TCS Quadratic Equation

QUADRATIC EQUATION – ZERO to HERO

* $ax^2+bx+c=0,\ a\neq 0$ (Quadratic Eqn)

* $D=b^2-4ac$
(discriminant)

$D>0 \Rightarrow$ 2 Real & distinct roots

$D=0 \Rightarrow$ 2 Real & equal roots.

$D<0 \Rightarrow$ 2 Imaginary roots.

QUADRATIC EQUATION – ZERO to HERO

Comment

* $ax^2+bx+c=0,\ a\neq 0$ (Quadratic Eqn)

* $D=b^2-4ac$ ✓ (discriminant)

$x^2-5x+6=0$

$x=2,3$ ✓

$x^2-6x+9=0$

$x=3,3$ ✓

$2+i$

$2-i$

✓ $D>0 \Rightarrow$ 2 Real & distinct roots

$D=0 \Rightarrow$ 2 Real & equal roots.

$D<0 \Rightarrow$ 2 Imaginary roots.

QUADRATIC EQUATION – ZERO to HERO

*1. $ax^2+bx+c=0$] Quad

* roots $= \dfrac{-b\pm\sqrt{D}}{2a}$

$D=b^2-4ac$ ✓

$x^2+5x+6=0$

$\alpha+\beta=-\dfrac{(-5)}{1}=5$

$\alpha\beta=6$

*2. $ax^2+bx+c=0$ roots α, β

Sum of roots $= \alpha+\beta=-\dfrac{b}{a}$

Product of roots $= \alpha\beta=\dfrac{c}{a}$

Important TCS Exam

QUADRATIC EQUATION – ZERO to HERO

Basics

I. $x^2 - 6x + 9 = 0$

II. $y^2 - 11y + 24 = 0$

$y^2 - 11y + 24 = 0$

$2 \times 2 \times 2 \times 3$

$-8, -3$

roots: $+8, +3$

$x^2 - 6x + 9 = 0$

roots

3×3

$-3, -3$

roots: $+3, +3$

Coeff x^2 Coeff of x^2

$\frac{+3}{1}, \frac{+3}{1}$

$+3, +3$

Clear

QUADRATIC EQUATION – ZERO to HERO

Basics

$9 - 18 + 9 = 0$

I. $x^2 - 6x + 9 = 0$ ✓

II. $y^2 - 11y + 24 = 0$

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$+3, +3$

Clear

QUADRATIC EQUATION – ZERO to HERO

I. $8x^2 + 26x + 15 = 0$

II. $4y^2 + 24y + 35 = 0$

I. $6x^2 + 77x + 121 = 0$

II. $y^2 + 9y - 22 = 0$

$ax^2 + bx + c = 0$

$8x^2 + 26x + 15 = 0$

$AC = 8 \times 15$

$= 2 \times 2 \times 2 \times 3 \times 5$

$+20, +6$

roots: $\frac{-20}{8}, \frac{-6}{8}$

$= -\frac{5}{2}, -\frac{3}{4}$

$4y^2 + 24y + 35 = 0$

$AC = 4 \times 35$

$= 2 \times 2 \times 5 \times 7$

$14, 10$

roots: $\frac{-14}{4}, \frac{-10}{4}$

$= \frac{-7}{2}, \frac{-5}{2}$ ✓

QUADRATIC EQUATION – ZERO to HERO

I. $8x^2 + 26x + 15 = 0$

II. $4y^2 + 24y + 35 = 0$

[I. $6x^2 + 77x + 121 = 0$

[II. $y^2 + 9y - 22 = 0$]

$ax^2 + bx + c = 0$

$6x^2 + 77x + 121 = 0$

$AC = 6 \times 121$

$2 \times 3 \times 11 \times 11$

66, 11

Roots: $\frac{-66}{6}, \frac{-11}{6}$

$-11, -11/6$ ✓

$ax^2 - bx + c = 0$ [roots +ve]

$ax^2 + bx + c = 0$ [roots -ve]

$y^2 + 9y - 22 = 0$ $AC =$

$AC = 22 = 2 \times 11$

$+11, -$

Roots: $-11, +2$

QUADRATIC EQUATION – ZERO to HERO

I. $x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$

II. $y^2 - 3y + 2 = 0$

Noise!

$= x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$

$: (x^2 - x) - (\sqrt{2}x + \sqrt{2}) = 0$

$x(x-1) - \sqrt{2}(x-1) = 0$

$(x-\sqrt{2})(x-1) = 0$

$x = \sqrt{2}, 1$

$x - \sqrt{2} = 0$

$x = \sqrt{2}$

$x - 1 = 0$

$x = 1$

$y^2 - 3y + 2 = 0$ roots

Slow ?? increase!

$ax^2 - bx + c = 0$ [roots +ve]

$AC = 1 \times 2$

$-1, -2$

roots = 1, 2 ✓

QUADRATIC EQUATION – ZERO to HERO

(a.) find the roots of the equation. 55m / 1min

$x^2 - 8\sqrt{3}x + 45 = 0.$

$AC = 45 = 3 \times 3 \times 5$

$3 = \sqrt{3} \times \sqrt{3}$

$\sqrt{3} \times \sqrt{3} \times 3 \times 5$

$-3\sqrt{3}, -5\sqrt{3}$

roots. $+3\sqrt{3}, +5\sqrt{3}$ ✓

QUADRATIC EQUATION – ZERO to HERO

Q) If p and q are the roots of the equation $x^2+px+q=0$ then $ax^2+bx+c=0$, $x^2+p'x+q=0$

a. $p=1, q=-2$ ✓
b. $p=0, q=2$
c. $p=-2, q=0$
d. $p=-2, q=1$

Sum of roots

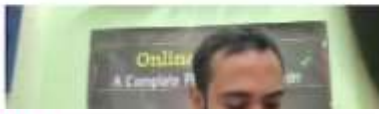
$p+q=-p$

$2p+q=0$

$2\times1-2=0$ ✓

Clear!

$2x-2+1=0$
$-4+1$
$-3x$



QUADRATIC EQUATION – ZERO to HERO

Q. If α and β are the roots of the equation $x^2-21x+4=0$, then find

$\sqrt{\alpha}+\sqrt{\beta}=5$ ✓

$\sqrt{\alpha}+\sqrt{\beta}=x$

Square

$(\sqrt{\alpha}+\sqrt{\beta})^2=x^2$

$\alpha+\beta+2\sqrt{\alpha\beta}=x^2$

$21+4=x^2$

$25=x^2$

$x=5$ (-5)

$\alpha+\beta=21$

$\alpha\beta=4$

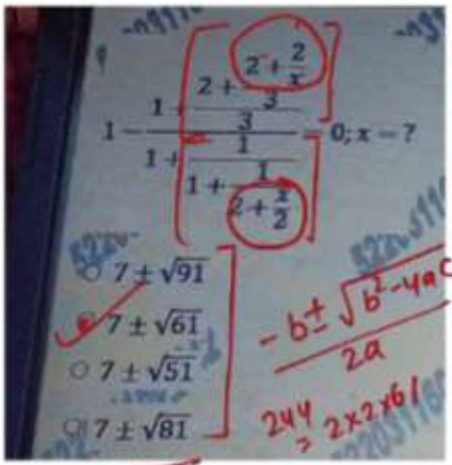
$2\sqrt{\alpha\beta}$

$2\sqrt{4}$

2×2

QUADRATIC EQUATION – ZERO to HERO

Imi



- $7\pm\sqrt{91}$
- $7\pm\sqrt{61}$
- $7\pm\sqrt{51}$
- $7\pm\sqrt{81}$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$244 = 2\times2\times61$

$\frac{14\pm\sqrt{244}}{2}$ $\frac{14\pm2\sqrt{61}}{2}$ $7\pm\sqrt{61}$

Imin

$2+\frac{2x+2}{3x}$

$\frac{3}{1}$

$\frac{6x+2x+2}{9x}=\frac{8x+2}{9x}$

$\frac{x}{1}$

$\frac{8x+2}{9x}\times\frac{4+x}{x+6}$

$\frac{1}{1+\frac{2}{4+x}}$

$\frac{4+x}{4+x+2}$

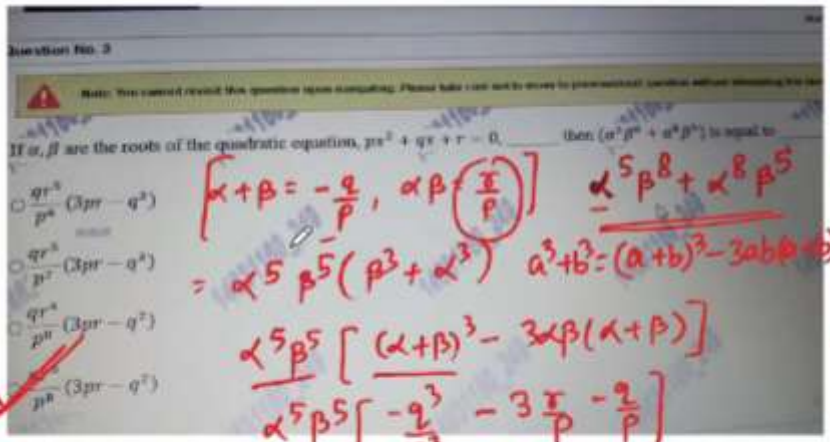
$\frac{4+x}{x+6}$ 4

So $36x+9x^2=8x^2+48x+2x+12$

$x^2-14x-12=0$

$\frac{14\pm\sqrt{196+4\times1}}{2a}$

QUADRATIC EQUATION – ZERO to HERO



$\alpha+\beta = -\frac{q}{p}$, $\alpha\beta = \frac{r}{p}$

$\alpha^5\beta^5(\beta^3+\alpha^3)$ $\quad a^3+b^3=(a+b)^3-3ab(a+b)$

$\alpha^5\beta^5\left[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)\right]$

$\alpha^5\beta^5\left[-\frac{q^3}{p^3} - 3\frac{r}{p}\cdot\frac{-q}{p}\right]$

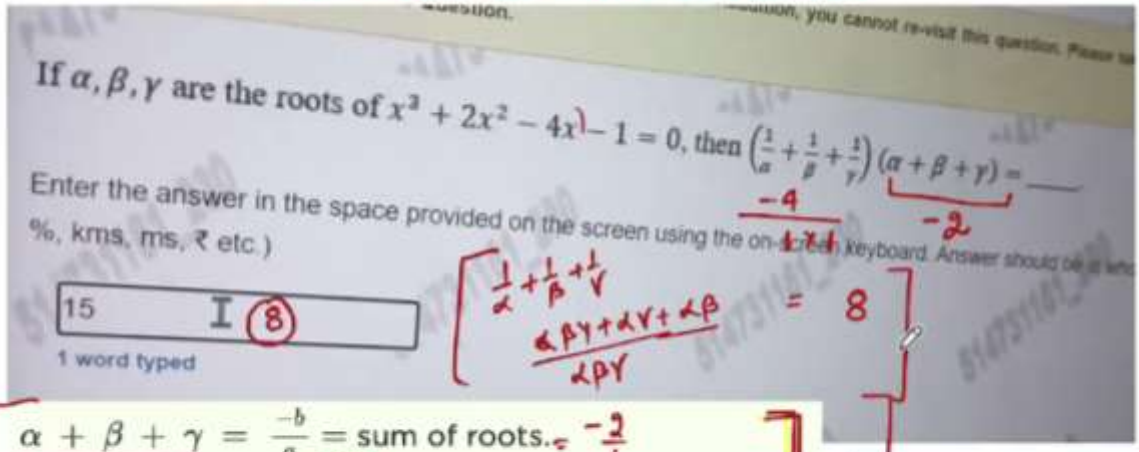
$= \alpha^5\beta^5\left[\frac{-q^3+3pqr}{p^3}\right]$

$\frac{q\alpha^5\beta^5}{p^3}\left[-q^2+3pr\right]$

$\frac{q\gamma^5}{p^5p^3}\left[3pr-q^2\right)$

$\frac{q\gamma^5}{p^8}(3pr-q^2)$

QUADRATIC EQUATION – ZERO to HERO



$\frac{-4}{-2} = 2$

$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$\frac{\alpha\beta+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma} = 8$

$\alpha + \beta + \gamma = \frac{-b}{a}$ = sum of roots. $= \frac{-2}{1}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ = sum of product of the roots.

$\alpha\beta\gamma = \frac{-d}{a}$ = product of the roots.

QUADRATIC EQUATION – ZERO to HERO

8. clear off-campus

If α and β are the roots of $x^2+x+2=0$, then $\frac{\alpha^{10}+\beta^{10}}{\alpha^{-10}+\beta^{-10}}$ is equal to:

(a) 4096 (b) 2048 (c) 1024 (d) 512

$\alpha^{-1} = \frac{1}{\alpha}$ ✓

$\alpha\beta = 2/1$ $\quad x^2+x+2=0$

$= \frac{\alpha^{10}+\beta^{10}}{\alpha^{-10}+\beta^{-10}}$

$\frac{\alpha^{10}+\beta^{10}}{\frac{\alpha^{10}+\beta^{10}}{\alpha^{10}\beta^{10}}}$ $\quad \alpha^{10}\beta^{10} = (\alpha\beta)^{10} = (2)^{10} = 2^5\times 2^5 = 32\times 32 = 1024$

$= \frac{\alpha^{10}+\beta^{10}}{\frac{1}{\alpha^{10}}+\frac{1}{\beta^{10}}}$

QUADRATIC EQUATION – ZERO to HERO

Q. If the roots of the equation $x^2 + (3k-36)x + k^2 - 24k + 144 = 0$ are reciprocal to each other, then find the value of k;

a. K= 11 or K=13
b. K=-11 or K=-13
c. K=12
d. K=-12

$ax^2 + bx + c$

$x^2 + (3k-36)x + k^2 - 24k + 144 = 0$

Product of roots = c/a

$1 = k^2 - 24k + 144$

$k^2 - 24k + 143 = 0$

$-11, -13$

roots, $+11, +13$

$11 \quad 13$

$3 \times \frac{1}{3} = 1$

$\alpha \times \frac{1}{\alpha} = 1$

QUADRATIC EQUATION – ZERO to HERO

Q. If the root of the equation $3ax^2 + 2bx + c = 0$ are in the ratio of 2:3, then;

a. $8ac = 25b$
b. $8ac = 9b^2$
c. $8b^2 = 9ac$
d. $8b^2 = 25ac$

$3ax^2 + 2bx + c = 0$

$2x, 3x$

Sum of roots : $(2x + 3x) = \frac{-2b}{3a}$

$5x = -\frac{2b}{3a} \Rightarrow x = \frac{-2b}{15a}$

$2x \cdot 3x = \frac{c}{3a}$

Test series clear

$26 \times \frac{4b^2}{225a^2} = \frac{c}{3a}$

$8b^2 = 25ac$

QUADRATIC EQUATION – ZERO to HERO

Q. A man born in 1900s realized that in 1980 his age was the square root of the year of his birth. What is his birth year;

a. 1929
b. 1949
c. 1936
d. 1946

Interesting problems

age = 60

1980 − 60 = 1920

clear

1980 : x

1980 − x

Square

$x = \sqrt{1980 - x}$

$x^2 = 1980 - x$

$x^2 + x - 1980 = 0$

45, −44

roots = −45, +44

1980 − 44 = 1936

2	1980
2	990
3	495
3	165
5	55
	11

QUADRATIC EQUATION – ZERO to HERO

Q. If the equation $X^2 - 4x + 5 = 0$ and $X^2 + ax + b = 0$ have a common root, find a and b?

Fill the answer in box below.

a = -4, b = 5

Serious TG 999 ✓ Live session whatsapp

roots

D < 0

$x^2 - 4x + 5 = 0$

$D = b^2 - 4ac$

$= (-4)^2 - 4 \times 1 \times 5$

$= 16 - 20$

$= -4$

a = -4, b = 5

$x^2 + ax + b = 0$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{1}{1} = \frac{-4}{a} = \frac{5}{b}$