A PROJECT REPORT

ON

STRUCTURAL VIBRATION ANALYSIS OF PLATES

BY

K.BADARI VISHAL 2018A4PS0807G

BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

ACADEMIC YEAR 2020-21

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Prepared in fulfillment of the
Study Oriented Project (SOP)
Course No. ME F266

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BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
ACADEMIC YEAR 2020-21

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Title of the Project: Study Oriented Project on Structural Vibration Analysis of Plates.

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Keywords: Vibration, Plates, Kirchoff's plate theory, Ritz's Method

Abstract: This report discusses the Structural Vibration analysis of 'Perforated plates' and the Kirchoff's plate theory involved in the analysis of plates. The report also discusses the Ritz's & Galerkin methods that are applied for the analysis of Plates followed by a code developed in 'MAPLE' software that is based on a 4 term approximation of Rayleigh-Ritz method for a simply supported plate.

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1. Introduction

A mathematically exact stress analysis of a thin plate subjected to loads acting normal to its surface requires a solution of the differential equations of three-dimensional elasticity. Since such a scenario is bound to encounter insurmountable mathematical difficulties, we utilize the Kirchoff's plate theory or often called 'Classical Plate Theory'.

1.1 Kirchoff's theory

Kirchoff's theory takes into consideration the following assumptions such that the originally 3D stress problems of elasticity are now reduced to 2D problems of plates:

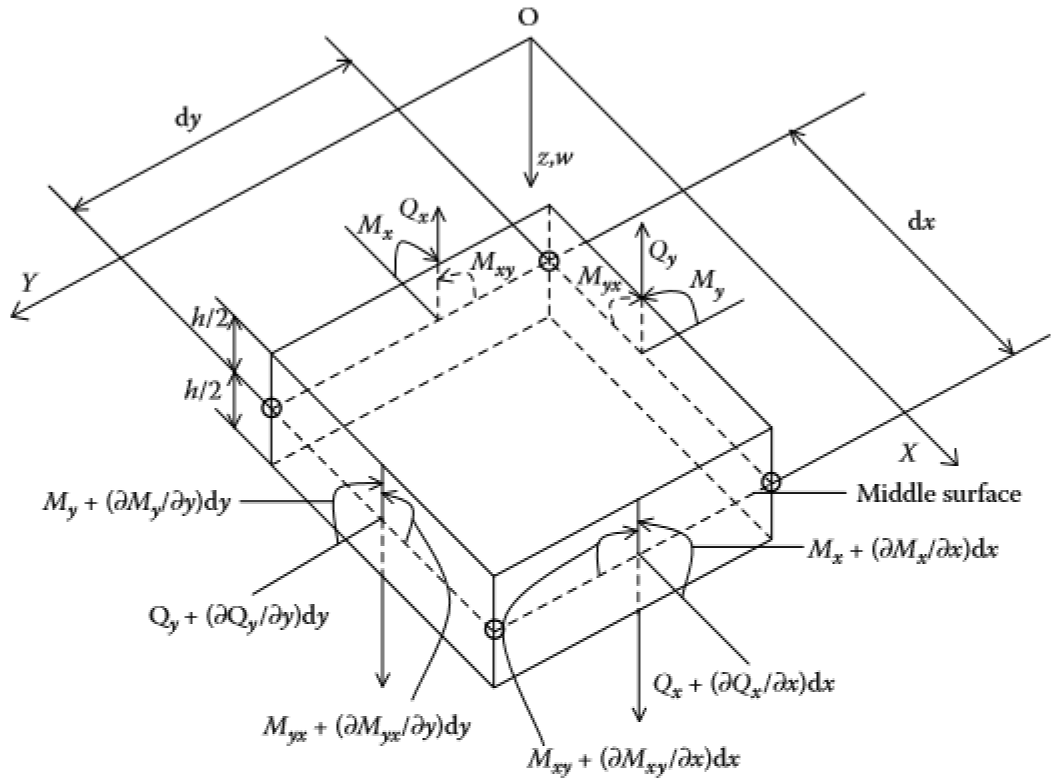
1. The material is homogeneous, isotropic and linearly elastic (it follows Hooke's law).
2. The plate is initially flat.
3. The middle surface of the plate remains unstrained during bending.
4. The constant thickness of the plate is small compared to its other dimensions (the smallest lateral dimension of the plate is at least 10 times larger than its thickness).
5. The transverse deflections *w(x,y)* are small compared to the plate thickness (A maximum deflection of one-tenth of the thickness is considered the limit of the small-deflection theory).
6. Slopes of the defected middle surface are small compared to unity.
7. Sections taken normal to the middle surface before deformation remain plane and normal to the deflected middle surface. Consequently, shear deformations are neglected.
8. The normal stress 'σ_z' in the direction transverse to the plate surface can be neglected.

1.2 Equations of the Plate

The bending moments and twisting moments are given by –

$$m_x = \int_{-(h/2)}^{+(h/2)} \sigma_x z \, dz \qquad \text{and} \qquad m_y = \int_{-(h/2)}^{+(h/2)} \sigma_y z \, dz.$$

$$m_{xy} = \int_{-(h/2)}^{+(h/2)} \tau_{xy} z \, dz \qquad \text{and} \qquad m_{yx} = \int_{-(h/2)}^{+(h/2)} \tau_{yx} z \, dz;$$



Figure

On substituting the normal stress which are given as –

$$\sigma_x = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right).$$

We arrive at –

$$m_x = -\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$
$$= -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) = D(\kappa_x + \nu\kappa_y)$$

$$m_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) = D(\kappa_y + \nu\kappa_x),$$

$$\boxed{D = \frac{Eh^3}{12(1-\nu^2)}}$$

Where D is flexural rigidity.

Based on the equilibrium equations and substitution of expressions for moments in them, we arrive at the non-homogeneous biharmonic equation, given by –

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{q}{D} = 0$$

Or in short

$$\nabla^4 w = \frac{q}{D}$$

1.3 Boundary Conditions

The summary for various boundary conditions (geometric, statical and mixed boundary conditions) are given as follows:

Type of Support at x =a	***Mathematical Expressions***
Simple support	$(w)_{x=a} = 0;\ (m_x)_{x=a} = \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)_{x=a} = 0$
Fixed edge	$(w)_{x=a} = 0;\ \left(\frac{\partial w}{\partial x}\right)_{x=a} = 0$
Free-edge	$(m_x)_{x=a} = \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)_{x=a} = 0$ $(v_x)_{x=a} = \left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x\, \partial y^2}\right]_{x=a} = 0$
Partially fixed edge	$(w)_{x=a} = 0$ $\left(\frac{\partial w}{\partial x}\right)_{x=a} = -(\rho^{*})^{-1} D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)_{x=a}$
Elastic support	$(m_x)_{x=a} = \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)_{x=a} = 0$ $(w)_{x=a} = \rho^{-1} D \left[\frac{\partial^2 w}{\partial x^2} + (2-\nu)\frac{\partial^3 w}{\partial x\, \partial y^2}\right]_{x=a}$
Elastic support and restraint	$(w)_{x=a} = \rho^{-1} D \left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x\, \partial y^2}\right]_{x=a}$ $\left(\frac{\partial w}{\partial x}\right)_{x=a} = -(\rho^{*})^{-1} D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)_{x=a}$

1.4 Solution in Rectangular coordinates

Considering the case for a simply supported plate, where having found *W(x,y)* as follows –

$$W_1(x,y) = (\overline{A}\sin\alpha_1 x + \overline{B}\cos\alpha_1 x)(\overline{C}\sin\alpha_2 y + \overline{D}\cos\alpha_2 y)$$
$$= A\sin\alpha_1 x\sin\alpha_2 y + B\sin\alpha_1 x\cos\alpha_2 y + C\cos\alpha_1 x\sin\alpha_2 y$$
$$+ D\cos\alpha_1 x\cos\alpha_2 y$$

$$W_2(x,\, y) = E\sinh\alpha_1 x\sinh\alpha_2 y + F\sinh\alpha_1 x\cosh\alpha_2 y$$
$$+ G\cosh\alpha_1 x\sinh\alpha_2 y + H\cosh\alpha_1 x\cosh\alpha_2 y$$

Combining above two equations, we get –

$$W(x,\, y) = A\sin\alpha_1 x\sin\alpha_2 y + B\sin\alpha_1 x\cos\alpha_2 y$$
$$+ C\cos\alpha_1 x\sin\alpha_2 y + D\cos\alpha_1 x\cos\alpha_2 y$$
$$+ E\sinh\alpha_1 x\sinh\alpha_2 y + F\sinh\alpha_1 x\cosh\alpha_2 y$$
$$+ G\cosh\alpha_1 x\sinh\alpha_2 y + H\cos\alpha_1 x\cosh\alpha_2 y$$

The boundary conditions for this case are given as –

$$\text{at } x = 0 \quad \text{and} \quad x = \text{a}, \quad \text{W} = \frac{\partial^2 \text{W}}{\partial x^2} = 0$$
$$\text{at } y = 0 \quad \text{and} \quad y = b, \quad \text{W} = \frac{\partial^2 \text{W}}{\partial y^2} = 0$$

Finding solutions separately for $\overline{W}(x)\; and\; \overline{W}(y)$, we arrive at the following solution for W –

$$W_{mn}(x,\, y) = A_{mn}\sin\frac{m\pi x}{a}\,.\,\sin\frac{n\pi y}{b}$$

And the frequency is given by –

$$\omega_{mn} = \pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\sqrt{\frac{D}{\rho h}}$$

The case with m = n = 1 corresponds to the case of fundamental natural frequency.

2. Rayleigh – Ritz Method

For the plate vibrating freely, we start with the assumption of the solution and hence obtain the maximum strain and kinetic energies which are given as follows:

$$T_{\max} = \frac{1}{2}\omega^2 \iint_R \rho h W^2 \mathrm{d}x\mathrm{d}y$$

$$U_{\max} = \frac{1}{2}\iint_R D\left[(\nabla^2 W)^2 + 2(1-\nu)\left\{W_{xy}^2 - W_{xx}W_{yy}\right\}\right]\mathrm{d}x\mathrm{d}y$$

Equating the above two expressions and applying the N-term approximation, we arrive at –

$$p\omega^2 = \frac{\iint_R \left[\left(\sum_{j=1}^{N} C_j\phi_j^{xx}\right)^2 + \left(\sum_{j=1}^{N} C_j\phi_j^{yy}\right)^2 + 2\nu\left(\sum_{j=1}^{N} C_j\phi_j^{xx}\right)\left(\sum_{j=1}^{N} C_j\phi_j^{yy}\right) + 2(1-\nu)\left(\sum_{j=1}^{N} C_j\phi_j^{xy}\right)^2\right]\mathrm{d}x\mathrm{d}y}{\iint_R \left(\sum_{j=1}^{N} C_j\phi_j\right)^2 \mathrm{d}x\mathrm{d}y}$$

where

$$\phi_j^{xx} = \frac{\partial^2 \phi_j}{\partial x^2}$$
$$\phi_j^{yy} = \frac{\partial^2 \phi_j}{\partial y^2}$$
$$\phi_j^{xy} = \frac{\partial^2 \phi_j}{\partial x \partial y}$$

and

$$p = \frac{12\rho(1-\nu^2)}{E}$$

Taking derivatives with respect to each coefficient and equating it to zeros followed by non-dimensionalizing the expression, we have the re-written form given by –

$$\sum_{j=1}^{N}(a_{ij} - \lambda^2 b_{ij})C_j = 0$$

where we get -

$$a_{ij} = \iint_{R'} H^3\left[\phi_i^{XX}\phi_j^{XX} + \phi_i^{YY}\phi_j^{YY} + \nu\left(\phi_j^{XX}\phi_j^{YY} + \phi_i^{YY}\phi_j^{XX}\right) + 2(1-\nu)\phi_i^{XY}\phi_j^{XY}\right]\mathrm{d}X\mathrm{d}Y$$

$$\lambda^2 = \frac{12\rho a^4(1-\nu^2)\omega^2}{Eh_o^2}$$

$$b_{ij} = \iint_{R^1} H\phi_i\phi_j \mathrm{d}X\mathrm{d}Y$$

We basically have a generalized eigenvalue problem, which can be solved for frequencies and mode shapes for free vibration of plates of various shapes and with different boundary conditions.

3. Code for simply supported plate

The code for the case of a simply supported plate is developed on MAPLE software. The problem is solved by a 4-term trigonometric approximation using Rayleigh – Ritz Method.

The dimensions (including thickness) and other plate properties considered are given as follows:

$$a = 0.138; b = 0.216; h = 0.002$$
$$\vartheta = 0.3; E = 210GPa = 210 * 10^9;\ \rho = 7850;$$

Based on the above values considered, the below parameters are found out:

$$m = \ \rho h = 15.7$$
$$D = \frac{E * h^3}{12(1-\vartheta^2)} = \ 153.8461538\ \ (which\ is\ the\ flexural\ rigidity)$$

As stated earlier, we take 4-term approximation for *W*. The coding interface is as follows:

Text Math | 2D Input | Times New Roman | 12

$h := 0.002;$
$$0.002 \quad \textbf{(1)}$$

$\upsilon := 0.3;$
$$0.3 \quad \textbf{(2)}$$

$E := 2.1 \cdot 10^{11};$
$$2.100000000\ 10^{11} \quad \textbf{(3)}$$

$\rho := 7850;$
$$7850 \quad \textbf{(4)}$$

$a := 0.138;$
$$0.138 \quad \textbf{(5)}$$

$b := 0.216;$
$$0.216 \quad \textbf{(6)}$$

$flex := \dfrac{E \cdot h^3}{12 \cdot (1 - \upsilon^2)};$
$$153.8461538 \quad \textbf{(7)}$$

$m := \rho \cdot h;$
$$15.700 \quad \textbf{(8)}$$

$W := c1 \cdot \left(\sin\left(\frac{\pi \cdot x}{a}\right)\right) \cdot \left(\sin\left(\frac{\pi \cdot y}{b}\right)\right) + c2 \cdot \left(\sin\left(\frac{\pi \cdot x}{a}\right)\right) \cdot \left(\sin\left(\frac{2 \cdot \pi \cdot y}{b}\right)\right) + c3 \cdot \left(\sin\left(\frac{\pi \cdot x}{a}\right)\right) \cdot \left(\sin\left(\frac{3 \cdot \pi \cdot y}{b}\right)\right) + c4 \cdot \left(\sin\left(\frac{2 \cdot \pi \cdot x}{a}\right)\right) \cdot \left(\sin\left(\frac{\pi \cdot y}{b}\right)\right);$

$$c1 \sin(7.246376812\,\pi x) \sin(4.629629630\,\pi y) + c2 \sin(7.246376812\,\pi x) \sin(9.259259260\,\pi y) + c3 \sin(7.246376812\,\pi x) \sin(13.88888889\,\pi y) + c4 \sin(14.49275362\,\pi x) \sin(4.629629630\,\pi y) \quad \textbf{(9)}$$

After finding the derivatives of W with respect to x and y, we calculate the kinetic and strain energies. For the ease of calculations the below code is written as such:

$$d1x := \frac{\partial}{\partial x}(W);$$

$$7.246376812\, c1 \cos(7.246376812\, \pi x)\, \pi \sin(4.629629630\, \pi y) + 7.246376812\, c2 \cos(7.246376812\, \pi x)\, \pi \sin(9.259259260\, \pi y) + 7.246376812\, c3 \cos(7.246376812\, \pi x)\, \pi \sin(13.88888889\, \pi y) + 14.49275362\, c4 \cos(14.49275362\, \pi x)\, \pi \sin(4.629629630\, \pi y) \quad \mathbf{(10)}$$

$$d2x := \frac{\partial}{\partial x}(d1x);$$

$$-52.50997690\, c1 \sin(7.246376812\, \pi x)\, \pi^2 \sin(4.629629630\, \pi y) - 52.50997690\, c2 \sin(7.246376812\, \pi x)\, \pi^2 \sin(9.259259260\, \pi y) - 52.50997690\, c3 \sin(7.246376812\, \pi x)\, \pi^2 \sin(13.88888889\, \pi y) - 210.0399075\, c4 \sin(14.49275362\, \pi x)\, \pi^2 \sin(4.629629630\, \pi y) \quad \mathbf{(11)}$$

$$d1y := \frac{\partial}{\partial y}(W);$$

$$4.629629630\, c1 \sin(7.246376812\, \pi x) \cos(4.629629630\, \pi y)\, \pi + 9.259259260\, c2 \sin(7.246376812\, \pi x) \cos(9.259259260\, \pi y)\, \pi + 13.88888889\, c3 \sin(7.246376812\, \pi x) \cos(13.88888889\, \pi y)\, \pi + 4.629629630\, c4 \sin(14.49275362\, \pi x) \cos(4.629629630\, \pi y)\, \pi \quad \mathbf{(12)}$$

$$d2y := \frac{\partial}{\partial y}(d1y);$$

$$-21.43347051\, c1 \sin(7.246376812\, \pi x)\, \pi^2 \sin(4.629629630\, \pi y) - 85.73388204\, c2 \sin(7.246376812\, \pi x)\, \pi^2 \sin(9.259259260\, \pi y) - 192.9012346\, c3 \sin(7.246376812\, \pi x)\, \pi^2 \sin(13.88888889\, \pi y) - 21.43347051\, c4 \sin(14.49275362\, \pi x)\, \pi^2 \sin(4.629629630\, \pi y) \quad \mathbf{(13)}$$

$$u1 := \int_0^a \int_0^b (d2x + d2y)^2 \, dx dy;$$

$$7636.334479\, c1\, c2 + 4917.770255\, c1^2 + 12664.16348\, c2^2 + 47377.76874\, c3^2 + 44594.38945\, c4^2 - 8593.557996\, c1\, c3 - 7530.561491\, c1\, c4 + 26070.49837\, c2\, c3 - 5846.743888\, c2\, c4 + 6579.640119\, c3\, c4 \quad \mathbf{(14)}$$

$$E := simplify\left(\frac{flex}{2} \cdot u1\right);$$

$$5.874103445\, 10^5\, c1\, c2 + 3.782900195\, 10^5\, c1^2 + 9.741664210\, 10^5\, c2^2 + 3.644443748\, 10^6\, c3^2 + 3.430337649\, 10^6\, c4^2 - 6.610429225\, 10^5\, c1\, c3 - 5.792739605\, 10^5\, c1\, c4 + 2.005422951\, 10^6\, c2\, c3 - 4.497495297\, 10^5\, c2\, c4 + 5.061261630\, 10^5\, c3\, c4 \quad \mathbf{(15)}$$

In the above code –

- d1x is the partial derivative of *W* with respect to x, d2x being the 2nd derivative of *W*.
- d1y is the partial derivative of *W* with respect to y, d2y being the 2nd derivative of *W*.
- *E* here in the code represents the strain energy.
- u1 was used to solve the integral so as to ease the calculation process.

In the code following below –

- *F* represents the kinetic energy.
- t1 was used to solve the integral so as to ease the calculation process.

$$t1 := \int_0^a \int_0^b (W)^2 \, dx dy;$$

$$0.007669020688\, c1\, c2 + 0.009233563836\, c1^2 + 0.006802762692\, c2^2 + 0.008075814119\, c3^2 + 0.008544342177\, c4^2 - 0.004861602287\, c1\, c3 - 0.004516761140\, c1\, c4 + 0.007888775467\, c2\, c3 - 0.001875718587\, c2\, c4 + 0.001189069393\, c3\, c4 \quad \textbf{(16)}$$

$$F := simplify\left(\frac{m}{2} \cdot t1\right);$$

$$0.06020181240\, c1\, c2 + 0.07248347610\, c1^2 + 0.05340168715\, c2^2 + 0.06339514085\, c3^2 + 0.06707308610\, c4^2 - 0.03816357796\, c1\, c3 - 0.03545657495\, c1\, c4 + 0.06192688740\, c2\, c3 - 0.01472439091\, c2\, c4 + 0.009334194735\, c3\, c4 \quad \textbf{(17)}$$

Taking derivatives with respect to the coefficients *c1*, *c2*, *c3* and *c4*, and clubbing them into rows, we have –

$$row1 := \frac{\partial}{\partial c1}(E) - \omega^2 \cdot \frac{\partial}{\partial c1}(F);$$

$$5.874103445\, 10^5\, c2 + 7.565800390\, 10^5\, c1 - 6.610429225\, 10^5\, c3 - 5.792739605\, 10^5\, c4 - \omega^2 (0.06020181240\, c2 + 0.1449669522\, c1 - 0.03816357796\, c3 - 0.03545657495\, c4) \quad \textbf{(18)}$$

$$row2 := \frac{\partial}{\partial c2}(E) - \omega^2 \cdot \frac{\partial}{\partial c2}(F);$$

$$5.874103445\, 10^5\, c1 + 1.948332842\, 10^6\, c2 + 2.005422951\, 10^6\, c3 - 4.497495297\, 10^5\, c4 - \omega^2 (0.06020181240\, c1 + 0.1068033743\, c2 + 0.06192688740\, c3 - 0.01472439091\, c4) \quad \textbf{(19)}$$

$$row3 := \frac{\partial}{\partial c3}(E) - \omega^2 \cdot \frac{\partial}{\partial c3}(F);$$

$$7.288887496\, 10^6\, c3 - 6.610429225\, 10^5\, c1 + 2.005422951\, 10^6\, c2 + 5.061261630\, 10^5\, c4 - \omega^2 (0.1267902817\, c3 - 0.03816357796\, c1 + 0.06192688740\, c2 + 0.009334194735\, c4) \quad \textbf{(20)}$$

$$row4 := \frac{\partial}{\partial c4}(E) - \omega^2 \cdot \frac{\partial}{\partial c4}(F);$$

$$6.860675298\, 10^6\, c4 - 5.792739605\, 10^5\, c1 - 4.497495297\, 10^5\, c2 + 5.061261630\, 10^5\, c3 - \omega^2 (0.1341461722\, c4 - 0.03545657495\, c1 - 0.01472439091\, c2 + 0.009334194735\, c3) \quad \textbf{(21)}$$

The solution for ω is further solved as follows:

- Club all the rows into a Matrix M with each row getting differentiated with respect to each of the coefficients.
- Take the determinant of the matrix and equal it to 0. We arrive at an equation for ω.
- Having found an equation, solve for ω further for obtaining the frequencies.

The below interface describes the code used for the above steps for solving.

$M := Matrix([[(diff(row1, c1)), (diff(row1, c2)), (diff(row1, c3)), (diff(row1, c4))], [(diff(row2, c1)), (diff(row2, c2)), (diff(row2, c3)), (diff(row2, c4))], [(diff(row3, c1)), (diff(row3, c2)), (diff(row3, c3)), (diff(row3, c4))], [(diff(row4, c1)), (diff(row4, c2)), (diff(row4, c3)), (diff(row4, c4))]]);$

$$[[7.565800390\ 10^5 - 0.1449669522\ \omega^2, 5.874103445\ 10^5 - 0.06020181240\ \omega^2, -6.610429225\ 10^5 + 0.03816357796\ \omega^2, -5.792739605\ 10^5 + 0.03545657495\ \omega^2], [5.874103445\ 10^5 - 0.06020181240\ \omega^2, 1.948332842\ 10^6 - 0.1068033743\ \omega^2, 2.005422951\ 10^6 - 0.06192688740\ \omega^2, -4.497495297\ 10^5 + 0.01472439091\ \omega^2], [-6.610429225\ 10^5 + 0.03816357796\ \omega^2, 2.005422951\ 10^6 - 0.06192688740\ \omega^2, 7.288887496\ 10^6 - 0.1267902817\ \omega^2, 5.061261630\ 10^5 - 0.009334194735\ \omega^2], [-5.792739605\ 10^5 + 0.03545657495\ \omega^2, -4.497495297\ 10^5 + 0.01472439091\ \omega^2, 5.061261630\ 10^5 - 0.009334194735\ \omega^2, 6.860675298\ 10^6 - 0.1341461722\ \omega^2]] \tag{22}$$

$$\xrightarrow{\text{determinant}} 1.782514943\ 10^{25} - 7.271485291\ 10^{18}\ \omega^2 + 5.247287015\ 10^{11}\ \omega^4 - 10823.10564\ \omega^6 + 0.00006368032486\ \omega^8 \tag{23}$$

$$\xrightarrow{\text{solve for omega}} [[\omega = -1761.314878], [\omega = 1761.314878], [\omega = -4218.965914], [\omega = 4218.965914], [\omega = -7245.579357], [\omega = 7245.579357], [\omega = -9826.476430], [\omega = 9826.476430]] \tag{24}$$

Having found the various solutions for ω, the respective frequencies are found out which are given as follows:

$$f1 := \frac{1761.314878}{2 \cdot 3.1416}$$

$$280.3213137 \tag{25}$$

$$f2 := \frac{4218.965914}{2 \cdot 3.1416}$$

$$671.4677098 \tag{26}$$

$$f3 := \frac{7245.579357}{2 \cdot 3.1416}$$

$$1153.167074 \tag{27}$$

$$f4 := \frac{9826.476430}{2 \cdot 3.1416}$$

$$1563.928640 \tag{28}$$

4. Verification on ANSYS

Simply supported plate conditions are used.
First we check for various metals
Plate dimensions are: *h=0.002, a=0.138,b=0.216*

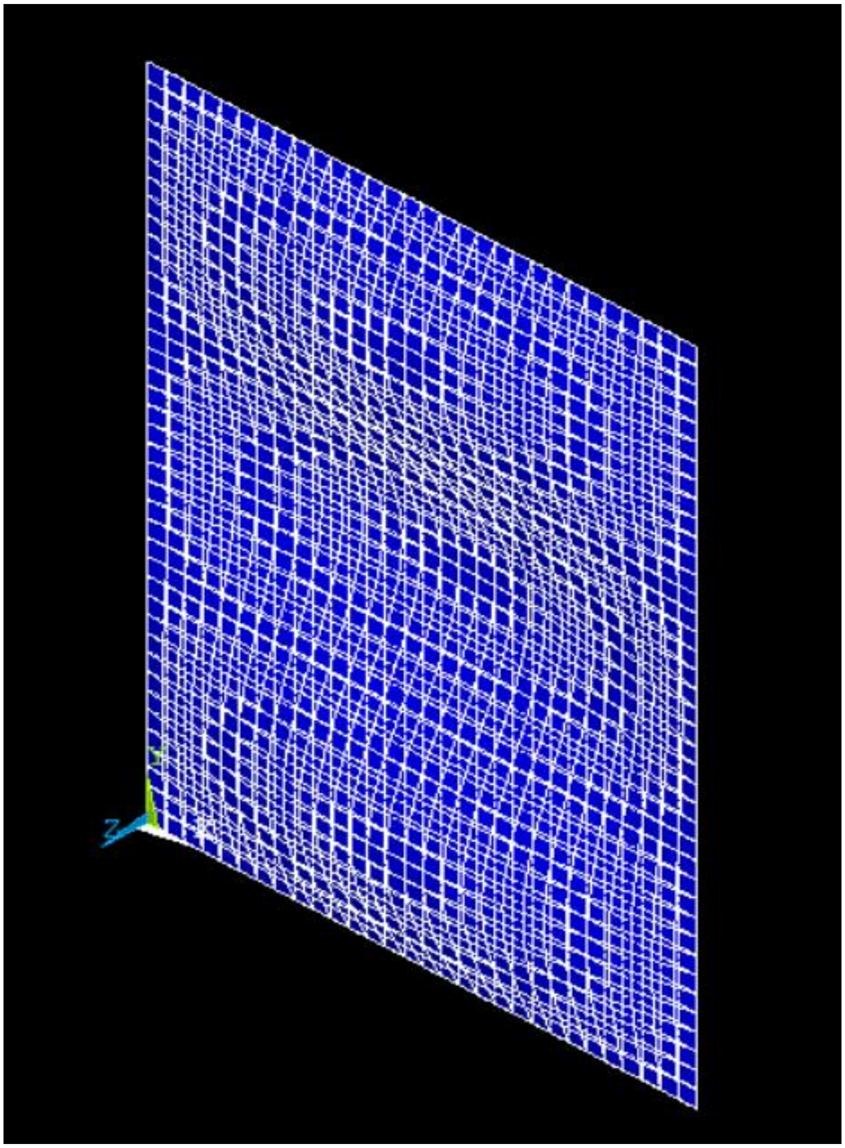
Metals	Natural frequency using MAPLE	Natural Frequency using APDL
Carbon steel	**f1 = 274.3621, f2 = 657.1933, f3 = 1128.652, f4 = 1530.593**	**f1 = 363.51 f2 = 679.44 f3 = 1137.7 f4 = 1205.9**
Cast Iron	**f1 = 197.0194, f2 = 471.9321, f3 = 810.4881, f4 = 1099.186**	**f1 = 249.68 f2 = 466.68 f3 = 781.44 f4 = 828.30**
Stainless Steel	**f1 = 269.2503, f2 = 644.9515, f3 = 1107.629, f4 = 1502.169**	**f1 = 345.27 f2 = 645.35 f3 = 1080.6 f4 = 1145.4**

Now we change the dimensions of the plate made of stainless steel.
Height is kept really small compared to the edges to satisfy thin plate requirement.

Dimensions	MAPLE	APDL
a = 0.138,b=0.216,h=0.002	**f1 = 269.2503, f2 = 644.9515, f3 = 1107.629, f4 = 1502.169**	**f1 = 345.27 f2 = 645.35 f3 = 1080.6 f4 = 1145.4**
a = 0.15,b = 0.2,h=0.002	**f1 = 327.65, f2 = 697.65, f3 = 1007.99, f4 = 1390.47**	**f1 = 324.26 f2 = 674.30 f3 = 946.66 f4 = 1257.6**
a = 0.3,b=0.4,h=0.002	**f1 = 81.91**	**f1 = 81.079**

	f2 = 174.41 **f3 = 251.99,** **f4 = 347.61**	**f2 = 168.63** **f3 = 236.74** **f4 = 314.55**

The mesh we obtained for modal analysis using ANSYS APDL is shown below.



5. Conclusion

The structural vibration analysis of plates was discussed starting with the Kirchoff's classical plate's theory followed by the equations of plate, boundary conditions and a solution method in rectangular coordinates.

The next study described the Rayleigh – Ritz method for solving the vibration analysis of plates given the appropriate boundary conditions.

The case of a simply supported plate was solved with a code developed on MAPLE software followed by verification on ANSYS software.

6. References

1. Chakraverty S., *"Vibration of Plates"*, Taylor & Francis Group.
2. Szliard R., *"Theories and applications of Plate Analysis"*, John Wiley & Sons.