frontiers in
SYSTEMS NEUROSCIENCE

TOPOLOGICAL ISOMORPHISMS OF HUMAN BRAIN AND FINANCIAL MARKET NETWORKS

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CONTENT

INTRODUCTION

- What is topological isomorphism?
- What is the objective of the study?

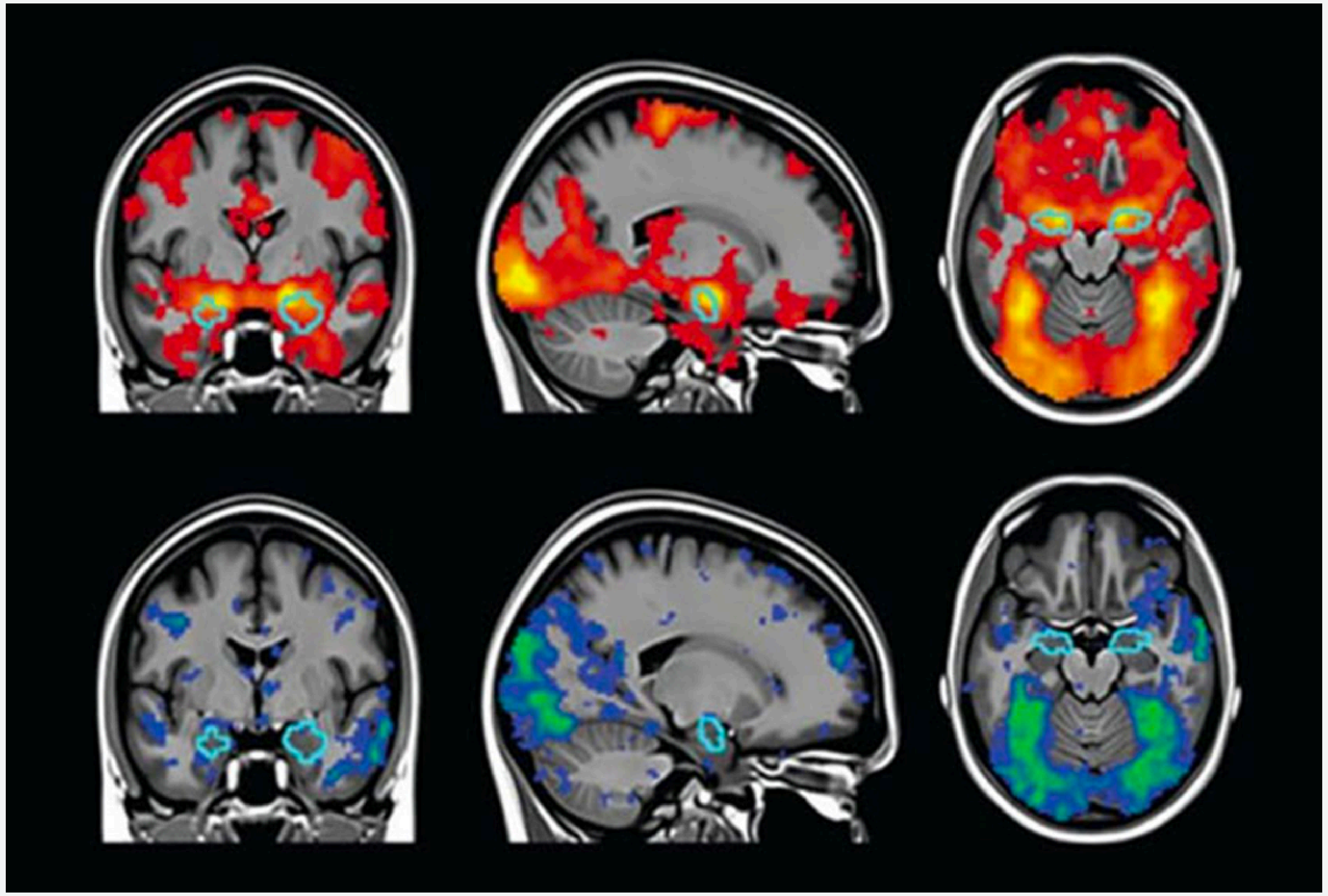
MATERIALS AND METHODS

fMRI DATA ACQUISITION AND PREPROCESSING

- The functional MRI data were acquired from 18 healthy volunteers recruited from the GlaxoSmithKline Clinical Unit Cambridge, in Addenbrooke’s Hospital, Cambridge.
- The images were acquired using a Siemens Magnetom Tim Trio whole body scanner operating at 3T.
- The subject were lying quietly in the scanner for about 9 min 50 s with eyes closed.

The following parameters were used:

- repetition time = 2000 ms; echo time = 30 ms
- flip angle = 78˚
- slice thickness = 3 mm plus 0.75 mm interslice gap
- 32 slices parallel to the inter-commissural (AC–PC) line
- image matrix size = 64 × 64
- within-plane voxel dimensions = 3.0 mm × 3.0 mm.



- The brain was divided into 90 anatomically defined regions.
- Each region represents a specific area of the brain with distinct functions.
- To analyze the brain activity in each region, the voxel time series data was summarized.
- This average activity level is called the "regional time series" for that specific region.
- By averaging the voxel time series within each region, the researchers obtained a single time series for each region, representing the overall activity level of that region over time.

MATERIALS AND METHODS

FINANCIAL DATA SOURCING AND PREPROCESSING

- Daily closing prices for 116 stocks from the New York stock exchange (NYSE) were obtained.
- For each stock, logarithmic daily returns as the time series was used.
- 90 stocks were sampled at random to ensure both the networks had 90 nodes.
- 90 time series simulating the evolution of 90 stock prices was generated according to the Black–Scholes model.

$$Y_i(t) = Y_0 \exp\left(\frac{\mu - \sigma^2}{2} + \sigma B(t)\right)$$

where $Y_i(t)$ is the price of the ith stock at time t, $Y_0 = 30$, $\mu = 0.0006$ is the drift rate, $\sigma = 0.024$ is the volatility of the stock's returns, and $B(t)$ follows Brownian motion.

MATERIALS AND METHODS

CONGRESSIONAL ROLL-CALL DATA

- In addition to fMRI and financial data, a social or political network based on correlations in voting patterns between US senators was constructed.
- These time series describe for each senator whether they voted for or against each bill over the duration of the 100th Congress (1987–1988).
- The correlation between pairs of senators was calculated by taking the mean value of their “voting agreement” over all bills. Voting agreement between two senators was defined as 1 for the bills where they voted the same, and 0 for the bills where they did not.

NETWORK CONSTRUCTION

- Network construction is performed using a statistical and graph theoretic approach.
- For brain functional networks, each node represents a different brain region, and edges represent statistical associations between the time series recorded by functional MRI at each region.
- Similarly, in financial networks, nodes represent stocks, and edges represent correlations between the logarithmic daily returns of the stocks.
- The degree of similarity between the time evolution of a pair of stocks or a pair of brain regions can then be measured by the correlation coefficient.
- It is then possible to draw a network of the system where the weight of each link corresponds to the correlation strength between each pair of nodes.

GRAPH THEORETICAL ANALYSIS

The *clustering coefficient* C_i of a node i is defined as the ratio of the number of triangular connections between the node's nearest neighbors to the maximal possible number of such triangular motifs. The overall clustering coefficient $C(G)$ of a graph G is defined as the average clustering coefficient of its N nodes:

$$C(G) = \frac{1}{N} \sum_{i \in G} C_i \tag{3}$$

The *cost-efficiency* $CE(G)$ is then defined as the global efficiency of a network minus its (arbitrary) topological cost or connection density, i.e., $CE(G) = (E(G) - \kappa)$.

The *modularity*, $Q(G)$, of a graph G quantifies the quality of a possible partition of the network into modules by measuring the fraction of the network's edges that fall inside modules compared to the expected value of this fraction if edges were distributed at random (Newman, 2004). This can be written as:

$$Q(G) = \frac{1}{2m} \sum_{i \neq j} (\mathcal{A}_{ij} = P_{ij}) \delta(M_i, M_j) \tag{6}$$

The *path length* L_{ij} between a pair of nodes i and j is defined as the minimum number of edges that need to be traversed to get from i to j. More commonly, one measures the average inverse path length, or *global efficiency*, $0 < E(G) < 1$, of a graph G which is defined as:

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{L_{ij}} \tag{4}$$

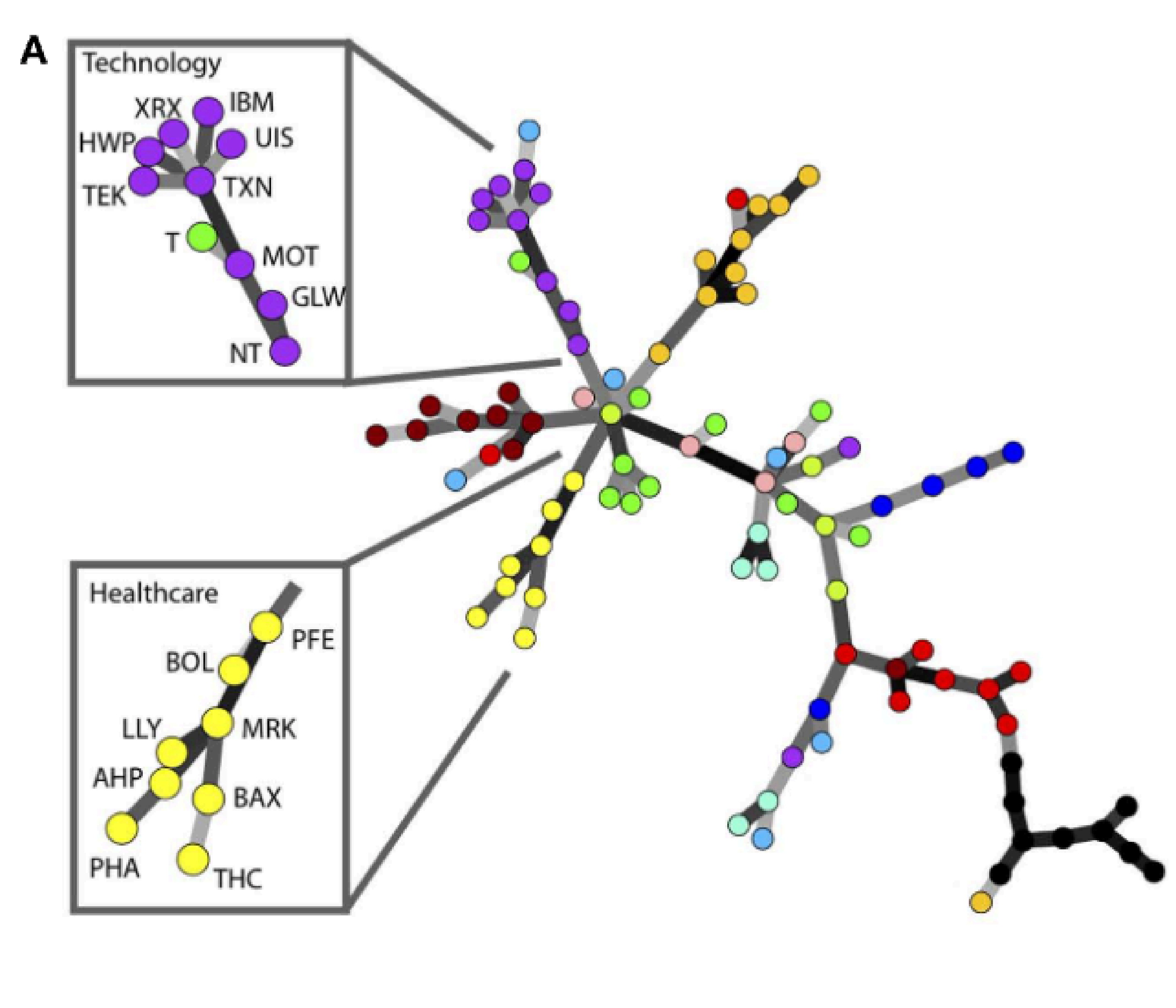
Small-worldness, σ, is a property of a network with high clustering, C, but low characteristic path length, L, compared to the clustering, C_R, and path length, L_R, of a comparable random graph with the same number of nodes and edges and the same degree distribution (Watts and Strogatz, 1998). It is calculated as:

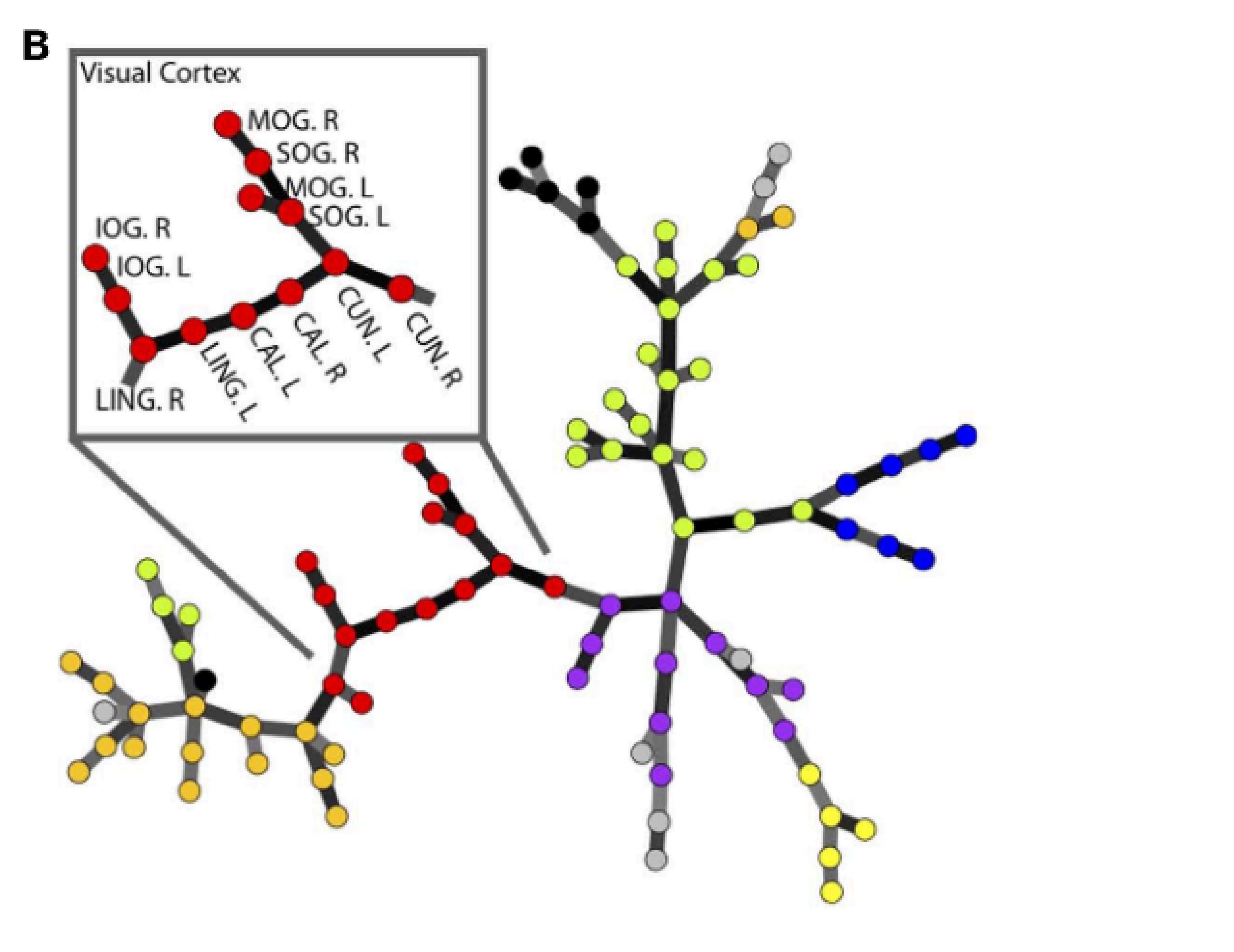
$$\sigma(G) = \frac{C/C_R}{L/L_R} \tag{5}$$

RESULTS

MINIMUM SPANNING TREES

- A minimum spanning tree is a concept in graph theory. Given a connected, undirected graph with weighted edges, a minimum spanning tree is a tree that spans all the vertices in the graph while minimizing the total weight of the edges.
- Key characteristics of a minimum spanning tree:
- **Connectedness:** The tree must connect all the vertices in the original graph.
- **Acyclic:** The tree must not contain any cycles.
- **Minimized Weight:** The sum of the edge weights in the tree must be as small as possible.
- In the network we have constructed the weights are the correlation coeffecients of the nodes.





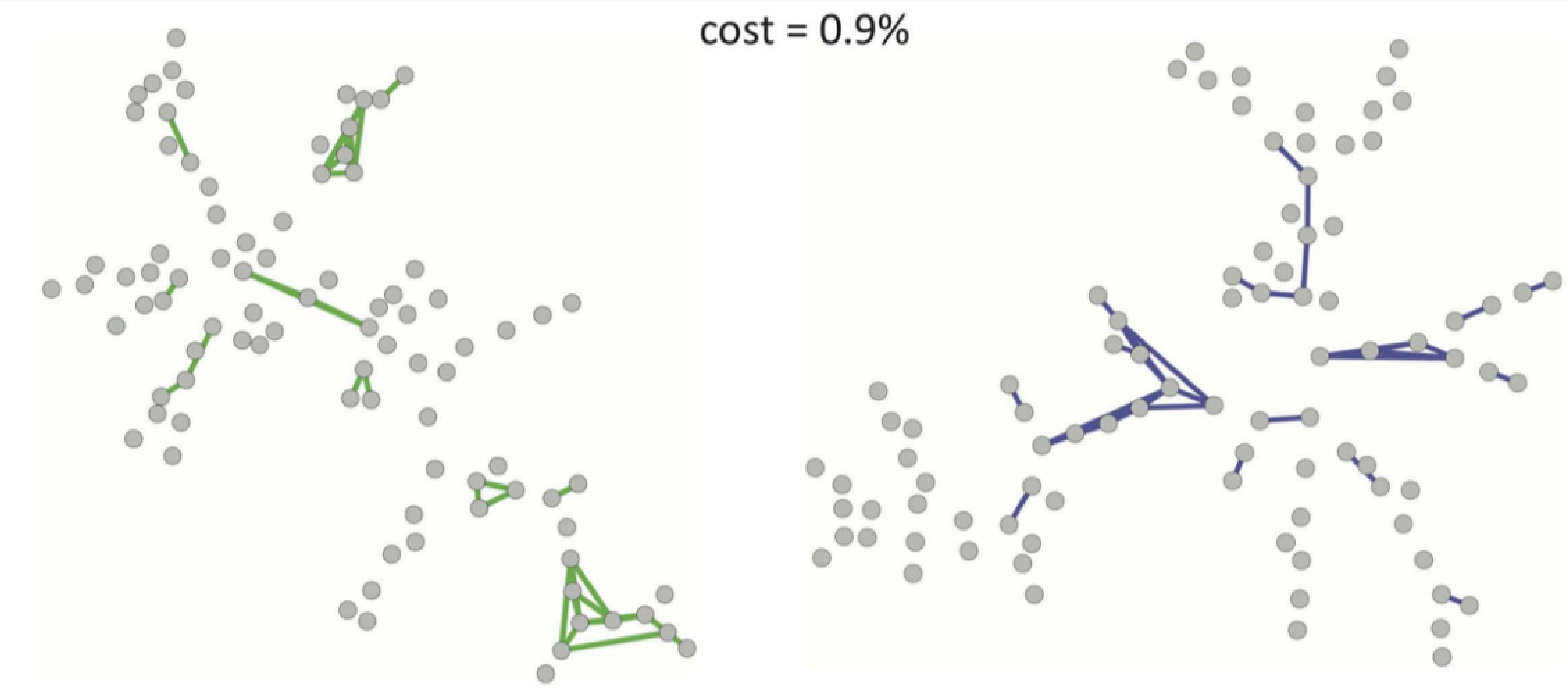
Financial Networks

Brain network

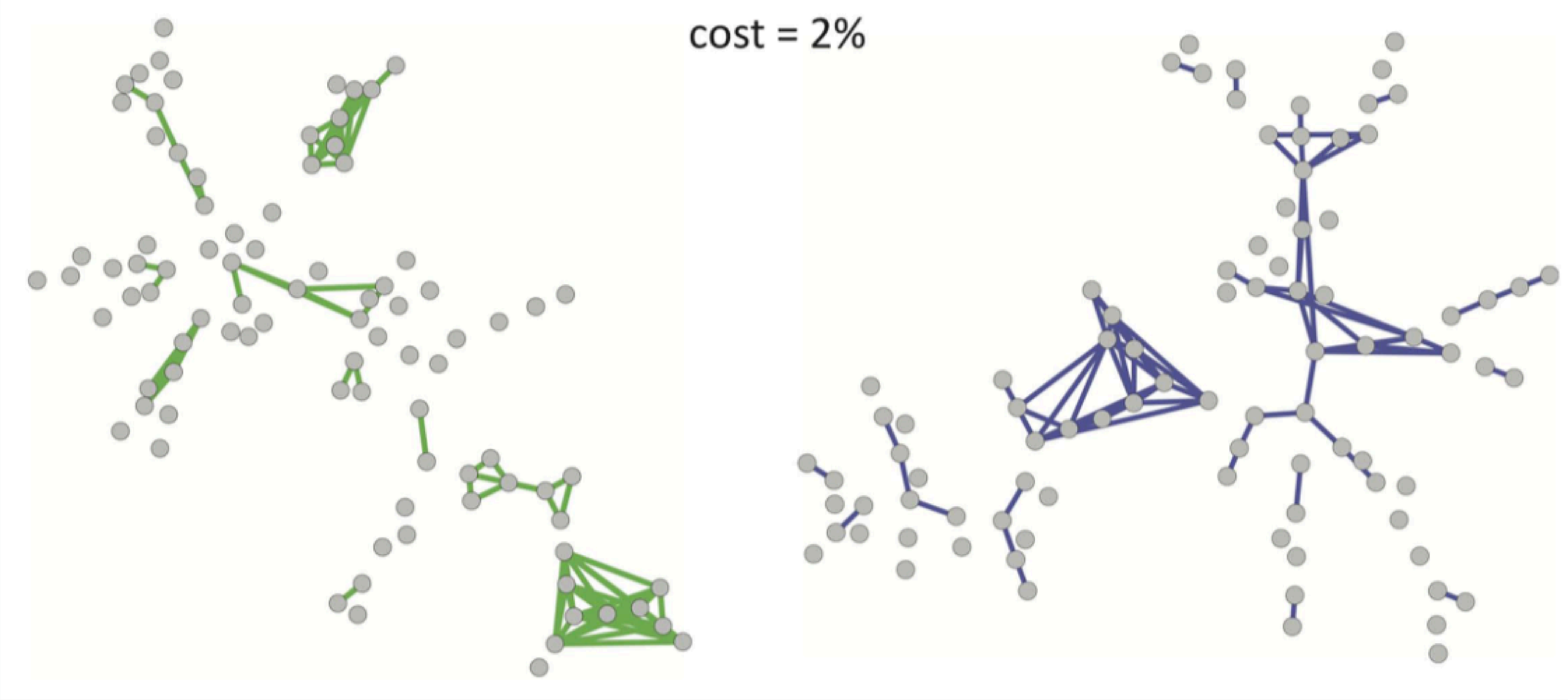
GRAPH GROWTH

- The growth of the brain and financial networks is studied by incrementally adjusting the connection density or cost of the networks.
- The skeleton of minimum spanning tree of the networks is used as a starting point, and links are gradually added in order of decreasing correlation strength to generate networks with varying connection density.
- Observations-
- Several loops, cyclic connections, or triangular motifs appear early on, when only a small fraction of all possible edges has been added.
- As the graph is grown, new edges seem to be preferentially added to the small clusters already present, instead of forming a backbone that connects as many of the nodes as possible.
- The network remains disconnected even after the addition of a very large number of edges.

financial networks **brain network**

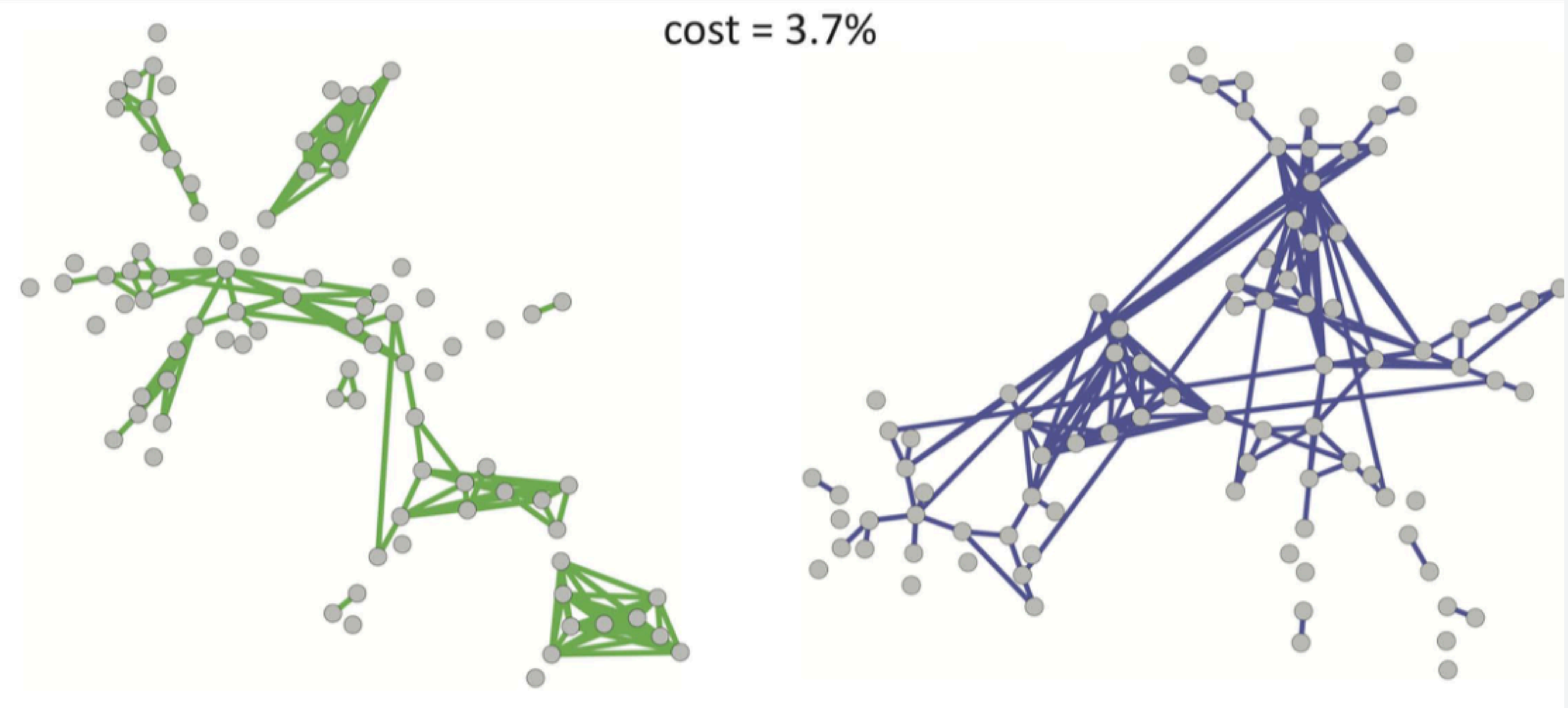


financial networks **brain network**



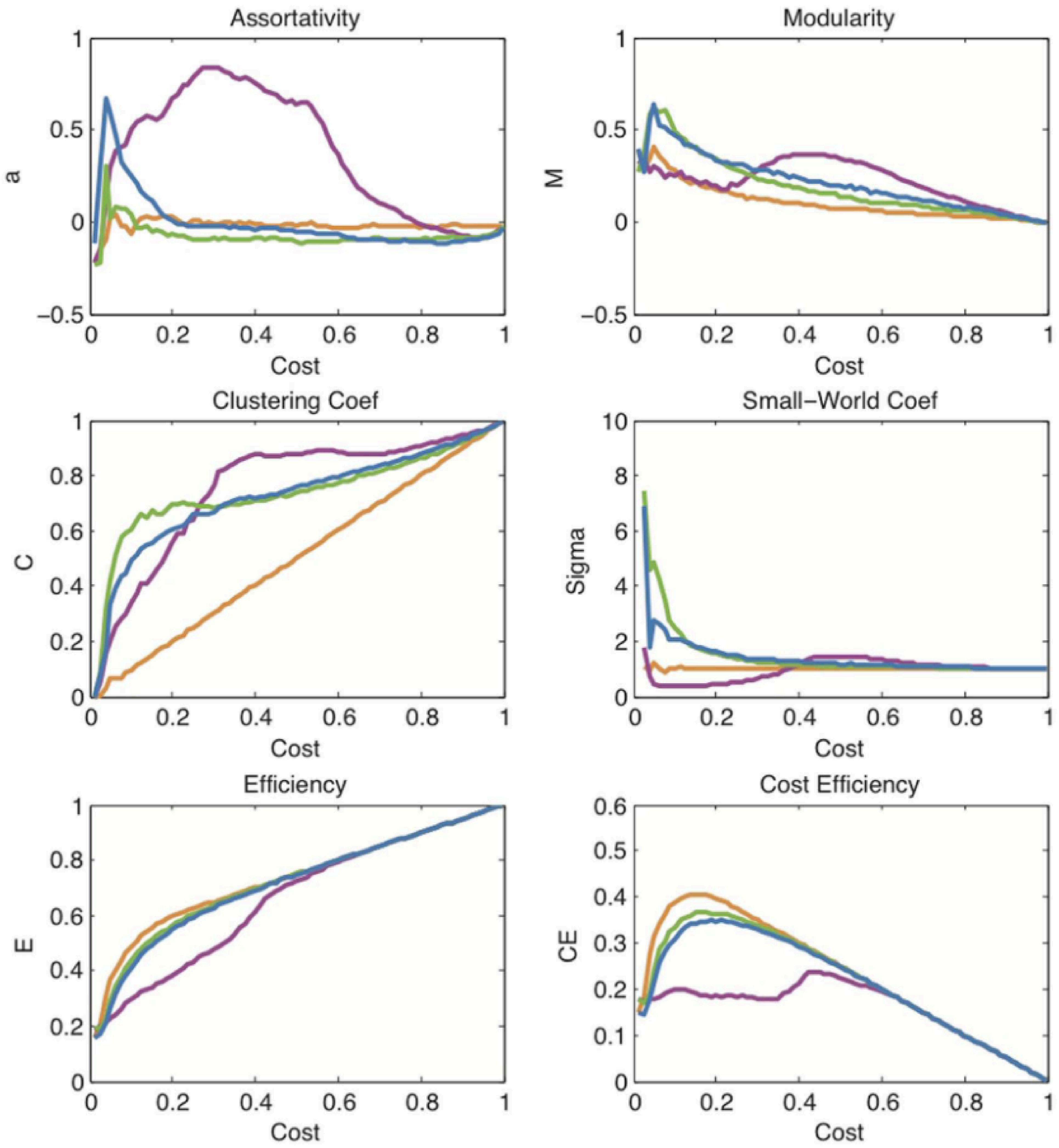
financial networks

brain network



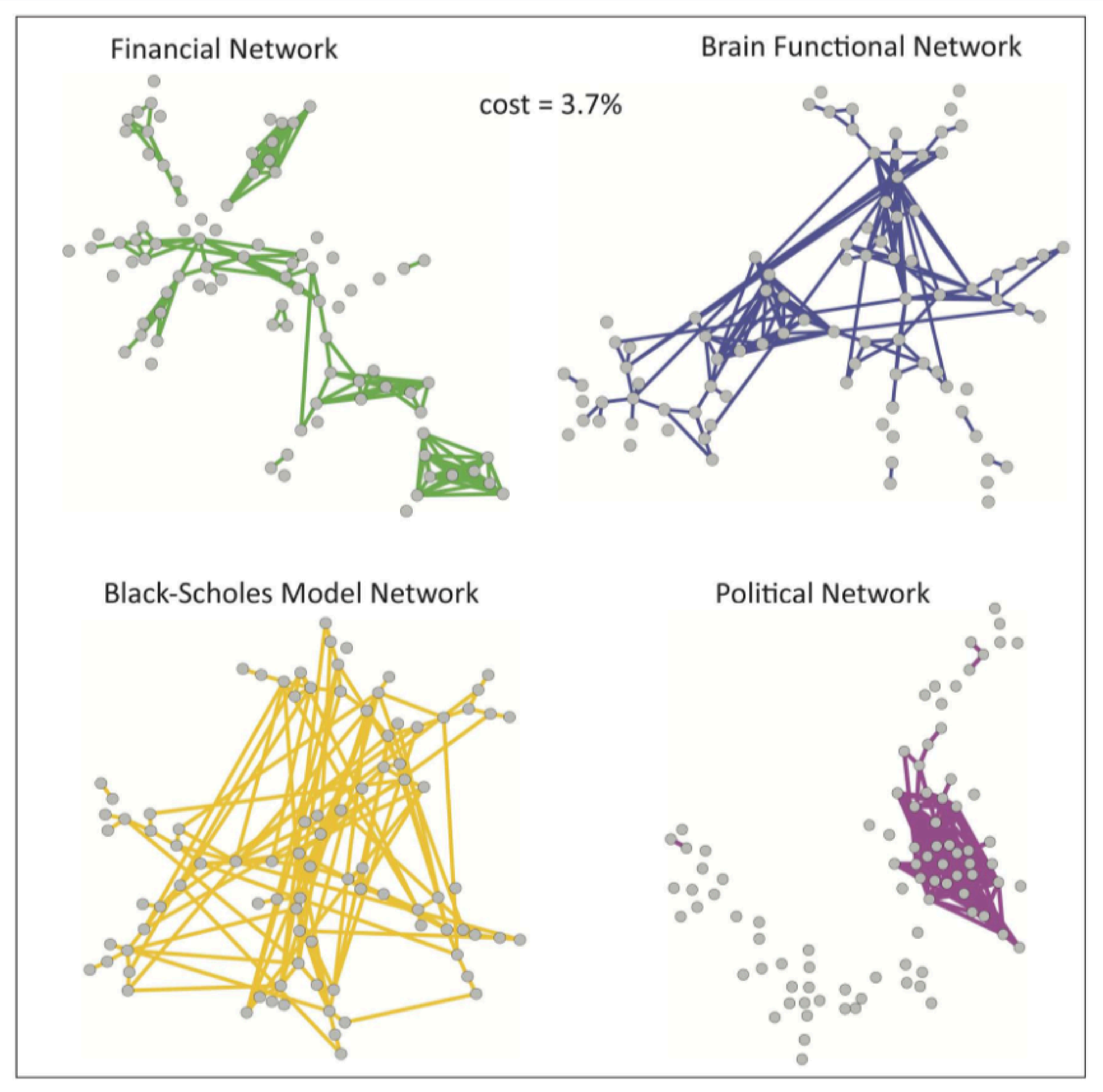
TRACKING NETWORK TOPOLOGY AS A FUNCTION OF CONNECTION DENSITY

- In order to better understand the networks, it is helpful to analyze and visualize their key topological properties.
- Both the brain and market networks demonstrate high clustering and high global efficiency, indicating that they have small-world properties.
- They both also have high modularity and positive cost-efficiency over a range of costs, with maximum cost-efficiency at a connection density of about 20%
- the political network is significantly less efficient and less small-world over a large range of costs for sparse networks. In addition, it reaches its maximally clustered and modular state at much higher connection densities than the brain and financial networks.
- The correlation structure observed in the Black-Scholes model does not match the characteristic pattern displayed by real financial data.



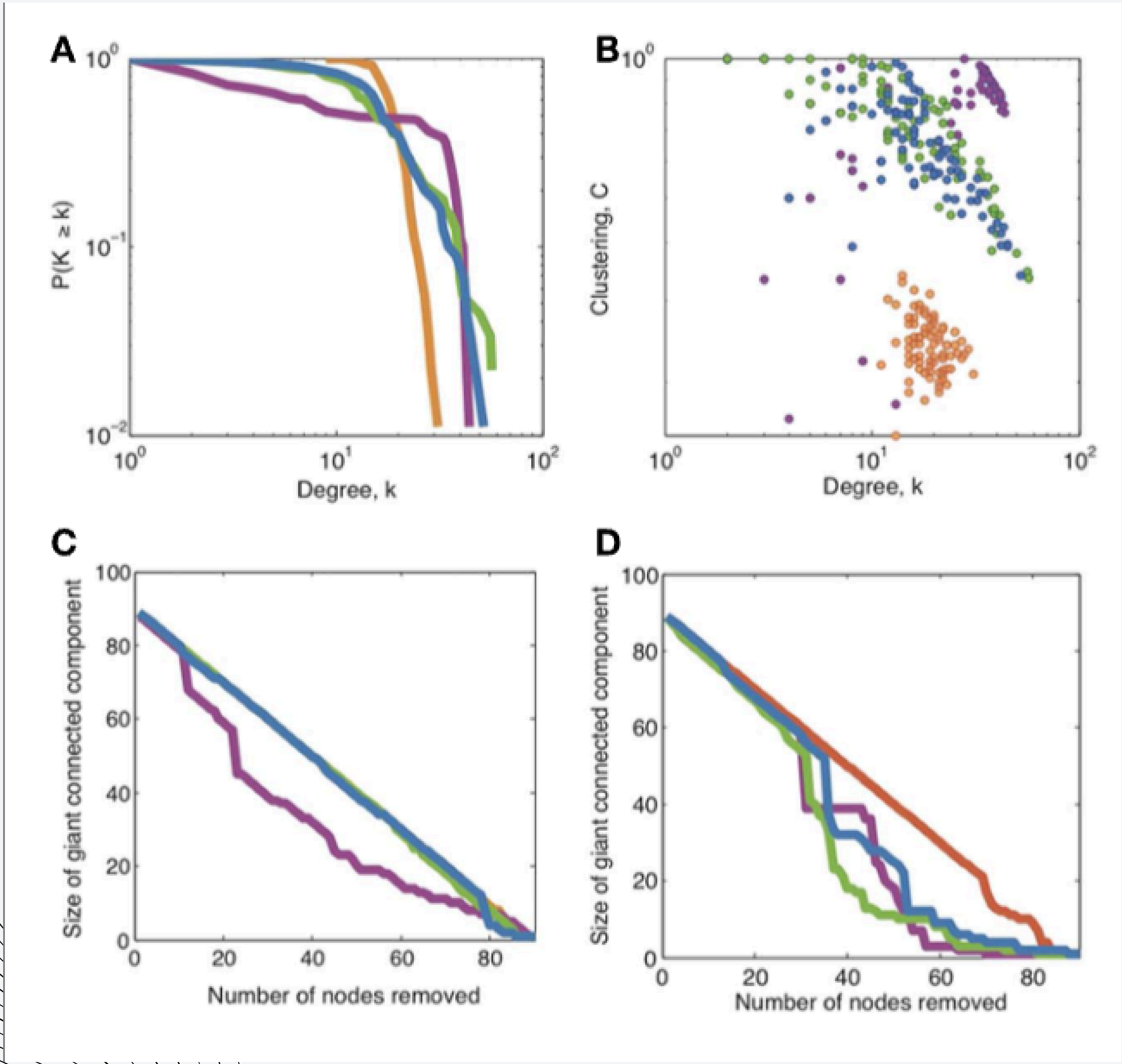
green- financial network
blue- brain network
magenta- political network
orange- black scholes network

NETWORK TOPOLOGY AT A PARTICULAR CONNECTION DENSITY



Although it is informative to consider network topology over the full range of possible connection densities, from the minimum (N − 1) edges of the MST to the maximum N × (N − 1)/2 edges of a fully connected network, it is also interesting to look at more detailed topological features of the networks, and for this purpose it is desirable to focus on a particular threshold.

NETWORK TOPOLOGY AT A PARTICULAR CONNECTION DENSITY



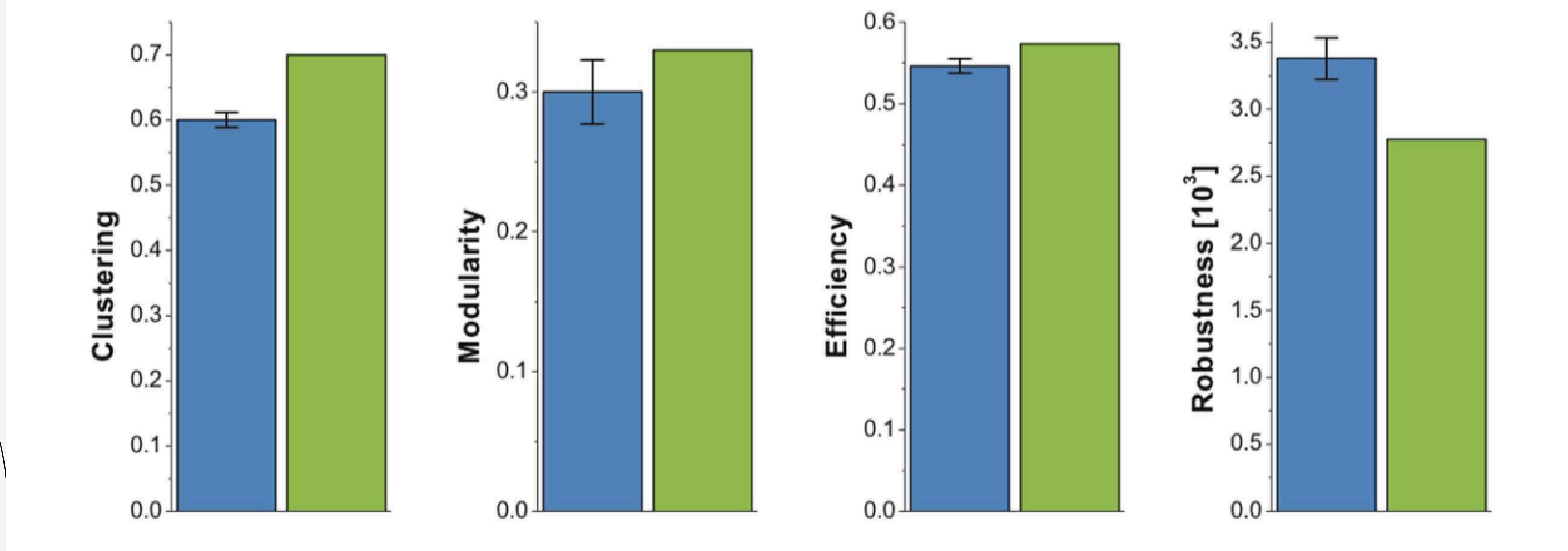
The cost efficiency curve of each system peaks at a particular cost, typically around κ = 20%

To allow a fair comparison between the different networks, the same cost is chosen for all networks. We chose this cost to be the one at which cost-efficiency is maximized in the brain functional network (κ = 21.2%)

green- financial network
blue- brain network
magenta- political network
orange- black scholes network

CONCLUSION

- The paper concludes that there are significant topological similarities between human brain networks and financial market networks, despite their obvious differences.
- Both brain and market networks exhibit non-random, small-world, modular, and hierarchical properties, with fat-tailed degree distributions indicating the presence of highly connected hubs.
- However, the financial market networks are more efficient and modular than the brain networks, but also less robust to systemic disintegration due to hub deletion.



green- financial network
blue- brain network