Lesson 3

Quiz, 8 questions

8/8 points (100%)

Congratulations! You passed!

Next Item

1 / 1
points

1.
If a random variable X follows a standard uniform distribution ($X \sim \mathrm{Unif}(0,1)$), then the PDF of X is $p(x) = 1$ for $0 \le x \le 1$.

We can use Monte Carlo simulation of X to approximate the following integral:
$\int_0^1 x^2 dx = \int_0^1 x^2 \cdot 1 dx = \int_0^1 x^2 \cdot p(x) dx = \mathrm{E}(X^2)$.

If we simulate 1000 independent samples from the standard uniform distribution and call them x_i^* for $i = 1, \ldots, 1000$, which of the following calculations will approximate the integral above?

- $\frac{1}{1000}\sum_{i=1}^{1000} x_i^*$
- $\frac{1}{1000}\sum_{i=1}^{1000} {x_i^*}^2$

Correct
If we want to approximate $\mathrm{E}(g(X))$ for some function $g()$, then we need to apply $g()$ to the samples and then average them. In this example we have $g(x) = x^2$.

- $\left(\frac{1}{1000}\sum_{i=1}^{1000} x_i^*\right)^2$
- $\frac{1}{1000}\sum_{i=1}^{1000} (x_i^* - \bar{x}^*)^2$ where $\bar{x}^*$ is the calculated average of the x_i^* samples.

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1 / 1
points

2.
Suppose we simulate 1000 samples from a $\mathrm{Unif}(0, \pi)$ distribution (which has PDF $p(x) = \frac{1}{\pi}$ for $0 \le x \le \pi$) and call the samples x_i^* for $i = 1, \ldots, 1000$.

If we use these samples to calculate $\frac{1}{1000}\sum_{i=1}^{1000} \sin(x_i^*)$, what integral are we approximating?

- $\int_{-\infty}^{\infty} \sin(x) dx$
- $\int_0^\pi \frac{\sin(x)}{\pi} dx$

Correct

This is $\mathrm{E}(\sin(X)) = \int_0^\pi \sin(x) \cdot p(x) dx$.

- $\int_0^1 \sin(x) dx$
- $\int_0^1 \frac{\sin(x)}{\pi} dx$

✓ 1 / 1 points

3.

Suppose random variables X and Y have a joint probability distribution $p(X, Y)$. Suppose we simulate 1000 samples from this distribution, which gives us 1000 (x_i^*, y_i^*) pairs.

If we count how many of these pairs satisfy the condition $x_i^* < y_i^*$ and divide the result by 1000, what quantity are we approximating via Monte Carlo simulation?

- $\Pr[X < Y]$

Correct

This is also $\mathrm{E}(I_{x<y}) = \int \int I_{x<y} \cdot p(x, y)\, dx\, dy$.

- $\Pr[\mathrm{E}(X) < \mathrm{E}(Y)]$
- $\mathrm{E}(XY)$
- $\Pr[X < \mathrm{E}(Y)]$

✓ 1 / 1 points

4.

If we simulate 100 samples from a $\mathrm{Gamma}(2, 1)$ distribution, what is the approximate distribution of the sample average $\bar{x^*} = \frac{1}{100} \sum_{i=1}^{100} x_i^*$?

Hint: the mean and variance of a $\mathrm{Gamma}(a, b)$ random variable are a/b and a/b^2 respectively.

- $\mathrm{Gamma}(2, 1)$
- $\mathrm{N}(2, 0.02)$

Correct

Due to the central limit theorem, the approximating distribution is normal with mean equal to the mean of the original variable, and with variance equal to the variance of the original variable divided by the sample size.

- $\mathrm{N}(2, 2)$

○ $\text{Gamma}(2, 0.01)$

1 / 1 points

5.
For Questions 5 and 6, consider the following scenario:

Laura keeps record of her loan applications and performs a Bayesian analysis of her success rate θ. Her analysis yields a $\text{Beta}(5, 3)$ posterior distribution for θ.

The posterior mean for θ is equal to $\frac{5}{5+3} = 0.625$. However, Laura likes to think in terms of the odds of succeeding, defined as $\frac{\theta}{1-\theta}$, the probability of success divided by the probability of failure.

Use R to simulate a large number of samples (more than 10,000) from the posterior distribution for θ and use these samples to approximate the posterior mean for Laura's odds of success ($\text{E}(\frac{\theta}{1-\theta})$).

Report your answer to at least one decimal place.

2.5

Correct Response
The posterior distribution of the odds (which you can plot with your samples if you create a new variable for the odds) is heavily skewed right, so the posterior mean for the odds (2.5) is much larger than the odds calculated from the posterior mean of θ $(0.625/0.375 \approx 1.667)$. The posterior median of the odds might be a better measure in this case.

```
theta = rbeta(9999, 5, 3)
alpha = theta / (1 - theta)
mean( alpha )
```

1 / 1 points

6.
Laura also wants to know the posterior probability that her odds of success on loan applications is greater than 1.0 (in other words, better than 50:50 odds).

Use your Monte Carlo sample from the distribution of θ to approximate the probability that $\frac{\theta}{1-\theta}$ is greater than 1.0.

Report your answer to at least two decimal places.

0.77

Correct Response
This is also equivalent to the posterior probability that $\theta > 0.5$.

```
theta = rbeta(9999, 5, 3)
alpha = theta / (1 - theta)
mean( alpha )
mean( alpha > 1.0 )
```

1 / 1
points

7.
Use a (large) Monte Carlo sample to approximate the 0.3 quantile of the standard normal distribution ($\mathrm{N}(0, 1)$), the number such that the probability of being less than it is 0.3.

Use the **`quantile`** function in R. You can of course check your answer using the **`qnorm`** function.

Report your answer to at least two decimal places.

-0.52

Correct Response

```
quantile( rnorm(9999, 0.0, 1.0) )
qnorm(0.3, 0.0, 1.0)
```

1 / 1
points

8.
To measure how accurate our Monte Carlo approximations are, we can use the central limit theorem. If the number of samples drawn m is large, then the Monte Carlo sample mean $\bar{\theta}^*$ used to estimate $\mathrm{E}(\theta)$ approximately follows a normal distribution with mean $\mathrm{E}(\theta)$ and variance $\mathrm{Var}(\theta)/m$. If we substitute the sample variance for $\mathrm{Var}(\theta)$, we can get a rough estimate of our Monte Carlo standard error (or standard deviation).

Suppose we have 100 samples from our posterior distribution for θ, called θ_i^*, and that the sample variance of these draws is 5.2. A rough estimate of our Monte Carlo standard error would then be $\sqrt{5.2/100} \approx 0.228$. So our estimate $\bar{\theta}^*$ is probably within about 0.456 (two standard errors) of the true $\mathrm{E}(\theta)$.

What does the standard error of our Monte Carlo estimate become if we increase our sample size to 5,000? Assume that the sample variance of the draws is still 5.2.

Report your answer to at least three decimal places.

0.032

Correct Response
This is just $\sqrt{5.2/5000}$.