Sequential Decision Making in Risk Management:

A Multi-Timestep Model for Probabilistic Risk Analysis of Managing Space Surveillance Systems

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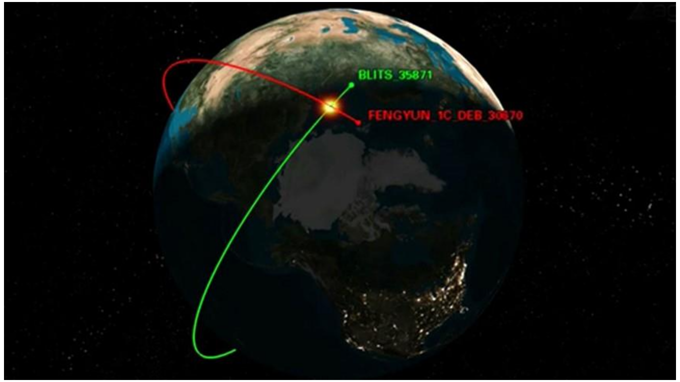
Bottom Line Up Front

- Space is a heavily-utilized environment
- Satellites have a risk of unintended collisions
- Space surveillance systems provide signals of potential collisions
- We use probabilistic risk analysis to quantify the risk reduction benefit of surveillance systems of various configurations
- Then, we use partially-observable Markov decision processes to compute an optimal policy for future sensor deployments

Source: Space.com

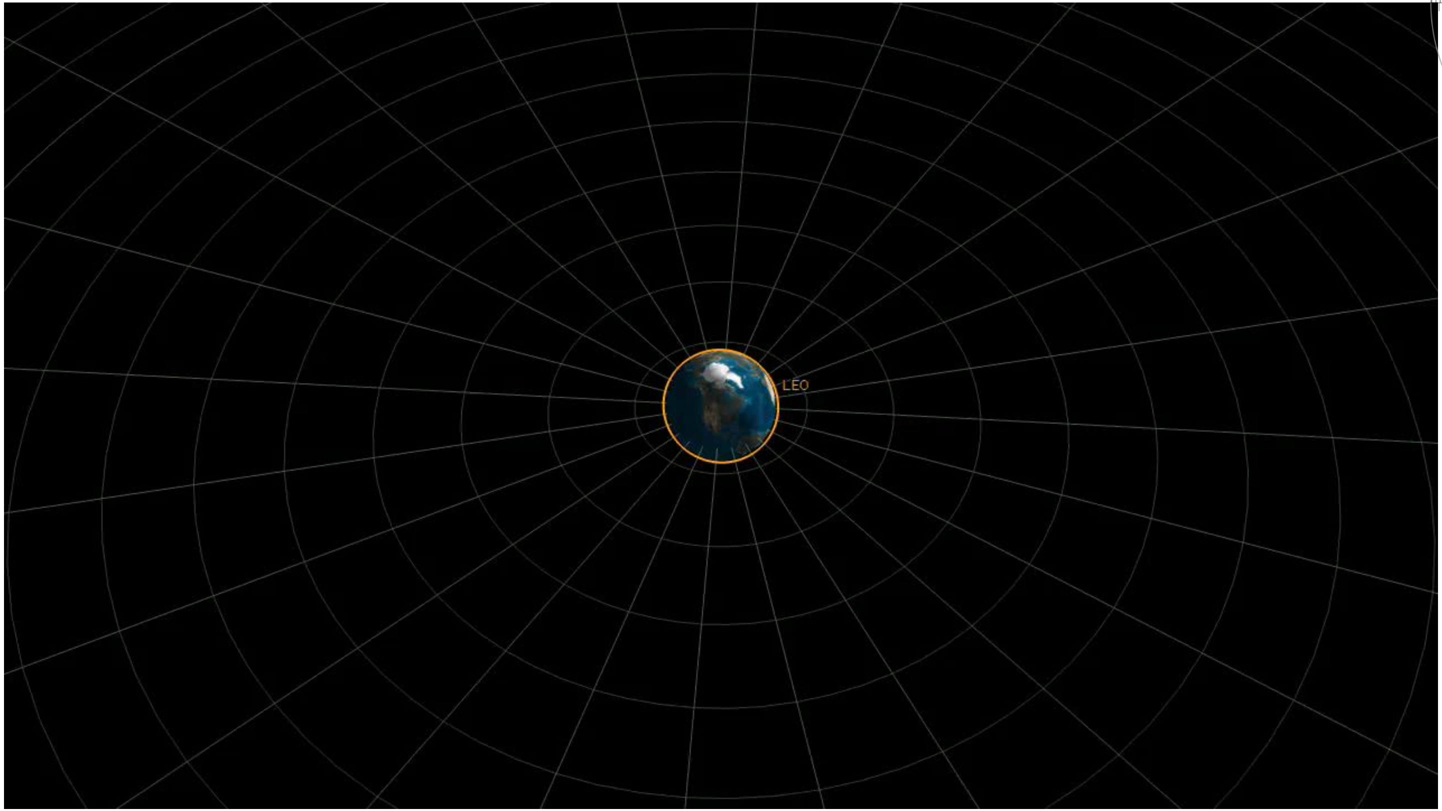


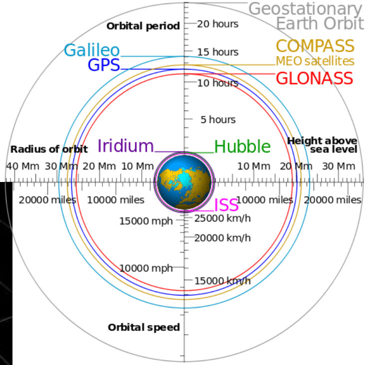
Background on Space, Value of Information, and Dynamic Programming

Why Do We Need Space?

- Precision navigation and timing
- Weather and climate monitoring
- Financial transactions
- Communications
  - Voice
  - Data
  - Entertainment
- Precision guided munitions
- Secure military communications
- Surveillance
- Missile defense
- Nuclear readiness
- Satellite industry revenue in 2013: $195.2B

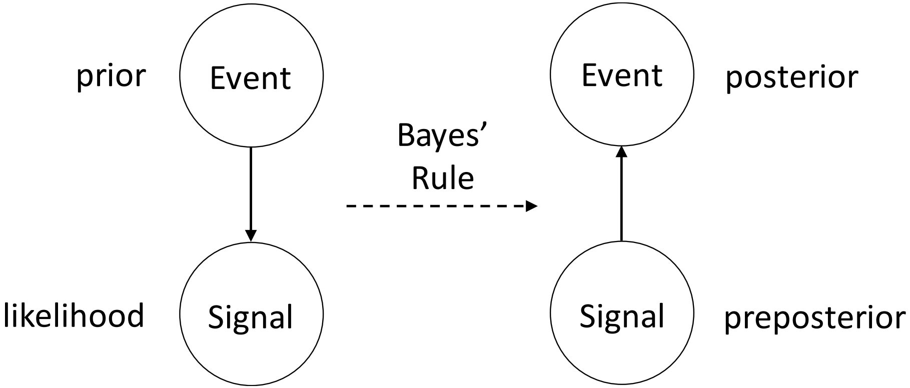
Space Environment: Orbit Visualization



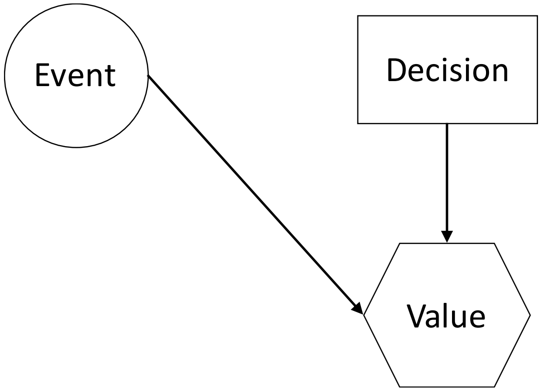


Sources: Guessproject, Cmglee

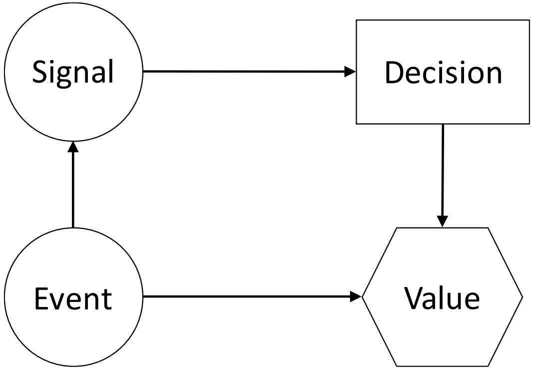
Value of Information (VOI)



Using Bayes' Rule to transform from the assessed form of a test to its equivalent inferential form

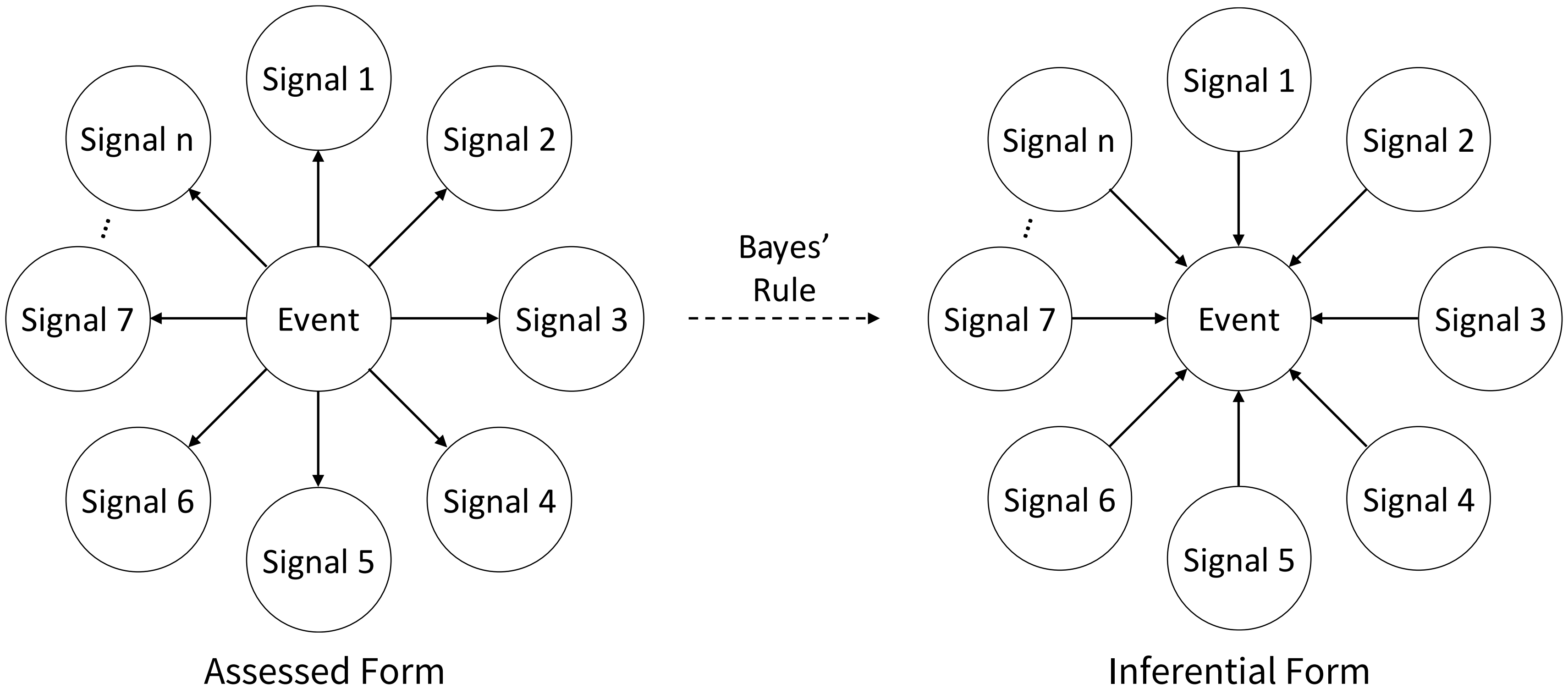


A simple decision with one uncertain event



Revised decision, given the use of new information

VOI with Multiple Detectors



Dynamic Programming

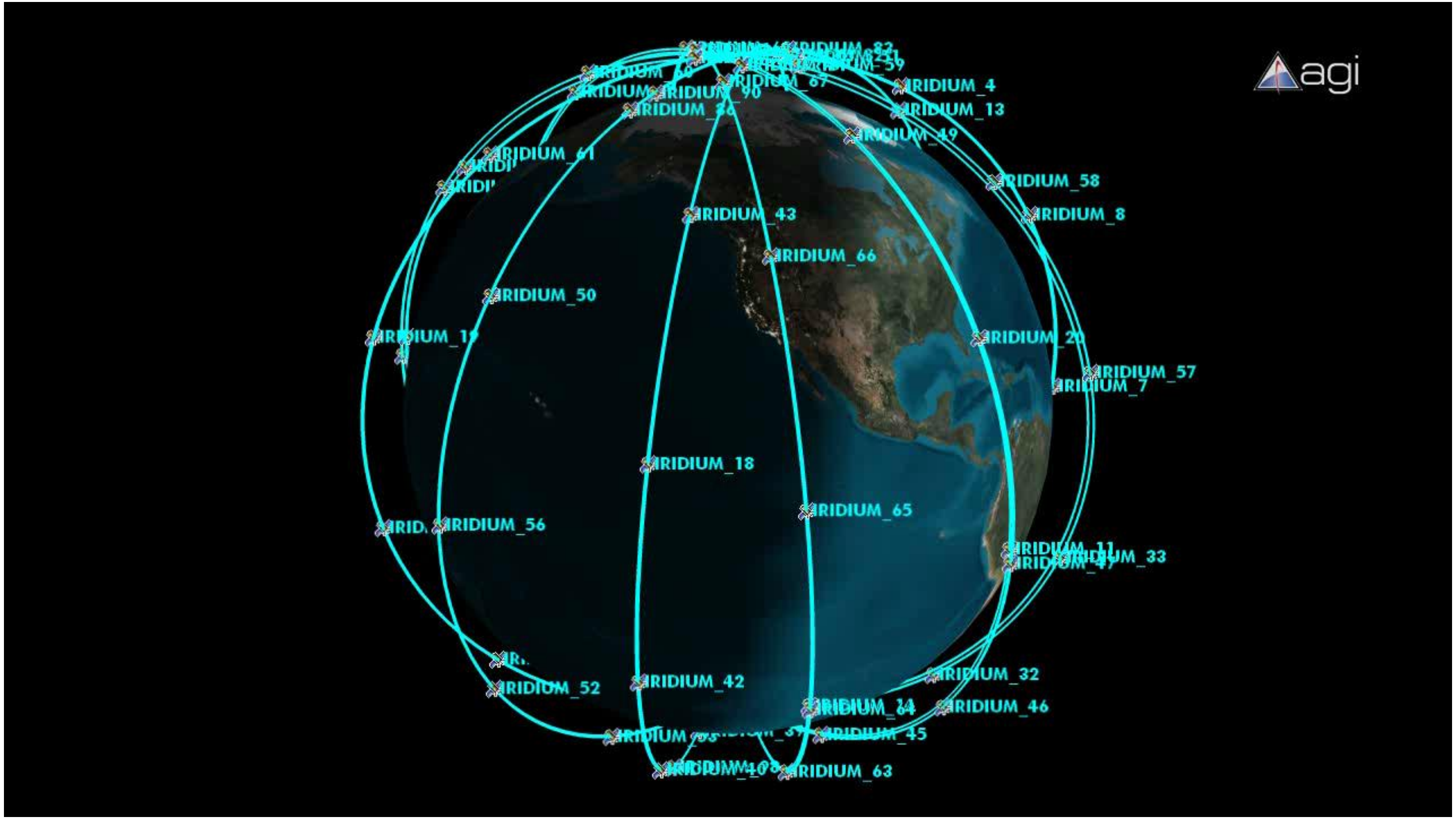
- Dynamic programming is a general method of characterizing stochastic processes
  - Typically, discrete state space, finite time horizon problems
    - Computational tractability
- Markov decision processes (MDPs)
  - Dynamic programs where stochastics obey the Markov Property
  - Markov Property: If we want the distribution of the next state given the current state, no additional information is gained if you are told the entire past history of the process.
  - Let $X_0, X_1, \ldots$ be a sequence of states. Then:

$$P(X_{t+1} = x_{t+1} | X_t = x_t) = P(X_{t+1} = x_{t+1} | X_0 = x_0, \ldots, X_t = x_t)$$

- Partially observable MDPs (POMDPs)
  - MDPs where the current state and/or state transition is not known

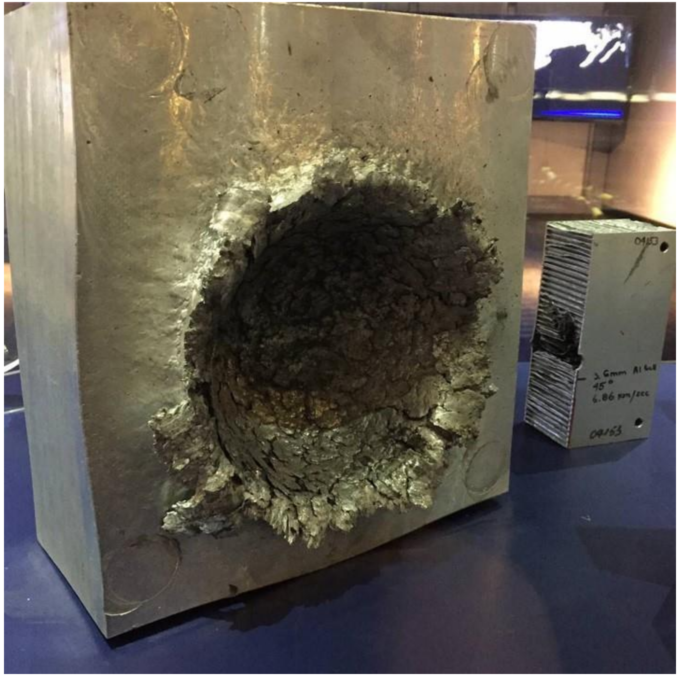
Unintentional Space Collisions and Space Surveillance

Motivating Threat – 2009 Iridium Cosmos Conjunction

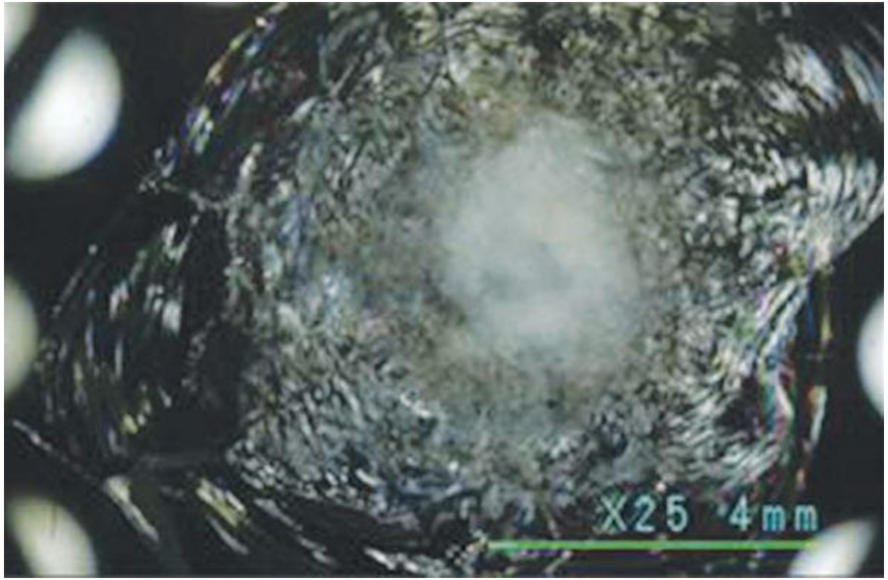


Source: AGI

Space Collision Risks: The Impacts



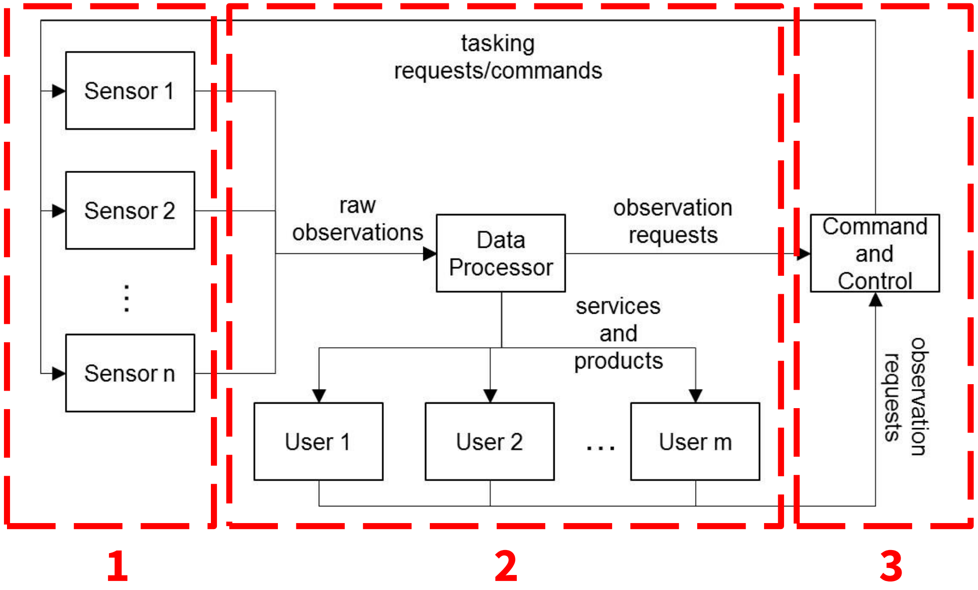
Impact crater from a 12mm plastic projectile, fired onto an aluminum plate at 15,000 mph



Example of window damage to the Space Shuttle Endeavour from debris particle impact

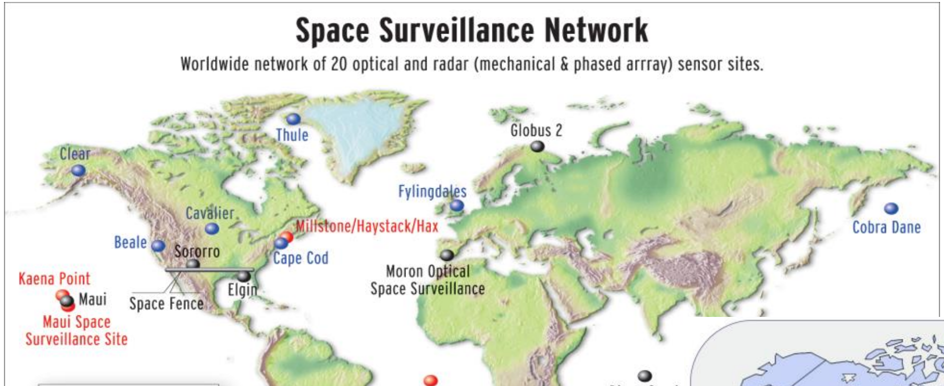
Source: NASA

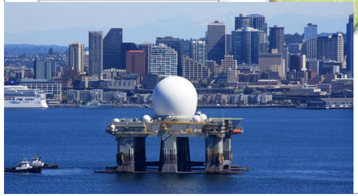
Generic Space Surveillance Network



- Subsystem 1: subsystem of sensors
  - Collects raw metric observations of objects of space
- Subsystem 2: data processor
  - Data products include: catalog updates, conjunction alerts, space weather
- Subsystem 3: command and control mechanism

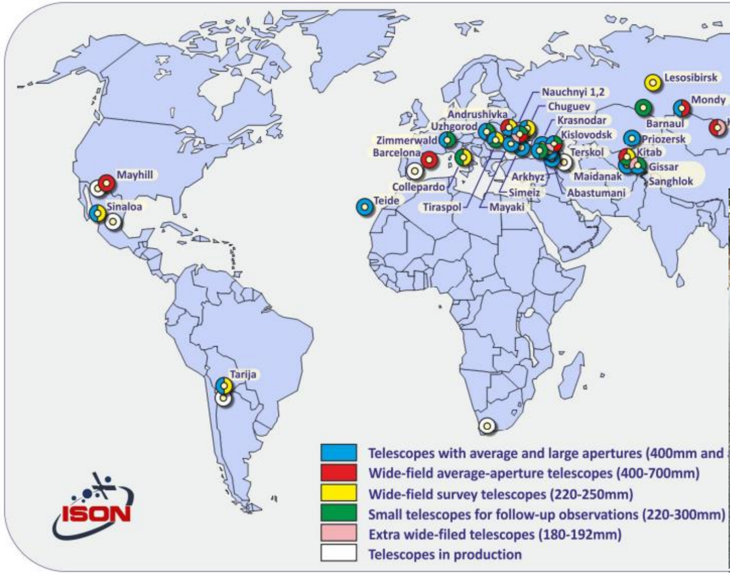
Space Surveillance Networks





United States Space Surveillance Network (USSSN)

International Scientific Optical Network (ISON)



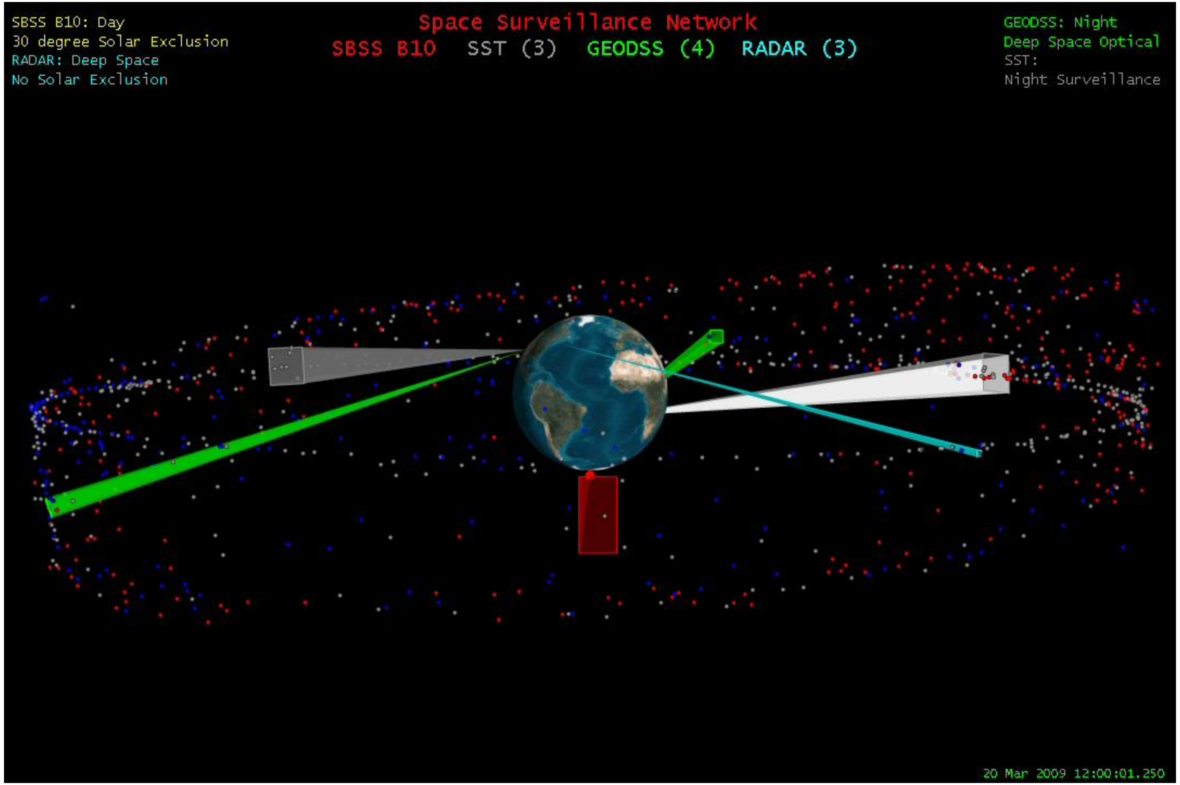
Sources: SpaceNews, ISON

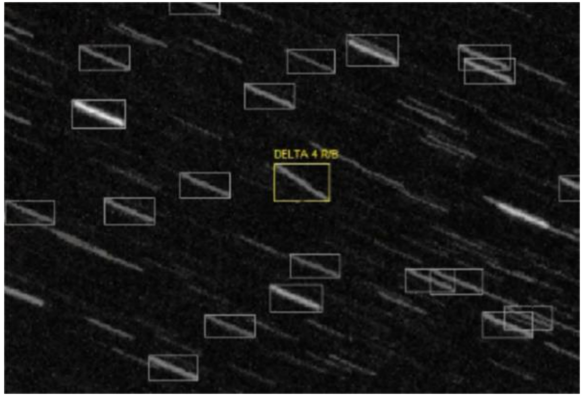
USAF SSA Watchfloors

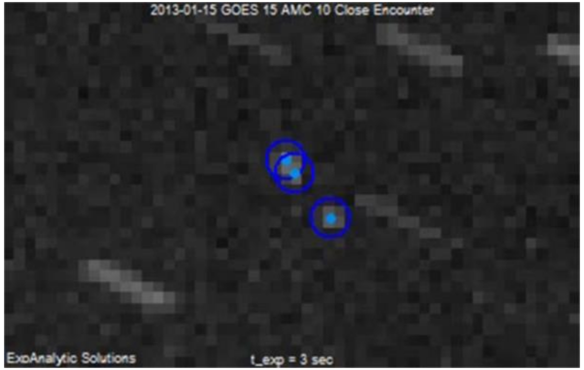
Applied Minds Concept for a Futuristic
Space Operations Center

Joint Space Operations Center
614 Air Operations Center

Space Surveillance Network Data Processing Chain

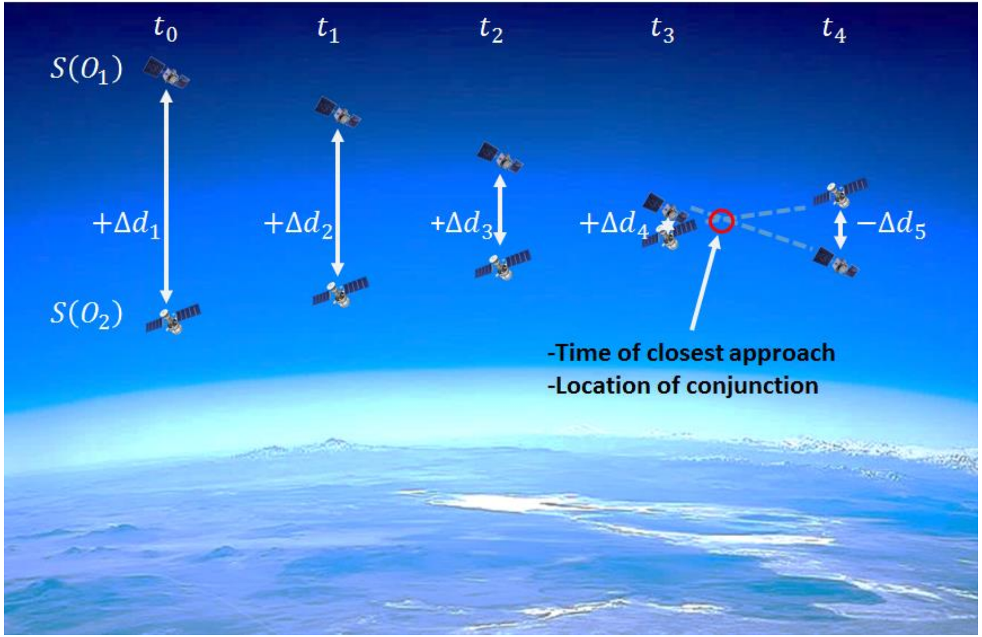




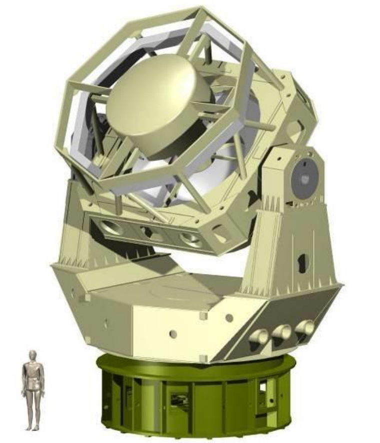


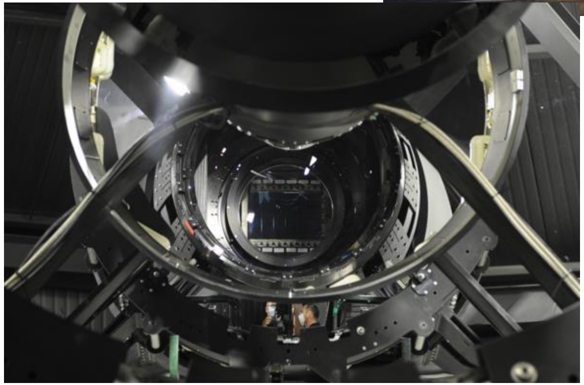
Conjunction Assessment

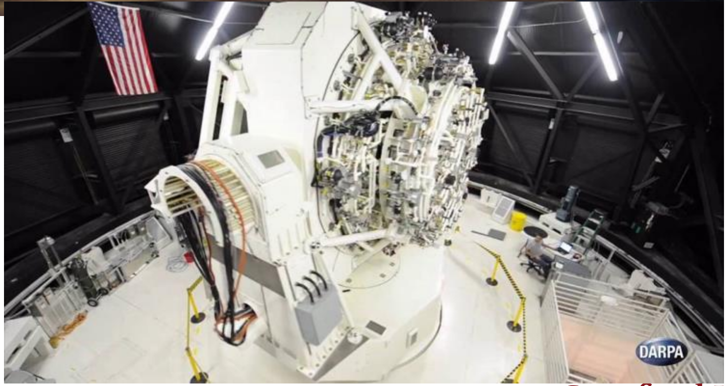
- Conjunction assessment by interpolating between measured positions of orbiting objects
- Result: estimated location and time of the collision



Deep Dive: The Space Surveillance Telescope (SST)

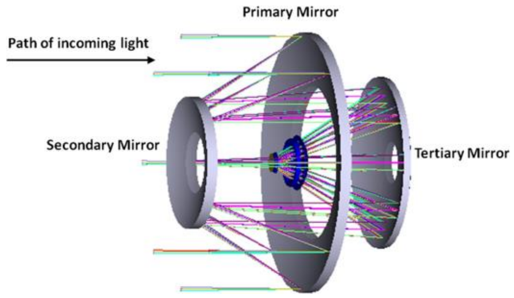


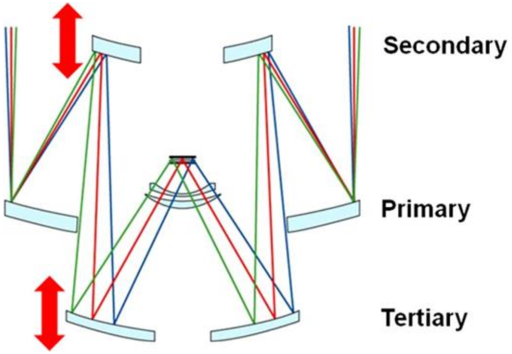




Deep Dive: The Space Surveillance Telescope (SST)

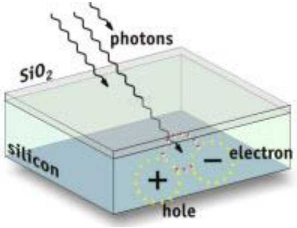
- DARPA experimental space telescope
- Optics: three-mirror Mersenne-Schmidt telescope
  - Built by L-3 Brashear
  - 3.5 meter primary mirror
  - F/1.0: Intended for wide field of view, rapid scan
  - FOV: 6 square degrees
- Imager: charge-coupled device
  - Built by MIT Lincoln Labs
  - Mosaic of 12 CCDs
  - $2k \times 4k$ pixels, each $15\mu m$ square
- Intended for GEO surveillance, LEO imaging

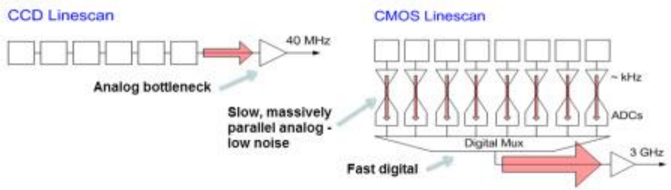


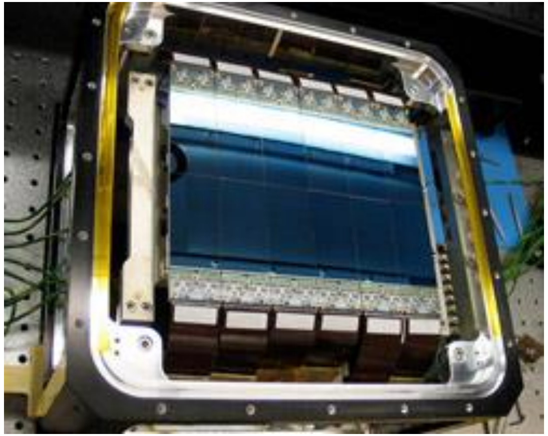


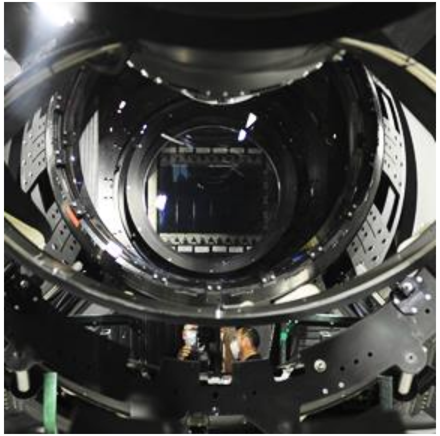
Deep Dive: The Space Surveillance Telescope (SST)

- Charge-coupled devices result in high quality, astronomical imaging, but at a cost

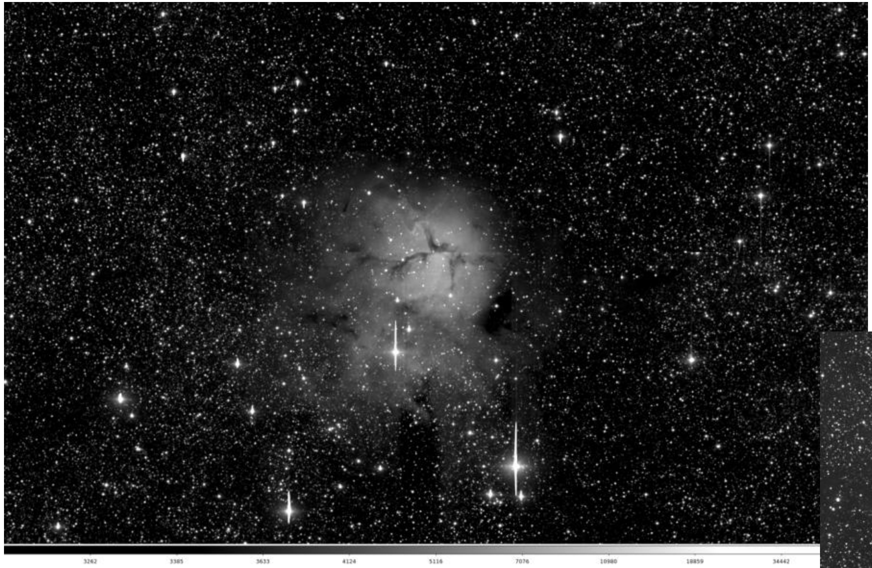








Deep Dive: The Space Surveillance Telescope (SST)



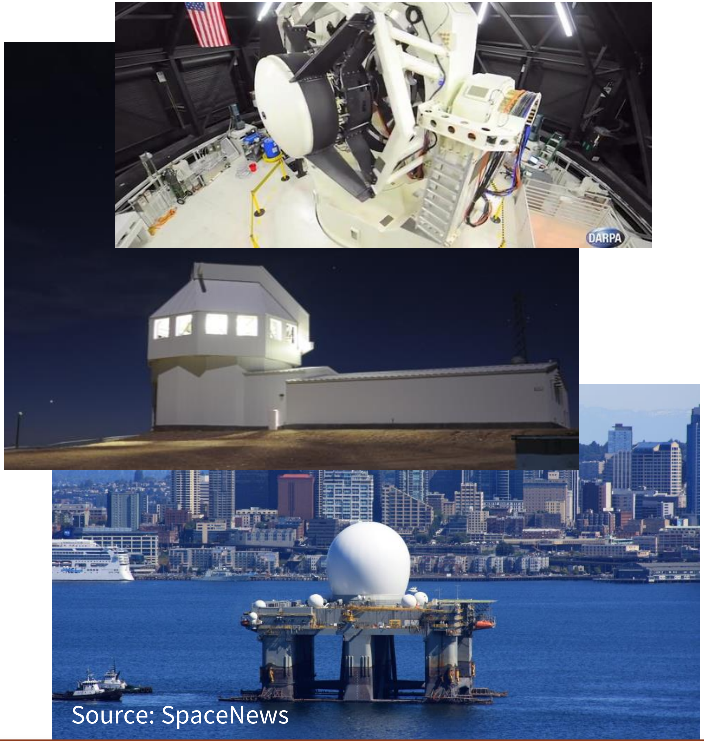
M20 Trifid Nebula, or NGC6514

Nebulae and Clusters of Stars: NGC6888

Deep Dive: The Space Surveillance Telescope (SST)

- Development Programmatics:
  - Total development budget: approximately $200M
  - Expected operations and maintenance costs: $10M/year
  - First light: 2011
  - Handoff to USAF and operationalization in 2016
  - Currently based in White Sands, NM
  - USAF will move the SST to Harold E. Holt Naval Communication Station in Western Australia, operating and maintaining the telescope jointly with the Royal Australian Air Force
  - SST will be a dedicated sensor in the USSSN
- Move from White Sands to Holt will be costly and disruptive to operations!
  - Potential system downtime: 1 year
  - Move cost alone: $1M
  - Installation and re-certification costs: $Millions

Space Situational Awareness: Large vs. Small Sensors



vs.

USSSN Costs Have Motivated the Rise and Proliferation of Sensor Networks Comprised of Small Telescopes

Research Questions

- Question 1: What are quantitative measures of the value of information and, risk mitigation of data loss due to collisions, for space surveillance networks?
  - Incorporates two decisions, tactical and strategic, from different decision-makers with different risk attitudes
- Question 2: What is an optimal policy for deploying new sensors, large or small, to minimize failure risk from satellite collision and thus data losses, over multiple time steps?

Single Timestep Bayesian Model

Bayesian Model for Evaluating Monitoring Systems

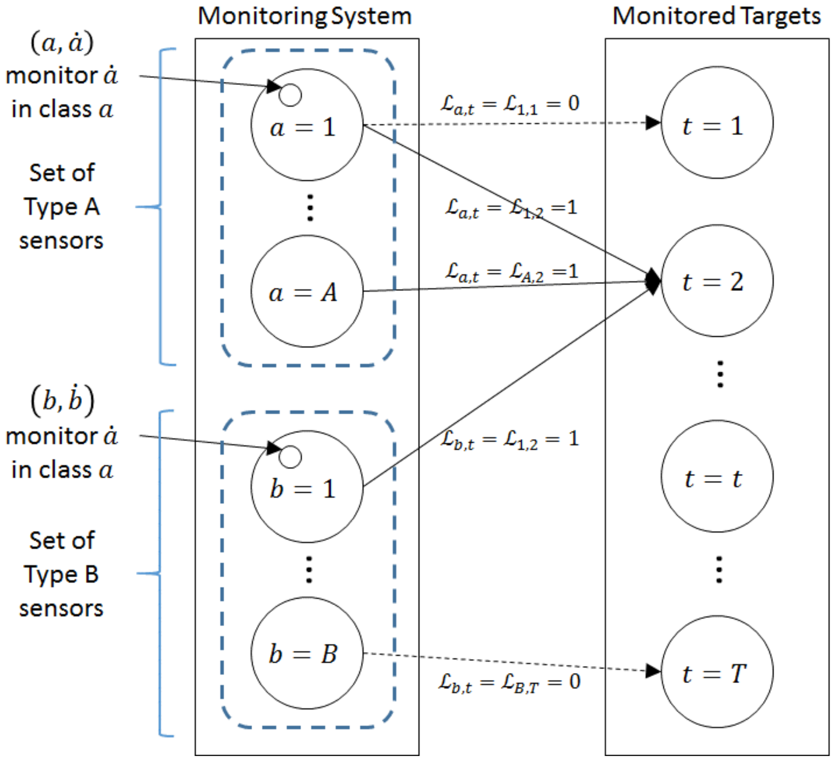
$$\mathcal{L}_{a,t} = \begin{cases} 1, & \text{if } \exists \text{ observation link between class } a \text{ and class } t \\ 0, & \text{if } \nexists \text{ observation link between class } a \text{ and class } t \end{cases}$$

$$\mathcal{L}_{b,t} = \begin{cases} 1, & \text{if } \exists \text{ observation link between class } b \text{ and class } t \\ 0, & \text{if } \nexists \text{ observation link between class } b \text{ and class } t \end{cases}$$

$$(a, \dot{a}) = \begin{cases} "X_{a,\dot{a}}" | X_{t,\dot{t}} & \text{true positive signal with probability } P("X_{a,\dot{a}}" | X_{t,\dot{t}}) \\ "X'_{a,\dot{a}}" | X'_{t,\dot{t}} & \text{true negative signal with probability } P("X'_{a,\dot{a}}" | X'_{t,\dot{t}}) \end{cases}$$

$$(b, \dot{b}) = \begin{cases} "X_{b,\dot{b}}" | X_{t,\dot{t}} & \text{true positive signal with probability } P("X_{b,\dot{b}}" | X_{t,\dot{t}}) \\ "X'_{b,\dot{b}}" | X'_{t,\dot{t}} & \text{true negative signal with probability } P("X'_{b,\dot{b}}" | X'_{t,\dot{t}}) \end{cases}$$

$$R = \prod_{a,b} \mathcal{L}_{a,t}\, \mathcal{L}_{b,t} (\dot{A}_a + 1)(\dot{B}_b + 1), \forall \mathcal{L}_{a,t} = 1 \text{ and } \mathcal{L}_{b,t} = 1$$



Computing Probability of r Correct Signals from R Sensors

- For risk event X, $\ddot{a}$ denotes number of positive indications among $\dot{A}_a$ sensors, and similarly for $\ddot{b}$
  - E.g. Sensor class a, comprised of $\dot{A}_a$ i roduce a combination of signals with $\ddot{a} = 0,1,2,$, binomially distrib
- For two a and b:

Prior

Likelihood for A-signals, binomial distributed

$$P\left(r \leq R \text{ positive indications}, X_{t,\dot{t}} \middle| \left(\dot{A}_a + \dot{B}_b\right) \text{ sensors}\right)$$

$$= \prod_{a,b} \mathcal{L}_{a,t}\,\mathcal{L}_{b,t} \left(\begin{array}{c} P\left(X_{t,\dot{t}}\right) \binom{\dot{A}_a}{\ddot{a}} P\left("X_{a,\dot{a}}"|X_{t,\dot{t}}\right)^{\ddot{a}} \left(1 - P\left("X_{a,\dot{a}}"|X_{t,\dot{t}}\right)\right)^{\dot{A}_a - \ddot{a}} \\ \times \binom{\dot{B}_b}{\ddot{b}} P\left("X_{b,\dot{b}}"|X_{t,\dot{t}}\right)^{\ddot{b}} \left(1 - P\left("X_{b,\dot{b}}"|X_{t,\dot{t}}\right)\right)^{\dot{B}_b - \ddot{b}} \end{array} \right),$$

$$\forall \mathcal{L}_{a,t} = 1 \text{ and } \mathcal{L}_{b,t} = 1$$

Likelihood for B-signals, binomial distributed

Computing Preposterior of Each Unique Si

- Preposterior probability of receiving r positive indicatio
s is:

Prior

$$P\left(r \leq R \text{ positive indications} \middle| \left(\dot{A}_a + \dot{B}_b\right) \text{ sensors}\right)$$

$$= \prod_{a,b} \mathcal{L}_{a,t}\,\mathcal{L}_{b,t} \left(\begin{array}{l} P\left(X_{t,\dot{t}}\right)\binom{\dot{A}_a}{\ddot{a}} P\left(\text{"}X_{a,\dot{a}}\text{"}\middle|X_{t,\dot{t}}\right)^{\ddot{a}} \left(1 - P\left(\text{"}X_{a,\dot{a}}\text{"}\middle|X_{t,\dot{t}}\right)\right)^{\dot{A}_a-\ddot{a}} \\ \quad \times \binom{\dot{B}_b}{\ddot{b}} P\left(\text{"}X_{b,\dot{b}}\text{"}\middle|X_{t,\dot{t}}\right)^{\ddot{b}} \left(1 - P\left(\text{"}X_{b,\dot{b}}\text{"}\middle|X_{t,\dot{t}}\right)\right)^{\dot{B}_b-\ddot{b}} \\ +P\left(X'_{t,\dot{t}}\right)\binom{\dot{A}_a}{\dot{A}_a-\ddot{a}} \left(1 - P\left(\text{"}X_{a,\dot{a}}\text{"}\middle|X_{t,\dot{t}}\right)\right)^{\ddot{a}} P\left(\text{"}X_{a,\dot{a}}\text{"}\middle|X_{t,\dot{t}}\right)^{\dot{A}_a-\ddot{a}} \\ \quad \times \binom{\dot{B}_b}{\dot{B}_b-\ddot{b}} \left(1 - P\left(\text{"}X_{b,\dot{b}}\text{"}\middle|X_{t,\dot{t}}\right)\right)^{\ddot{b}} P\left(\text{"}X \right. \end{array} \right),$$

$$\forall \mathcal{L}_{a,t} = 1 \text{ and } \mathcal{L}_{b,t} = 1$$

Likelihood for A and B signals, joint binomial distributed

1 – Prior

Complement of likelihood for A and B signals, binomial distributed

Computing Posteriors for Failure for Individual Targets

- Posterior probability … le target $\dot{t}$ wi…

Probability of r correct positive indications

Likelihood for A and B signals, joint binomial distributed

Prior

$$\ldots\{(a,\dot{a}),(b,\dot{b})\}) = \begin{cases} P(X_{t,\dot{t}}) & \text{if } \mathcal{L}_{a,t} = 0 \text{ a}\ldots \\ P' & \text{if } \mathcal{L}_{a,t} = 1 \text{ a}\ldots \end{cases}$$

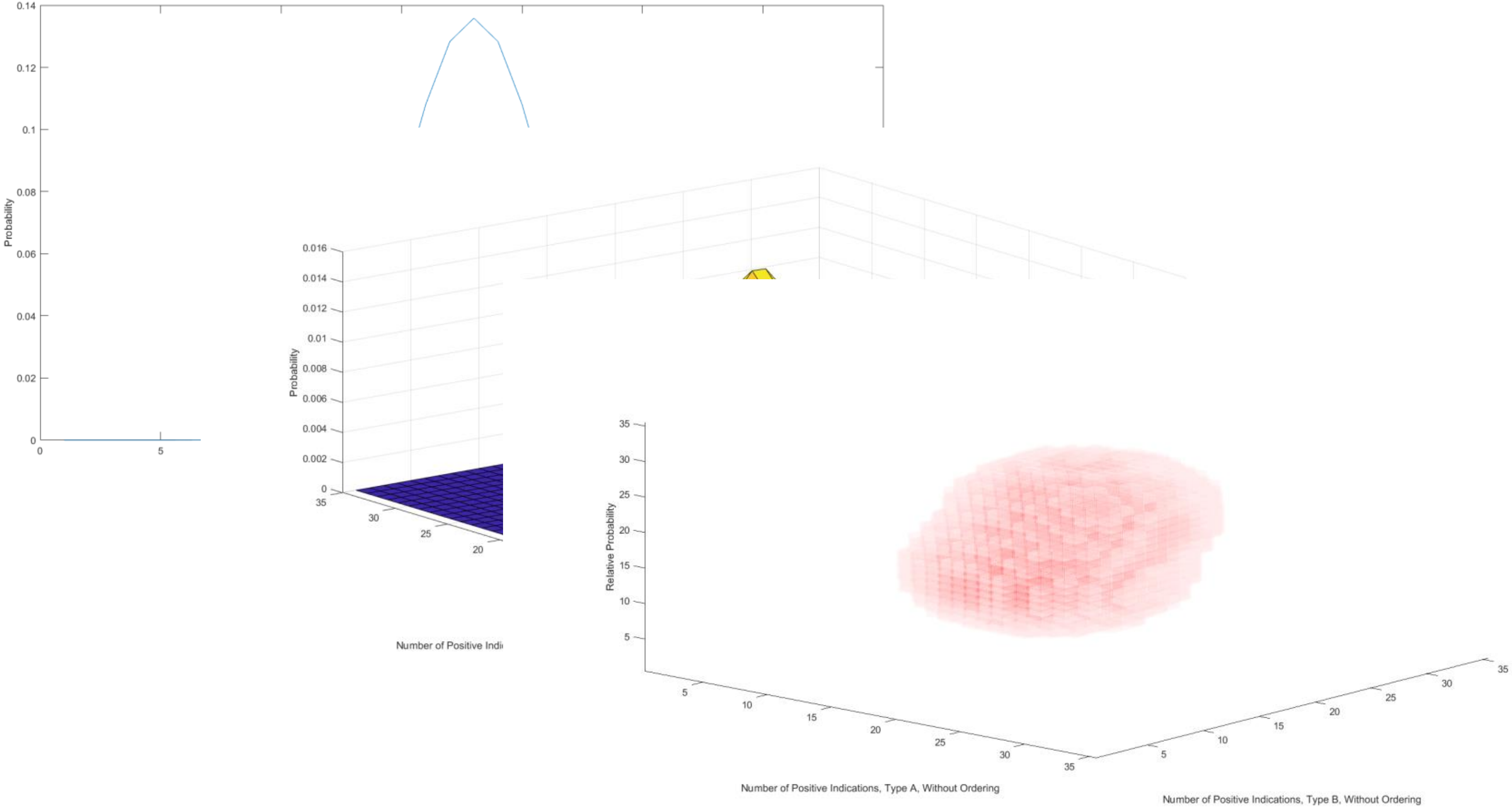
where

$$P' = \frac{P(r \leq R \text{ positive indications}, X_{t,\dot{t}} | (\dot{A}_a + \dot{B}_b) \text{ sensors})}{P(r \leq R \text{ positive indications} | (\dot{A}_a + \dot{B}_b) \text{ sensors})},$$

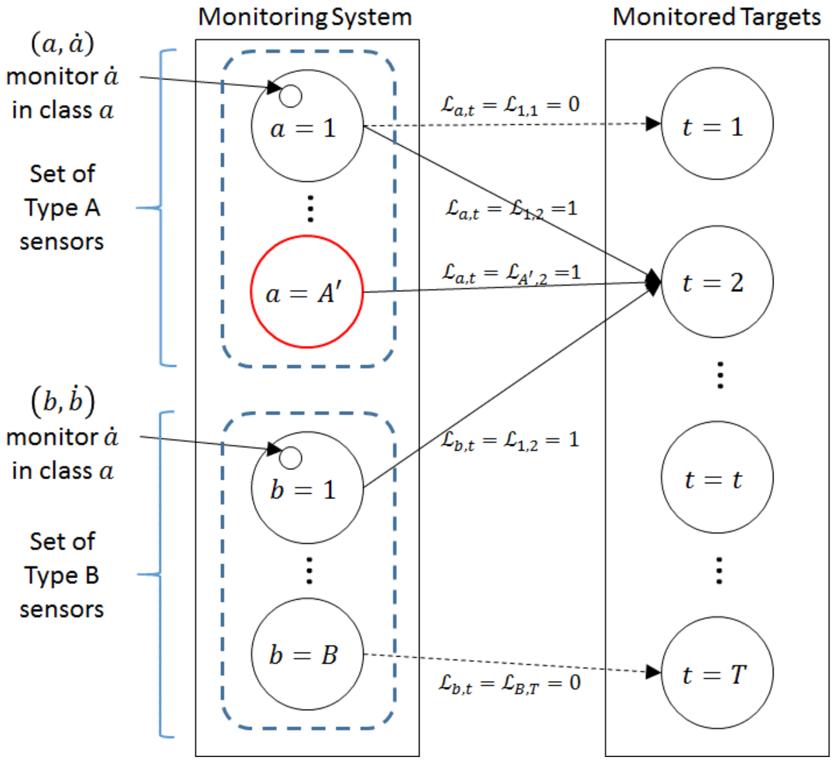
$$\forall a = 1,2,\ldots,A \text{ and } \forall b = 1,2,\ldots,B$$

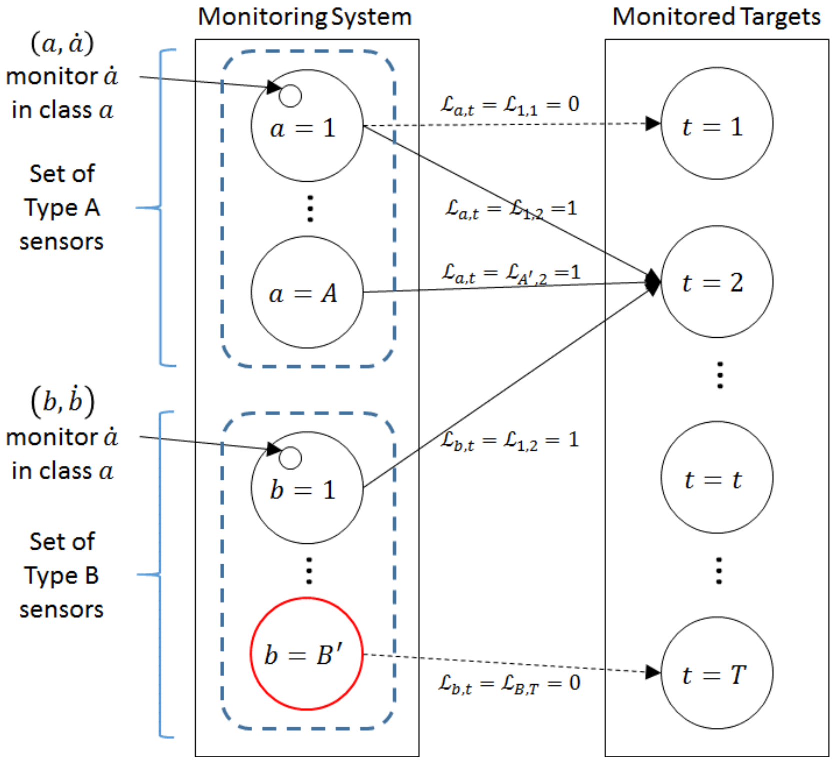
Preposterior

Preposteriors for Many Sensor Types



Alternatives: Addition of Large (Type A) or Small (Type B) Sensors





Alternatives: Addition of Large (Type A) or Small (Type B) Sensors

- Added value of modifying the baseline monitoring system with additional Type A or Type B sensors

$$V_A = CE[EU'_A; \gamma_2] - CE[EU'; \gamma_2]$$

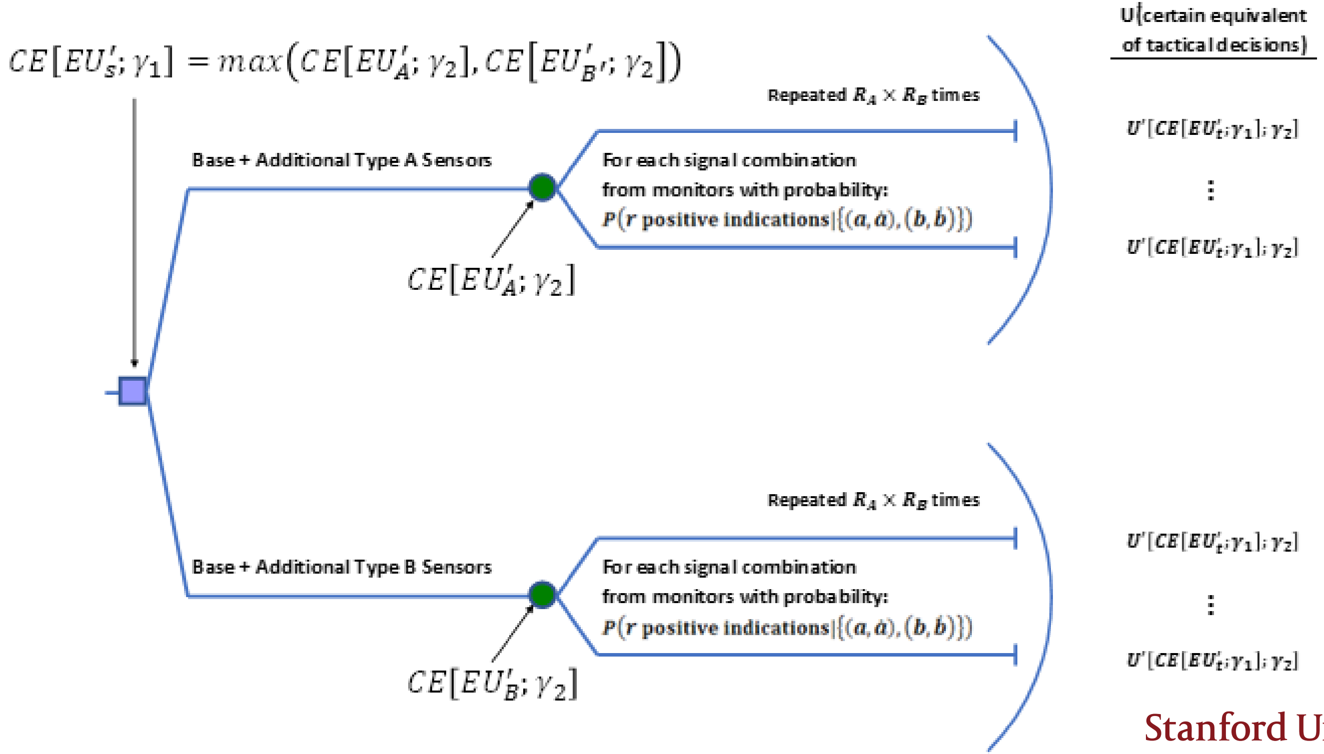
$$V_B = CE[EU'_B; \gamma_2] - CE[EU'; \gamma_2]$$

- Strategic decision for optimal policy given by:

$$max(V_A, V_B)$$

Alternatives: Addition of Large (Type A) or Small (Type B) Sensors

- Equivalently, this decision tree is sufficient:



Multi-Timestep Sensor Deployment Model

Problem Setup

- Given the single-timestep model, what is the optimal policy for managing the monitoring system, **over multiple time periods**?
- Problem characteristics:
  - Discrete state space, defined by current composition of the system plus the risk environment (unintended space collisions)
  - States do not transition deterministically, and current state is not known with certainty
  - Probability of unintended collision transitions adhere to the Markov Property
    - Assume Markov-1
- Given these problem characteristics, a reasonable analytical approach: partially-observable Markov decision processes (POMDP)

POMDP Setup

- State space
  - $x_t = (s_{A_t}, s_{B_t}, r_t)$, where the t is indexed $t \in \{1,2,\ldots,(s_{A_{max}} \times s_{B_{max}} \times r_{max})\}$
    - Number of Type A sensors in a time step denoted $s_{A_t} \in \{0,1,\ldots s_{A_{max}}\}$
    - Number of Type B sensors in a time step denoted $s_{B_t} \in \{0,1,\ldots s_{B_{max}}\}$
    - Annual collision probability of a satellite denoted r_t, given the debris environment (modeled with three possible realizations, "decreased", "unchanged", and "increased" from the previous time step)
  - Total size of state space is $(s_{A_{max}} \times s_{B_{max}} \times r_{max})$

POMDP Setup

- Action
  - Sequential decision over T on whether to add additional Type A or Type B sensors
  - Given by
    - $\mu_{A_t} \in \{0,1, \ldots \mu_{A_{max}}\}$
    - $\mu_{B_t} \in \{0,1, \ldots \mu_{B_{max}}\}$
  - In any time step, only one of μ_{A_t} or μ_{B_t} may be nonzero, since the decision maker is choosing between Type A or Type B sensors

POMDP Setup

- State transitions
  - Non-deterministic and partially observable
  - Each sensor type treated independently
  - Transition function:

$$x_{t+1} = \left(s_{A_{t+1}}, s_{B_{t+1}}, r_t\right)$$
$$= \left(\mu_{A_t} \times Beta(\alpha_A, \beta_A) + s_{A_t}, \mu_{B_t} \times Beta(\alpha_B, \beta_B) + s_{B_t}, r_{t+1}\right)$$

  - Probability of satellite collision is a discrete probability distribution denoted as $r_t \in \{0,1,\ldots,r_{max}\}$, with transition probabilities

r_t	$P(r_{t+1} = 0 \vert r_t)$	$P(r_{t+1} = 1 \vert r_t)$	$\cdots$
0	p_{00}	p_{01}	$\cdots$
1	p_{10}	p_{11}	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$

POMDP Setup

- Stage costs
  - Addition of new sensors μ_{A_t} and μ_{B_t} incur a cost of $c_{A_1} > 0$ and $c_{B_1} > 0$
  - Posterior benefit of adding μ_{A_t} and μ_{B_t} new sensors: $c_{A_2} < 0$ and $c_{B_2} < 0$, where

$$c_{A_2} = V_A = CE[EU'_A; \gamma_2] - CE[EU'; \gamma_2]$$
$$c_{B_2} = V_B = CE[EU'_B; \gamma_2] - CE[EU'; \gamma_2]$$

  - Cost function $g(s_{A_t}, s_{B_t}, r_t)$ derived by choosing the optimal action which minimizes costs in within a time step, over all time steps

$$g(s_{A_t}, s_{B_t}, r_t) = min\begin{pmatrix}(c_{A_1}|\mu_{A_t}, \mu_{B_t} + c_{A_2}|s_{A_t}, s_{B_t}, r_t), \\ (c_{B_1}|\mu_{A_t}, \mu_{B_t} + c_{B_2}|s_{A_t}, s_{B_t}, r_t)\end{pmatrix}$$

POMDP Setup

- Objective: find the policy $u_t = \mu\left(s_{A_t}, s_{B_t}, r_t\right)$ that minimizes the expected average cost V_t
  - Optimal sensor fielding policy: $\mu_{A_t} \in \left\{0,1, \ldots \mu_{A_{max}}\right\}$ and $\mu_{B_t} \in \left\{0,1, \ldots \mu_{B_{max}}\right\}$ that minimizes total cost J^* for any state x_t
- Value iteration algorithm:
  - Set $V_0 = 0$
  - For $t = 1{:}T$ for sufficiently large T
    - $V_{t+1}(x) = \min_{u_t} EV\left[g\left(s_{A_t}, s_{B_t}, r_t\right) + V_t\left(x_t, \mu_{A_t}, \mu_{B_t}\right)\right]$, the minimum cost resulting from the action set u_t, iterated over t
    - Optimal policy: $\mu_t^*(x) \in \operatorname{argmin}_{u_t} EV\left[V_{t+1}^*\left(x_t, \mu_{A_t}, \mu_{B_t}\right)\right]$, computed by tabulating the action set u_t which results in the lowest cost, given any state

POMDP Setup

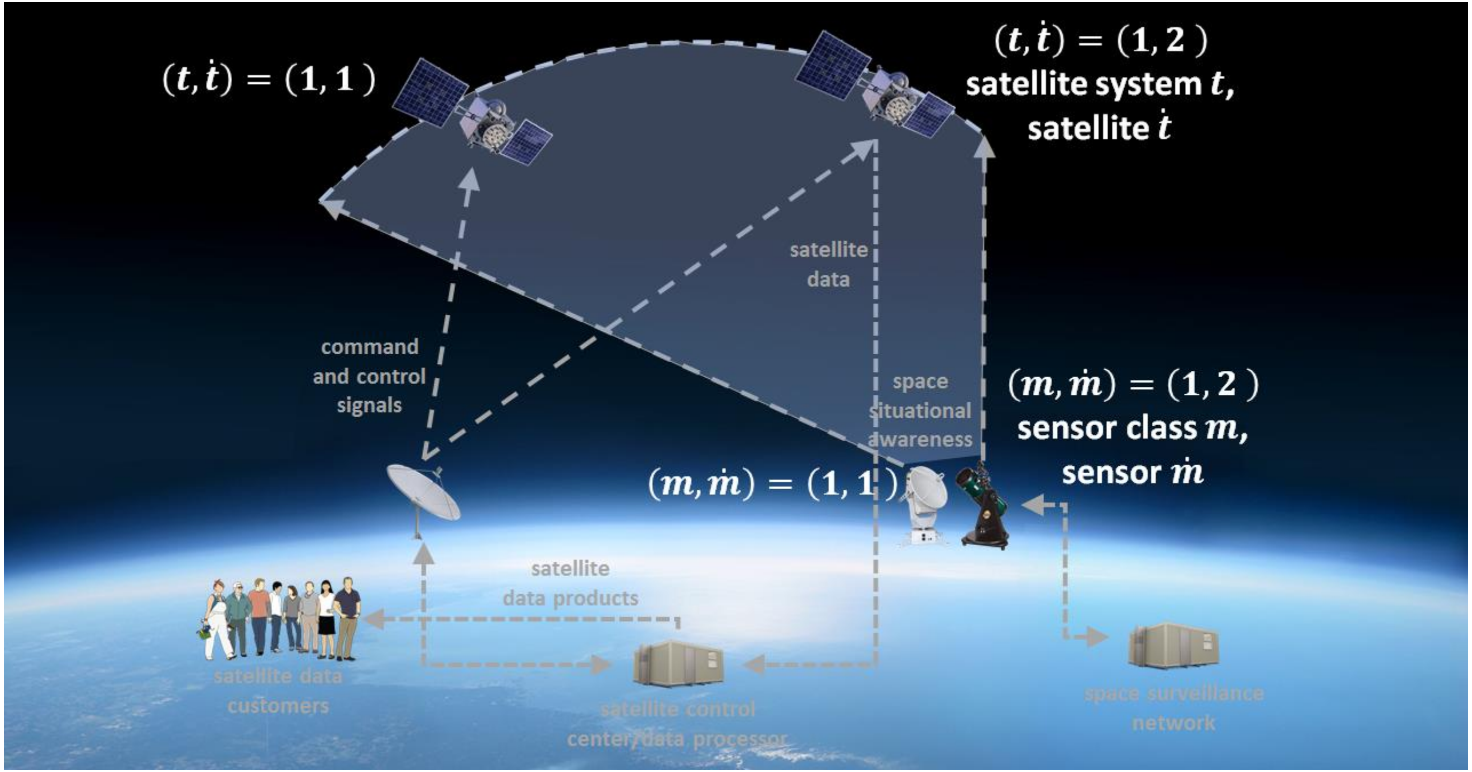
- Optimal costs
  - Optimal marginal cost in a time step denoted J^*, is given by $V_{t+1}(x) - V_t(x)$

$$J_t^* = V_{t+1}(x) - V_t(x)$$

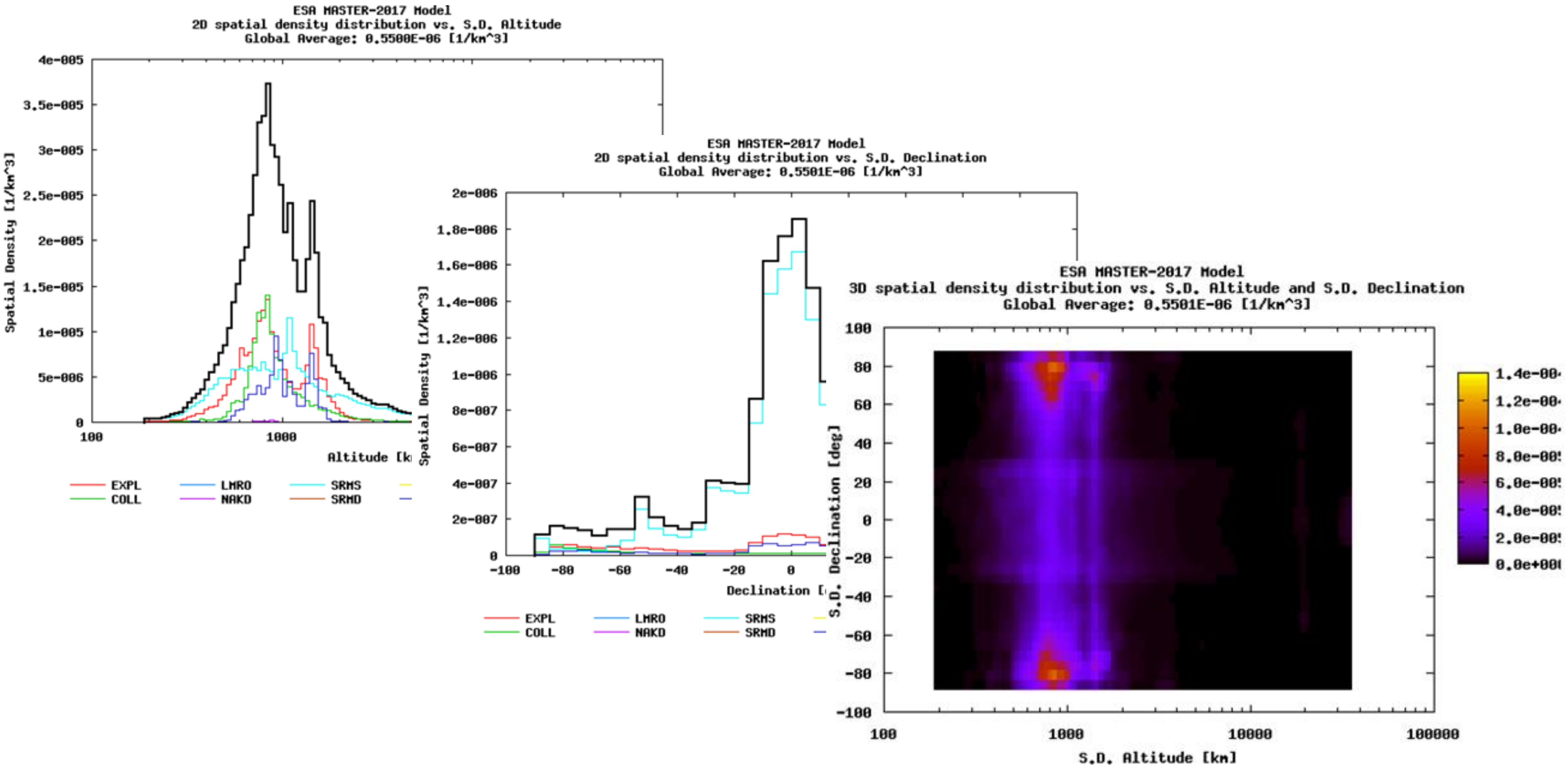
  - J^* will converge to 0 for any state x
    - With an optimal policy strategic decision maker fields sensors in an optima
  - Over sufficiently large number of time steps, the value of signals from the collection of optimally-deployed sensors converge toward clairvoyance
    - Therefore, marginal changes in J^* will converge toward 0

Model Applied to Space Surveillance Systems

Model Applied to Space Surveillance Systems



Characterizing Population Spatial Density

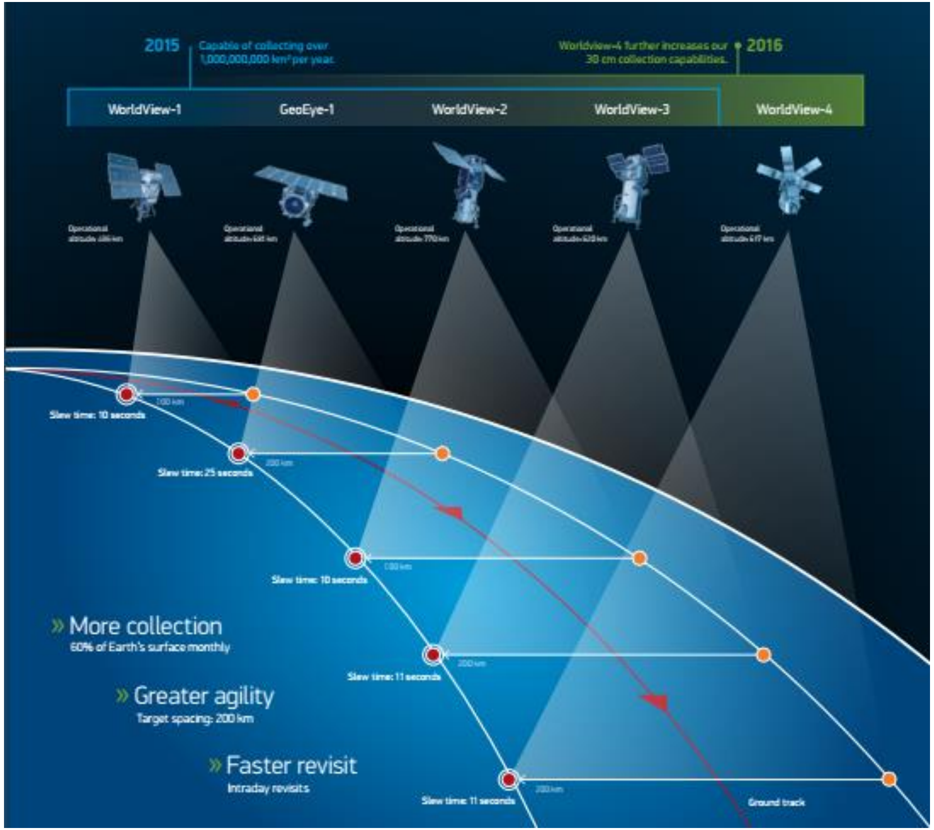


The DigitalGlobe System

Source: DigitalGlobe

Satellite Name	Mass (kg)	Perigee (km)	Ecc.	Inc. (deg)	Period (min)
GeoEye-1	1,955	671	1.06E-03	98.1	98.3
Worldview 1	4,500	491	2.19E-04	97.3	94.5
Worldview 2	2,800	765	1.40E-04	98.5	100.2
Worldview 3	2,800	612	1.43E-04	98.0	96.9
Worldview 4	2,485	617	2.15E-04	98.0	96.9

Summary of DigitalGlobe constellation



DigitalGlobe architectural overview

Applied Example: Fictionalized Version of Current USSSN

List of USSSN sensors used in our probabilistic risk analysis

US Space Surveillance Network Sensor	Short Name	US Space Surveillance Network Sensor	Short Name
Dedicated Sensors		**Collateral Sensors**	
GEODSS*-Socorro	G-S	Advanced Electro-Optical System	AEOS
GEODSS-Maui	G-M	Haystack Ultrawideband Satellite Imaging Radar	HUSIR
GEODSS-Diego Garcia	G-D	Haystack Auxiliary Radar	HAX
SST	SST	Millstone Hill Radar	MHR
Globus II	GII	ALTAIR	ALTAIR
AN/FPS-85	AN	ALCOR	ALCOR
Space Fence-Kwajalein	SF-K	Ascension Range Radar	ARR
Space Fence-Australia	SF-A	Ground-Based Radar Prototype	GBR-P
SBSS	SBSS		

*Ground-Based Electro-Optical Deep Space Surveillance

Fictionalized version:
15 large sensors
15 small sensors

Inputs to Model, Space Surveillance of DigitalGlobe

Sensor Class	True Pos	False Neg	False Pos	True Neg
Large SSN sensors observing LEO	0.8	0.2	0.2	0.8
Small SSN sensors observing LEO	0.6	0.4	0.4	0.6
Additional large sensors	0.8	0.2	0.2	0.8
Additional small sensors	0.6	0.4	0.4	0.6

Probabilities of collision detection, by sensor class

POMDP Inputs

- State space
  - Type A sensors: $s_{A_t} \in \{15,16,\ldots 20\}$
  - Type B sensors: $s_{B_t} \in \{15,16,\ldots 40\}$
- Markov transitions
  - Probability of satellite collision is a discrete probability distribution

r_t	$P(r_{t+1}=0\|r_t)$	$P(r_{t+1}=1\|r_t)$	$P(r_{t+1}=2\|r_t)$
0	0.4444	0.0000	0.5555
1	0.2000	0.0000	0.8000
2	0.0851	0.1064	0.8085

  - where

$$P(X_{ik}|r_t=0) = decreased\ state = 0.000176075$$
$$P(X_{ik}|r_t=1) = nominal\ state = 0.000184708$$
$$P(X_{ik}|r_t=2) = increased\ state = 0.000187229$$

POMDP Inputs

- Total state space $x_t = \left(s_{A_t}, s_{B_t}, r_t\right)$, with these states:

$\boldsymbol{x_t}$	$\boldsymbol{\left(s_{A_t}, s_{B_t}, r_t\right)}$
$x_t = 1$	$(15,15,0)$
$x_t = 2$	$(15,15,1)$
$x_t = 3$	$(15,15,2)$
$x_t = 4$	$(15,16,0)$
$x_t = 5$	$(15,16,1)$
$x_t = 6$	$(15,16,2)$
$\vdots$	$\vdots$
$x_t = 466$	$(20,40,0)$
$x_t = 467$	$(20,40,1)$
$x_t = 468$	$(20,40,2)$

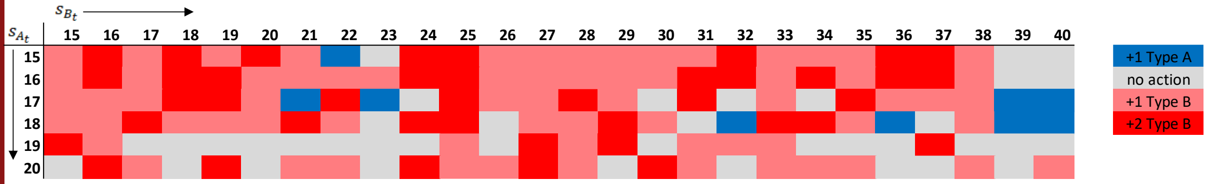
POMDP Inputs

- States and transitions:
  - Maximum number of Type A and Type B sensors that can be added to the existing surveillance system: $\mu_{A_t} \in \{0,1\}$ or $\mu_{B_t} \in \{0,1,2\}$
  - Transitions modeled using Beta distributions with

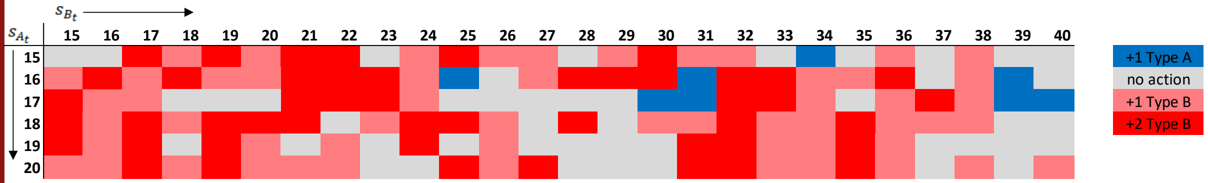
$$s_{A_{t+1}} = \mu_{A_t} \times Beta(4,2) + s_{A_t}$$
$$s_{B_{t+1}} = \mu_{B_t} \times Beta(4,1) + s_{B_t}$$

- POMDP model run for $T = 50$ timesteps, primarily due to runtime limitations
  - Nominal runtime for each simulation approximately 12 hours

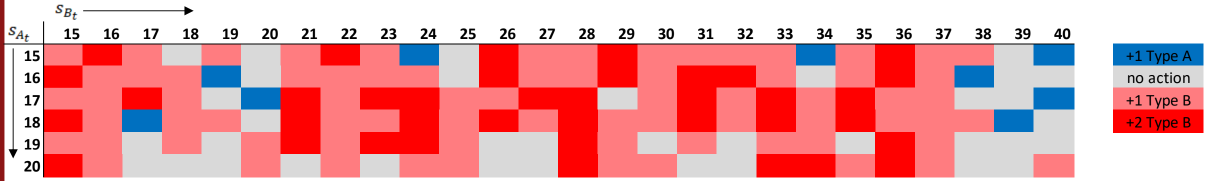
POMDP Model Results: Baseline



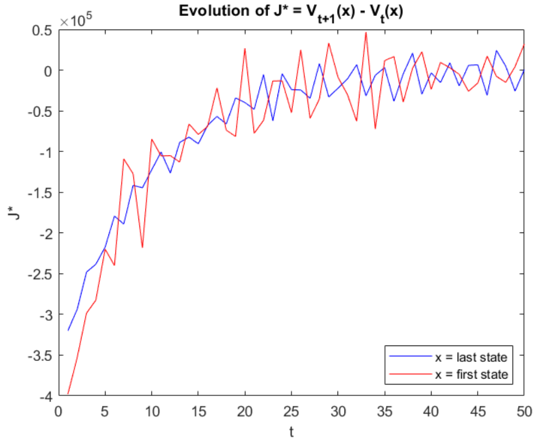
Policy matrix for $(s_{A_t}, s_{B_t}, r_t) = (:,:,0)$

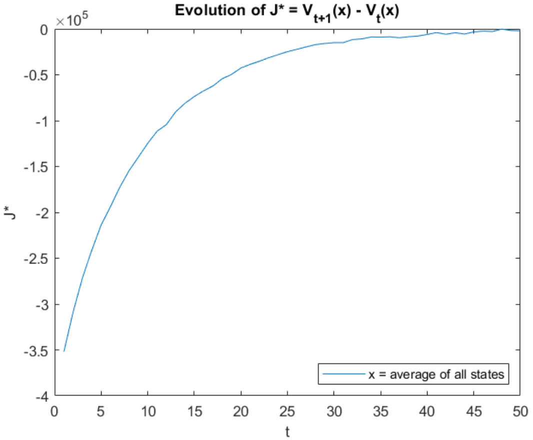


Policy matrix for $(s_{A_t}, s_{B_t}, r_t) = (:,:,1)$



Policy matrix for $(s_{A_t}, s_{B_t}, r_t) = (:,:,2)$





Conclusions

- Satellites face the risk of unintended collisions, largely driven by the debris population in space
- Networks of space surveillance sensors provide signals on possible collisions
- We use POMDP models to compute optimal ordering policies for additional space surveillance sensors
- Model abstraction can be used to compute optimal ordering policies for many different monitoring systems
  - Cyber monitoring, health monitoring, seismic monitoring, etc.