Bilayer Shell Simulation

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***Abstract*— The ultimate goal for this simulation is to be able to successfully simulate a bilayer shell with mismatched stretch values. A stretch is applied to a circular substrate layer and an unstretched kirigami layer is put on top. The results from the discrete shell simulation is to then be compared against finite element analysis results from Abaqus.**

I. Background

The need to create increasingly complex robots is hindered by conventional fabrication techniques. Three-dimensional fabrication is difficult, costly, and difficult to scale up, especially if the robot being fabricated is complex. Thus, the need for two-dimensional fabrication is paramount in order to easily produce complex robots. Kirigami techniques provide us a way to map three dimensional shapes to two dimensional maps. By cutting a two-dimensional substrate in a certain way, it is possible to create a more complex three-dimensional shape. Kirigami manufacturing is also faster, cheaper, and much simpler than rapid prototyping, pop-up, and the related origami techniques [1]. In the example of soft robotics in general, kirigami techniques can be used to create robots that are able to morph their structure [2]. This allows for many more degrees of freedom than in typical, hard robotics, and thus allows for soft robotics to be used in a wide variety of situations. A study similar to what this project aims to produce was able to successfully simulate the behavior of a single layer [3]. This study was able to produce saddle and dome-like 3D configurations by applying a stretch to a portion of the shell. Another study focused on the simulation of the effects of pre-stretch on rectangular and square bilayer structures [4]. By varying the ratio of width and height in their bilayers, researchers were able to produce many interesting shapes. However, no simulations regarding circular shapes have been performed, which is what prompted this project.

Thorough simulations regarding soft robotics have historically been difficult to execute, largely due to the many degrees of freedom and nonlinear material effects that are characteristic of soft materials [5]. However, as computer processing speeds improve and more nodes can be simulated, accurate simulations for bi-layered actuators are becoming a more feasible goal. Current research on the subject involves the use of nonlinear relaxation being used for kinematic simulation, where the structure is represented as a network of springs, masses, and beams [5]. Composite beam theory has been used to model actuators before, however, it has not been applied to the kirigami shape this project seeks to model [6]. The researchers also noted that a physically correct simulation does not necessarily imply one that is reflective of reality, which stresses the importance of using experimental results in conjunction with simulation. More recently, finite element simulation was used on thin, elastic sheets various strategic cuts meant to generate linear actuation [7]. In-plane stress and out-of-plane displacement was simulated via FEM software and displayed for sheets with cuts that varied in count and geometry. While the planar geometries displayed in this publication are certainly more complex than a simple mass and beam system, in the end they are still solely rectangular sheets. The kirigami cuts made in the middle of the sheets cause deformation and actuation of the middle section, but not the boundaries. There is still minimal work done on sheets that are circular in shape, perhaps even with pieces cut out.

II. Description of the Overall Goal

The simulation aims to determine the equilibrium shape of a bilayer shell, with the bottom substrate layer being stretched, and the top kirigami layer being unstretched. The remainder of this section is dedicated to describing the composition of the bilayer shell being modeled, as well as the physical processes that motivated the creation of this simulation.

A. Substrate Layer

The substrate layer of the structure is a clear, circular cutout of VHB tape. Previous research has established that the use of a pre-stretch (stretching one or both layers prior to attaching them to one other) can produce various structural models [8]. It is assumed that the substrate maintains its volume when stretched, meaning that it has a Poisson's ratio of 0.5. The substrate layer is stretched, and a λ value can be assigned to this stretch, where λ is the stretched length divided by the unstretched length. Thus, a λ of one means that the substrate has not been stretched. The applied λ introduces stretching energy to the system, and the amount of stretch affects the final three-dimensional shape of the structure.

B. Kirigami Layer

The kirigami layer of the structure is a gray piece of VHB tape that has been cut using kirigami principles. The kirigami layer is unstretched, so it should mainly experience bending energies and forces when applied to the substrate layer. The three-dimensional shape of the structure comes from the mismatch in λ values between the two layers.

C. Combining the Two Layers

As mentioned before, the kirigami layer is placed on a substrate layer that has been stretched. For this simulation, it is assumed that only a radial stretch will be applied to the substrate layer, so the stretch can be represented by a single λ instead of a λ in the x direction and a λ in the y direction. When the substrate layer is released, it will try to go to its original undeformed state. However, the kirigami layer on top prevents the substrate from just shrinking. Instead, the structure takes a curved three-dimensional shape. The stretching energy of the substrate and the bending energy of the substrate and kirigami layers are what the simulation will be modeling.

III. Key Equations

The provided bending and stretching energy equations will need to be modified in order to accurately model the system the code aims to simulate. One key thing to mention is that almost all the properties of the system will need to become functions of time. This is to ensure that the simulation converges.

A. Stretching Energy

When simulating the shell, the natural curvature of the shell as well as the mismatch in swelling (or stretch in this case) must be taken into account [9]. Curvature, $1/K$, can be related to the stretch applied to the system.

$$E_s = \frac{1}{2} k_s^{substrate} (\lambda - 1)^2 \tag{1}$$

The λ -1 term accounts for the stretch applied to the substrate layer. When λ equals 1, there is no stretching energy. When λ is not equal to one, the substrate has a stretching energy applied to it. k_s is the stretching stiffness defined as:

$$k_s = \frac{\sqrt{3}}{2} Y^{substrate} t^{substrate} l_k^2 \tag{2}$$

l_k, the reference length of each edge, needs to be calculated before the simulation begins. The mesh created by MATLAB assumes that the substrate layer has already been stretched, which means that the reference lengths are deformed. Thus, the undeformed reference length, l^∞, needs to be found. The formula for undeformed reference length is:

$$l^\infty = \frac{l^{deformed}}{\lambda} \tag{3}$$

B. Bending Energy

$$E_b = \frac{1}{2} k_b (\theta - \bar{\theta})^2 \tag{4}$$

The $(\theta - \bar{\theta})^2$ term accounts for the preferred curvature of the shell. In its equilibrium position, $\theta = \bar{\theta}$, which gives zero net bending energy. The preferred curvature can be interpreted as a combination of pressure and torque, and if desired, can be used to determine the buckling point, or the threshold of bending energy required to cause the shell to crumple [10]. If the system is not in its equilibrium position, then it has a bending energy associated with it. k_b, the bending stiffness, can be found using:

$$k_b = \frac{2}{\sqrt{3}} Y_{kirigami} I_{effective} \tag{5}$$

In the combined section of the structure, the moment of inertia is no longer that of just the substrate. Traditional composite beam theory was used to find the effective moment of inertia of the combined section. This is possible because the thicknesses and Young's Moduli of the substrate and kirigami are known.

However, it is important to note that the natural curvature of the system can be calculated before the simulation starts. $\bar{\theta}$ can be found using the equation:

$$E_b = \frac{1}{2} k_b \bar{\theta}^2 \tag{6}$$

It is assumed that all the energy in the stretching of the substrate layer is converted into bending energy of the combined section. Thus, $\bar{\theta}$ can be found equating the stretching energy to the bending energy:

$$\bar{\theta} = \sqrt{\frac{k_s (\lambda - 1)^2}{k_b}} \tag{7}$$

C. Simplifications

In order to simplify the code for the mid-term progress report, the thickness for both the kirigami and the substrate regions was assumed to be constant. Since it is assumed that the substrate has a Poisson's ratio of 0.5, it is possible to calculate its change in thickness as a function of λ. This calculation will be considered in the final version of the simulation.

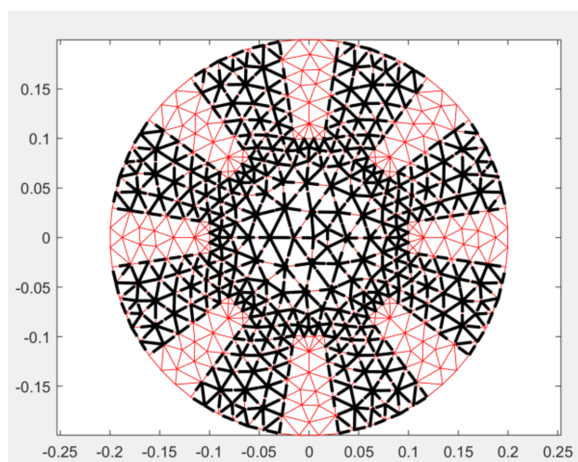

Figure 1:Triangular mesh of shell generated by MATLAB at the beginning of the simulation.

IV. Description of Simulation Code

A. Mesh Creation

The circular bilayer shell is discretized into a finite number of nodes which are connected by straight edges, forming a system of triangular elements, as shown below in Figure 1. The combined kirigami-substrate region is denoted by black edges, while the region that is solely composed of the substrate is denoted by red edges. Additionally, each pair of triangles sharing an edge is considered to be a bending element, with the share edge referred to as the hinge. The angle across each bending element's hinge is used to help determine the shell's bending energy and is gradually changed linearly with time to push the simulation forwards. Once the mesh has been generated, the unique edges are identified and stored, as well as each bending element. In addition, a vector is created to identify and store which bending elements are part of the bilayer region, and which are solely substrate, since there is no bending energy associated with the non-bilayer region.

B. *Description of Model Concept*

Unlike simpler models, each iteration of the simulation does not solely push time forwards. Both the reference length of each edge as well as the bending angle of each bending element are linearly modified with time so that they reach their equilibrium values at the end of the simulation. As denoted in equation (3), for each given edge, the undeformed reference length can be determined as a function of original (deformed) reference length and lambda. Thus, the simulation is constructed so that at the end of the simulation, the reference length between two given nodes is equivalent to the value calculated by equation (3).

The significance of this is that at any given point in time, the distance between two given nodes is known beforehand. What is not known, however, is where those two nodes will be located in relation to other nodes. The simulation aims to determine the minimum energy state that the shell would seek to fit to in reality. Similarly, the bending angle of the bending elements is stepped linearly with time so that at the end of the simulation, the angle of a given bending element is equivalent to that calculated by equation (7).

C. *Execution of Model*

In order to prevent rigid body motion, it is necessary to constrain a node. The simulation determines the node closest to the origin and sets the free nodes to be all nodes except that one. Additionally, it is necessary to introduce some external forces so that the simulation can properly execute and reflect reality. The bending direction of a shell of any shape is determined immediately after symmetry is broken [11]. Thus, it is necessary to introduce an external force that provides that lapse in symmetry that is necessary for the simulation to reflect reality. To do this, we introduce a gravitational force to provide a direction for the shell to converge in. The gravitational force is not a significant part of the simulation and is slowly tapered off with time. It exists just to provide an effect at the beginning, similarly to how imperfections with the physical fabrication of the shell would provide an effect in an experiment. Additionally, a weak viscous force is introduced to remove some of the shell's energy and help the simulation converge.

Once the setup work is created, the simulation proceeds through a preset number of iterations, which is determined by the final time value as well as the time step value. The simulation makes use of the Newton - Raphson method, and only considers an iteration complete when the error falls below a predetermined tolerance value, which is considered convergence. During each iteration, the reference length and bending angle are linearly modified so that they approach the infinity value. The equations in the previous section are used to find the bending and stretching energy for each element, which is then translated into a force and jacobian that modify the location of the free to move nodes. At the end of each iteration, the resulting 3D mesh is plotted. This proceeds until the iterations are completed, or the error propagates to a point where the simulation is no longer able to converge.

V. MATLAB RESULTS

The simulation was run with a Young's Modulus of 368988 Pascals for the kirigami layer, a Young's Modulus of 116250 Pascals for the substrate layer, a thickness of 0.001 meters for both layers, an inner radius of 0.008 meters, an outer radius of 0.04 meters, 5 strips, and a dTheta value of 0.3 radians. These parameters were kept constant across simulations.

The simulation was run with 8 different lambda values: 1.03, 1.06, 1.09, 1.12, 1.15, 1.18, 1.21, and 1.24. Beyond a lambda value of 1.24, the simulation would fail to converge well before meaningful results could be obtained.

Below in figures 1 and 2, the isometric and bird's eye view for the lambda values of 1.03, 1.06, 1.15, and 1.24 are show. These values were chosen to give a representative sample of the results obtained.

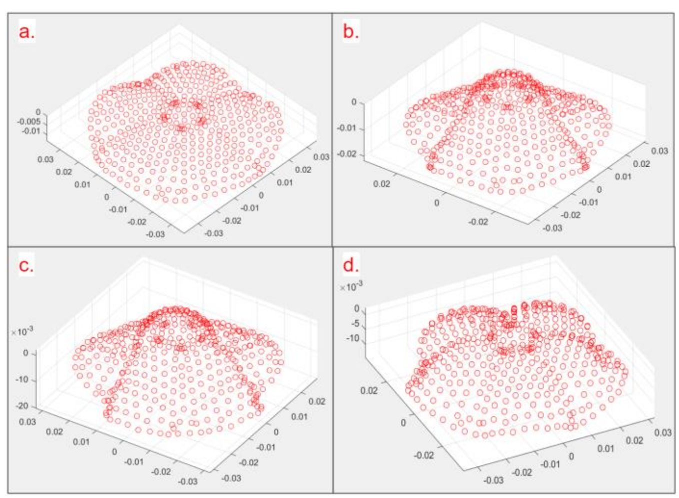

Figure 2: Isometric views of the simulation results. (a) 1.03 (b) 1.06 (c)1.15 (d)1.24

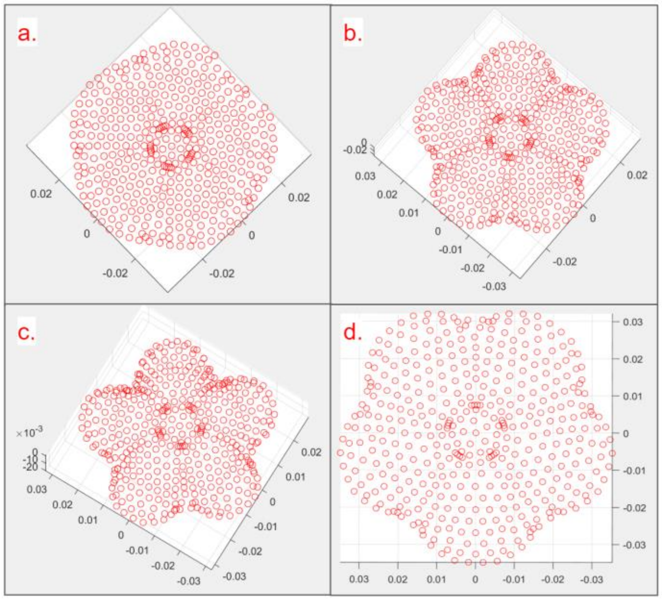

Figure 3: Bird's eye view of simulation results. (a) 1.03 (b) 1.06 (c) 1.15 (d) 1.24

As the plots show, all the chosen lambda values except for 1.03 gave the same shape. The simulation for λ = 1.24 was cut short because the simulation's error was starting to grow beyond the point of convergence. As λ increased, the simulation took less time to get to its final configuration, but error increased quickly. Conversely, lower λ values took longer to get to their final shape, but had lower error values, This suggests that the simulation is overexaggerating the effects of λ. Additionally, the flower-like appearance of the shell shows that the regions in between the kirigami cuts is being ignored. Future iterations of this simulation will have to adjust the energy balance so that changing λ has a more significant impact on the final shape of the shell.

VI. FINITE ELEMENT ANALYSIS

To compare the validity of the previous discrete shell simulation, we also build the same model using the finite element method.

One layer – hereafter referred to as the substrate layer – is radially stretch upon which a second stress-free layer is affixed. This results in residual stress in the material, causing the structure to assume a 3D shape upon release of the boundary constraints.

Our method is to generate the nodes, then mesh it and generate a PDE function. An input file with be generate by MATLAB including all the para-

meters and boundary conditions. Then We input that input file into Abaqus and Abaqus will act like the solver to solve the PDE function. Then I read the data exported by Abaqus and use MATLAB to analysis it.

For the model, I first tried to use the linear elastic model that describe the relation between the stretch and stress:

$$\sigma = \varepsilon * E * (1 - v) \quad (8)$$

E is the Young's modulus of the substrate layer.

The relation of the strain, ε, and the stretch, λ, is

$$\varepsilon = \lambda - 1 \quad (9)$$

The result turns out not that good, and there are many cases that the I can't get a convergent result.

Then I switch the model to using Mooney-

Rivlin. The C1 and C2 is measured by DMA. The D1 is derived using the relation between shear modulus, bulk modulus and Youngs' modulus. Then the result is much robust.

We use MATLAB will do the fitting job, trying to fit a radius to the result. We compare that radius using a parameter called quality. The closer the value of the quality to 0, the more likely we can get a hemisphere shape. Some of my result is showed in fig 2.

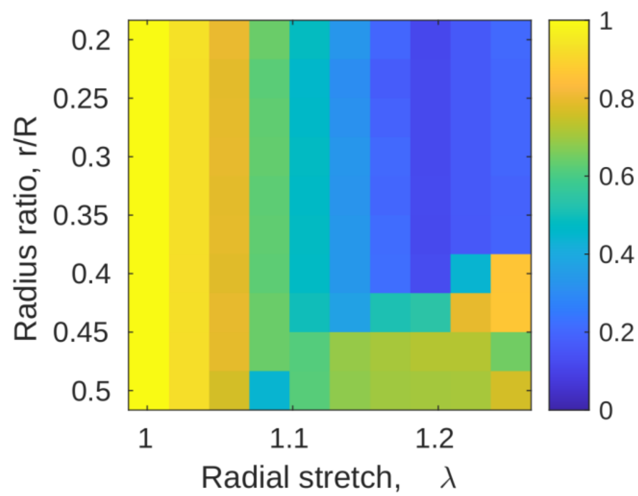

fig 4. some simulation results

And I can get a hemi-sphere shape whose quality is about 0.1.

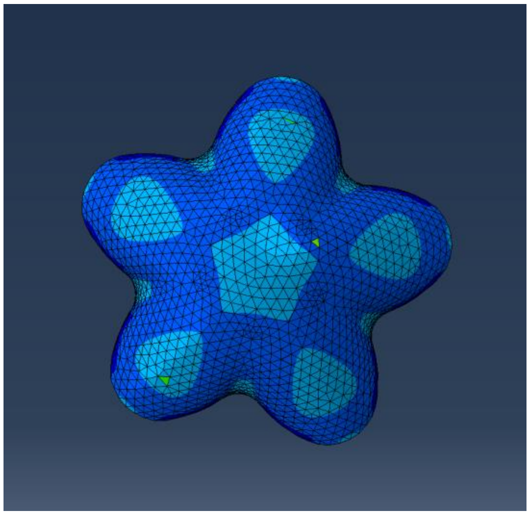

fig 5. FEA result of the hemisphere shape

Here comes my conclusion:

1. The number of the strips on the kirigami layer is the most important parameters. I have success with a robust result when number of the stripe equals to 3,4,5,6. This can be fitted with the circle case.
2. The cutout angle and the ratio between the inter radius and outer radius can be used to adjust the final shape. But currently the best result I can get is 0.1.
3. The comparison about the cutout angle and the ratio between the inter radius and outer radius are showed in fig 5. In these cases the number of cut is regulated at 5.

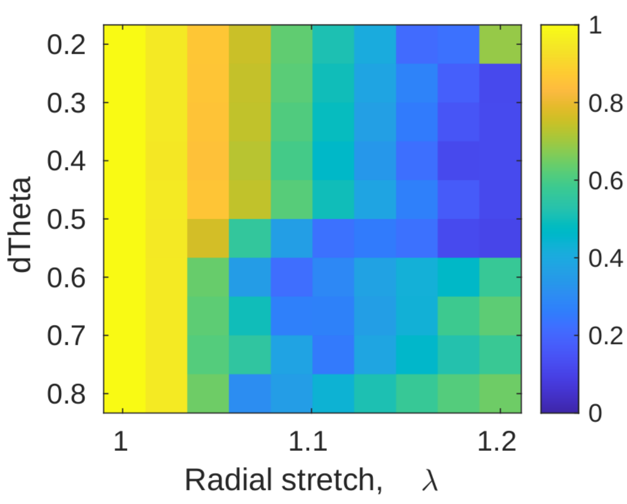

fig 5.a. the cutout angle influence on the quality

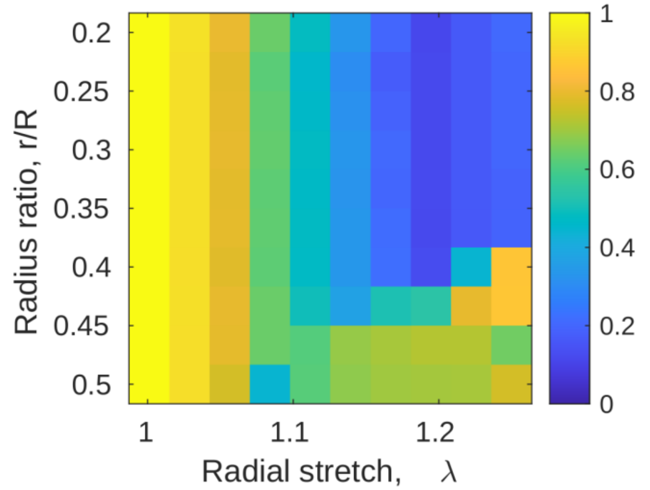

fig 5.b. the ratio influence to the quality

4. For the future, I may change the shape of the cut on the kirigami layer trying to get a perfect hemisphere shape.

VII. Final Discussion

The figure below compares the quality of the results from the discrete shell simulation with the quality of the FEA results. Quality is a metric that compares the shape of the shell to a hemisphere. A quality of 1 means that the shell is far away from being a hemisphere. A quality of 0 means the shell is very close to a hemisphere. Quality is found by comparing the x, y, and z spans of the shell to the radius of a hemisphere and the root mean squared error of the shell's coordinates and a hemisphere's coordinates.

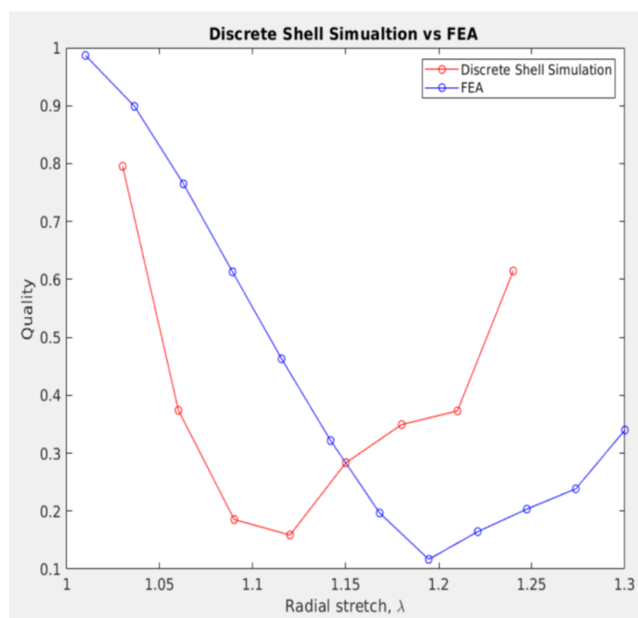

Figure 6: Quality of the shell simulation plotted against the quality of the FEA results

As figure 6 shows, the quality of the λ of the shell simulation is about 0.08 behind that of the FEA results. As stated before, this suggests that the simulation is over exaggerating the effects of the stretch. The simulation and FEA results are similar qualitatively; they both have similar shapes. Looking at this graph, $\lambda = 1.03$ has a quality of about 0.8, meaning that is not very close to a hemisphere. This quality value matches with the result shown in figure 2a.

REFERENCES

[1] Ji, Mingyang, et al. "Near-Infrared Light-Driven, Highly Efficient Bilayer Actuators Based on Polydopamine-Modified Reduced Graphene Oxide." Wiley Online Library, John Wiley & Sons, Ltd, 9 July 2014.

[2] Neville, R., Scarpa, F. & Pirrera, A. Shape morphing Kirigami mechanical metamaterials. Sci Rep 6, 31067 (2016). https://doi.org/10.1038/srep31067

[3] Pezzulla, Matteo, et al. "Morphing of geometric composites via residual swelling." Soft Matter 11.29 (2015): 5812-5820.

[4] Caruso, Noe A., et al. "Spontaneous morphing of equibiaxially pre-stretched elastic bilayers: the role of sample geometry." International Journal of Mechanical Sciences 149 (2018): 481-486.

[5] Lipson, Hod. "Challenges and opportunities for design, simulation, and fabrication of soft robots." Soft Robotics 1.1 (2014): 21-27.

[6] Nardinocchi, P., and E. Puntel. "Swelling-induced wrinkling in layered gel beams." Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 473.2207 (2017): 20170454.

[7] *Dias, Marcelo A., et al. "Kirigami actuators." Soft matter 13.48 (2017): 9087-9092.*

[8] DeSimone, Antonio. "Spontaneous bending of pre-stretched bilayers." Meccanica 53.3 (2018): 511-518.

[9] Pezzulla, Matteo, et al. "Curvature-driven morphing of non-Euclidean shells." Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 473.2201 (2017): 20170087.B. Smith, "An approach to graphs of linear forms (Unpublished work style)," unpublished.

[10] Pezzulla, Matteo, et al. "Curvature-induced instabilities of shells." Physical review letters 120.4 (2018): 048002.\

[11] Pezzulla, Matteo, et al. "Geometry and mechanics of thin growing bilayers." Soft Matter 12.19 (2016): 4435-4442.W.-K. Chen, *Linear Networks and Systems* (Book style). Belmont, CA: Wadsworth, 1993, pp. 123–135.