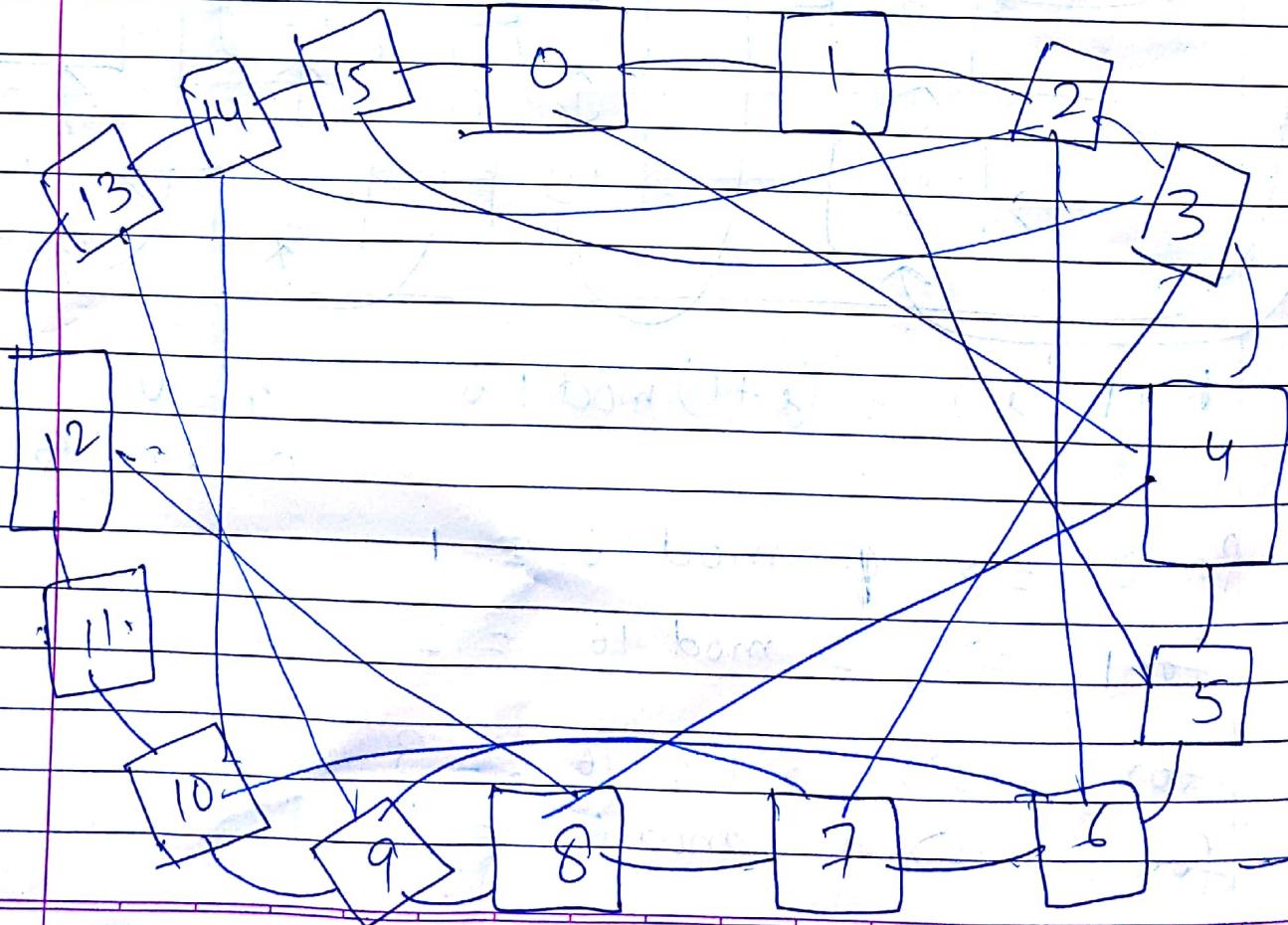
for 1 $\Rightarrow 0 \bmod 16 = 0$

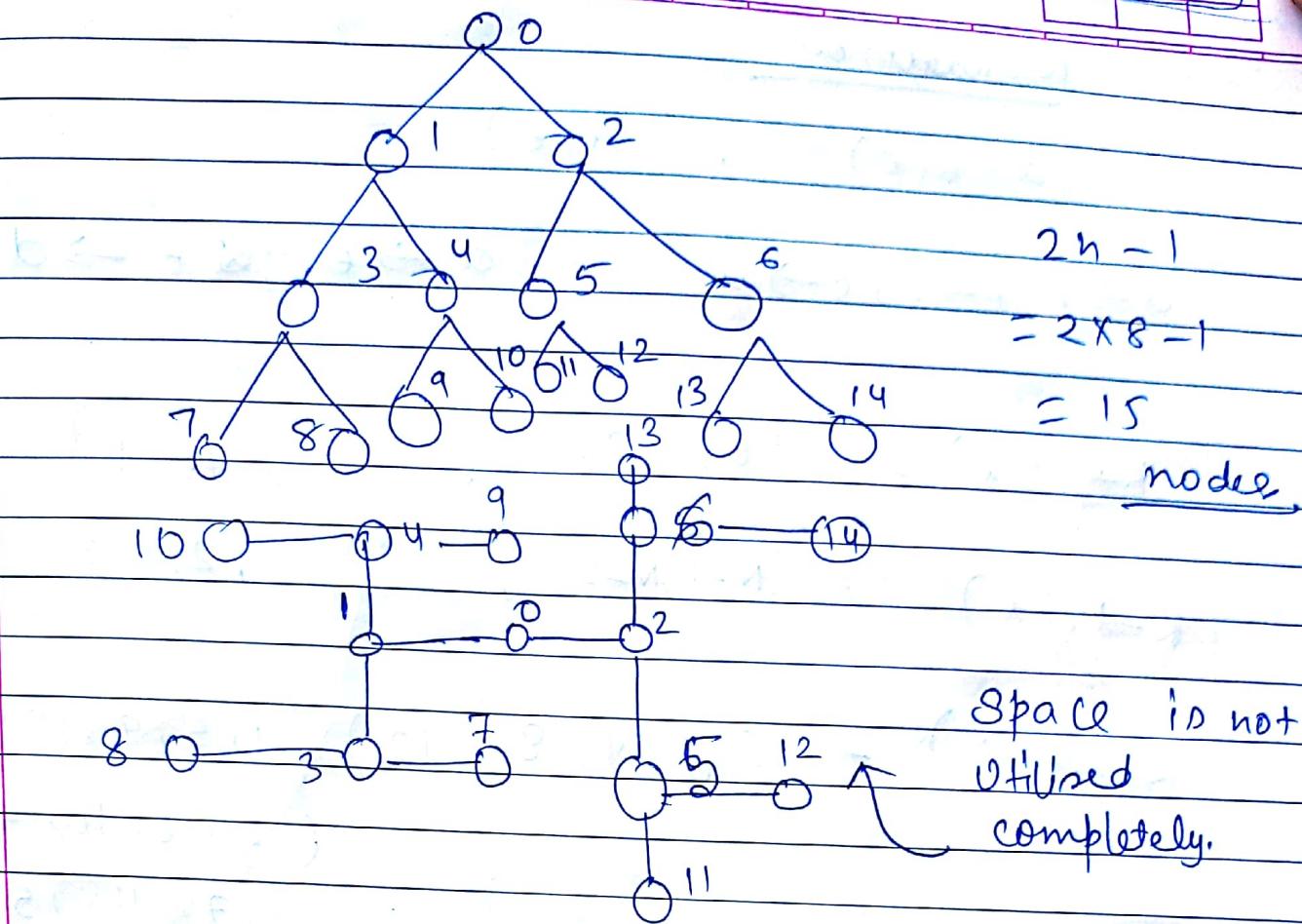
$$R+4(i) = (i+4) \bmod 16$$

for 0 $\Rightarrow 4 \bmod 16 = 4$

$$R-4(i) = (i-4) \bmod 16$$

for 0 $\Rightarrow -4 \bmod 16$





original Illiac Routing function is implemented for 64 PE (processing element).

R+1

R-1

R+4

R-4

This is not suitable for fabrication



Permutation:

(a, b, c) , (d, e)

a → b, b → c, c → a

d → e & e → d

R+1(i) = (0, 1, 2, ..., N-1)

R-1(i) = (N-1, N-2, ..., 2, 1, 0)

R+ γ (i) = (0, 4, 8, 12) (1, 5, 9, 13)

(2, 6, 10, 14)

(3, 7, 11, 15)

= $\sum_{i=0}^{\gamma-1} \pi(i, i+\gamma, i+2\gamma, \dots, i+N-\gamma)$

$\gamma-1$

R- γ = $\pi_{i=0}^{\gamma-1}$

$$l = a + (n-1)d$$

$$= i + (\gamma-1)\gamma$$

$$= i + \gamma^2 - \gamma$$

$$= i + N - \gamma$$

$\gamma, 2\gamma, 3\gamma$
 $\gamma + (n-1)\gamma$ $\gamma + \gamma(\gamma-1)$

Barrel Shifter.

$$B+j(i) = (i + 2^j) \pmod{N}$$

$$B-j(i) = (i - 2^j) \pmod{N}$$

$$0 \leq i \leq N-1, 0 \leq j \leq n-1 \text{ and}$$

$$n = \log_2 N$$

$$B+0 = R+1, B-0 = R-1, B+n/2 = R+r$$

$$B-n/2 = R-s$$

$$B \leq (\log_2 N)/2$$

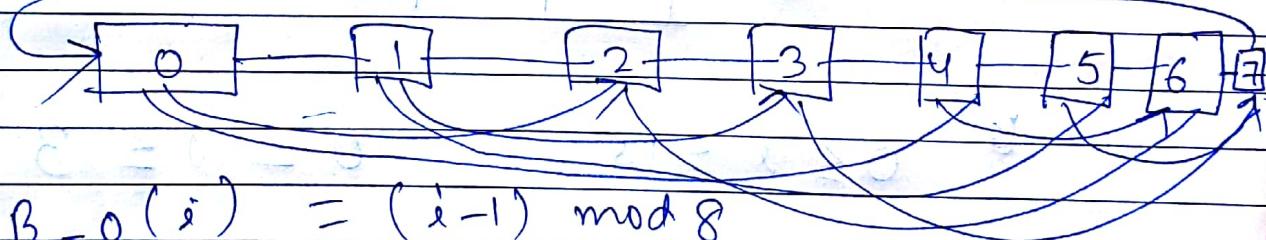
CIS.

~~$$B \pm j(i) = (i \pm 2^j) \pmod{N}$$~~

$$N=8$$

$$, 0 \leq i \leq 7, 0 \leq j \leq 2$$

$$B+0(i) = (i+1) \pmod{8}$$

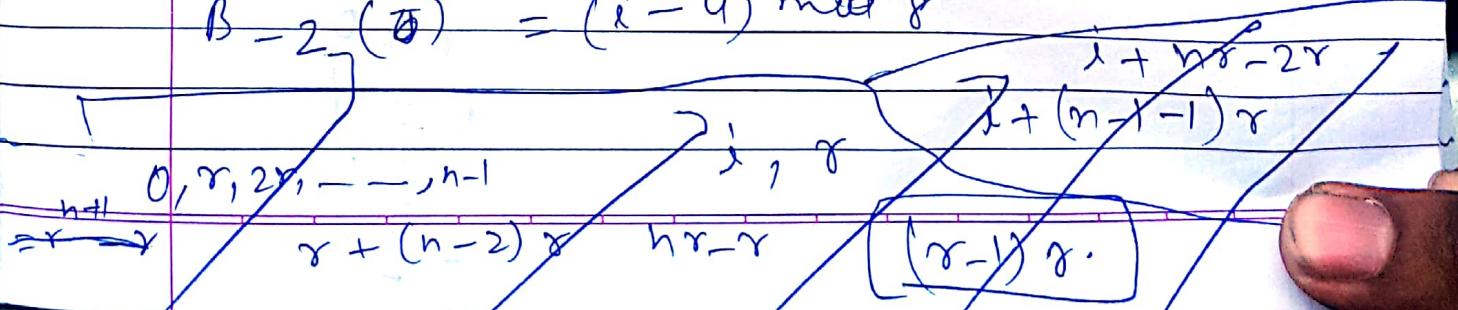


$$B-0(i) = (i-1) \pmod{8}$$

$$B+1(i) = (i+2) \pmod{8}$$

$$B+2(i) = (i+4) \pmod{8}$$

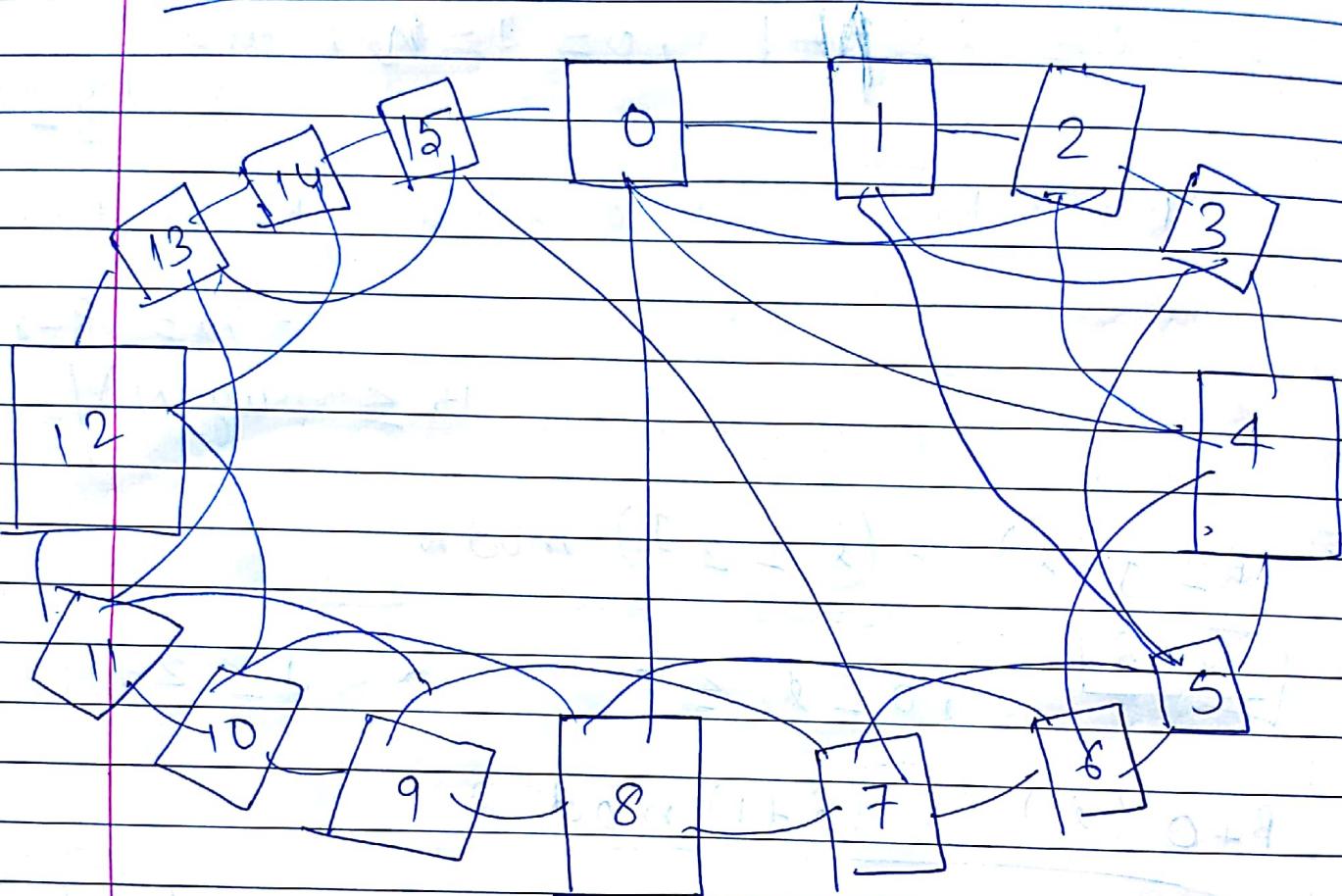
$$B-2(i) = (i-4) \pmod{8}$$



Redraw for $N = 16$ in the form of a chordal ring.

$$B_{\pm j}(i) = (i \pm 2^j) \bmod N$$

$$N = 16$$



$$0 \leq i \leq 15, \quad 0 \leq j \leq 3$$

$$B+0(i) = (i + 2^j) \bmod 16$$

$$B-0(i) = (i - 1) \bmod 16$$

$$B \pm 0$$

$$B \pm 1$$

$$B \pm 2$$

$$B \pm 3$$

~~Prove that the set of illiac routing functions is a subset of Barrel Shifter.~~

$$R \pm r(i) = (i \pm r) \bmod N$$

$$r = \sqrt{N}, 0 \leq i \leq N-1$$

Barrel:

$$B \pm j(i) = (i \pm 2^j) \bmod N$$

$$0 \leq i \leq N-1,$$

$$0 \leq j \leq n-1$$

$$n = \log_2 N$$

$$\text{Let } N=4$$

$$\text{or } r=2, 0 \leq i \leq 3$$

$$R \pm 2(i) = (i \pm 2) \bmod 4 \quad B \pm 1(i) = (i \pm 2^j) \bmod N$$

$$\text{Let } N=4$$

$$0 \leq i \leq 3$$

$$0 \leq j \leq 1$$

$$R \pm 1(i) = (i \pm 1) \bmod N$$

$$R \pm r(i) = (i \pm r) \bmod N, r = \sqrt{N}$$

$$B \pm j(i) = (i \pm 2^j) \bmod N$$

$$B \pm 0(i) = (i \pm 1) \bmod N = R \pm 1(i)$$

$$B \pm 1(i) = i \pm 2$$

$$n = \log_2 N$$

$$N = 2^n$$

$$\sqrt{N} = 2^{n/2}$$

$$B \pm n/2 (i) = (i \pm 2^{n/2}) \bmod N$$

$$= (i \pm \sqrt{N}) \bmod N$$

$$= (i \pm r) \bmod N$$

$$= R + r(i)$$

~~Task~~

Express the Barrel Shift Routing function in the form of Permutation.

~~Q3~~

Q. Draw the Barrel Shifter with 256 PEs by showing ~~at least~~ all the connections ~~of atleast~~ ~~from~~ at least from one PE.

$$B \pm J(i) = (i \pm 2^J) \bmod N$$

$$0 \leq i \leq N-1, 0 \leq J \leq n-1$$

$$0 \leq i \leq 2^{55}, 0 \leq J \leq 7$$

$$n = \log_2 N$$

$$= \log_2 \frac{256}{8}$$

$$B \pm 0(i) = (i \pm 1) \bmod 256$$

$$= \log_2 2^2$$

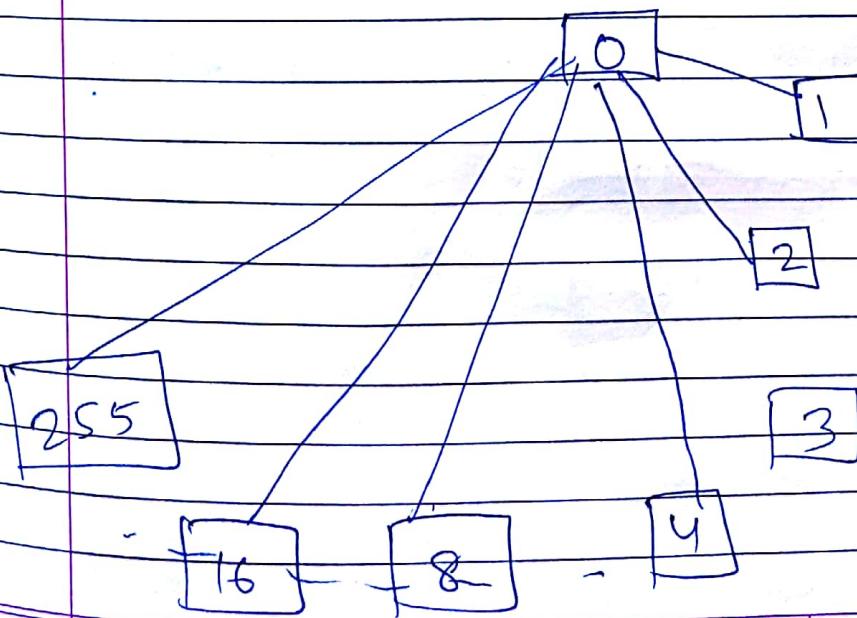
$$\text{for } i=0 \Rightarrow (\pm 1) \bmod 256$$

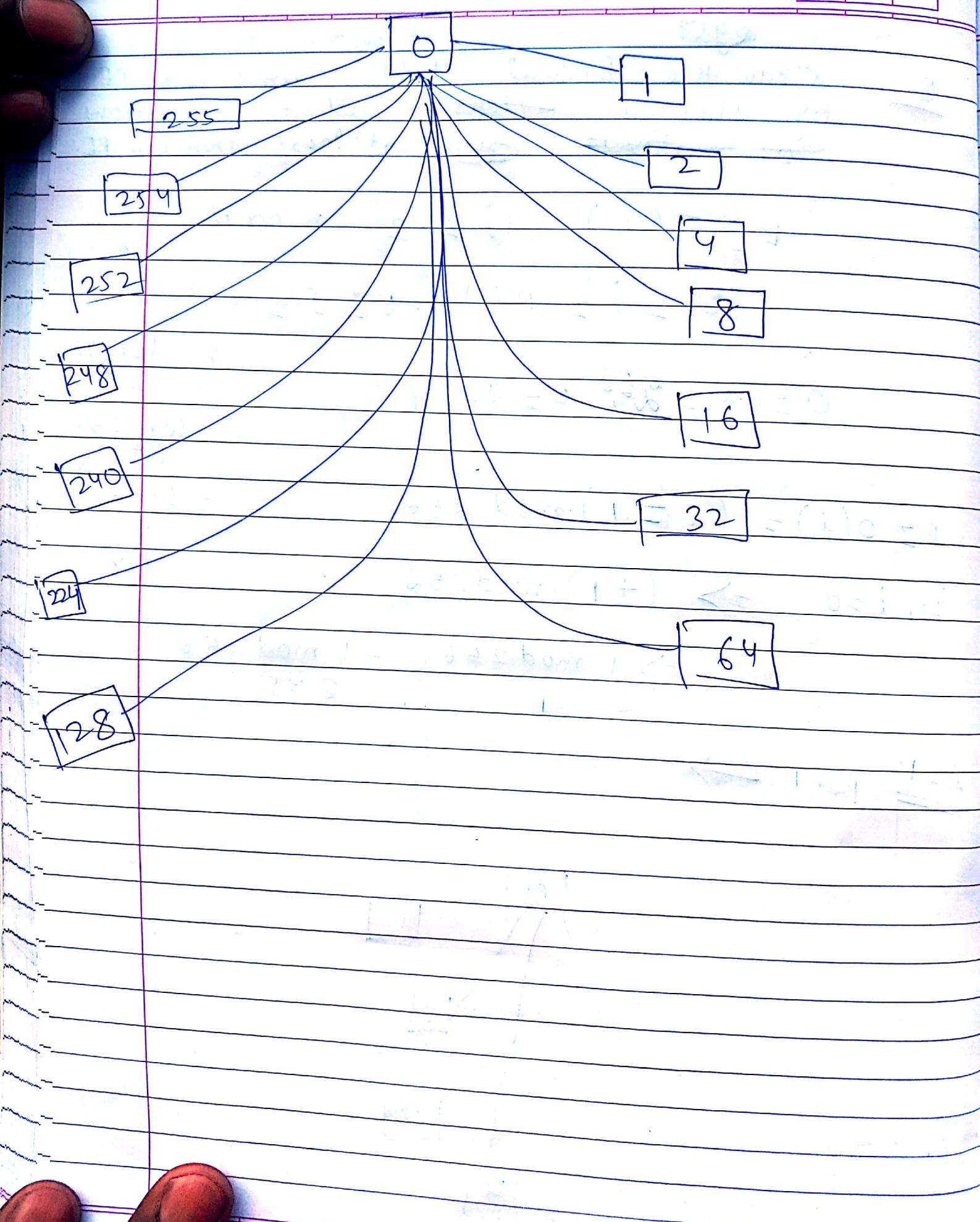
$$= 8$$

$$\Rightarrow 1 \bmod 256, -1 \bmod 256$$

$$1, 255$$

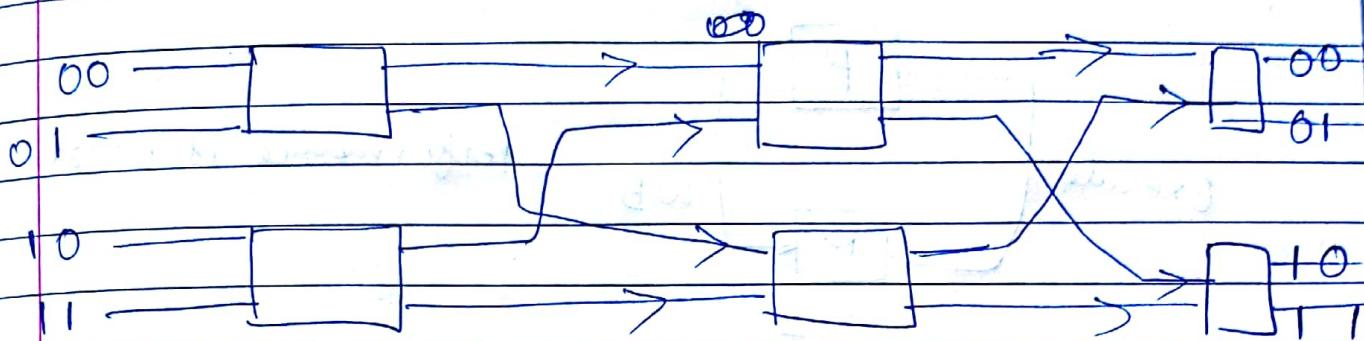
$$i=1' B \pm 1 \rightarrow$$



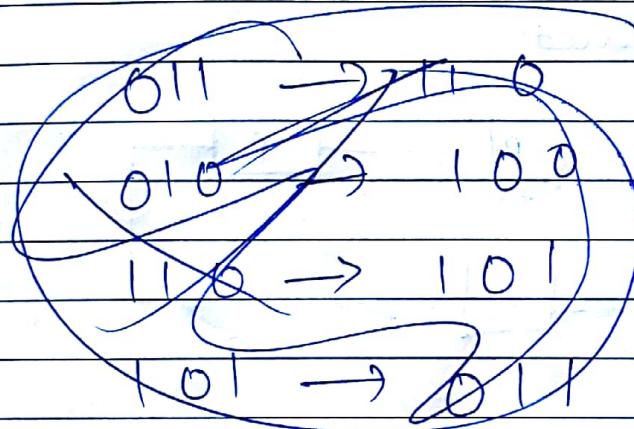


/disprove

Prove that the multi stage Omega network can't perform shift permutation.



2 Stage 2x2 Omega N/w



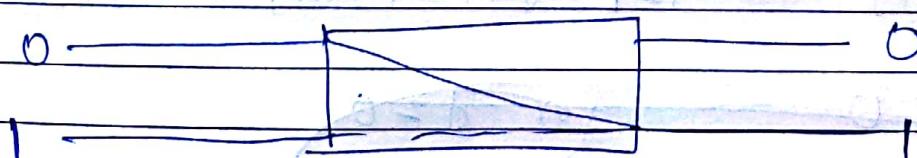
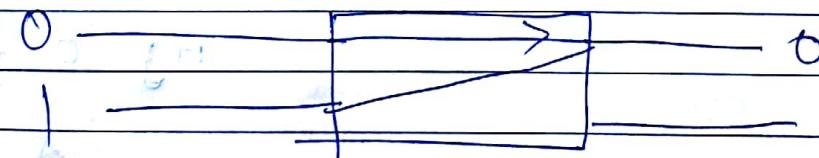
$$00 \rightarrow 00$$

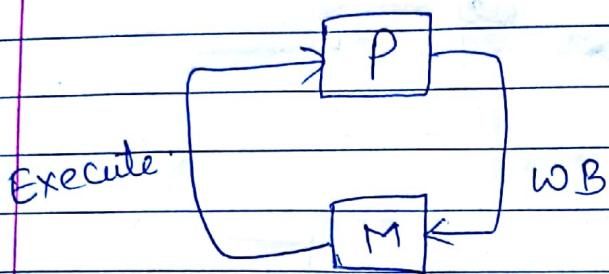
$$01 \rightarrow 10$$

$$10 \rightarrow 01$$

$$11 \rightarrow 11$$

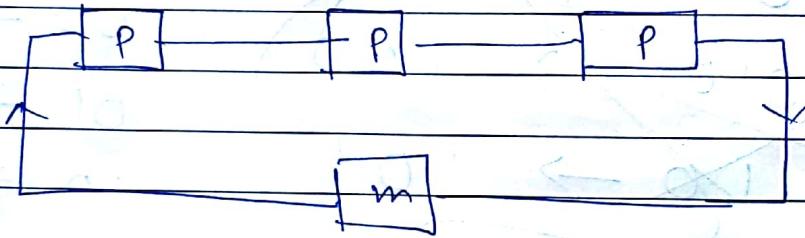
NOT Permitted.



Systolic System

performance is poor.

By choosing collection of processor, performance can be increased.



Graeffe's Root squaring.

$$A_j = 0 \text{ for } j < 0$$

$$\& j > n$$

$$C_j = A_j^2 - 2A_{j-1}A_{j+1} + 2A_{j-2}A_{j+2} - 2A_{j-3}A_{j+3}$$

$$\text{for } j=0$$

$$C_0 = A_0^2$$

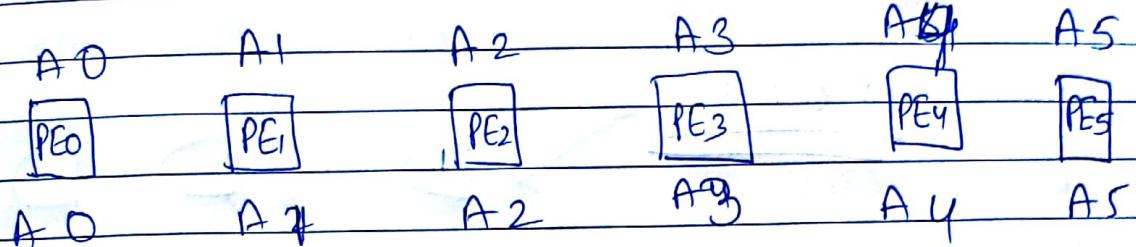
$$C_1 = A_1^2 - 2A_0A_2$$

$$C_2 = A_2^2 - 2A_1A_3 + 2A_0A_4$$

$$C_3 = A_3^2 - 2A_2A_4 + 2A_1A_5 - [2A_0A_6]$$

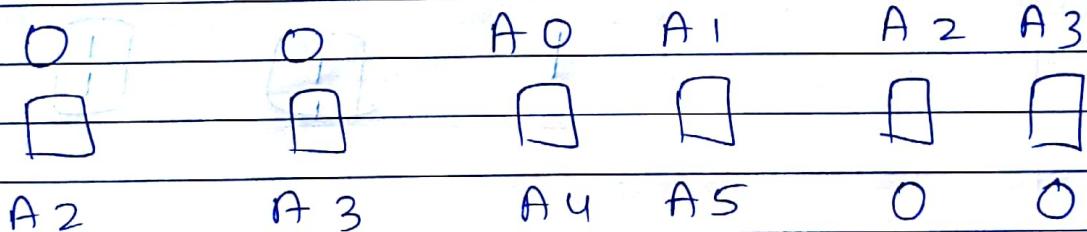
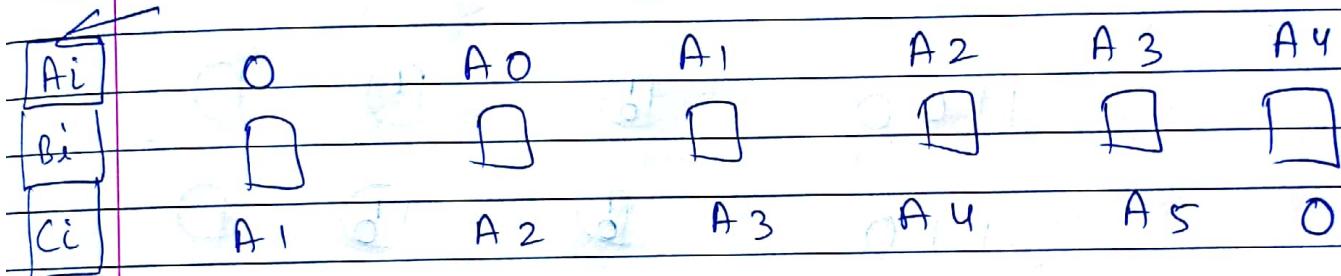
$$C_4 = A_4^2 - 2A_3A_5$$

$$C_5 = A_5^2$$

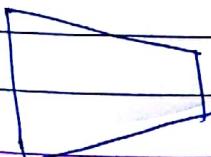


left shift

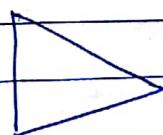
right shift



n=5



n=6



for odd value
blunt edge

for even value
sharp edge

palindrome \rightarrow $w w^r$

form

w^r is reverse of w .

110011 is a palindrome,

110011

\rightarrow D P P

11001

D D D

1100

D D D

110

D D D

11

D D P

H

H P

P

another form:

$w \subset w^r$

10001

Page No.:

Buffered reader : org:

[SOC]

System on chip.

Difference b/w CPU & CPU.