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MAE 259B

Homework 2: Chapter 8 – Plate Simulation

The purpose of this homework assignment is to compare the results of running a plate simulation against the results of a simple beam simulation and the Euler Bernoulli Beam theory answer. Seeing as to how beam simulations were the focus of homework 1 for this class, this report will not talk about how to write the beam simulation. This report will instead focus on the plate simulation and how the results of this simulation compare to the other two methods.

Key Formulas

$$E_{elastic} = \sum_{k=1}^{N_{edge}} E_s^k + \sum_{k=1}^{N_{hinge}} E_b^k$$

$$E_s^k = \frac{1}{2} k_s \left(\frac{\|e^k\|}{l_k} - 1\right)^2$$

$$E_b^k = \frac{1}{2} k_b (\theta_k - \bar{\theta}_k)^2$$

$$k_s = \frac{\sqrt{3}}{2} Y h l_k^2$$

$$k_b = \frac{2}{\sqrt{3}} \frac{Y h^3}{12}$$

The above formulas give the general equations for the stretching and bending energies of the plate. Stretching can happen between any two nodes that are connected by an edge, while bending can happen between edges that are shared between two triangular elements. Thus, all edges are capable of stretching energy, but not all edges are capable of bending energy. The formulas described the Newton-Raphson method will not be covered in this report as they are the same between plates and beams.

Parameters of this Problem

$$l = 0.1\ m$$

$$w = 0.01\ m$$

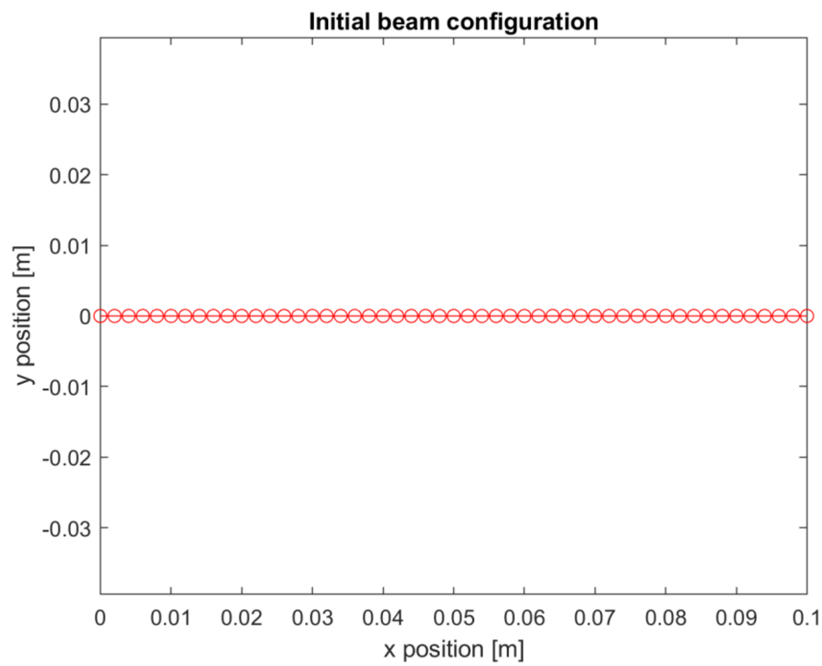
$$h = 0.002\ m$$

$$Y = 10^7\ Pa$$

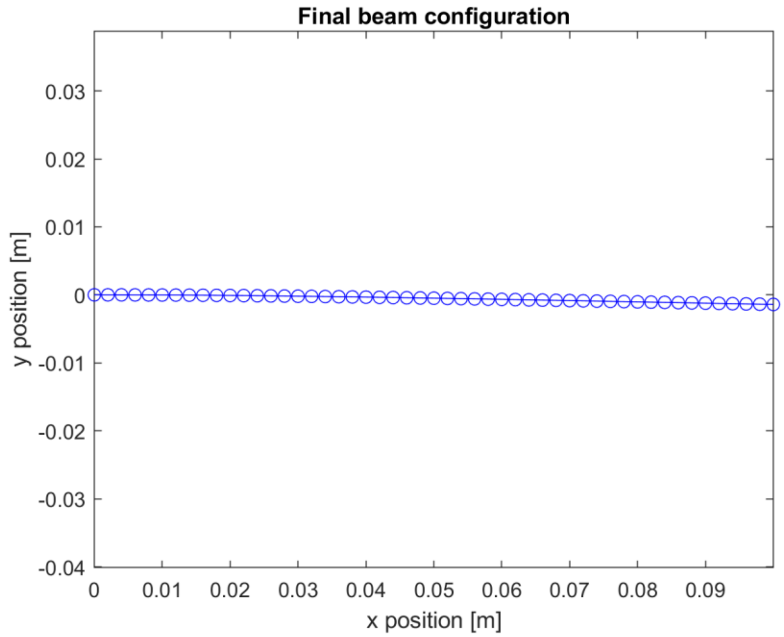
$$\rho = 1000\ kg/m^3$$

Beam Simulation

The beam simulation was run from t = 0s to t = 2s with dt = 0.001s and 51 nodes. The beam's initial configuration is given below:



The final beam configuration after running the simulation is given below:



As the figure above shows, the beam has very little deflection when only subjected to its own weight. The tip deflection is -0.0014 meters.

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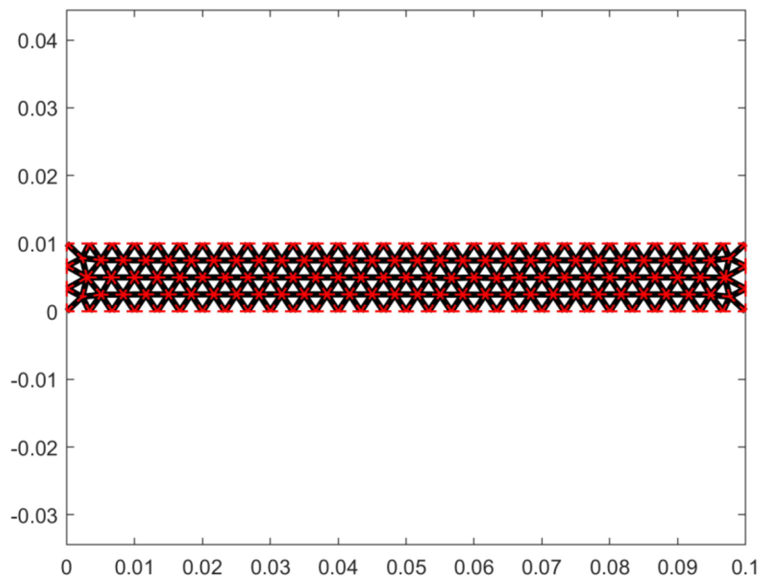
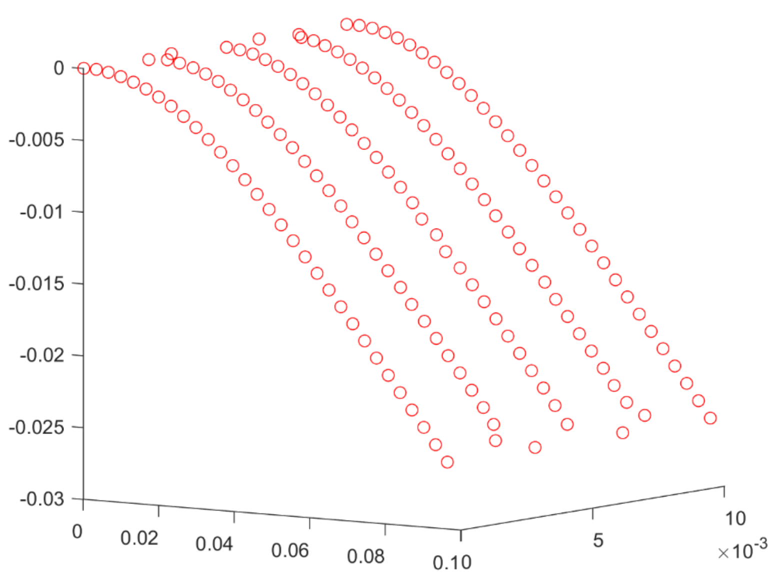
Plate Simulation

The plate simulation was run from t = 0s to t = 2s with dt = 0.001s. The plate's initial configuration is given below:



The final configuration of the plate after running the simulation:



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Euler Bernoulli Beam Theory

From Euler-Bernoulli Beam theory, the maximum deflection is -0.0014 meters, which is the same as the beam simulation.

Comparing the Three Results

The deflection from the beam simulation and the beam theory calculation match. This was to be expected because our beam simulation from homework one also matched the Euler-Bernoulli Beam theory answer. The plate simulation, however, gave us a much larger deformation than the other two. I believe that this was caused by an error in calculating the stretching stiffness or the bending stiffness of the plate. This also could have been caused by the mesh size chosen for the experiment. A larger mesh means less accuracy, but quicker computation time. A smaller mesh means higher accuracy, but a much longer computation time.

Decreasing the mesh size of the plate had a significant impact on the computation time of the simulation. This is because the number of edges and elements increases very quickly as the number of nodes increases. Larger and larger matrices are created in order to properly model the system. Decreasing the mesh size also made the simulation more accurate.

Increasing the mesh size had the opposite effect on the simulation. The simulation took much less time to complete when I increased the mesh. However, doing so also made the simulation less accurate.