STATISTICAL QUALITY CONTROL

(A case study demonstrating process control using control charts for quantitative and qualitative characteristics, and product control using sampling plans, for some industrial and manufacturing processes.)

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Instructor's Remarks:

3. STATISTICAL QUALITY CONTROL (8 CASES)

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4.2 PRODUCT CONTROL – CONSTRUCTING AND ANALYZING OC CURVE, AOQ CURVE AND ASN CURVE FOR A SINGLE SAMPLING PLAN UNDER CORRECTIVE SAMPLING

Statistical Quality Control

Statistical Quality Control (SQC) is one of the most important application of statistical techniques in industry. These techniques are based on theory of probability and sampling, and are used in almost all kinds of industries.

A product may have several aspects of quality as well as an overall quality which is something more than the sum of its individual quality aspects – a property technically known as *Synergy.*

Quality means a level/standard of the product which, in turn, depends on four factors namely - materials, manpower, machines and management.

Quality control is a powerful productivity technique for effectiveness diagnosis of lack of quality of materials, processes, machines or end products. Quality control, therefore, covers all the factors and processes of production which may be broadly classified as follows:

- **Quality of materials**: Material of good quality will result in smooth processing and thereby reducing the waste and increasing the output.
- **Quality of manpower**: Trained and qualified personnel will give increased efficiency due to better quality production through the application of skill and also reduce production cost and waste.
- **Quality of machines**: Better quality equipment will result in efficient work due to lack of scarcity of breakdowns and thus reduce cost of defectives.
- **Quality of management**: A good management is imperative for increase in efficiency, harmony in relations, and growth of business and markets.

The basis of statistical quality control is the degree of variability in the size or the magnitude of a given characteristic of the product. These variations are broadly classified as being due to two causes:

- **Chance Causes**: The variation due to these causes is beyond the control of human hand and cannot be prevented or eliminated under any circumstances.
- **Assignable causes**: The assignable causes may creep in at any stage of the process, right from the arrival of the raw materials to the final delivery of goods.

Quality control is divided into two parts:

When we want to ensure that the proportion of defective items in the manufactured product is not too large, this is called **process control** and this is achieved by control charts.

Product Control means controlling the quality of the product by critical examination at strategic points and this is achieved through sampling inspection plans.

Process Control

Process control involves controlling and maintaining the quality of manufactured product so that it conforms to standard quality standards, which is achieved through the technique called control charts pioneered by W.A. Shewhart. A control chart is a pictorial way of understanding the variations in quality with respect to a certain attribute. A standard control chart has three horizontal lines viz. upper control limit (UCL) – maximum allowed level of quality characteristic, central line (CL) – average or desired level of quality characteristic and lower control limit (LCL) – minimum allowed level of quality characteristic. A process is said to be in state of control if:

- All sample points fall within control limits.
- The points are randomly scattered i.e. they do not follow any trend indicating Assignable causes.

Association between Testing of Hypothesis and Control Charts:

H_0: The process is under statistical control.

H_1: The process is not under statistical control.

Test Statistic: Control Chart

Type I Error: Regarding process to be out of statistical control when it is actually under statistical control.

Type II Error: Regarding process to be in statistical control when it is actually out of statistical control.

Thus, widening the control band leads to Type II Error while narrowing the control band leads to Type I Error.

Process of Drawing Statistical Control Charts:

Firstly one identifies a quality characteristic and then consider a statistic t that specifies it. The control limits can then be drawn as follows:

Central Line: (t)

Lower Control Limit: $(t) - 3 \times (t)$

Upper Control Limit: $(t) + 3 \times (t)$

which is due to the area property of normal distribution that chances of the value lying beyond 3 standard deviation steps is very negligible. Of course, when the underlying process isn't normal the argument may not hold.

Control Charts for Variables

These are the control charts for quality characteristics which can be measured quantitatively.

Control Charts for Mean ($\bar{X}$ – Chart) and Range (R – Chart)

Let X_{ij} j = 1,…, be measurements on the i^{th} sample or subgroup i = 1,…,k. Suppose X_{ij} ~ $(\mu,^2)$.

The $\bar{X}$ chart tries to see if average measurement is under control or not, i.e. the statistic is sample mean. We calculate:

$$\bar{X}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{ij}$$

Then as discussed the control limits are:

Central Line: $(\bar{X}_i) = \mu$

Lower Control Limit: $\mu - 3 \times SE(\bar{X}_i) = \mu - \frac{3\sigma}{\sqrt{n_i}}$

Upper Control Limit: $\mu + 3 \times SE(\bar{X}_i) = \mu + \frac{3\sigma}{\sqrt{n_i}}$

Clearly when standards are given, i.e. μ and σ^2 are known, we can execute the limits directly, otherwise we replace the standards with their estimates based on the data.

Consider another statistic which specifies the dispersion in the quality characteristic called Range and defined as:

$$R_i = \max X_{ij} - \min X_{ij}.$$

It can be shown that $(R_i) = d_2\sigma$ where d_2 is a constant depending upon n. Hence we can write:

$$SE(\bar{X}_i) = \frac{3E(R_i)}{d_2\sqrt{n_i}}$$

$$SE(\bar{X}_i) = \frac{3R}{d_2\sqrt{n_i}} \dots \quad (1)$$

where R is the population range.

We can estimate μ and R as follows:

$$\hat{\mu} = \bar{\bar{X}} = \frac{1}{k}\sum_{i=1}^{k} \bar{X}_i = \text{overall sample mean}$$

$$\hat{R} = \bar{R} = \frac{1}{k}\sum_{i=1}^{k} \bar{R}_i = \text{overall sample range.}$$

Hence we can write the control limits as follows:

Central Line: $E(\bar{X}_i) = \hat{\mu} = \bar{X}$

Lower Control Limit: $\bar{X} - \frac{3\bar{R}}{d_2} * \frac{1}{\sqrt{n_i}}$

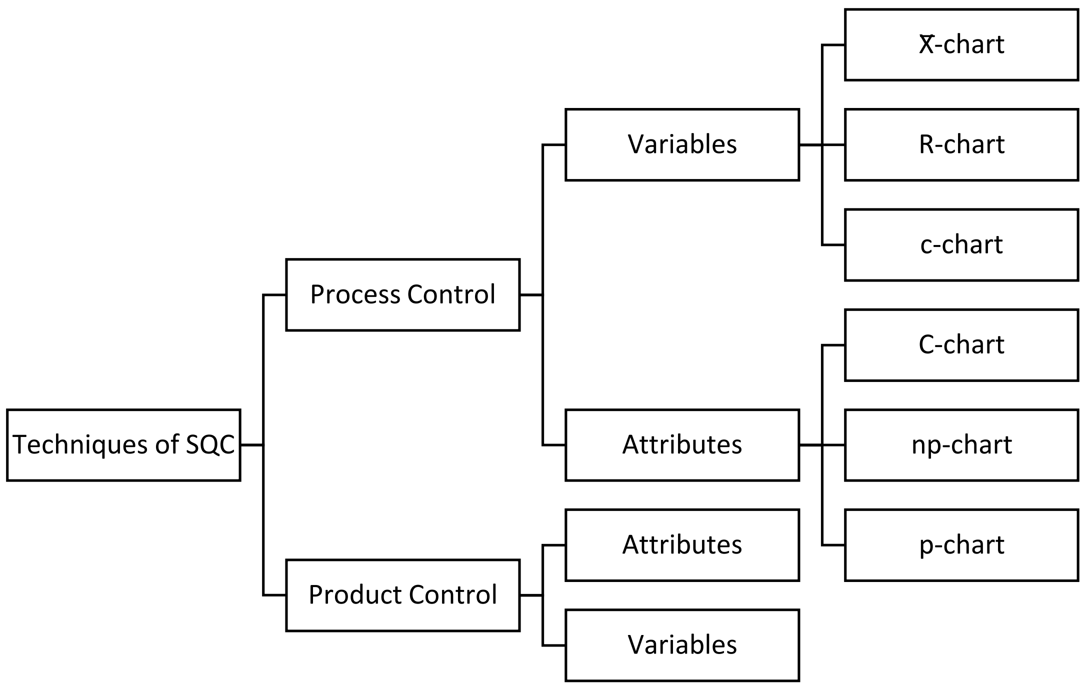
Upper Control Limit: $\bar{X} + \frac{3\bar{R}}{d_2} * \frac{1}{\sqrt{n_i}}$

It is also important to see that variability within the sample is within the permissible limits. For this purpose we use the R chart, where the statistic is sample range which is defined as follows:

$$R_i = \max X_{ij} - \min X_{ij}$$

We need the expectation and standard error of the statistic for calculating the control limits, which are obtained as $(R_i) = R$, i.e. population range and $(R_i) = cR$ where c is a constant depending upon sample size. Clearly if the standard, i.e. the population range (R) is given then we can execute the limits, otherwise we replace R by overall sample range, i.e. $\bar{R}$.

A tabular representation of the quality control processes is:



Control Charts for Mean ($\bar{X}$–Chart) and Standard Deviation (s –Chart)

R Chart is used to see if the variability in the measurement is within the permissible limits or not? We can use standard deviation as well to quantify this. s-Chart actually does this, we try to obtain the control limits using sample standard deviation as the statistic which is defined as:

$$s_i = \sqrt{\left(\frac{1}{n}\sum_{j=1}^{n_i} X_{ij} - \bar{X}_i\right)}$$

We need (s_i) and (s_i) for calculating the control limits. The control limits are obtained as follows:

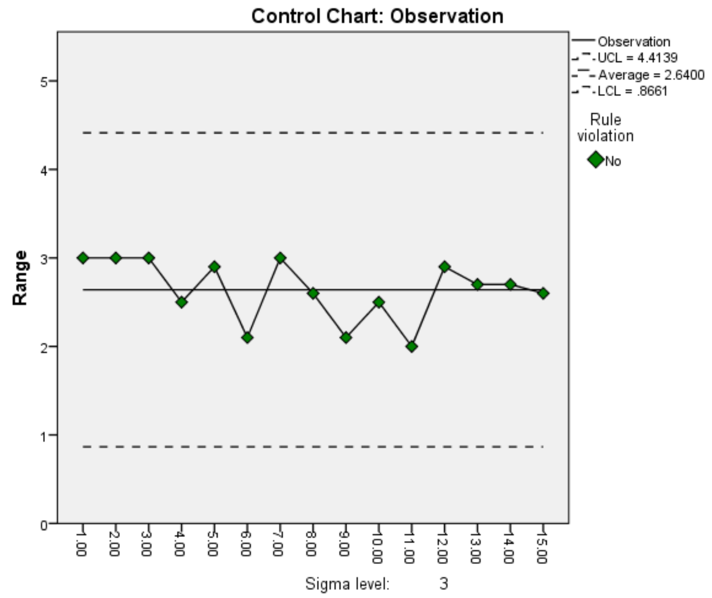
$$CL = \bar{s} = \frac{1}{k}\sum_{i=1}^{k} s_i$$

$$LCL = B_3\bar{s}$$

$$UCL = B_4\bar{s}$$

Case 1: Consider the "diameter" dataset which has data on diameter of a fibre for 15 different samples with each sample of size 14. Comment on the state of process control using $\overline{X}$ – R chart or $\overline{X}$ – s chart.

Data is coming from 15 samples, each of size 14. We will proceed by making control charts as follows:



Conclusion

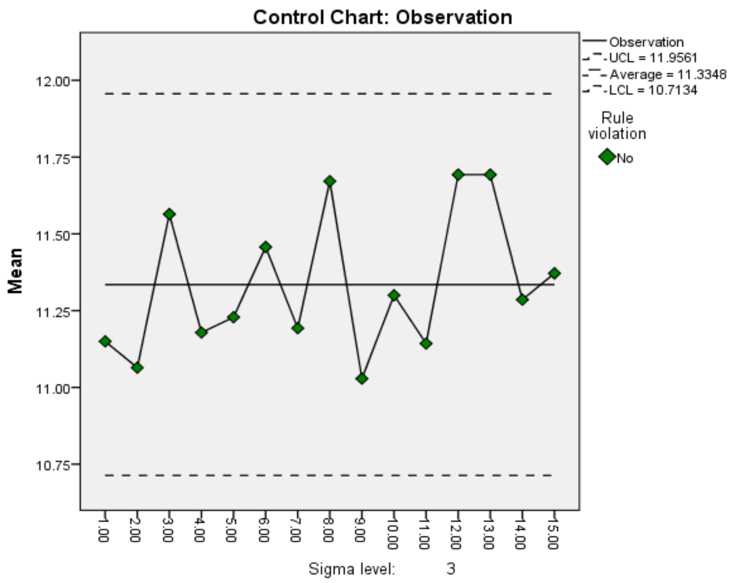
UCL: 4.4139

CL: 2.64

LCL: 0.8861

Range of various samples lies within the control limit.

Now we will proceed to make control chart for mean and standard deviation.



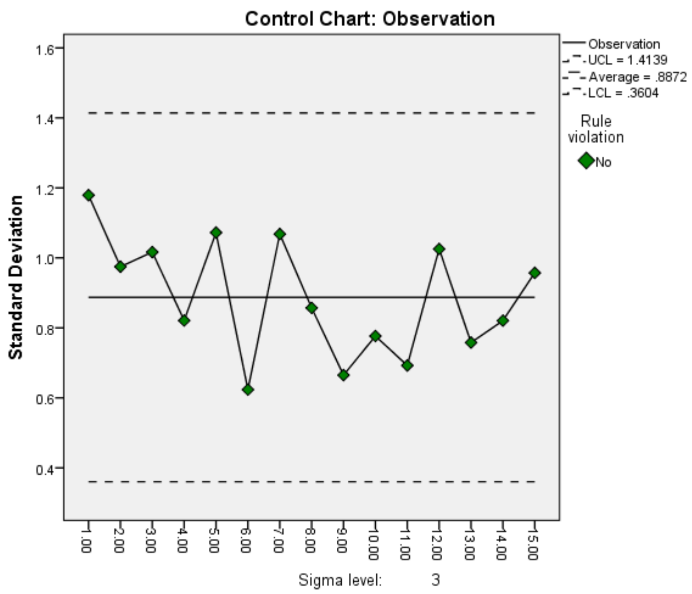
Conclusion

UCL: 11.9561

CL: 11.3348

LCL: 10.7134

From **$\overline{X}$ chart** we conclude that, mean of all the samples lies within control limit, hence the process is under statistical control.



Conclusion

UCL: 1.4139

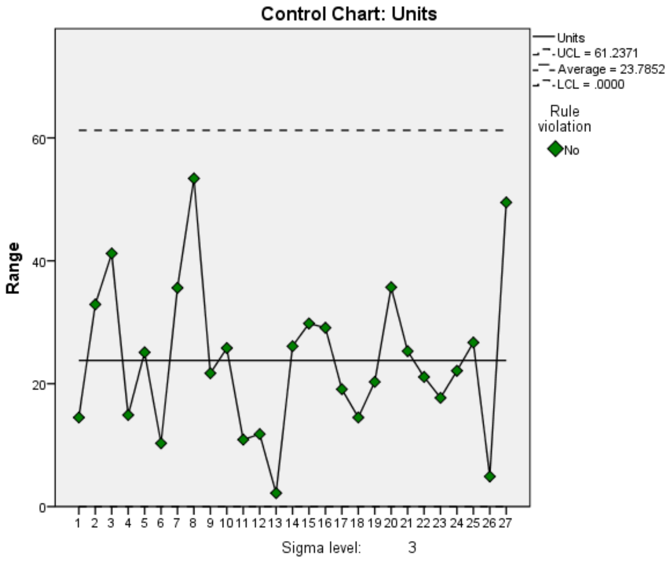
CL: 0.8872

LCL: 0.3604

From **s chart** we conclude that, standard deviation of all the samples lies within control limit, hence the process is under statistical quality control.

Case 2: Consider the "weight" dataset which has data on weight of a container for 27 different samples with each sample of size 3. Comment on the state of process control using $\overline{X} - R$ chart or $\overline{X} - s$ chart.

Sample No	Unit 1	Unit 2	Unit 3	Sample No	Unit 1	Unit 2	Unit 3
1	24.0	23.1	37.6	14	60.9	49.8	34.8
2	17.2	47.4	14.5	15	47.1	22.7	17.3
3	18.2	54.6	13.4	16	4.3	33.4	11.6
4	20.8	12.5	27.4	17	23.8	42.9	23.8
5	17.0	42.1	21.8	18	20.5	34.8	20.3
6	20.8	29.5	19.2	19	28.3	15.9	36.2
7	19.7	55.3	33.7	20	60.2	24.5	41.5
8	22.9	12.9	66.3	21	38.0	37.0	62.3
9	28.5	20.9	42.6	22	30.0	21.1	8.9
10	16.1	41.4	15.6	23	38.9	21.2	34.5
11	13.4	19.0	8.1	24	8.0	9.7	30.1
12	25.6	37.4	28.1	25	38.4	11.7	25.4
13	14.2	14.4	12.2	26	15.2	14.0	10.3
				27	6.4	55.9	39.5



Conclusion

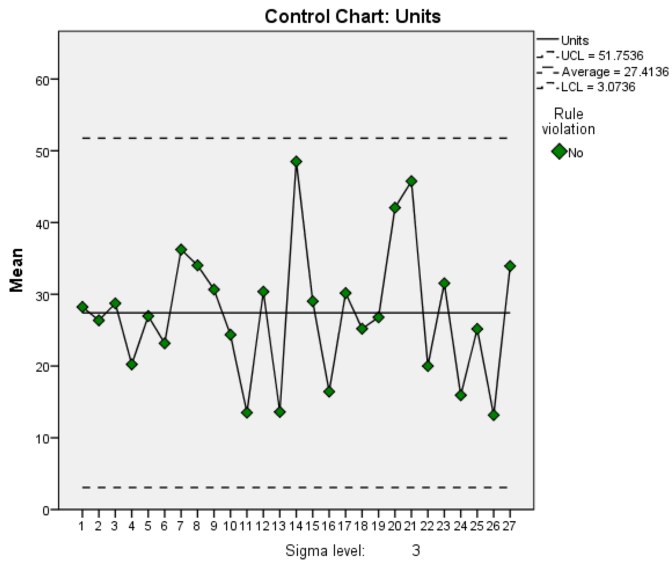
UCL: 61.2371

CL: 23.7852

LCL: 0.0

Range of various samples lies within the control limit.

Now we will proceed to make control chart for mean and standard deviation.



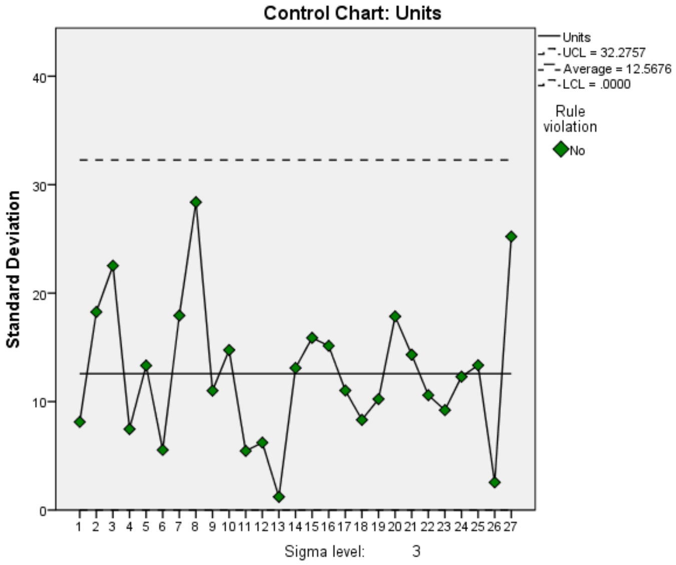
Conclusion

UCL: 51.7536

CL: 27.4136

LCL: 3.0736

From **$\overline{X}$ chart** we conclude that, mean of all the samples lies within control limit, hence the process is under statistical control.



Conclusion

UCL: 32.2757

CL: 12.5676

LCL: 0.0

From **s chart** we conclude that, standard deviation of all the samples lies within control limit, hence the process is under statistical quality control.

Control Charts for Attributes

Attributes are the quality characteristics which can't be measured quantitatively. We study the following control charts for attributes:

Control Chart for Fraction Defectives ($\boldsymbol{p}$ –Chart) and No. of Defectives ($\boldsymbol{np}$ Chart)

Let d_i be the no. of defectives in a sample of size ni then proportion of defectives is:

$$p_i = \frac{d_i}{n_i} \ ; \ i = 1, \dots, k$$

We want to construct control chart to see if proportion of defectives is within permissible limits of not. We need the following:

$$E(p_i) \ = \ P \ \ and\ SE(p_i) \ = \ \sqrt{P(1-P)n_i}$$

where P is the population proportion of defectives. If P is known then use that, otherwise estimate P as overall proportion of defectives, i.e.

$$\hat{P} = \bar{p} = \frac{1}{k}\sum_{i}^{k} p_i$$

Hence the control limits are given by

$$E(p_i) \ \pm \ 3SE(p_i)\ i.e.(\bar{p} \ \pm \ 3\sqrt{pqn_i})$$

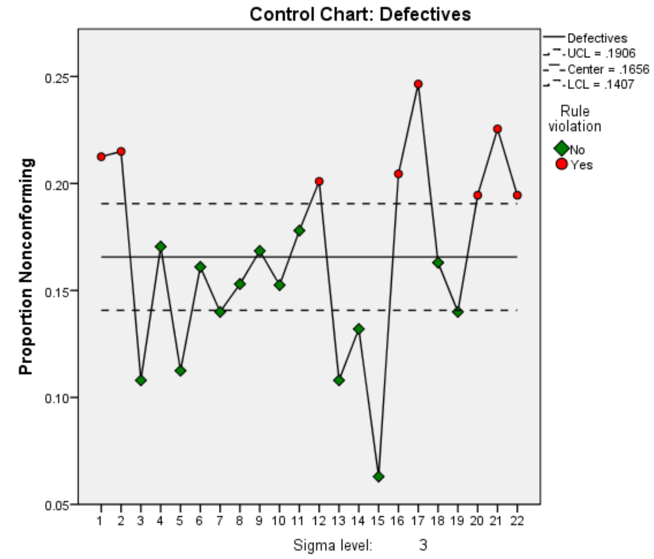
We can instead use directly the number of defects as well, the control limits will be by

$$E(d_i) \ \pm \ 3SE(d_i)\ i.e.(n_i\bar{p} \ \pm \ 3\sqrt{n_i pq})$$

In fact p –Chart and d – Chart give exactly same information but when same size varies p –Chart is preferred as the central line remains constant unlike in d –Chart.

Case 3: Consider the “leather_belts” dataset which has data on no. of defectives in 22 lots each containing 2000 leather belts. Draw a control chart for fraction defectives in order to assess the state of process control.

Lot Number	No. of Defectives	Lot Number	No. of Defectives
1	425	12	402
2	430	13	216
3	216	14	264
4	341	15	126
5	225	16	409
6	322	17	493
7	280	18	326
8	306	19	280
9	337	20	389
10	305	21	451
11	356	22	389



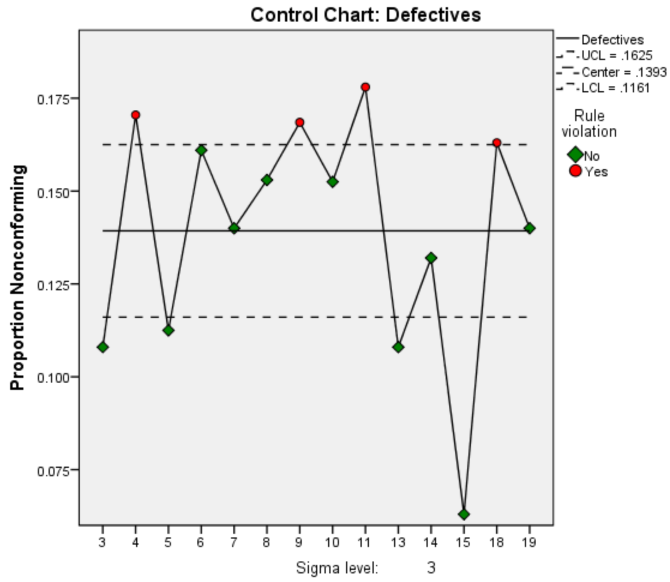
Conclusion

UCL: 0.1906

CL: 0.1656

LCL: 0.1407

From **p chart** we conclude that, proportion of defectives in lot no. 1, 2, 12, 16, 17, 20, 21 and 22 is higher than UCL. At this stage process is **out of control**, hence we will remove these lots and make our control chart again until the process comes under control.



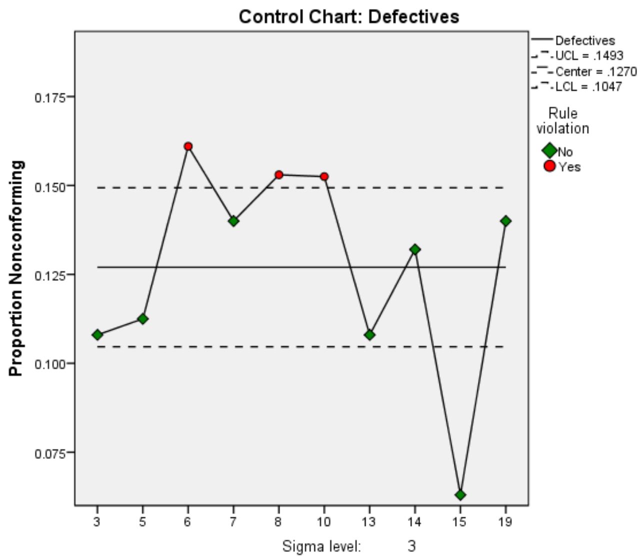
Conclusion

UCL: 0.1625

CL: 0.1393

LCL: 0.1161

From **p chart** we conclude that, proportion of defectives in lot no. 4, 9, 11 and 18 is higher than UCL. At this stage process is **out of control**, hence we will remove these lots and make our control chart again until process comes under control.



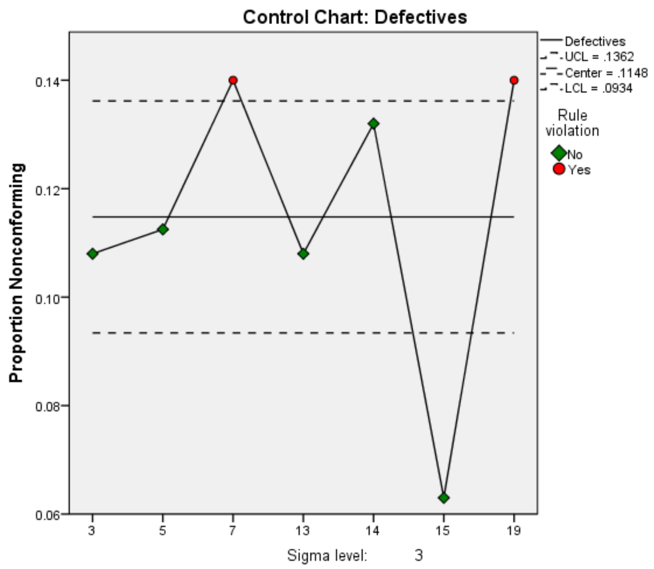
Conclusion

UCL: 0.1493

CL: 0.1270

LCL: 0.1047

From **p chart** we conclude that, proportion of defectives in lot no. 6, 8 and 10 is higher than UCL. At this stage process is **out of control**, hence we will remove these lots and make our control chart again until process comes under control.



Conclusion

UCL: 0.1362

CL: 0.1148

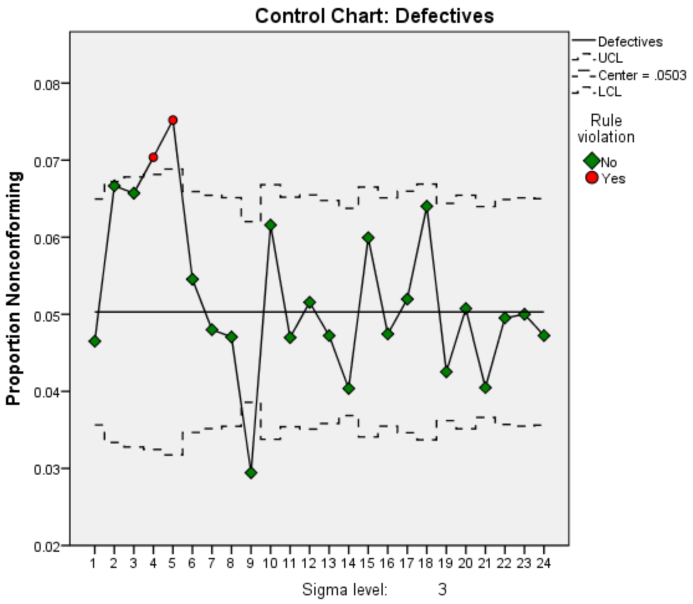
LCL: 0.0934

From **p chart** we conclude that, proportion of defectives in lot no. 7 and 19 is still higher than UCL. At this stage we see number of samples have reduced to 7 and process is still **out of control**.

Hence based on this lot we cannot setup the control limits for the future reference. And we stop at this stage.

Case 4: Consider the “cricket_gloves” dataset which has data on no. of defectives in 24 lots each containing different no. of cricket gloves. Draw a control chart for fraction defectives in order to assess the state of process control.

Lot Number	Lot Size	No. of Defectives	Lot Number	Lot Size	No. of Defectives
1	2000	93	13	2054	97
2	1500	100	14	2378	96
3	1400	92	15	1635	98
4	1350	95	16	1960	93
5	1250	94	17	1751	91
6	1760	96	18	1562	100
7	1875	90	19	2163	92
8	1955	92	20	1872	95
9	3125	92	21	2297	93
10	1575	97	22	2019	100
11	1937	91	23	1960	98
12	1862	96	24	1990	94



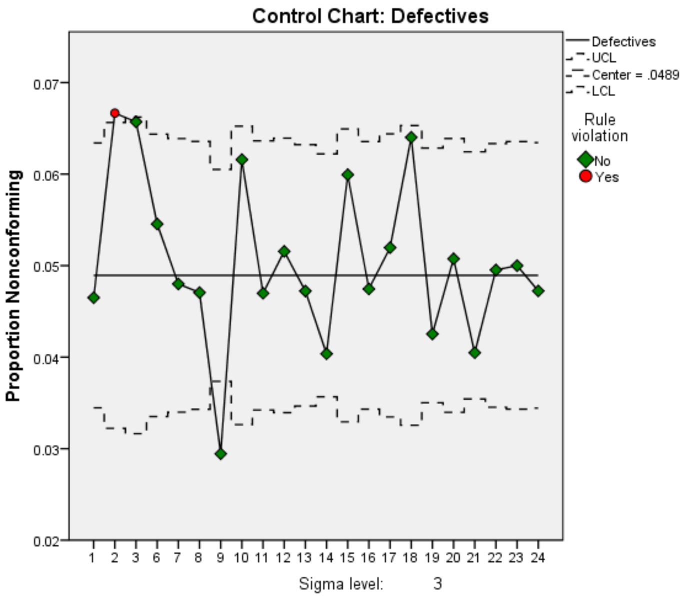
Conclusion

UCL: Variable Upper Control Limit

CL: 0.0503

LCL: Variable Lower Control Limit

From **p chart** we conclude that, proportion of defectives in lot no. 4 and 5 is higher than UCL. At this stage process is **out of control**, hence we will remove these lots and make our control chart again until process comes under control.



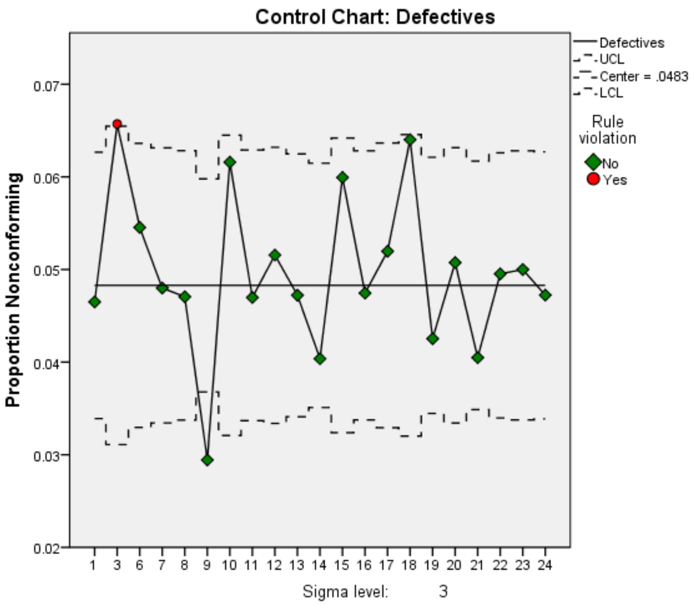
Conclusion

UCL: Variable Upper Control Limit

CL: 0.0503

LCL: Variable Lower Control Limit

From **p chart** we conclude that, proportion of defectives in lot no. 2 is higher than UCL. At this stage process is **out of control**, hence we will remove this lot and make our control chart again until process comes under control.



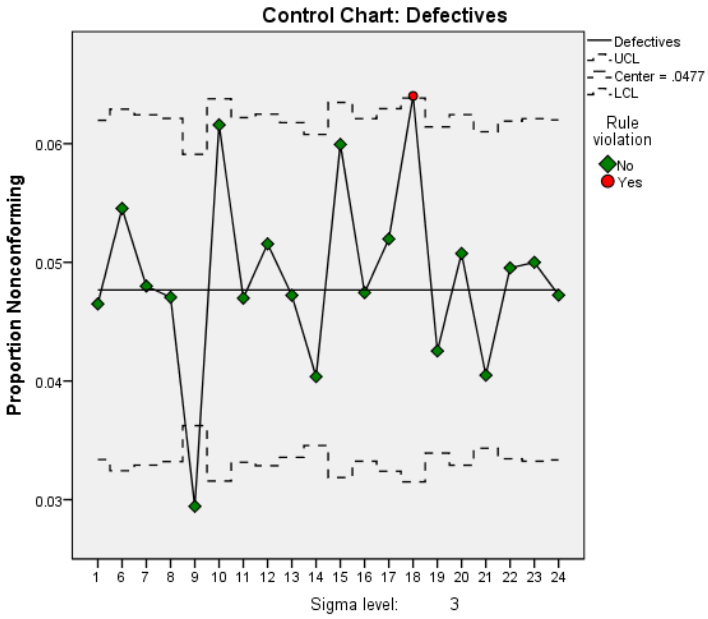
Conclusion

UCL: Variable Upper Control Limit

CL: 0.0483

LCL: Variable Lower Control Limit

From **p chart** we conclude that, proportion of defectives in lot no. 3 is higher than UCL. At this stage process is **out of control**, hence we will remove this lot and make our control chart again until process comes under control.



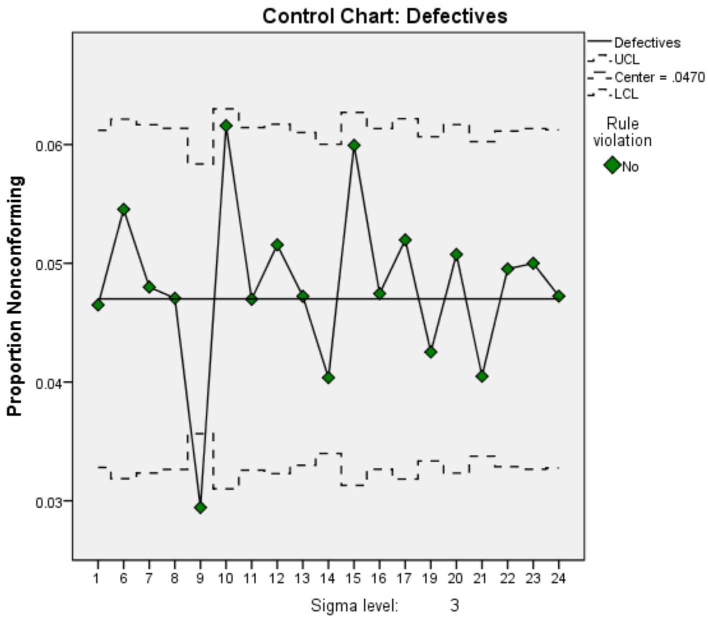
Conclusion

UCL: Variable Upper Control Limit

CL: 0.0477

LCL: Variable Lower Control Limit

From **p chart** we conclude that, proportion of defectives in lot no. 18 is higher than UCL. At this stage process is **out of control**, hence we will remove this lot and make our control chart again until process comes under control.



Conclusion

UCL: Variable Upper Control Limit

CL: 0.0470

LCL: Variable Lower Control Limit

From **p chart** we conclude that, proportion of defectives in all the lots lies within control limits. At this stage process becomes **under statistical control**, hence we can use these control limits as a standard limits for future process control.

Control Chart for No. of Defects/sample (c –Chart) and No. of Defects/unit (u–Chart)

An article which doesn't conform to one or more of the specifications is termed as defective while any instance of article's lack of conformity to specifications is a defect. In many manufacturing and inspection situations the sample size n is large (since the opportunities for the defects to occur are numerous) and the probability p of occurrence of a defect in any one spot is very small such that np is finite. In such situations from the statistical theory we know that the pattern of variations in data can be represented by Poisson distribution, i.e. the variable no. of defects per unit follows a Poisson distribution.

Let c_i be the no. of defects in the i^{th} sample of size n_i, i = 1… k which follows a Poisson distribution with parameter λ. We use c_i as the statistic and obtain the control limits using:

$$E(c_i) = \lambda$$

$$SE(c_i) = \sqrt{\lambda}$$

When λ is unknown we estimate it as overall average no. defects, i.e.

$$\hat{\lambda} = \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i$$

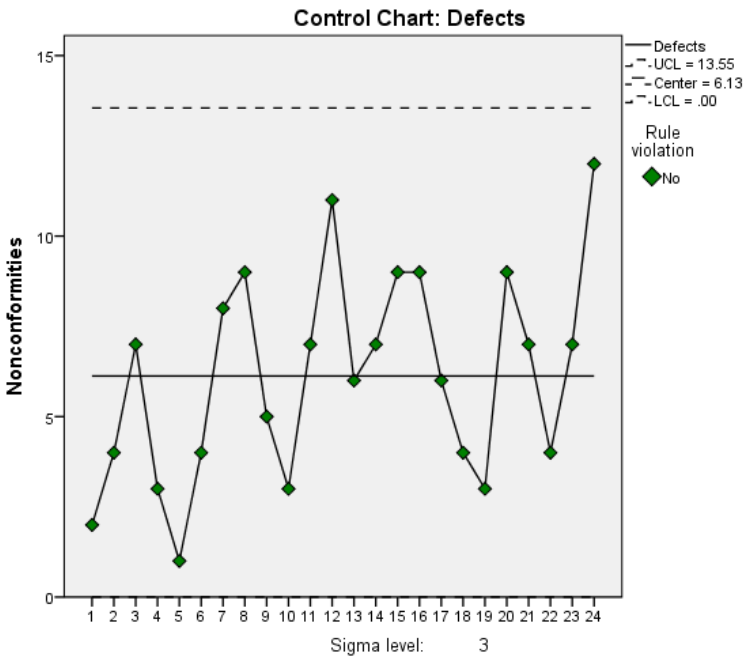
When the sample size is varying we generally use $u_i = c_i/n_i$, i.e. average no. defects per unit for the i^{th} sample.

$$E(u_i) = \frac{\lambda}{n_i}$$

$$SE(c_i) = \frac{\sqrt{\lambda}}{n_i}$$

Case 5: Consider the "welding" dataset which has data on defects in the process of welding, such as pinholes, cracks etc. Draw a control chart for the no. of defects and comment on the state of process control.

Unit no.	Date	Time	No. of defects	Unit no.	Date	Time	No. of defects
1	1.12.2014	8:00 AM	2	13	2.12.2014	12:00 PM	6
2	1.12.2014	9:00 AM	4	14	2.12.2014	1:00 PM	7
3	1.12.2014	10:00 AM	7	15	2.12.2014	2:00 PM	9
4	1.12.2014	11:00 AM	3	16	2.12.2014	3:00 PM	9
5	1.12.2014	12:00 PM	1	17	3.12.2014	8:00 AM	6
6	1.12.2014	1:00 PM	4	18	3.12.2014	9:00 AM	4
7	1.12.2014	2:00 PM	8	19	3.12.2014	10:00 AM	3
8	1.12.2014	3:00 PM	9	20	3.12.2014	11:00 AM	9
9	2.12.2014	8:00 AM	5	21	3.12.2014	12:00 PM	7
10	2.12.2014	9:00 AM	3	22	3.12.2014	1:00 PM	4
11	2.12.2014	10:00 AM	7	23	3.12.2014	2:00 PM	7
12	2.12.2014	11:00 AM	11	24	3.12.2014	3:00 PM	12



Conclusion

UCL: 13.55

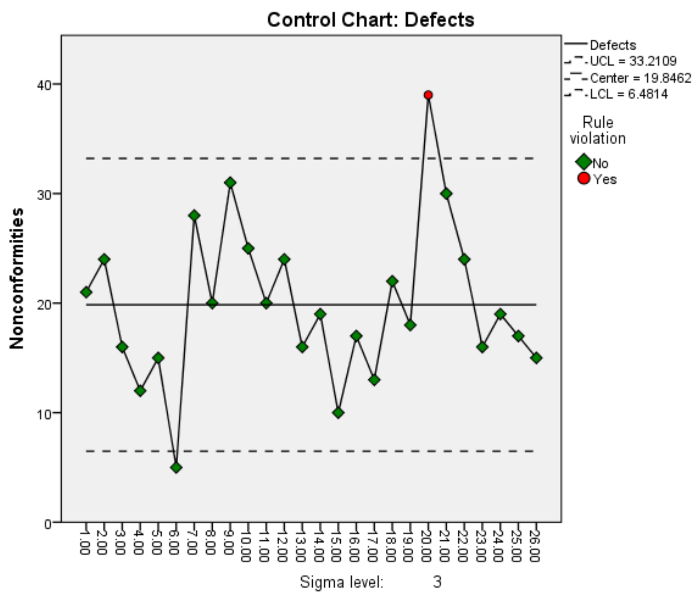
CL: 6.13

LCL: 0

From **c chart** we conclude that, number of defectives/samples lies within control limits. Process is **under statistical control**, hence we can use these control limits as a standard limits for future process control.

Case 6: Consider the "circuit_board" dataset which has data on no. of defects observed in 26 successive samples of 100 printed circuit boards. Setup a control chart for non-conformities. Does the process appear to be in statistical control?

Sample No.	No. of Defects	Sample No.	No. of Defects	Sample No.	No. of Defects
1	21	10	25	19	18
2	24	11	20	20	39
3	16	12	24	21	30
4	12	13	16	22	24
5	15	14	19	23	16
6	5	15	10	24	19
7	28	16	17	25	17
8	20	17	13	26	15
9	31	18	22		



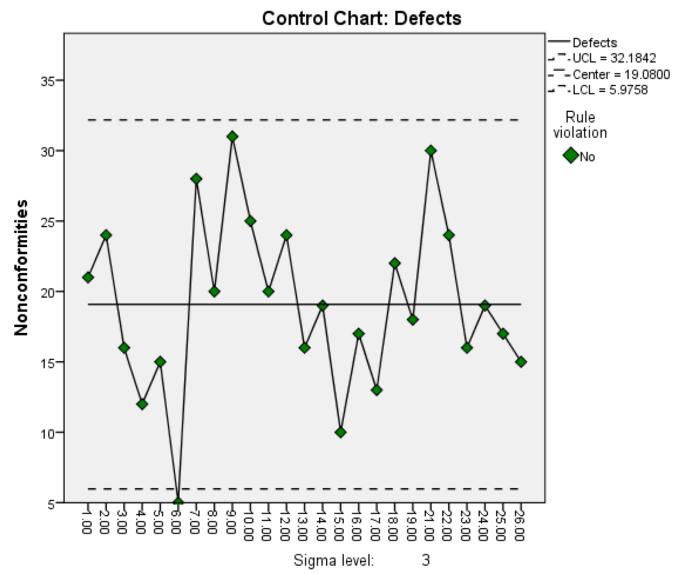
Conclusion

UCL: 33.21

CL: 19.84

LCL: 6.4814

From **c chart** we conclude that, number of defectives/samples lies outside control limits for the 20th sample. Process is **out of control**, hence we remove this sample and re draw our control chart.



Conclusion

UCL: 32.1842

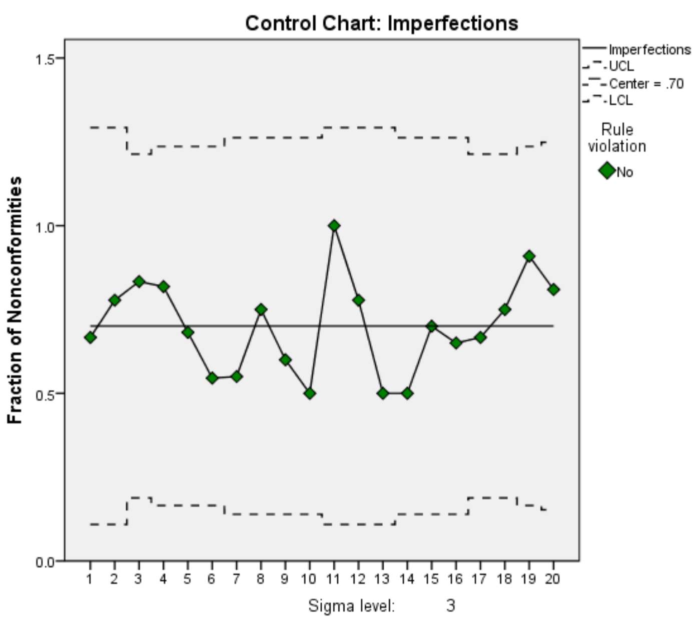
CL: 19.08

LCL: 5.9758

From **c chart** we conclude that, number of defectives/samples lies within control limits. Process is **under statistical control**, hence we can use these control limits as a standard limits for future process control.

Case 7: Consider the "paper_rolls" dataset which has data on no of imperfections in finished rolls of paper observed by a paper mill during inspection of production output for 20 days. Setup a control chart for nonconformities per roll of paper. Does the process appear to be in statistical control?

Day	No. of Rolls	No. of imperfections	Day	No. of Rolls	No. of imperfections
1	18	12	11	18	18
2	18	14	12	18	14
3	24	20	13	18	9
4	22	18	14	20	10
5	22	15	15	20	14
6	22	12	16	20	13
7	20	11	17	24	16
8	20	15	18	24	18
9	20	12	19	22	20
10	20	10	20	21	17



Conclusion

UCL: Variable Upper Control Limit

CL: 0.0470

LCL: Variable Lower Control Limit

From **u chart** we conclude that, number of defectives/unit lies within control limits. Process is **under statistical control**, hence we can use these control limits as a standard limits for future process control.

Product Control

Product control involves controlling the quality of product by critical examination at strategic points and this is achieved through Sampling Inspection Plans pioneered by Dodge and Roming. In sampling inspection plans a sample is drawn from a lot and is examined for defectives, if the no. of defectives are less than some specified threshold then one accepts the lot otherwise it is rejected (acceptance sampling) or sends for 100% inspection and all the defectives are replaced with standard ones (rectifying or corrective sampling). Also note that sometimes more than one samples can also be drawn to arrive at a decision. We are going to analyze a single sampling plan under corrective sampling.

Single Sampling Plan under Corrective Sampling

As discussed in a single sampling plan under corrective sampling a sample is drawn from a lot and is examined for defectives, if the no. of defectives are less than some specified threshold then one accepts the lot while replacing the defective pieces found in sample by non-defective ones, otherwise it is sent for 100% inspection and all the defectives are replaced with standard ones (rectifying or corrective sampling).

We define and calculate the concepts of acceptance quality level,

Incoming Quality: The quality of a lot, i.e. the proportion of defectives p say in a lot is termed as its incoming quality.

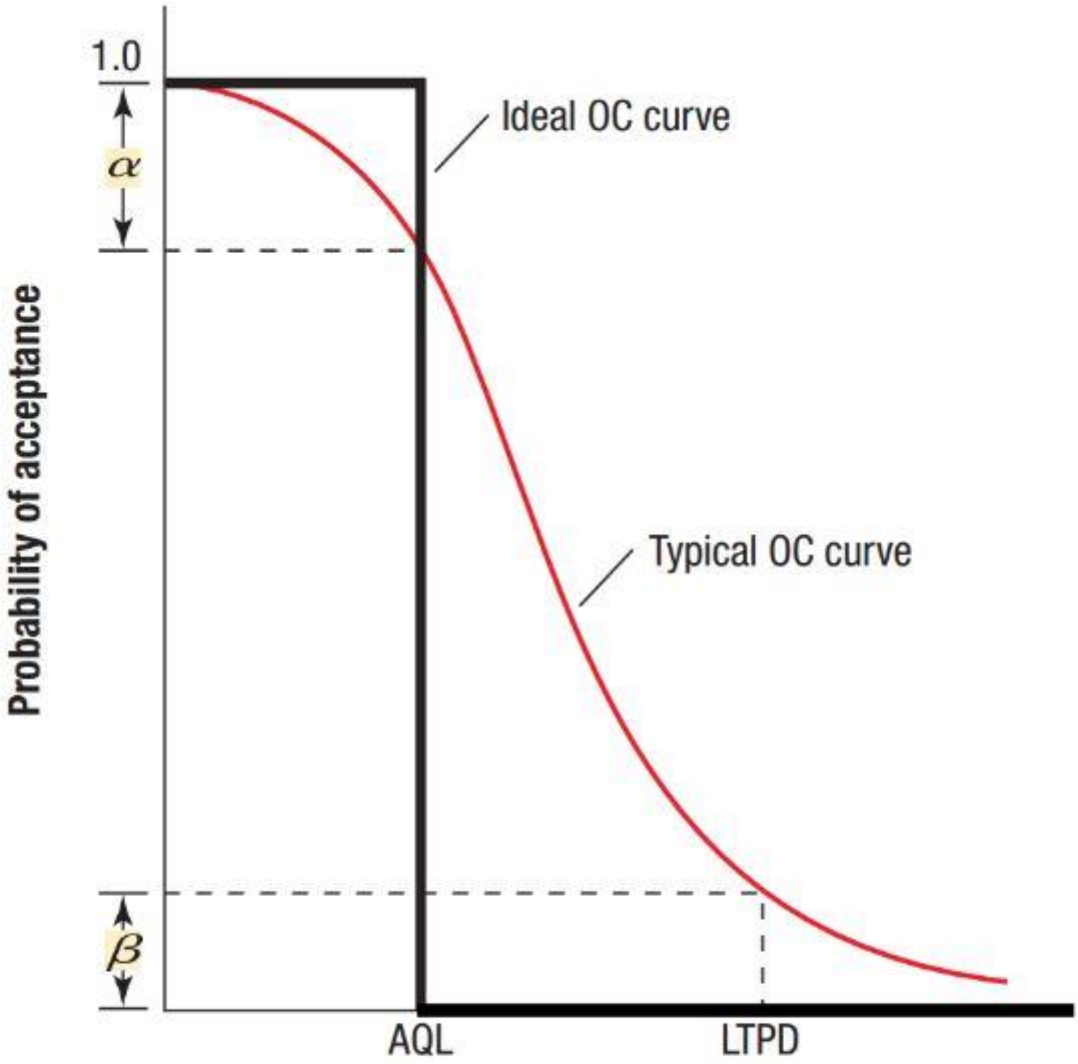
Acceptance Quality Level (AQL): A lot with relatively small fraction defective say p_1 that the consumer do not wish to reject more often than a small proportion of times is referred to as “Good” quality lot, i.e. (Rejecting a lot of quality p_1) = $\alpha \Rightarrow$ (Accepting a lot of quality p_1) = $1 - \alpha$, where α a very small positive number then p is termed as acceptance quality.

Lot Tolerance Proportion Defective (LTPD): The lot tolerance proportion defective usually denoted as p_t, is the lot quality which is considered to be bad by the consumer. The consumer is not willing to accept a lot with proportion of defectives p_t or greater.

It is usually of importance to know the probability of accepting a lot of a specific incoming quality p. It can be approximated using Poisson distribution. Suppose c is the threshold on no. of defectives, and p is the incoming quality then:

$$P_a \coloneqq P(Accepting\ a\ lot\ of\ quality\ of\ quality\ p) = \sum_{x=0}^{c} \frac{(np)^x e^{-np}}{x!}$$

The behaviour of P_a with the values of p is termed as Operating Characteristic (OC) of the sampling plan which describes how well a sampling plan discriminates between good and bad lots. Undoubtedly, every manager wants a plan that accepts lots with a quality level better than the AQL 100 percent of the time and accepts lots with a quality level worse than the AQL 0 % of the time. This ideal OC curve for a single-sampling plan is plotted in black. However, such performance can be achieved only with 100 percent inspection. A typical OC curve for a single-sampling plan, plotted in red, shows the probability α of rejecting a good lot (producer’s risk) and the probability β of accepting a bad lot (consumer’s risk). Consequently, managers are left with choosing a sample size n and an acceptance number to achieve the level of performance specified by the AQL, α, LTPD, and β.



Average Outgoing Quality (AOQ): Sometimes the consumer is guaranteed a certain quality level after inspection regardless of what quality level is being maintained by the producer. Let the producer's fraction defective, i.e. lot quality before inspection be p. This is termed as incoming quality. The fraction defective after inspection is known as outgoing quality of the lot. The expected fraction defective remaining in the lot after application of sampling plan is termed as Average Outgoing Quality.

For single sampling plan under corrective sampling AOQ is given by the following formula:

$$AOQ = \frac{p(N-n)P_a}{N}$$

where N is the size of lot, n is the size of sample and P_a is the probability of acceptance of lot with incoming quality p. It is derived as follows:

1. The outgoing quality of the lot when it is accepted is given by the proportion of defectives in the uninspected units $(N - n)$ as the incoming quality is p, the number of defectives in uninspected part of the lot will be $(N - n)$. Hence the fraction defectives is $(N-n)$/N. This can happen with probability P_a.
2. If the lot is rejected then under corrective sampling we go for 100% inspection and then replace all the defectives with good ones and hence there are no defectives, i.e. outgoing quality is zero. This happens with probability $1 - P_a$.

Hence we have,

$$AOQ = \frac{p(N-n)P_a}{N} + 0(1-P_a)$$

The graphical representation of AOQ against p is known as AOQ Curve and the maximum value of AOQ with respect to p is known as AOQ Limit (AOQL).

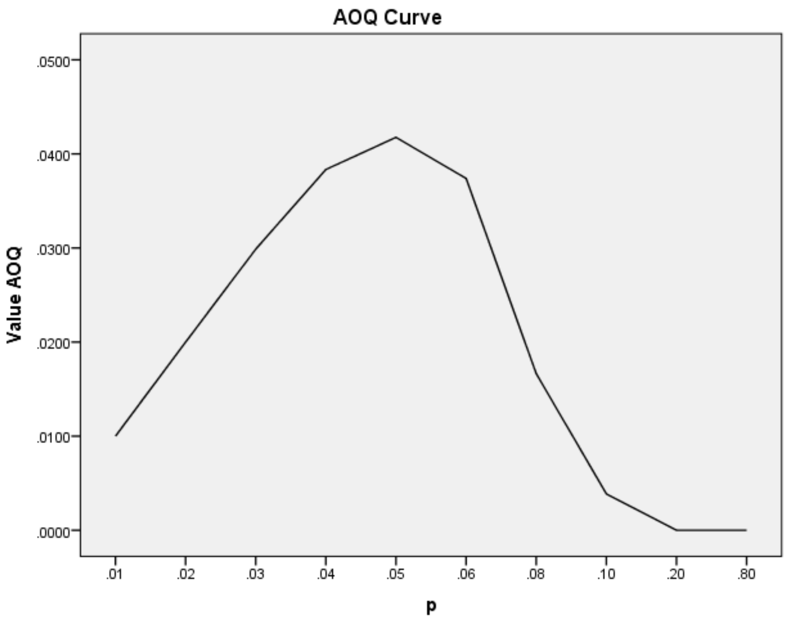
$$AOQL \ = \max_{p} [AOQ]$$

Average Sample Number (ASN): Average Sample Number is the expected no. of units examined in a sampling plan.

Clearly in a single sampling plan with corrective sampling one examines n units with probability 1 and $(N - n)$ units with probability $(1 - P_a)$. Hence ASN is $n + (N - n)(1 - P_a)$.

Case 8: From a lot consisting of 2200 items, a random sample of 225 items is drawn. If the criteria for acceptance of the lot is to have 14 or less defectives. Under a single sampling plan with corrective sampling plot AOQ, ASN & OC Curves, also obtain AOQL.

p	P(a)	AOQ	ASN
0.01	1	0.01	225
0.02	0.999926	0.02	225.15
0.03	0.995848	0.0299	233.2
0.04	0.958534	0.0383	306.9
0.05	0.835244	0.0418	550.39
0.06	0.623271	0.0374	969.04
0.08	0.208077	0.0166	1789.05
0.1	0.038602	0.0039	2123.76
0.2	0	0	2200
0.8	0	0	2200



Conclusion

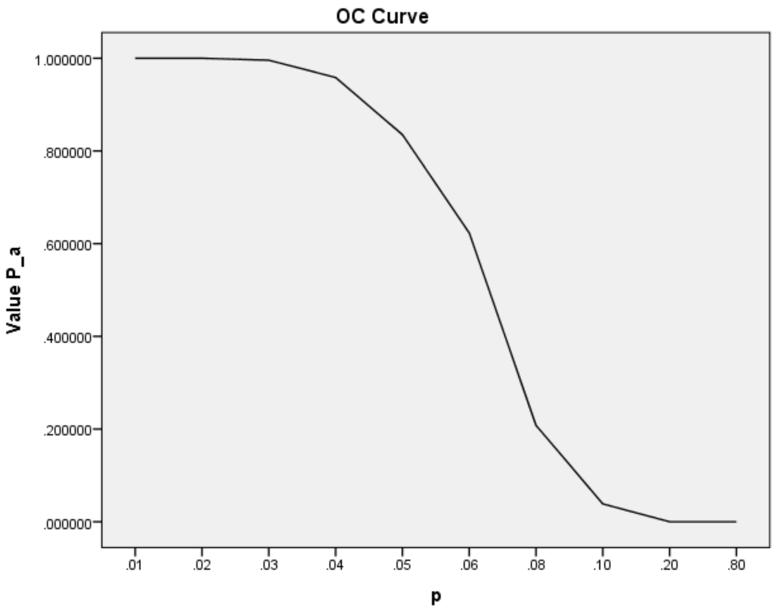
Curve on the left side is AOQ curve.

From the curve, the maximum value of AOQ curve is approximately 0.041.

Descriptive Statistics

	N	Maximum
AOQ	10	.0418
Valid N (listwise)	10	

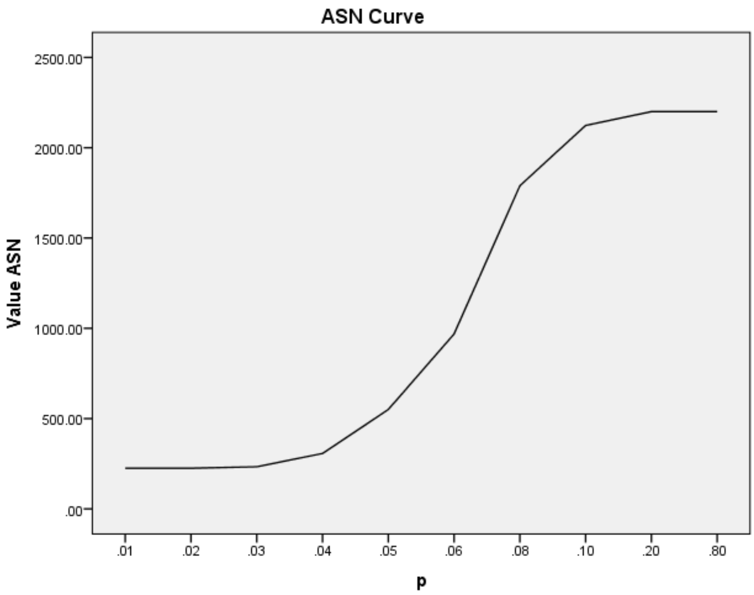
Conclusion: AOQL is maximum values of AOQ curve, hence AOQL=0.0418.



Conclusion

Curve on the left side is OC curve.

We see, as the proportion of defectives increases for a lot, probability of accepting that lot decreases.



Conclusion

Curve on the left side is ASN curve.

We see, as the proportion of defectives increases for a lot, sample size required for inspection increases.