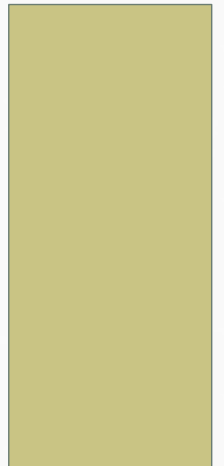


# BEST-FIRST SEARCH AND ALGORITHM A\*

JEFFREY L. POPYACK



# GRAPHSEARCH REVIEW

Key statement in **Graphsearch**:

**Insert(s', OPEN):**

- Insert at front: ***Depth-First Search***
- Insert at rear: ***Breadth-First Search***

# GRAPHSEARCH WITH HEURISTICS

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**Insert(s', OPEN):** use heuristics to guide

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  - **heuristic = constant**  
(just put at front, since all have same value)
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- Insert where it probably belongs: **Best-First Search**

# GRAPHSEARCH WITH HEURISTICS

Key statement in **Graphsearch**:

**Insert(s', OPEN):** use heuristics to guide

- Insert where it *probably belongs*: **Best-First Search**
- In general, don't know for sure.
- Use a good heuristic: (low: good; high: bad)
- **OPEN** is ordered from low to high

# ALGORITHM A

## IDEA: ("Algorithm A")

Let **f(n)** be an *estimate* of the TOTAL COST (path-length, distance) from **start** to a **goal** if you go through node **n**.

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## EXAMPLE:

**start** = Drexel

**goal** = Disney World

current location =  
Charlottesville, VA

**Total Distance if  
going through C'ville:**

$\text{dist}(\text{Drexel}, \text{C'ville}) +$   
 $\text{dist}(\text{C'ville}, \text{DisneyWorld})$

**= 250 + 800 = 1050**





# ALGORITHM A

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Let  $f(n)$  be an *estimate* of the TOTAL COST (path-length, distance) from **start** to a **goal** if you go through node  $n$ .

$f(n)$  = Total Distance  
if going through  $n$ :

$f(\text{C'ville}) =$

$$250 + 800 = 1050$$

$f(\text{Kansas City}) =$

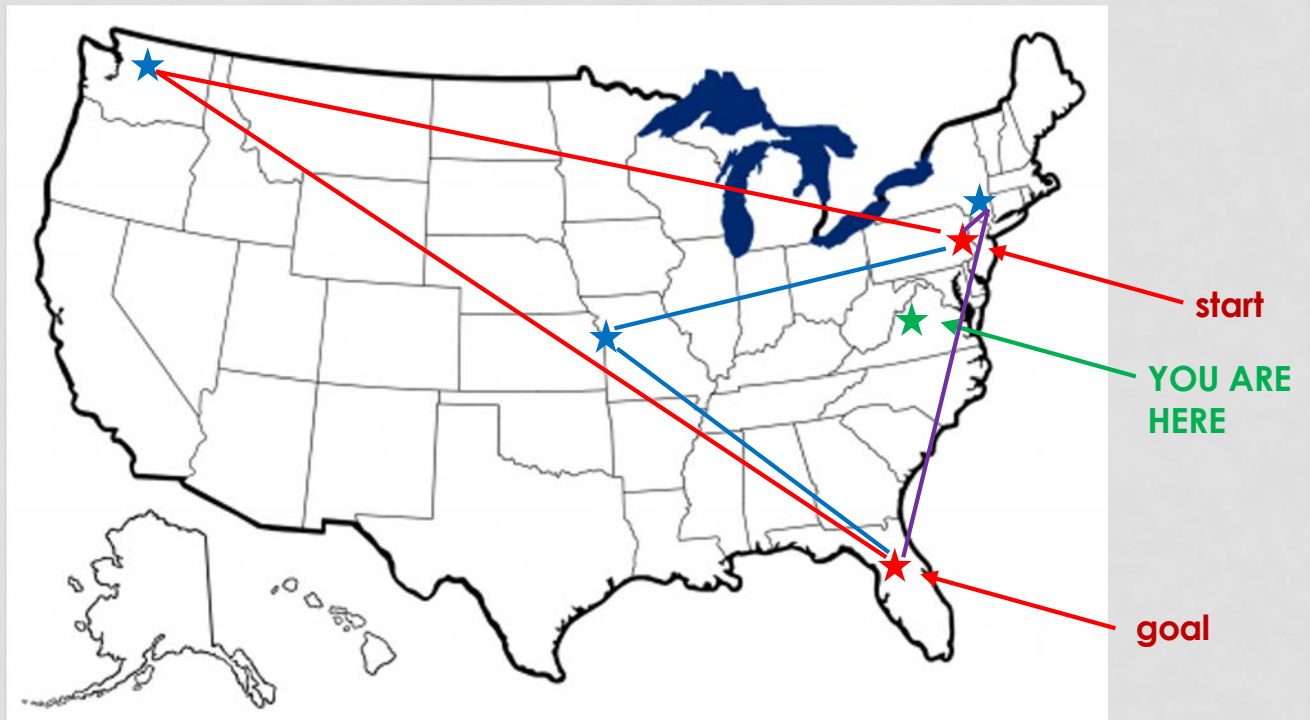
$$1125 + 1250 = 2375$$

$f(\text{New York City}) =$

$$95 + 1100 = 1195$$

$f(\text{Seattle}) =$

$$2800 + 3100 = 5900$$



$f(\text{Seattle}) > f(\text{Kansas City}) > f(\text{New York City}) > f(\text{C'ville})$

# ALGORITHM A

## Algorithm A:

Let **f(n)** be an *estimate* of the TOTAL COST (path-length, distance) from **start** to a **goal** if you go through node **n**.

$$\mathbf{f(n) = depth(n) + h(n)}$$

where **h(n)** is an *estimate* of distance from **n** to a goal

and **depth(n)** is the *actual (known) distance* traveled so far, from **start** to **n**.

# ALGORITHM A\*

## Algorithm A\*: ("A-Star")

Let  $f(n)$  be an *estimate* of the TOTAL COST

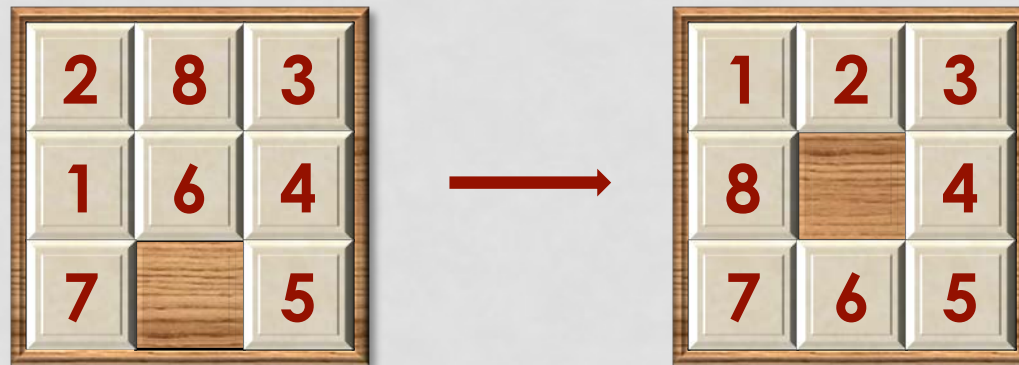
$f(n) = \text{depth}(n) + h(n)$ , as in **Algorithm A**,  
*but with a guarantee that  $h(n)$  does not ever over-estimate the distance from  $n$  to a goal.*

***Use of Algorithm A\* is guaranteed to find a shortest-path solution!***

# EXAMPLE: 8-PUZZLE

Recall: for the 8-Puzzle, the rules are:

- actions: move the *empty spot* UP, DOWN, LEFT or RIGHT
- preconditions: the destination spot is on the board.



For Best-First Search, we will use **Algorithm A**, with

$$f(n) = \text{depth}(n) + h(n),$$

and the heuristic

**$h(n)$  = number of tiles out of place** (not including empty cell).

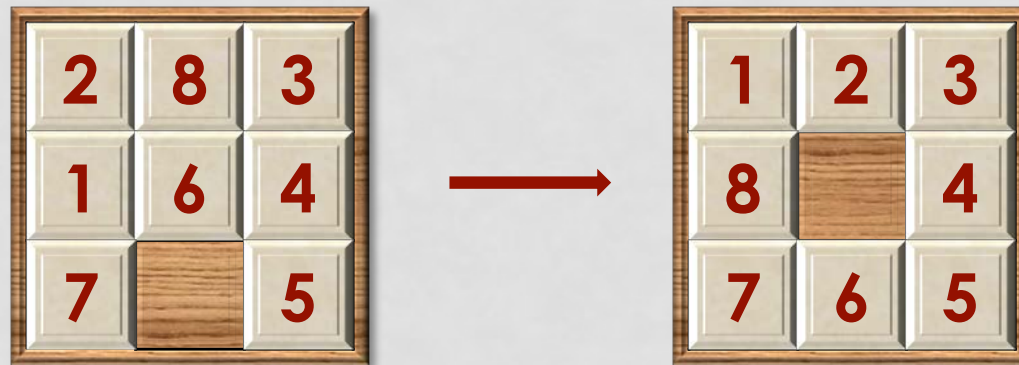
# EXAMPLE: 8-PUZZLE

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and the heuristic

**$h(n)$  = number of tiles out of place** (not including empty cell).



**$h(\text{start}) = 4$**  (Since 1,2,6,8 are out of position, others OK).

It will take at least 4 moves to get all tiles in position (goal!) because on a single move, at most 1 tile can move into position. In short,  **$h()$**  never overestimates the distance to the goal.

This is an **admissible heuristic** for **Algorithm A\***

OPEN: { **A** }  
CLOSED: { }

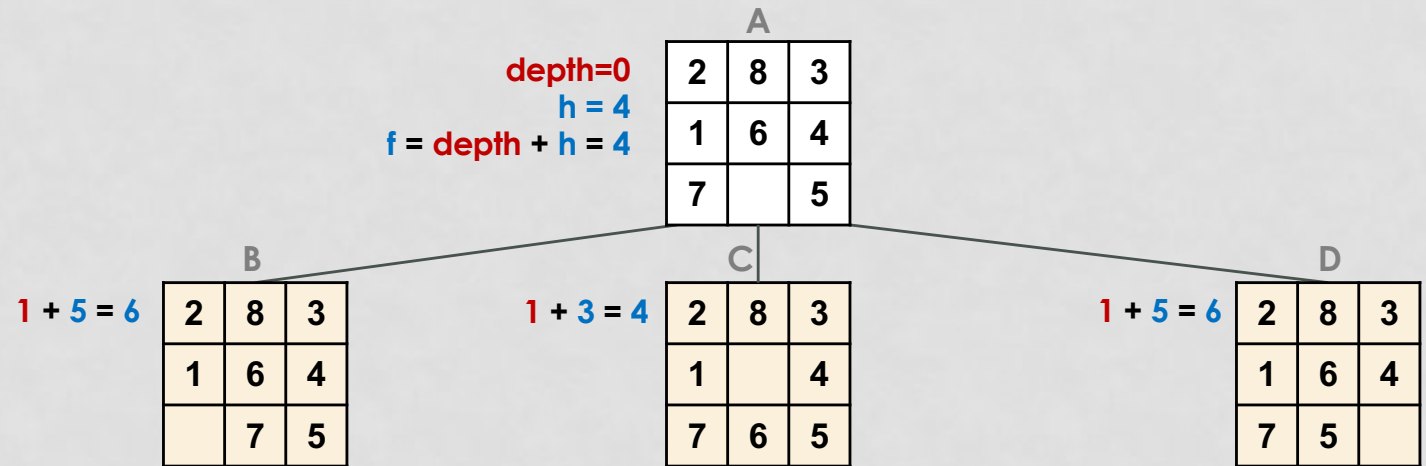
$$\begin{aligned} \text{depth} &= 0 \\ h &= 4 \\ f &= \text{depth} + h = 4 \end{aligned}$$

A

2	8	3
1	6	4
7		5

OPEN: { A }  
CLOSED: { }

OPEN: { C B D }  
CLOSED: { A }

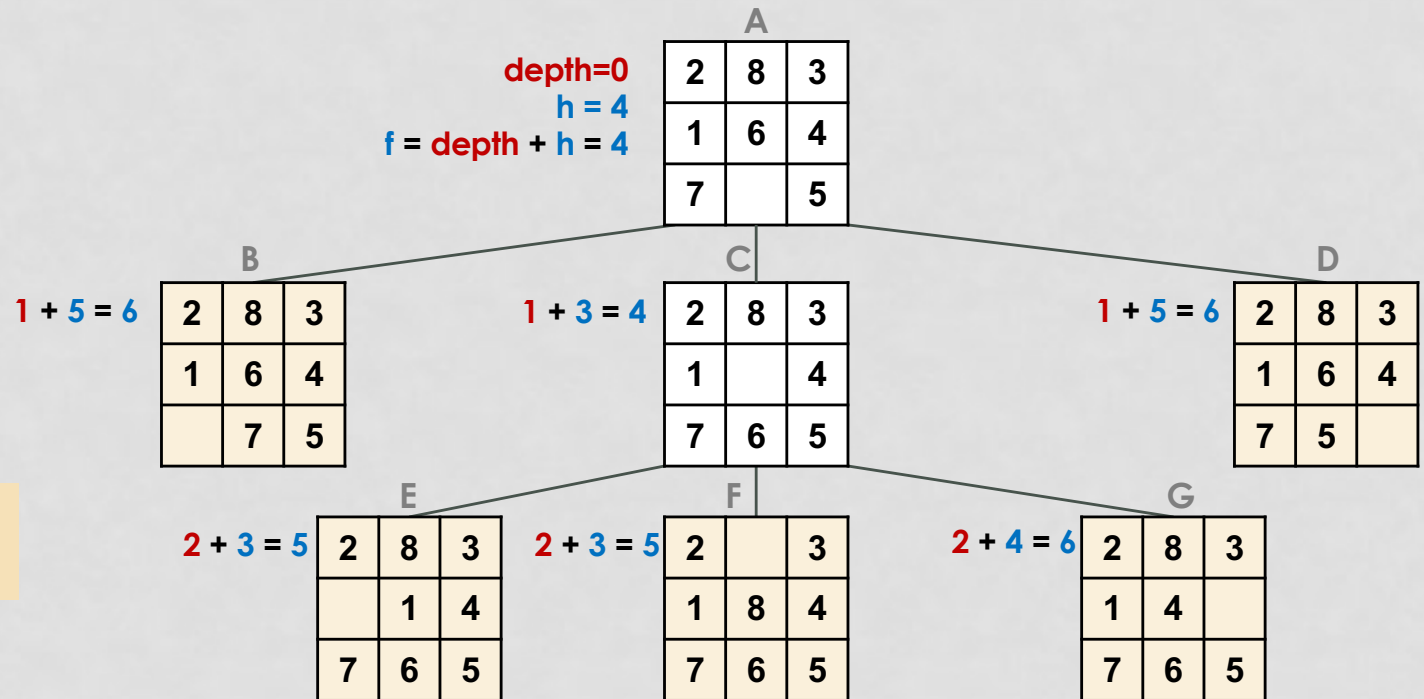




OPEN: { A }  
CLOSED: { }

OPEN: { C B D }  
CLOSED: { A }

OPEN: { E F G B D }  
CLOSED: { A C }



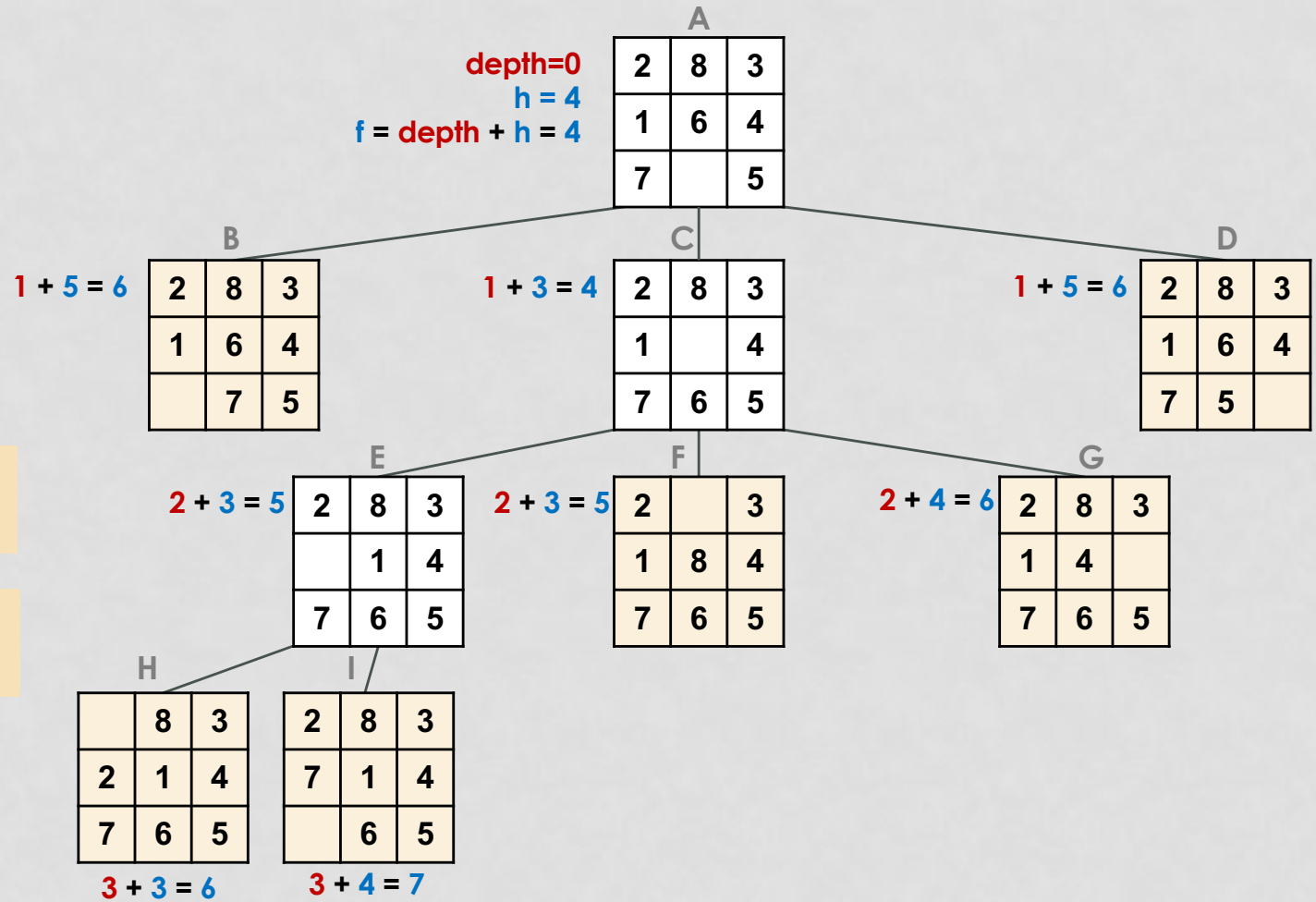


OPEN: { A }  
CLOSED: { }

OPEN: { C B D }  
CLOSED: { A }

OPEN: { E F G B D }  
CLOSED: { A C }

OPEN: { F H G B D I }  
CLOSED: { A C E }



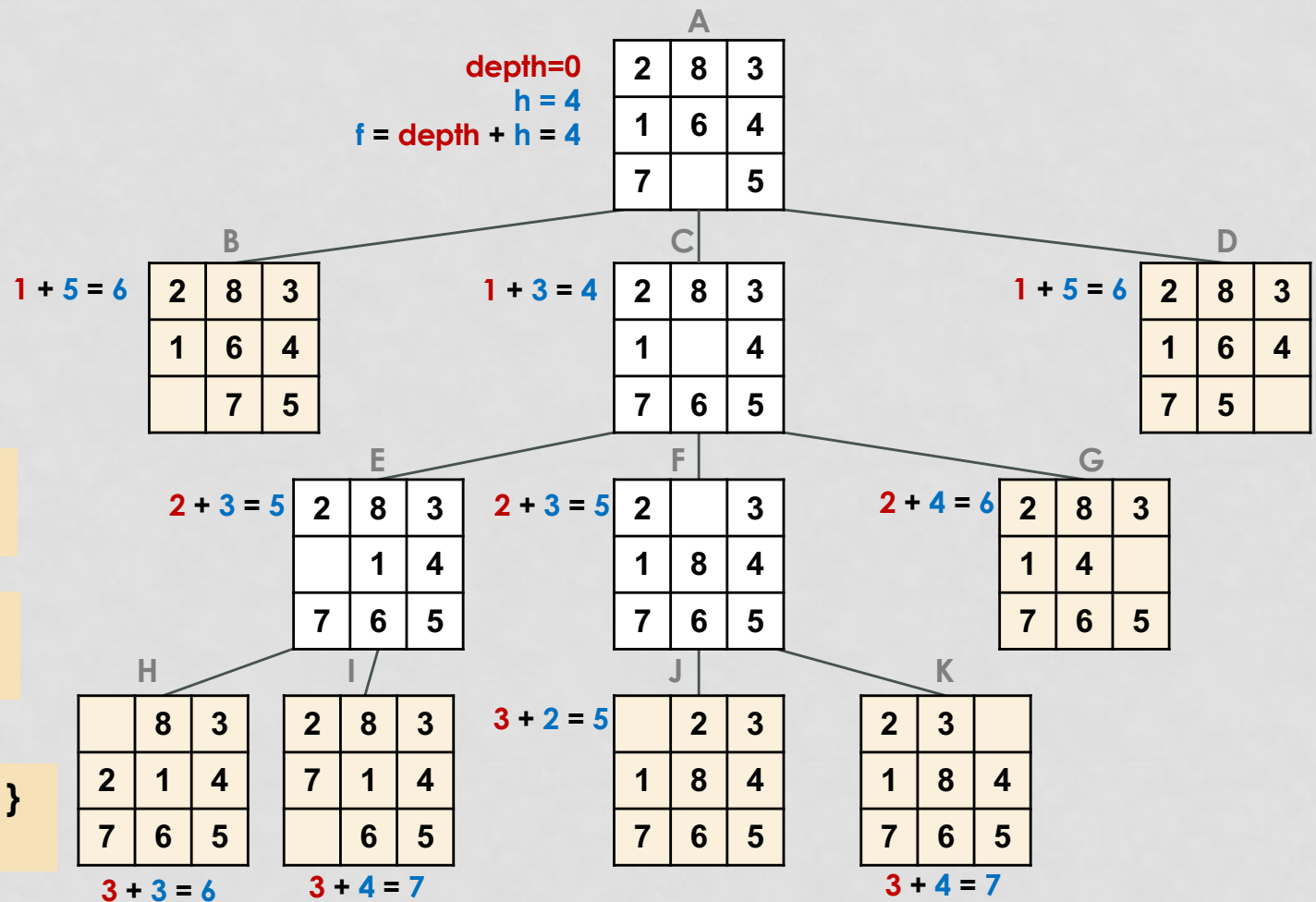
OPEN: { A }  
CLOSED: { }

OPEN: { C B D }  
CLOSED: { A }

OPEN: { E F G B D }  
CLOSED: { A C }

OPEN: { F H G B D I }  
CLOSED: { A C E }

OPEN: { J H G B D K I }  
CLOSED: { A C E F }



OPEN: { A }  
CLOSED: { }

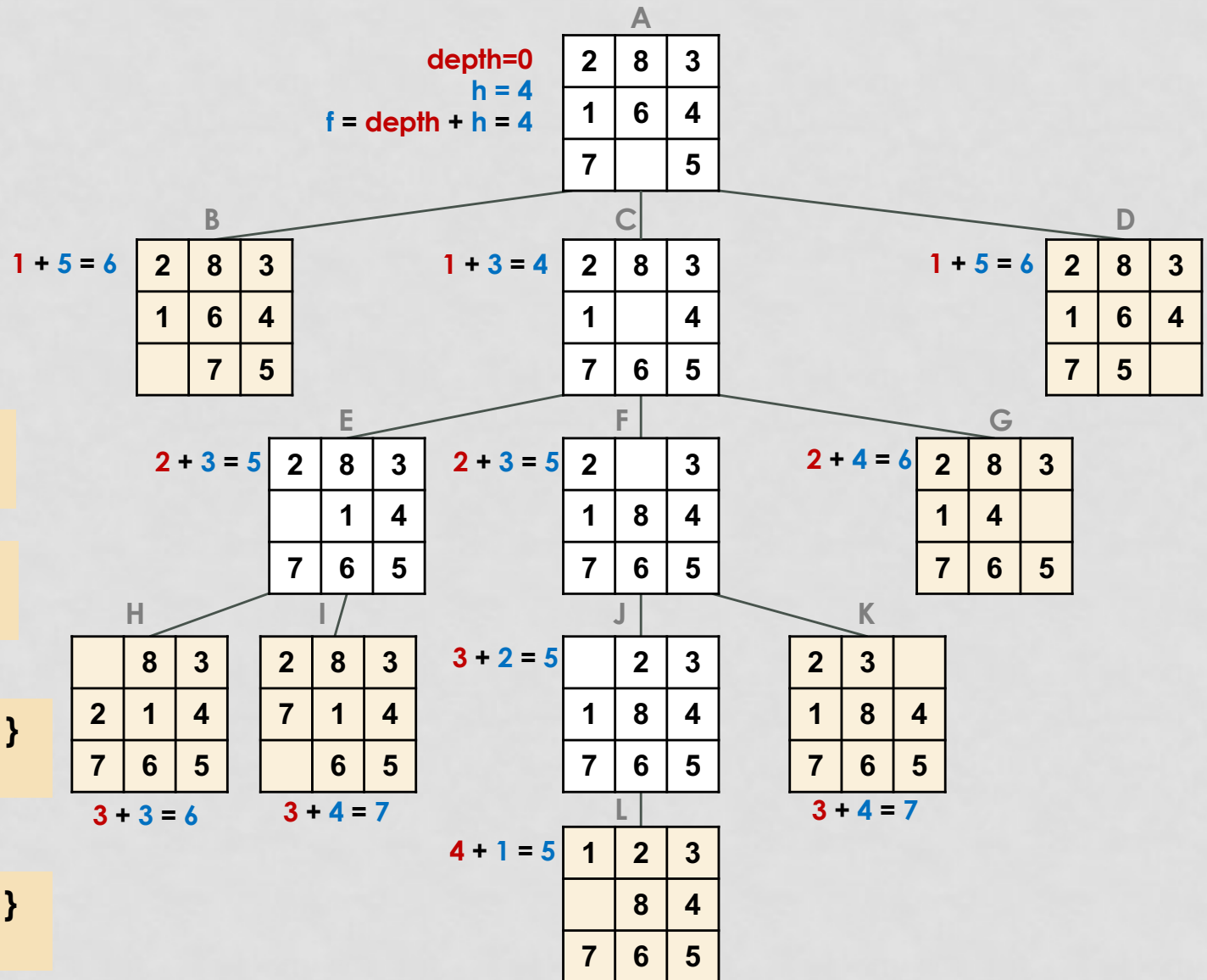
OPEN: { C B D }  
CLOSED: { A }

OPEN: { E F G B D }  
CLOSED: { A C }

OPEN: { F H G B D I }  
CLOSED: { A C E }

OPEN: { J H G B D K I }  
CLOSED: { A C E F }

OPEN: { **L** H G B D K I }  
CLOSED: { A C E F **J** }



OPEN: { A }  
CLOSED: { }

OPEN: { C B D }  
CLOSED: { A }

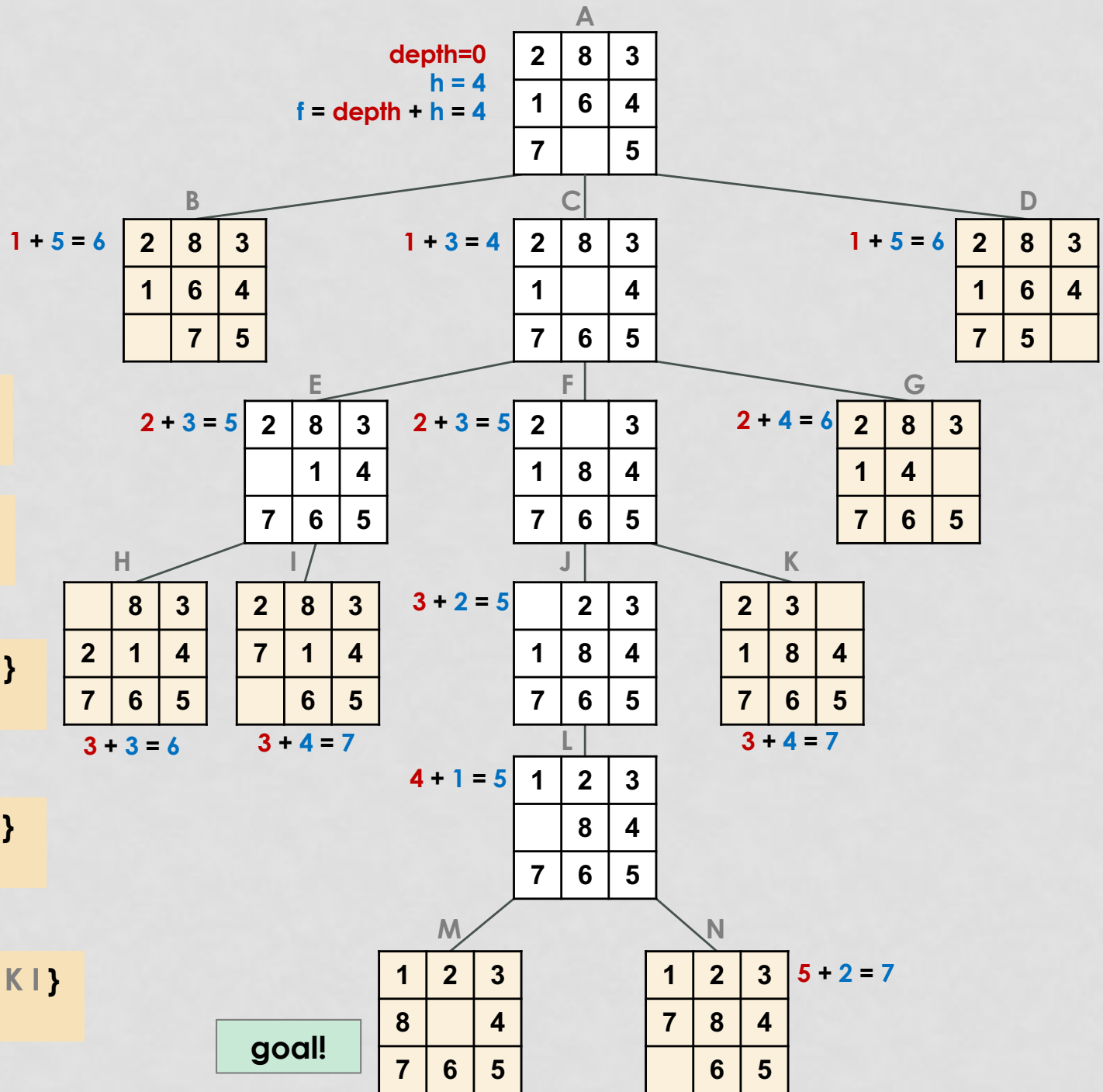
OPEN: { E F G B D }  
CLOSED: { A C }

OPEN: { F H G B D I }  
CLOSED: { A C E }

OPEN: { J H G B D K I }  
CLOSED: { A C E F }

OPEN: { L H G B D K I }  
CLOSED: { A C E F J }

OPEN: { M H G B D N K I }  
CLOSED: { A C E F J L }



# ALGORITHM A\*

## Best-First Search, using Algorithm A\*

- finds a shortest-path solution to a goal
- requires fewer node expansions than breadth-first search (an admissible heuristic guarantees you won't go deeper than a goal node, so you can't expand more nodes than breadth-first search.)
- requires fewer node expansions than depth-first search (unless you were lucky – a good heuristic will let you choose properly which nodes to examine next, rather than rely on luck.)
- should have improved performance with an improved heuristic.