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BT–4 / M-19

MATHEMATICS–III

Paper–AS – 201N

Time allowed : 3 hours]   *[Maximum marks : 75*

Note :- *Attempt all questions.*

Unit–I

1. (a) Expand $f(x) = x \sin x$ as a Fourier Series in the interval $0 < x < 2\pi$.   8

   (b) Obtain Fourier series for the function   7

   $f(x) = \pi x$ for $0 \leq x \leq 1$
   $= \pi (2-x)$ for $1 \leq x \leq 2$

2. (a) Find the Fourier Sine transform of   7

   $$f(x) = \begin{cases} x, & \text{for} \quad 0 < x < 1 \\ 2-x, & \text{for} \quad 1 < x < 2 \\ 0, & \text{for} \quad x > 2 \end{cases}$$

   (b) State and prove the Convolution theorem for Fourier Transforms.   8

Unit–II

3. (a) Solve the partial differential equation $pxy + pq + qy = yz$ by using Charpit's method.   8

   (b) Solve the partial differential equation   7
   $(D^2 + DD' - 6D'^2) z = y \cos x$

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4. Using Simplex method, solve the following Linear Programming Problem: 15

   Maximize $Z = 5x_1 + 3x_2$

   Subject to Constraints

   $$x_1 + x_2 \leq 2$$

   $$5x_1 + 2x_2 \leq 10$$

   $$3x_1 + 8x_2 \leq 12$$

   $$x_1, x_2 \geq 0$$

Unit–III

5. (a) If $u = \log \tan \left(\dfrac{\pi}{4} + \dfrac{\theta}{2} \right)$, then prove that: 8

   (i) $\tanh \dfrac{u}{2} = \tan \dfrac{\theta}{2}$

   (ii) $\cosh u = \sec \theta$.

   (b) Separate $\tan^{-1} (x + iy)$ in to real and imaginary parts. 7

6. (a) Show that the function $f(z) = \sqrt{|xy|}$, is not analytic at the origin, even though Cauchy - Riemann equations are satisfied thereof. 8

   (b) Evaluate, using Cauchy's integral formula. 7

   (i) $\displaystyle \int_c \dfrac{\sin^2 z}{\left(z - \dfrac{\pi}{6} \right)^3} dz$, where c is the circle $|z| = 1$

(ii) $\int_c \dfrac{e^{2z}}{(z+1)^4} dz$, where c is the circle $|z| = 3$

Unit–IV

7. (a) Three machines M_1, M_2 and M_3 produce identical items of their respective output 5%, 4% and 3% of items are faulty. On a certain day M_1, M_2 and M_3 produced 25%, 30 % and 45% of the total output. An item selected at random is found to be faulty; what are the chances that it was produced by the machine M_3.   8

(b) Find the moment generating function of the exponential distribution   7

$$f(x) = \frac{1}{c} e^{-x/c}, \ 0 \le x \le \infty, \ c > 0$$

Hence find its mean and standard deviation.

8. (a) The mean of a binomial distribution is 6 and variance 4, Calculate n, p and q.   7

(b) Fit a Poisson distribution to the following: 8

x :	0	1	2	3	4
y :	46	38	22	9	1

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