Mid-Term Progress Report

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***Abstract*— The ultimate goal for this simulation is to be able to successfully simulate a bilayer shell with mismatched stretch values. A stretch is applied to a circular substrate layer and an unstretched kirigami layer is put on top. The results from the discrete shell simulation is to then be compared against finite element analysis results from Abaqus.**

I. Introduction

For the simulation, the goal for the mid-term progress report was to have the overall structure of the code completed. Since this simulation is trying to model a bilayer structure, the concepts covered in class had to be modified to account for the different material properties throughout the structure.

II. Description of the Overall Goal

The simulation aims to determine the equilibrium shape of a bilayer shell, with the bottom substrate layer being stretched, and the top kirigami layer being unstretched. The remainder of this section is dedicated to describing the composition of the bilayer shell being modeled, as well as the physical processes that motivated the creation of this simulation.

A. Substrate Layer

The substrate layer of the structure is a clear, circular cutout of VHB tape. It assumed that the substrate maintains its volume when stretched, meaning that it has a Poisson's ratio of 0.5. The substrate layer is stretched, and a λ value can be assigned to this stretch, where λ is the stretched length divided by the unstretched length. Thus, a λ of one means that the substrate has not been stretched. The applied λ introduces stretching energy to the system, and the amount of stretch affects the final three-dimensional shape of the structure.

B. Kirigami Layer

The kirigami layer of the structure is a gray piece of VHB tape that has been cut using kirigami principles. The kirigami layer is unstretched, so it should mainly experience bending energies and forces when applied to the substrate layer. The three-dimensional shape of the structure comes from the mismatch in λ values between the two layers.

C. Combining the Two Layers

As mentioned before, the kirigami layer is placed on a substrate layer that has been stretched. For this simulation, it is assumed that only a radial stretch will be applied to the substrate layer, so the stretch can be represented by a single λ instead of a λ in the x direction and a λ in the y direction. When the substrate layer is released, it will try to go to its original undeformed state. However, the kirigami layer on top prevents the substrate from just shrinking. Instead, the structure takes a curved three-dimensional shape. The stretching energy of the substrate and the bending energy of the substrate and kirigami layers are what the simulation will be modeling.

III. Key Equations

The provided bending and stretching energy equations will need to be modified in order to accurately model the system the code aims to simulate. One key thing to mention is that almost all the properties of the system will need to become functions of time. This is to ensure that the simulation converges.

A. Stretching Energy

$$E_s = \frac{1}{2} k_s^{substrate} (\lambda - 1)^2 \qquad (1)$$

The λ -1 term accounts for the stretch applied to the substrate layer. When λ equals 1, there is no stretching energy. When λ is not equal to one, the substrate has a stretching energy applied to it. k_s is the stretching stiffness defined as:

$$k_s = \frac{\sqrt{3}}{2} Y^{substrate} t^{substrate} l_k^2 \qquad (2)$$

l_k, the reference length of each edge, needs to be calculated before the simulation begins. The mesh created by MATLAB assumes that the substrate layer has already been stretched, which means that the reference lengths are deformed. Thus, the undeformed reference length, l^{∞}, needs to be found. The formula for undeformed reference length is:

$$l^{\infty} = \frac{l^{deformed}}{\lambda} \qquad (3)$$

B. Bending Energy

$$E_b = \frac{1}{2} k_b (\theta - \bar{\theta})^2 \qquad (4)$$

The $(\theta - \bar{\theta})^2$ term accounts for the preferred curvature of the shell. In its equilibrium position, $\theta = \bar{\theta}$, which gives zero net bending energy. If the system is not in its equilibrium position, then it has a bending energy associated with it. k_b, the bending stiffness, can be found using:

$$k_b = \frac{2}{\sqrt{3}} Y_{kirigami} I_{effective} \qquad (5)$$

In the combined section of the structure, the moment of inertia is no longer that of just the substrate. Traditional composite beam theory was used to find the effective moment of inertia of the combined section. This is possible because the thicknesses and

Young's Moduli of the substrate and kirigami are known.

However, it is important to note that the natural curvature of the system can be calculated before the simulation starts. $\bar{\theta}$ can be found using the equation:

$$E_b = \frac{1}{2} k_b \bar{\theta}^2 \quad (6)$$

It is assumed that all the energy in the stretching of the substrate layer is converted into bending energy of the combined section. Thus, $\bar{\theta}$ can be found equating the stretching energy to the bending energy:

$$\bar{\theta} = \sqrt{\frac{k_s(\lambda-1)^2}{k_b}} \quad (7)$$

C. Simplifications

In order to simplify the code for the mid-term progress report, the thickness for both the kirigami and the substrate regions was assumed to be constant. Since it is assumed that the substrate has a Poisson's ratio of 0.5, it is possible to calculate its change in thickness as a function of λ. This calculation will be considered in the final version of the simulation.

IV. Description of Simulation Code

A. Mesh Creation

The circular bilayer shell is discretized into a finite number of nodes which are connected by straight edges, forming a system of triangular elements, as shown below in Figure 1. The combined kirigami-substrate region is denoted by black edges, while the region that is solely composed of the substrate is denoted by red edges. Additionally, each pair of triangles sharing an edge is considered to be a bending element, with the share edge referred to as the hinge. The angle across each bending element's hinge is used to help determine the shell's bending energy and is gradually changed linearly with time to push the simulation forwards. Once the mesh has been generated, the unique edges are identified and stored, as well as each bending element. In addition, a vector is created to identify and store which bending elements are part of the bilayer region, and which are solely substrate, since there is no bending energy associated with the non-bilayer region.

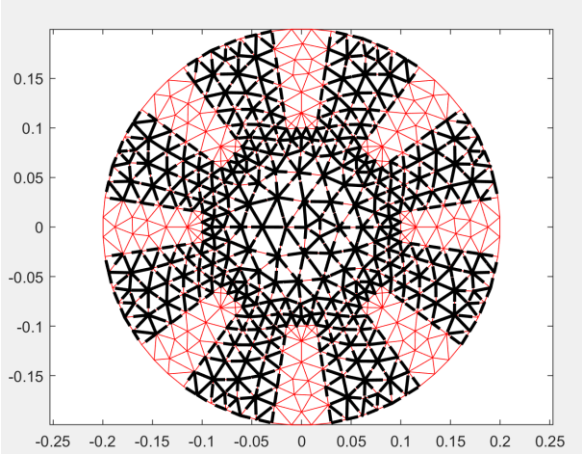

Figure 1:Triangular mesh of shell generated by MATLAB at the beginning of the simulation.

B. Description of Model Concept

Unlike simpler models, each iteration of the simulation does not solely push time forwards. Both the reference length of each edge as well as the bending angle of each bending element are linearly modified with time so that they reach their equilibrium values at the end of the simulation. As denoted in equation (3), for each given edge, the undeformed reference length can be determined as a function of original (deformed) reference length and lambda. Thus, the simulation is constructed so that at the end of the simulation, the reference length between two given nodes is equivalent to the value calculated by equation (3).

The significance of this is that at any given point in time, the distance between two given nodes is known beforehand. What is not known, however, is where those two nodes will be located in relation to other nodes. The simulation aims to determine the minimum energy state that the shell would seek to fit to in reality. Similarly, the bending angle of the bending elements is stepped linearly with time so that at the end of the simulation, the angle of a given bending element is equivalent to that calculated by equation (7).

C. Execution of Model

In order to prevent rigid body motion, it is necessary to constrain a node. The simulation determines the node closest to the origin and sets the free nodes to be all nodes except that one. Additionally, it is necessary to introduce some external forces so that the simulation can properly execute and reflect reality. First, a gravitational force is introduced to provide a direction for the shell to converge in. The gravitational force is not a significant part of the simulation and is slowly tapered off with time. It exists just to provide an effect at the beginning, similarly to how imperfections with the physical fabrication of the shell would provide an effect in an experiment. Additionally, a weak viscous force is introduced to remove some of the shell's energy and help the simulation converge.

Once the setup work is created, the simulation proceeds through a preset number of iterations, which is determined by the final time value as well as the time step value. The simulation makes use of the Newton - Raphson method, and only considers an iteration complete when the error falls below a predetermined tolerance value, which is considered convergence. During each iteration, the reference length and bending angle are linearly modified so that they approach the infinity value. The equations in the previous section are used to find the bending and stretching energy for each element, which is then translated into a force and jacobian that modify the location of the free to move nodes. At the end of each iteration, the resulting 3D mesh is plotted. This proceeds until the iterations are completed, or the error propagates to a point where the simulation is no longer able to converge.