# MS&E 260

## Introduction to Operations Management

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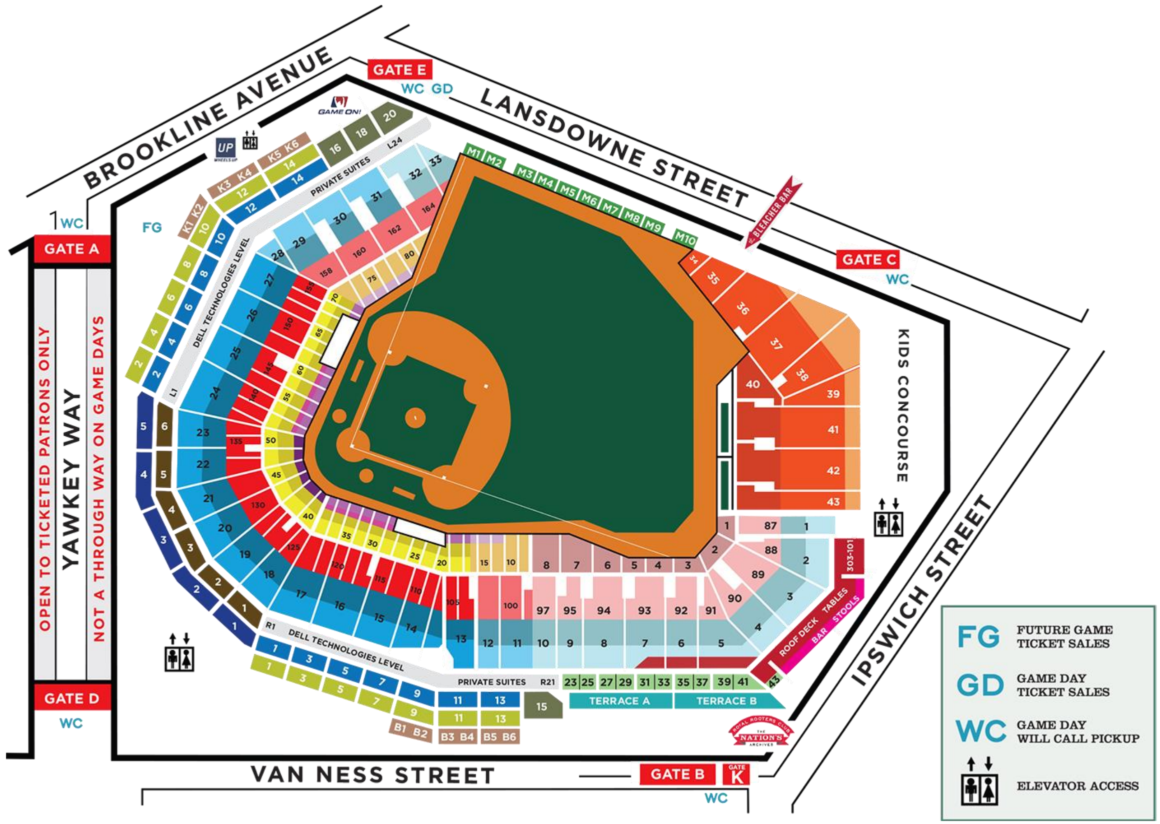
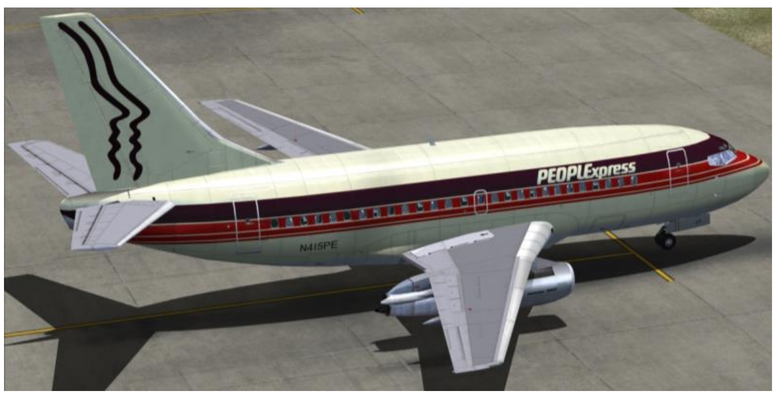
# What is Revenue Management?

- Revenue Management: The science of selling the right item to the right person (at the right price)
- Given:
  - Limited capacity
  - Uncertain demand
- What are the levers?
  - Allocation
  - Pricing



# The Origins of Revenue Management

- American Airlines implements DINAMO (Dynamic Inventory and Maintenance Optimizer)
- Led to Ultimate Super Saver Fares in 1985
- Revenues increased 14.5% and profits increased 47.8%
- Drove low-fare competitor PEOPLExpress out of business in 1986




vs.


"American Airlines, in a move to stimulate traffic in the January doldrums, said yesterday that it would cut fares by up to 74 percent from regular coach fares for travel between Jan. 8 and Feb. 10. … Tickets must be bought within three days of making a reservation, but no later than Jan. 20, and 50 percent of the ticket is not refundable. The ultimate super saver now has an advance purchase requirement of 30 days. The new fares, which are expected to be matched by other major carriers, also require customers to stay over at least one Saturday night before departure."
New York Times, December 21, 1985

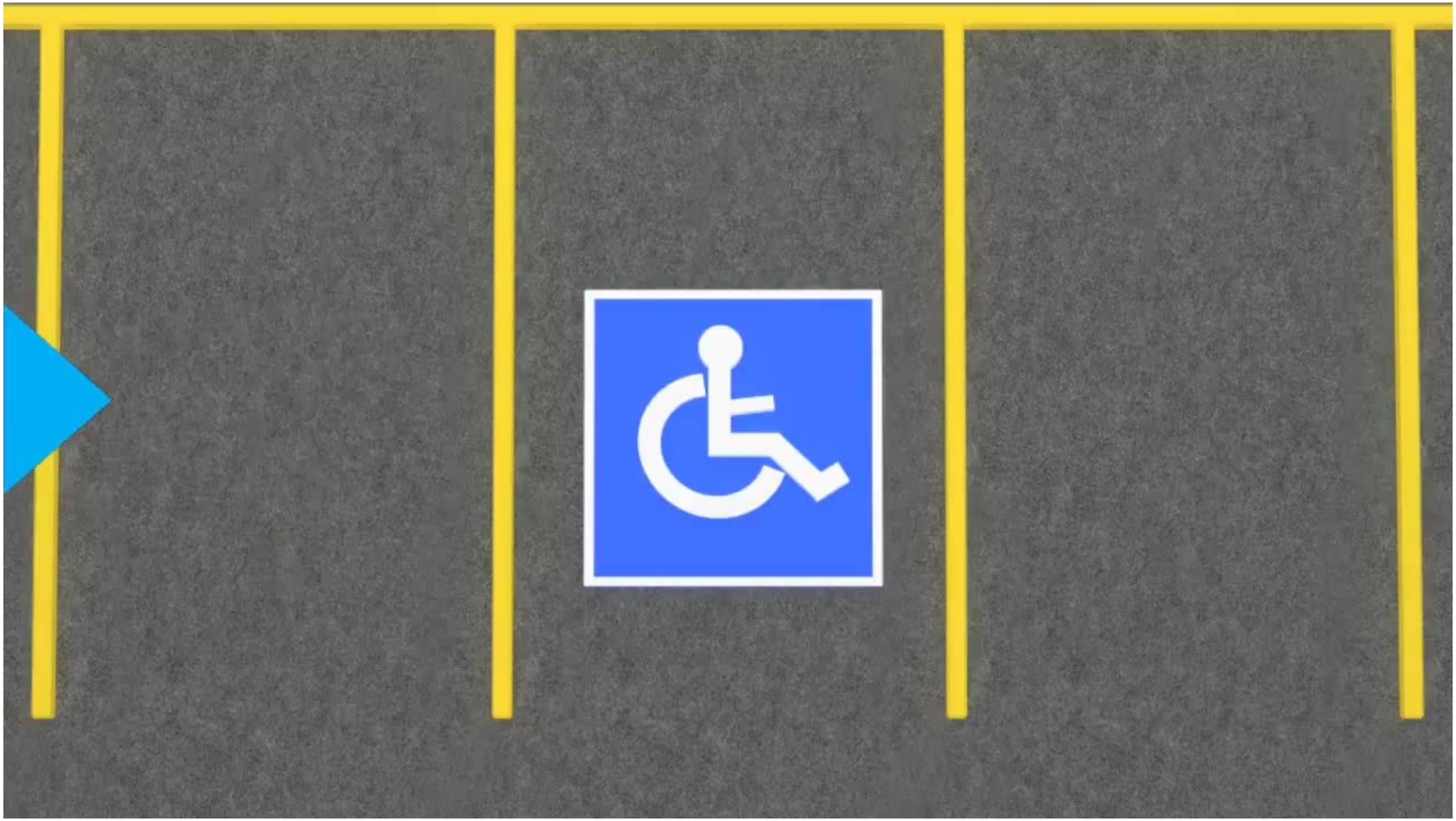
# Where is Revenue Management Utilized?

- Mature Use
  - Airlines
  - Freight and Cargo
  - Cruise lines
  - Car Rental
  - Hotels
  - Apparel
  - Retail
- Emergent Use
  - Advertising
  - Entertainment
  - Oil and Gas markets
  - E-commerce
  - Insurance
  - Public policy

# Revenue Management Example: Sporting Event Tickets

# Revenue Management Example: Taxis (Uber and Lyft)

# Revenue Management Example: Parking Meters



# Revenue Management Example: Disneyland

# Consumers and Perceptions of Fairness

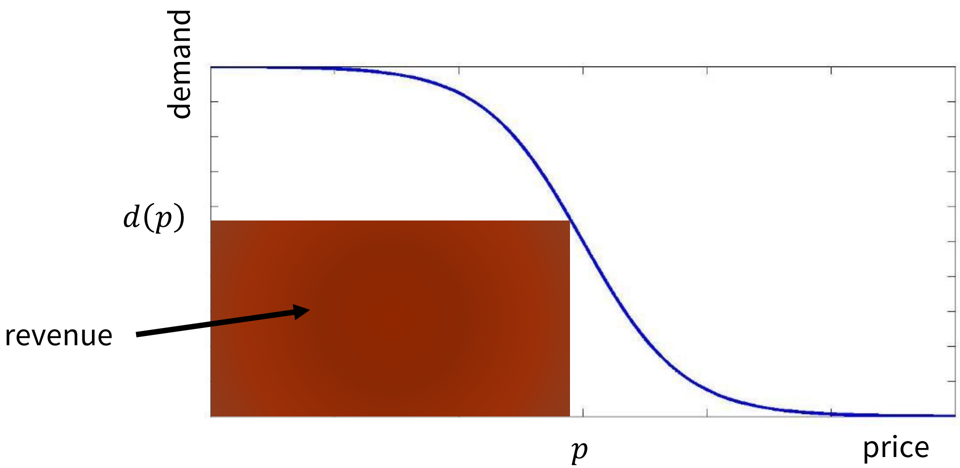
- Amazon's test of dynamic pricing strategies
  - "This is a very strange business model, to charge customers more when they buy more or come back to the site more. ... This is definitely not going to earn customer loyalty."
  - "I find this extremely sneaky and unethical."
  - "I will never buy another thing from those guys!!!"

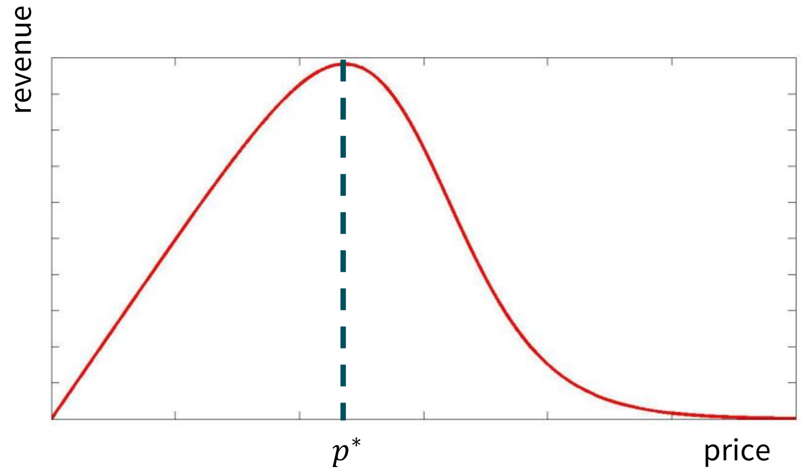
# Pricing

# Basic Price Optimization

- Revenue maximization:

$$\begin{aligned} &\text{max} \quad && p \times d(p) \\ &\text{s.t.} \quad && p \geq 0 \end{aligned}$$



# Example: Electric Stand Mixer

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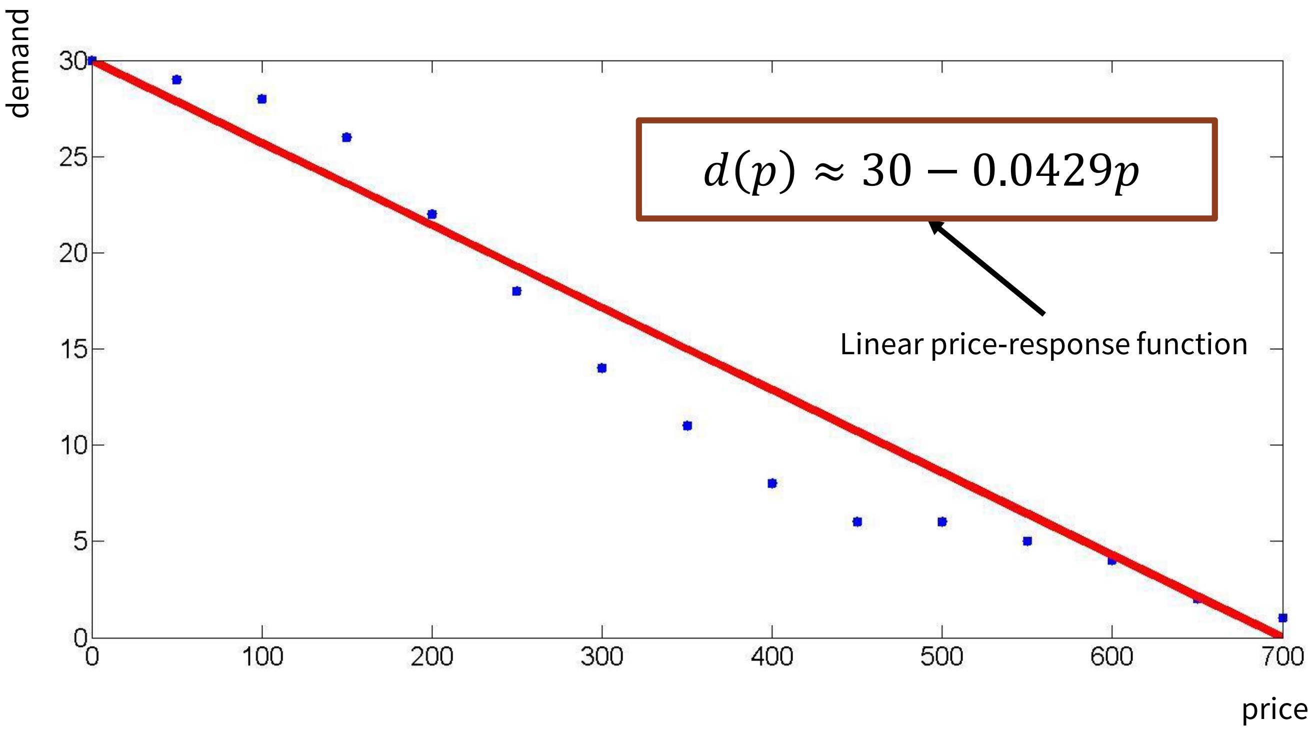


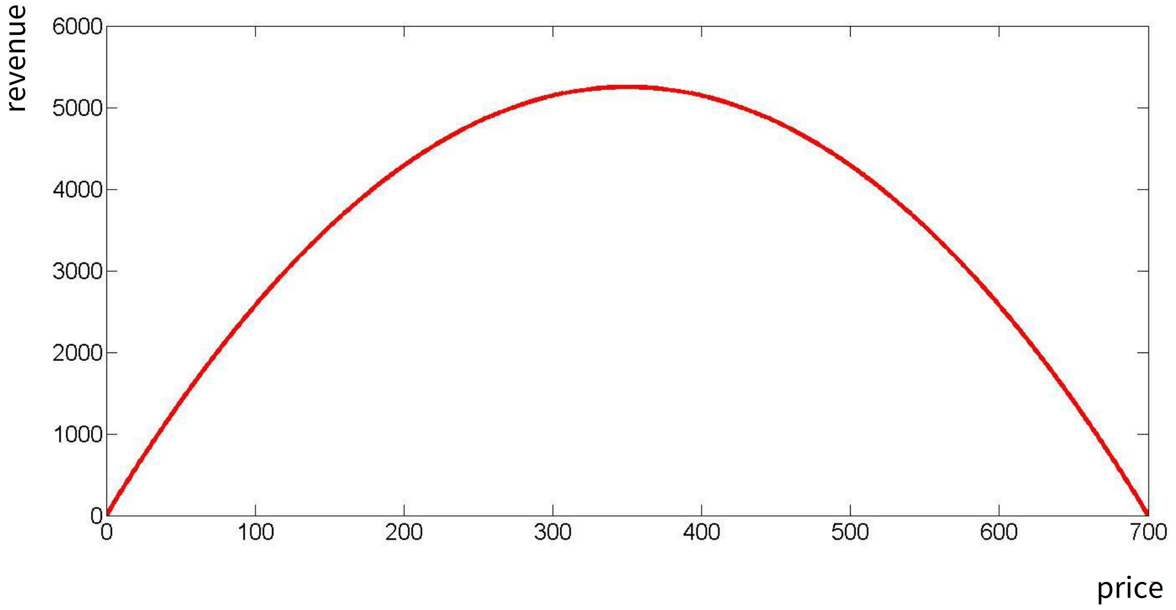
# Example: Electric Stand Mixer

- Revenue maximization:

| | |
|---|---|
| max | $p(30 - 0.0429p)$ |
| s.t. | $p \geq 0$ |

$$p^* = \$349.65$$

$$revenue^* = \$5{,}244.75$$

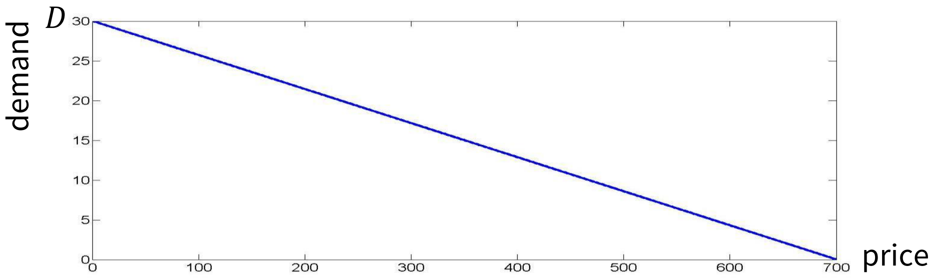


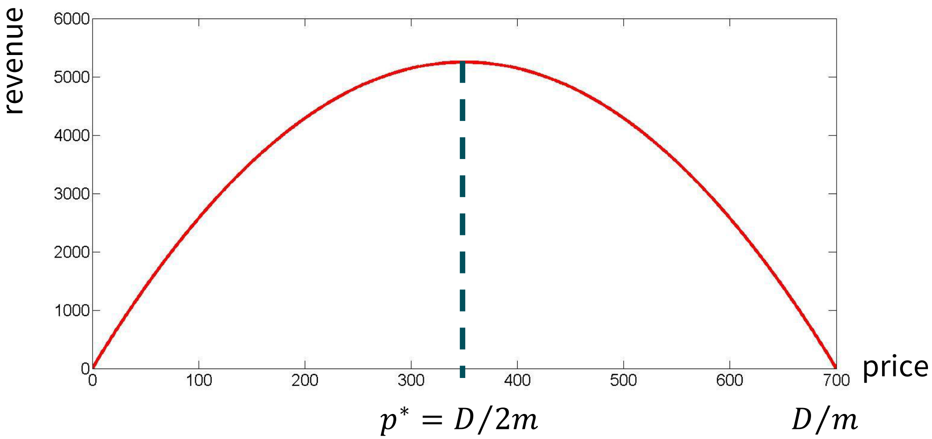
# Basic Price Optimization: Linear Price-Response Function

$$d(p) = D - mp$$

- Revenue optimizing price:

  $p^* = D/2m$
- Demand at $p^*$: $D/2$
- Revenue at $p^*$: $D^2/4m$
- Selling to only half the market!
- Is there a way to optimally sell to a bigger share of the market?
  - Yes, by segmenting the market!

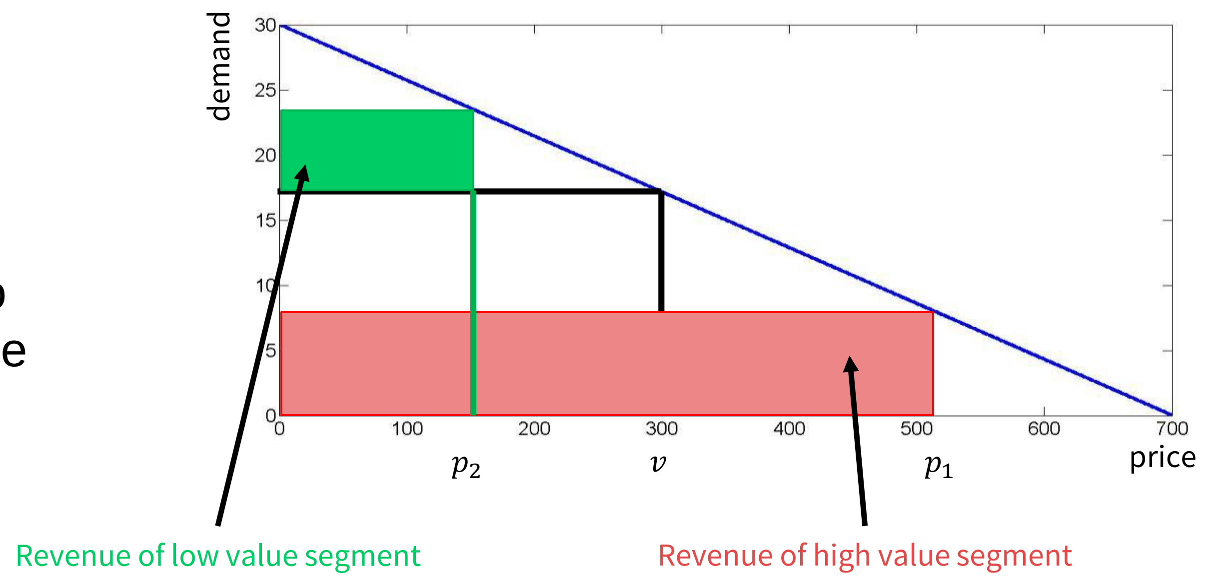


# 2-Segment Price Differentiation

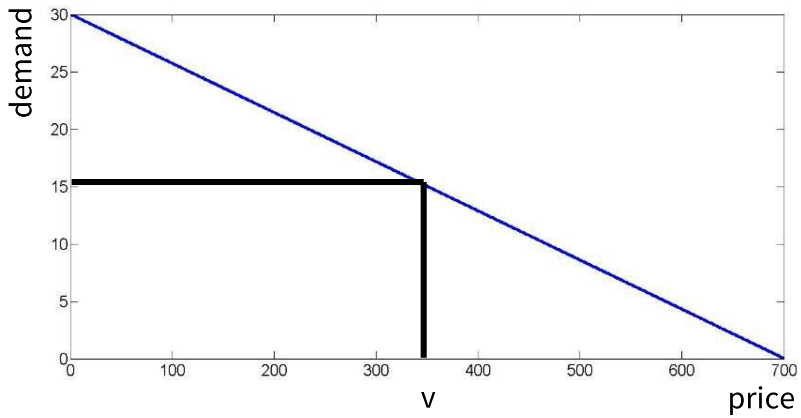
- Two segments:
  - $v$: segmentation threshold
  - $p_1$: price for high-valuation segment
  - $p_2$: price for low-valuation segment
- Question: How to optimize for $p_1$ and $p_2$?

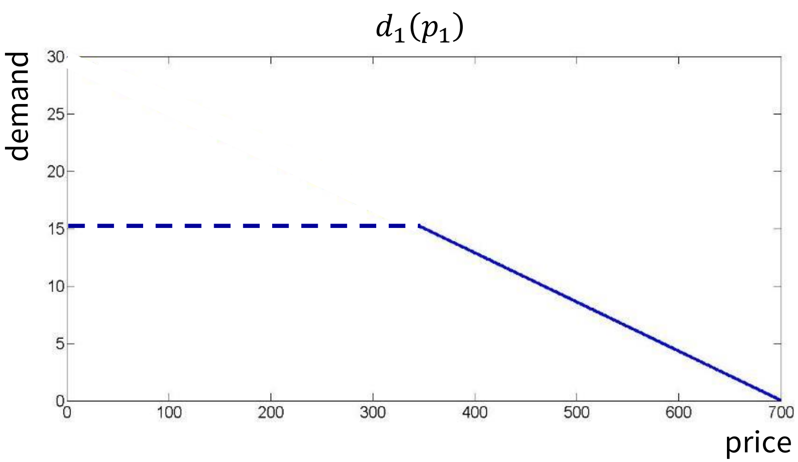
Remark: We assumed here that we perfectly segment the market so NO customer with value $> v$ buys the product with the lower price.

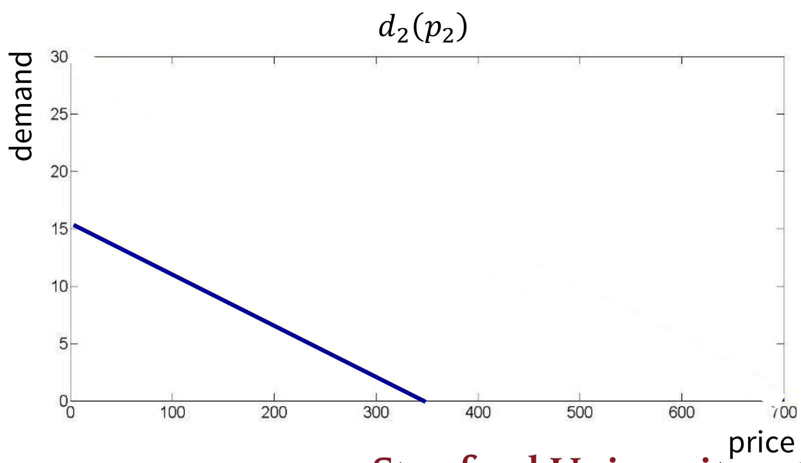


# 2-Segment Price Optimization

- Two segments:
  - $d_1(p_1)$: price-response function of high-valuation segment
  - $d_2(p_2)$: price-response function of low-valuation segment
- Find optimal price in each segment separately!
  - Only true if segments are independent and there are no capacity constraints



# Example Electric Stand Mixer

- Segmentation threshold $= \$280$
- High-valuation segment revenue maximization:

$$\begin{array}{ll} \text{max} & p_1 min\{18, (30 - 0.0429p_1)\} \\ \text{s.t.} & p_1 \geq 280 \end{array}$$

$$\begin{aligned} p_1^* &= \$349.65 \\ d(p_1^*) &= 15 \\ revenue_{p_1^*} &= \$5{,}244.76 \end{aligned}$$

- Low-valuation segment revenue maximization:

$$\begin{array}{ll} \text{max} & p_2(12 - 0.0429p_2) \\ \text{s.t.} & p_2 \leq 280 \\ & p_2 \geq 0 \end{array}$$

$$\begin{aligned} p_2^* &= \$139.86 \\ d(p_2^*) &= 6 \\ revenue_{p_2^*} &= \$839.16 \end{aligned}$$

Revenue increased by $840!

# Tactics of Price Differentiation

- Group pricing
  - Collective buying power can influence prices
- Channel pricing
- Couponing and self-selection
- Product versioning
  - Modify product slightly to make it superior or inferior

# Limits of Price Differentiation

- Imperfect segmentation
  - Difficult to determine the exact valuations
- Cannibalization
  - Costumers of high-valuation segment have strong incentives to buy at the lower price
- Arbitrage
  - A third-party intermediary buys at low price, and sells it to high-valuation segment costumers below the segment price

# Dynamic Pricing

# Dynamic Pricing Problem

- You allocate 2 days to sell a single item
- Every day one potential buyer arrives:
  - On day $i = 1{,}2$ buyer $i$ arrives to the store and their willingness to pay, $v_i$, is drawn independently $v_i \sim U[0{,}100]$
- How do you price the item?

# Dynamic Pricing Problem, Solution

- The problem is to find $p_1$ and $p_2$ (prices for each period) to maximize the revenue
- Let $v_1$ and $v_2$ be the valuations (willingness to pay) of the buyers at period 1 and 2 respectively
- We solve:

$$max_{p_1,p_2} p_1 \times P(v_1 \geq p_1) + P(v_1 < p_1) \times p_2 \times P(v_2 \geq p_2)$$
$$= max_{p_1,p_2} p_1 \times \left(1 - \frac{p_1}{100}\right) + \frac{p_1}{100} \times p_2 \times \left(1 - \frac{p_2}{100}\right)$$

- The second term is the expected revenue in the second period
- So given assuming we know $p_1$ we solve:

$$max_{p_2} \frac{p_1}{100} \times p_2 \times \left(1 - \frac{p_2}{100}\right) \Rightarrow p_2 = 50$$

- Now we can solve for $p_1$:

$$max_{p_1} p_1 \times \left(1 - \frac{p_1}{100}\right) + \frac{p_1}{100} \times 50 \times \left(1 - \frac{50}{100}\right) \Rightarrow p_1 = 62.5$$