Today's Content :-

→ Modular operator
→ Modular arithmetic
→ 1 Hard problem.

int range → $(-2 * 10^9 \text{ to } 2 * 10^9)$
long rang → $(-8 * 10^{18} \text{ to } 8 * 10^{18})$

% → modules / Remainder

Dividend = divisor * quotient + remainder

$10 \% 4 = 2 \Rightarrow 4 * \left(\frac{10}{4}\right) + r \Rightarrow 8 + r = 10, \; r = 2$

$13 \% 5 = 3 \Rightarrow 5 * \left(\frac{13}{5}\right) + r \Rightarrow 10 + r = 13, \; r = 3$

$100 \% 7 \Rightarrow 7 * \left(\frac{100}{7}\right) + r \Rightarrow 100 \Rightarrow 98 + r = 100,$
$r = 2$

$150 \% 7 \Rightarrow 7 * \left(\frac{150}{7}\right) + r \Rightarrow 147 + r = 150,$
$r = 3$

$-60 \% 9 \Rightarrow 9 * \left(\frac{-60}{9}\right) + r = -60,$

$-54 + r = -60$

$r = -6$

$$\left\{ \begin{array}{c} -\infty \\ | \\ +\infty \end{array} \right\} \quad \% \, 10 \longrightarrow \{ \underset{\text{min}}{0}, \; \underset{\text{max}}{9} \}$$

min max

$$\left\{ \begin{array}{c} -\infty \\ | \\ +\infty \end{array} \right\} \longrightarrow \underbrace{\% \, M}_{\text{limit range}} = \{ 0 \quad m-1 \}$$

min max

limit range
- consistent hashing
- hashmap/dict
- Cryptography

Conceptually

Remainder = Dividend - $\big($divisor * $\underbrace{\text{Quotient}\big)}_{}$

greatest multiple of divisor <= dividend.

10 % 4 = 10 - 8 ⇒ **2**

13 % 5 = 13 - 10 ⇒ 3

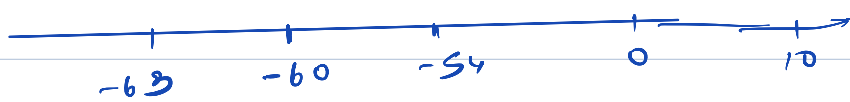
100 % 7 = 100 - 98 ⇒ 2

150 % 7 ⇒ 150 - 147 ⇒ 3

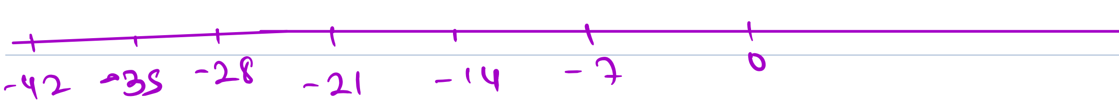
-60 % 9 ⇒ -60 - (greatest multiple of 9 <= -60)

-60 - (-63) ⇒ -60 + 63 ⇒ 3.

-63 -54



-40 % 7 ⇒ -40 - { greatest multiple of 7 <= -40 }



$-40 - \{-42\} \Rightarrow -40 + 42$

$\Rightarrow 42 - 40 \Rightarrow \underline{2}$

As per your language :-

$-80 \% 9$ → language → -8

→ concept → $-80 - \{$greatest multiple of $9 <= -80\}$

$\Rightarrow -80 - (-81) \Rightarrow \underline{1}$

$-40 \% 9$ → language → -4

→ concept → $-40 -$ (greatest multiple of $9 <= -40\}$

$-40 - (-45) \Rightarrow \underline{5}$

$-60 \% 9$ → language → -6

→ concept → $-60 -$ (greatest multiple of $9 <= -60\}$

$-60 - (-63) \Rightarrow \underline{3}$

```
if (x<0) {
    {x % p + p }
}
```

→ only by adding p we can get expected no.

Modular arithmetic

$$\boxed{(a+b)\,\%\,m = (\underbrace{a\,\%\,m}_{(0,\,m-1)} + \underbrace{b\,\%\,m}_{(0,\,m-1)})\,\%\,m}$$

$a=6,\ b=13,\ m=7 \Rightarrow (6\,\%\,7 + 13\,\%\,7)\,\%\,7$

$(19\,\%\,7) \Rightarrow 5$

$\Rightarrow (6+6)\,\%\,7$

$\Rightarrow 12\,\%\,7$

$\Rightarrow \underline{5}$

$a=4,\ b=5,\ m=6 \Rightarrow (4\,\%\,6 + 5\,\%\,6)\,\%\,6$

$\Rightarrow (4+5)\,\%\,6 \Rightarrow \underline{3}$

$\underbrace{(9\,\%\,6)}_{3}$

$$\boxed{(a*b)\,\%\,m \Rightarrow \{\underbrace{a\,\%\,m}_{(0,\,m-1)} * \underbrace{b\,\%\,m}_{(0,\,m-1)}\}\,\%\,m}$$

$a=6,\ b=7,\ m=4 \Rightarrow (6\,\%\,4 * 7\,\%\,4)\,\%\,4$

$42\,\%\,4$

$\Rightarrow 2$

$(2*3)\,\%\,4$

$\underline{2}$

$(a-b)\ \%\ m$
$(a/b)\ \%\ m$ } advance batch.

Problems :-

given $a, n, p \rightarrow$ calculate $a^n \% p$.

a	n	p
3	4	7

$\Rightarrow (3^4)\ \%\ 7 \Rightarrow 81\ \%\ 7 \Rightarrow 4$

```
power (int a, int n, int p)
  // no overflow
  for (int i=1; i<=n; i++) {
      a = a * a
  }
  return a % p
}
```

wrong.

given $a, n=5,$
$a^5 \% p$

i	// a	a
1	a = a * a	a^2
2	a = a * a	a^4
3	a = a * a	a^8

```
power (int a, int n, int p) {   // a^n % p
    // assume no overflow.
    int ans = 1
    for (i = 1; i <= n; i++) {
        ans = a * a
    }
    return a % p
}
```

→ wrong

```
power (int a, int n, int p) { // a^n % p
    long ans = 1
    for (i = 1; i <= n; i++) {
        ans = (ans * a) % p
    }
    return ans % p
}
```

ans = (ans % p * a % p) % p

constraints :-

$1 <= n <= 10^5$

$1 <= a <= 10^9$

$1 <= p <= 10^9$

$a = 10, n = 14, p = 25$

$(10^{14}) \% 25$

i	ans
1	10

T.C → O(N)   2   0
S.C → O(1)   3

Break → 10:10pm to 10:20pm.

Divisibility of 3 :- Sum of digits are divisible by 3.

= (789) % 3 ✓   (8215) % 3 ✗   (3458) % 3 ✗

$10 \% 3 = 1$
$10^2 \% 3 = 1$
$10^3 \% 3 = 1$
$10^x \% 3 = 1$

(3458)
$= 3 * 10^3 + 4 * 10^2 + 5 * 10^1 + 8 \times 10^0$

$$(3 * 10^3 + 4 * 10^2 + 5 * 10^1 + 8 \times 10^0) \% 3$$

$$((3 * \underbrace{10^3) \% 3}_{1} + (4 * \underbrace{10^2) \% 3}_{1} + (5 * \underbrace{10^1) \% 3}_{1} + (8 * \underbrace{10^0) \% 3}_{1}) \% 3$$

$$(3 \% 3 + 4 \% 3 + 5 \% 3 + 8 \% 3) \% 3$$

→ $(3 + 4 + 5 + 8) \% 3$

→ $(20) \% 3 \Rightarrow \underline{2}$

* Divisibility of 4 :- (last 2 digits % 4)

$\{a_3 a_2 a_1 a_0\} \% 4 \rightarrow 5432$

$5 * 1000 + 4 * 100 + 3 * 10 + 2$

$\{a_3 * 10^3 + a_2 * 10^2 + a_1 a_0\} \% 4$

$$\left(\underbrace{(a_3 * 10^3) \% 4}_{0} + \underbrace{(a_2 * 10^2) \% 4}_{0} + \underbrace{a_1 a_0 \% 4}\right) \% 4$$

Last 2 digits are divisible by 4 or not.

Divisible by 9 $\Rightarrow$ { Sum of digits % 9 }

$\underbrace{\text{Sum of digits}}$ → Todo.

Ques) Given 1 number in arr[], calculate number % P.

Each arr[i] contains single digit of a no.

N = 5

	0	1	2	3	4
arr[5] =	7	8	9	6	2

$0 <= arr[i] <= 9$

Constraints :-

$1 <= N <= 10^5$

$1 <= P <= 10^9$

N = 3, 9 9 9 : $10^3 - 1$

N = 4, 9 9 9 9 : $10^4 - 1$

N = 10, $10^{10} - 1$

N = 20, $10^{20} - 1$

N = 10^5, $10^{10^5} - 1$

	0	1	2	3	4
arr[5] =	3,	2,	6,	4,	9

$9 * 10^0$

$4 * 10^1$

$6 * 10^2$

$3 * 10^4$ $2 * 10^3$

$(3 * 10^4 + 2 * 10^3 + 6 * 10^2 + 4 * 10^1 + 9 * 10^0) \% P.$

$$(3*10^4 + 2*10^3 + 6*10^2 + 4*10^1 + 9*10^0)\ \%\ p.$$

$$((3*10^4)\ \%\ p + (2*10^3)\ \%\ p + (6*10^2)\ \%\ p + (4*10^1)\ \%\ p + (9*10^0)\ \%\ p)\ \%\ p$$

$$(3\ \%\ p * 10^4\ \%\ p)\ \%\ p$$

$$+$$

$$(2\ \%\ p * 10^3\ \%\ p)\ \%\ p$$

$$+$$

$$(6\ \%\ p * 10^2\ \%\ p)\ \%\ p$$

$$+$$

$$(4\ \%\ p * 10\ \%\ p)\ \%\ p$$

$$+$$

$$(9\ \%\ p * 10^0\ \%\ p)\ \%\ p$$

// arr[6] = arr[0], arr[1], arr[2], arr[3], arr[4], arr[5]

$arr[5] * 10^0$

$arr[4] * 10^1$

$arr[3] * 10^2$

$arr[2] * 10^3$

$arr[1] * 10^4$

$arr[0] * 10^5$

$(arr[0]*10^5 + arr[1]*10^4 + arr[2]*10^3 + arr[3]*10^2 + arr[4]*10^1 + arr[5]*10^0)\%p$

$(arr[0]\%p * 10^5\%p)\%p$

$(arr[1]\%p * 10^4\%p)\%p$

$(arr[2]\%p * 10^3\%p)\%p$

$10^2\%p$

$10^1\%p$

$(arr[5]\%p * 1\%p)\%p$

Save 10 :-

$p = 3$

$ans = 1$

$ans = (ans * 10)\%p = 10\%p \Rightarrow 1$

$ans = (ans * 10)\%p = (10^2)\%p \Rightarrow 1$

$ans = (ans * 10)\%p = (10^3)\%p$.

Pseudo Code :-

```
// given arr[N] & P
{
  long fans = 0
  long ans = 1
  for (i = N-1; i >= 0; i--)
  {
    fans = fans + { (arr[i] % P) * (ans % P) } % P
    fans = fans % P.
    ans = (ans * 10) % P
  }
  return fans;
}
```

$10 \% P \rightarrow 10^2 \% P \rightarrow 10^3 \% P \rightarrow 10^4 \% P.$

$10 \% P \downarrow x$

$10^2 \% P$

$\downarrow$

$(10 * 10) \% P$

$\downarrow$

$(10 \% P * 10 \% P) \% P$

$(x * 10) \% P.$