

**Solutions to Exercises from  
‘Types and Programming Languages’**

ABSTRACT. Solutions to exercises from the book ‘Types and Programming Languages’.

## Contents

Preface	v
Chapter 1. Introduction	1
Chapter 2. Mathematical Preliminaries	3



## Preface

Solutions to exercises from the book ‘Types and Programming Languages’.



## CHAPTER 1

# **Introduction**





## CHAPTER 2

### Mathematical Preliminaries

**EXERCISE:** Suppose we are given a relation  $R$  on a set  $S$ . Define the relation  $R'$  as follows:

$$R' = R \cup \{(s, s) \mid s \in S\}.$$

That is,  $R'$  contains all the pairs in  $R$  plus all pairs of the form  $(s, s)$ . Show that  $R'$  is the reflexive closure of  $R$ .

**SOLUTION:** Clearly,  $R \subseteq R'$ , and  $R'$  is a reflexive relation on  $S$ . Now, suppose  $R'' \subsetneq R'$  is the reflexive closure of  $R$ . Then, there exists  $(s, t) \in R'$ , for some  $s, t \in S$ , such that  $(s, t) \notin R''$ . If  $s = t$ , then  $(s, s) \notin R''$ , which is a contradiction. On the other hand, if  $s \neq t$ , then  $(s, t) \in R \subseteq R''$ , again a contradiction. We thus conclude  $R$  is indeed the reflexive closure of  $R$ , and we are done.