Problems on Remainder theorem

Remainder theorem

Q1. $\dfrac{66^{66}}{67} = ?$ what is Remainder? $= 1$

$\dfrac{9}{4} = 1$ dividend

$\dfrac{66}{67} = \dfrac{+66}{-1}$ $(-1)^{66} = 1$ ✓

$(-1)^{66} = 1$ ✓

$\dfrac{3}{7} = 7 \quad ^{+3}_{-4}$ ✓ $\dfrac{4}{7} = \dfrac{+4}{-3}$

divisor 4 $)\,9\,(2 \to$ Quotient

$\dfrac{8}{1}$ $-3 - 4 = ?$ 1 (Remainder)

$7 \times 0 + 3$ ✓

$9 = 4 \times 2 + 1$

or (2) -3

Q2. $\left(\dfrac{27}{14}\right)^{58} = ?$ what is Remainder? $\Rightarrow 1$ ✓

dividend = divisor × Quo + Rem

$(-1)^{58} = 1$ ✓

$\dfrac{27}{14} \Big\langle {}^{+13\checkmark}_{-1}$

$\dfrac{27}{14}$

$7)\ 3\ (0$
$\dfrac{0}{3}$ ✓

$\left(\dfrac{3}{7}\right) = {}^{+3}_{-4}$

$-3, -4$ ✓

$18)\ 49\ (2$
$\dfrac{36}{13}$ ✓

$3 = 7 \times 0 + 3$
$0 + 3 = 3$

Remainder:

$\dfrac{49}{18} \Big\langle {}^{-5\checkmark}_{+13\ \checkmark}$

$-5 - 13$ ✓

$49 = 18 \times 3 + (-5)$

divide $\quad 54 - 5 = 49$ ✓

Remainder

$27 = 14 \times 2 + (-1)$

Q3. $\dfrac{{}^{(+1)}39 \times 75^{(-1)}}{19} = ?$ ✓

$19 \times 2 = 38$
$19 \times 4 = 76$

$\dfrac{-1}{19} \quad \dfrac{-1}{18}$

$(+1) \times (-1) = -1$ ✓

$75 = \dfrac{19 \times 4 + (-1)}{76 - 1} = 75$ ✓

$18 + 1 = 19$

$\boxed{-1, +18}$ ✓

Q4. $\dfrac{{}^{(-5)}63 \times {}^{(-5)}97 \times {}^{(+3)}88 \times {}^{(+2)}189 \times {}^{(-5)}148}{17} = ?$

$17)\ 148\ (9$
$\ \ \ 153$

can of

$17 \times 4 = 68$
$17 \times 4 - 5$
$17 \times 4 + (-5)$ ✓
$17 \times 3 + (\ )$

(a) -2
(b) $+15$
(c) both

$= \dfrac{-5 \times -5 \times (+3) \times (+2) \times (-5)}{17}$

$17 \times 6 + (-5)$
$17 \times 5 + (3)$

$= \dfrac{-125 \times 6}{17} = \dfrac{-750}{17}$ ✓ $= -2$

$\dfrac{17 \times 9 + (-5)}{}$

$17)\ 750\ (44$
$\ \ \ \dfrac{68}{70}$
$\ \ \ \dfrac{68}{2}$

Q5. $-1\left(\dfrac{37^{819}+2}{38}\right)$ What is Remainder? ✓   TCS NQT 2020

✓ $\dfrac{37^{819}}{38}+\dfrac{2}{38}$ (30sec)

$(-1)^{819}+2$ ⟹ $-1+2=\dfrac{1}{38}$ ✓

$\dfrac{37}{38}$: $\begin{cases}+37\\ \boxed{-1}\end{cases}$

Remainder $= 1$ ✓

Q.

Q6. $\dfrac{7^{113}}{50}$ What is the Remainder? imp. ✓

Rem $\dfrac{\searrow}{7}$ ✓ ✓ $\dfrac{(7^2)^{56}\times 7^1}{50}$ ∿

$\dfrac{113}{2}=56+1$

$\left(\dfrac{(49)^{56}\times 7}{50}\right)$

$(-1)^{56}\times 7 = 1\times 7$
$= 7$

$\dfrac{7}{50}$

★★★ ✓

$(7^2)^{56}$

$7^{112}\times 7^1 = 7^{113}$

$\dfrac{49}{50} : \begin{cases}-1\\ +49\end{cases}$

Q7. $\left(\dfrac{3^{148}}{5}\right)$ = what is the Remainder? ✓

$(3^2)=9$

$=\dfrac{(3^2)^{74}}{5}$

$=\dfrac{(9)^{74}}{5}$ $(-1)^{74}=1$ ✓

$\dfrac{148}{2}=74$

$\dfrac{9}{5}: \begin{cases}+4\\ \boxed{-1}\end{cases}$

Q8. $\dfrac{12^6-14}{11}$ = ? what is the remainder?

$\left(\dfrac{12^6}{11}-\dfrac{14}{11}\right)$ ⟹ $(1)^6-3 = 1-3= \dfrac{-2}{11}$ $\begin{cases}-2\\ +9\end{cases}$ ✓

Q9. $\boxed{\dfrac{11^{77}}{7}}$ what is the remainder? $\underset{+2\;\checkmark}{\overset{30\,sec}{\text{Inter}}}$

$\longrightarrow \dfrac{(4)^{77}}{7} = \dfrac{(2^2)^{77}}{7} = \dfrac{2^{154}}{7}\;\;\checkmark$ $\boxed{-1\;\text{or}\;+1}$ $\dfrac{11}{7} = 4)$

$\underset{(1)}{\underbrace{}}$

$\underset{+1\;\text{or}\;-1}{\underbrace{}} \left(\dfrac{(2^3)^{51}\times 2^1}{7}\right) \gg \dfrac{(1)^{51}\times 2}{7} = \dfrac{2}{7} = \boxed{2+\text{...}}$ $\dfrac{11}{7} = 4$

$3)\,\overline{154}\,(51$
$\quad\;15\,1$
$\quad\quad\;\;1$ $\dfrac{8}{7} = 1$

Q10. $\dfrac{2^{100}+3^{100}+4^{100}+5^{100}}{7} = ?$ Find the remainder?

$$(2^m)^n = \underline{2^{mn}}$$

$2^{3\times 51}\times 2^1$

$2^{153}\times 2^1 = \underline{2^{154}}$

Q10. $\dfrac{2^{100}+3^{100}+4^{100}+5^{100}}{7} = ?$ find the Remainder?

$\Rightarrow \dfrac{2^{100}}{7} + \dfrac{3^{100}}{7} + \dfrac{4^{100}}{7} + \dfrac{5^{100}}{7}$ $\underset{2^7}{\underbrace{+1,-1}}$ $\dfrac{8}{7} = 1$

$= \dfrac{(2^3)^{33}\times 2^1}{7} + \dfrac{(3^3)^{33}\times 3^1}{7} + \dfrac{(4^3)^{33}\times 4^1}{7}$ $\begin{matrix}64\\63\\\hline\end{matrix}$ $2^3 = \dfrac{8}{7} = 1\;\dfrac{125}{126}$

$\qquad\qquad + \dfrac{(5^3)^{33}\times 5^1}{7}$ $(2^m)^n = 2^{mn}$

$= \dfrac{(+1)^{33}\times 2}{7} + \dfrac{(-1)^{33}\times 3}{7} + \dfrac{(+1)^{33}\times 4}{7} + \dfrac{(-1)^{33}\times 5}{7}$ $(2^3)^{33} = 2^{99}\times 2^1$
$\qquad\qquad\qquad\qquad\qquad\qquad = \underline{2^{100}}\;\checkmark$

$= 2 + (-1)\times 3 + 4 + (-1)\times 5 = \left(\dfrac{-2}{7}\right)\left(\dfrac{-2}{+5}\right)\Big\}\;\checkmark$

$\quad\underbrace{2 - 3 + 4 - 5}$