MS&E 260

Introduction to Operations Management

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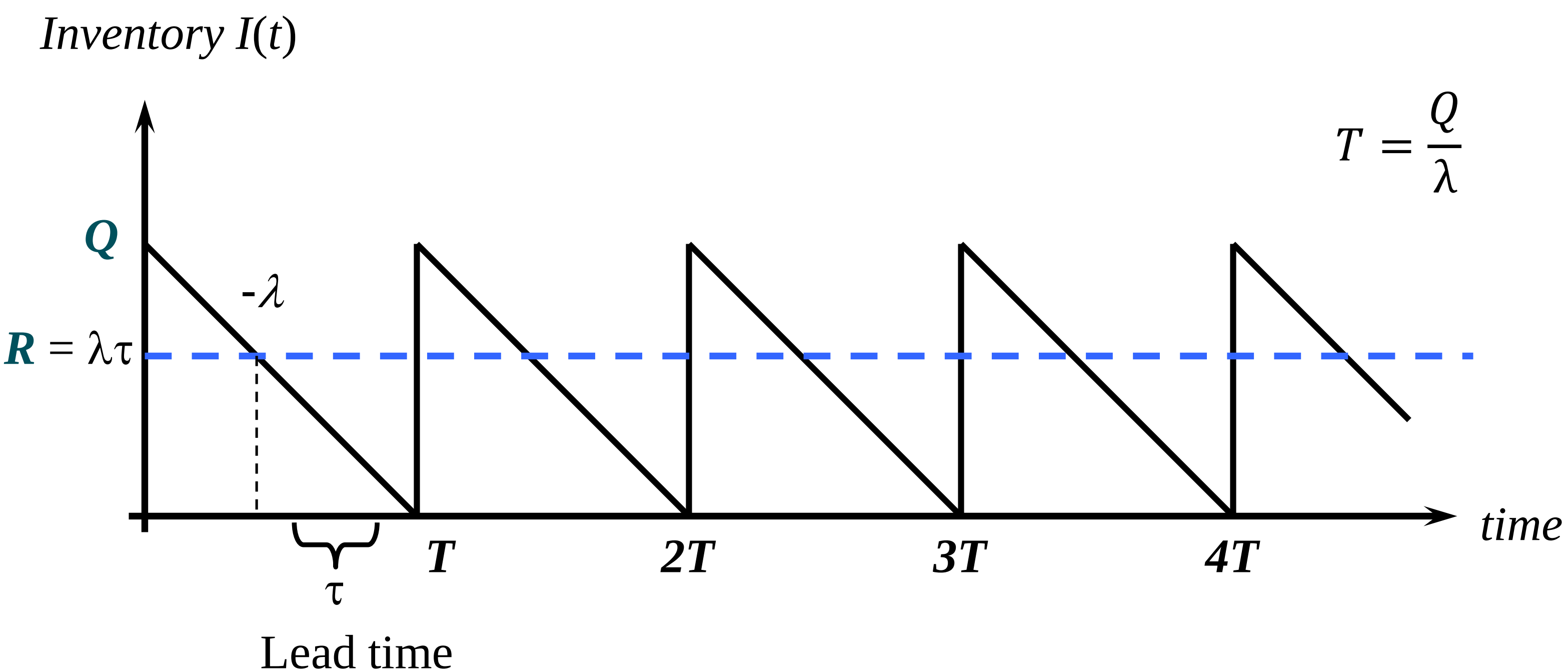
Newsvendor in Action

Inventory Management with Uncertain Demand

Inventory Control Subject to Uncertain Demand

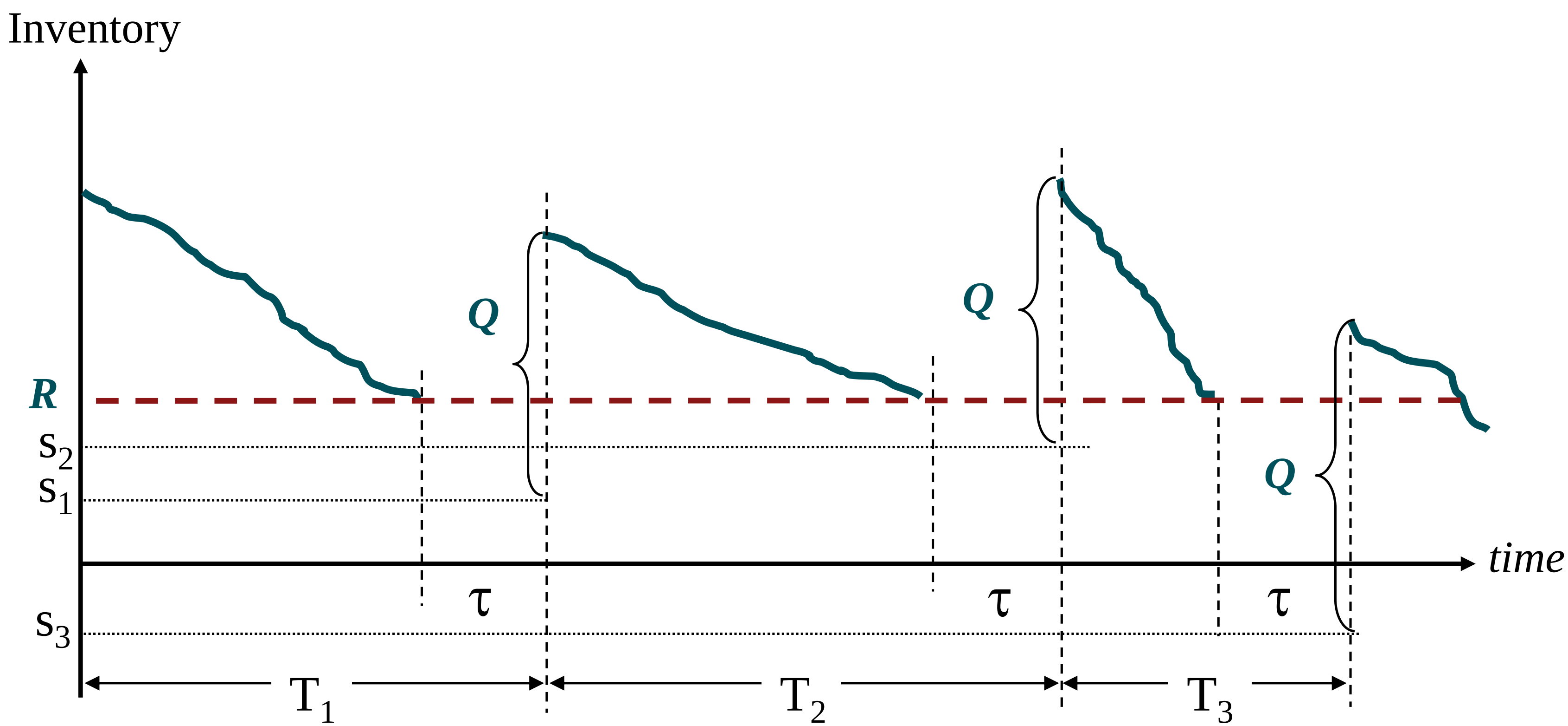
- Inventory Systems with Uncertain (Stochastic) Demand
  - Newsvendor Model (single period)
  - **Continuous Review (Q,R) Model (multiple periods)**
    - **Per-Unit Backorder Penalty**
    - Service Level (Fill Rate)

Recap: Basic EOQ



- Place an order when the inventory level is R. The order arrives after τ time periods
  - Q was the only decision variable
  - R could be computed easily because demand was deterministic

Uncertain Demand



- Both Q and R are decision variables
- Cycle time is no longer constant

(Q,R) Decisions

- We choose R to **meet the demand during lead time**
  - Service levels: Protect against uncertainties in demand (or lead time)
  - Balance the costs: stock-outs and inventory
- Tradeoff in Q: Fixed cost versus holding cost
- Objective:
  - Minimize expected
    - fixed cost + holding cost + stockout (backorder) cost

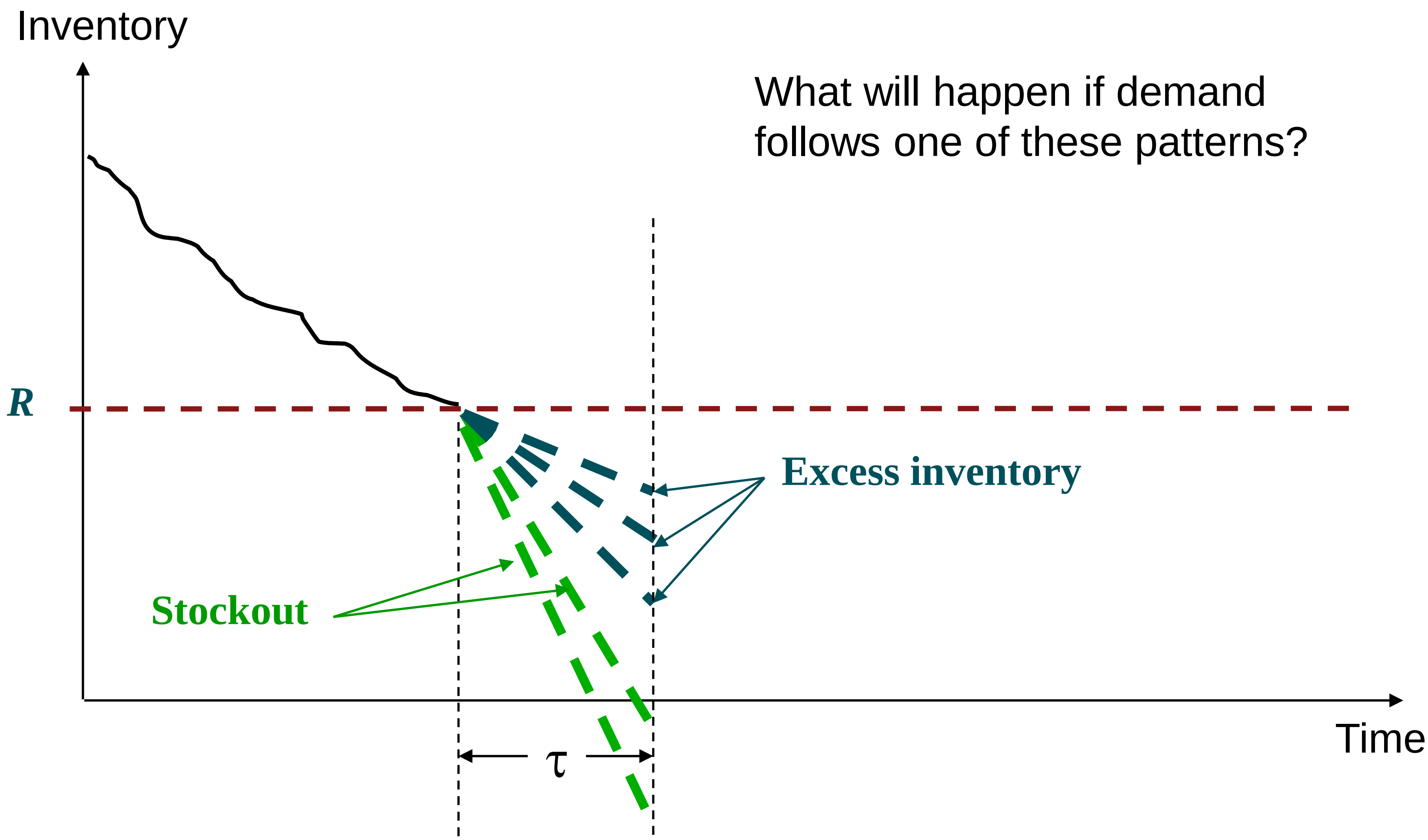
Q tradeoffs

R tradeoffs

(Q,R) Model Assumptions

- Inventory levels are reviewed continuously
- Single product or no product interactions
- **Demand** is **random** and **stationary**. Expected demand is λ per unit time
- Lead time is τ
  - Time elapsed is from the time an order is placed until it arrives
- The relevant costs are:
  - K Setup cost per order
  - h Holding cost per unit per unit time
  - c Purchase price (cost) per unit
  - p Penalty cost per unit of unsatisfied demand

Demand During Lead Time

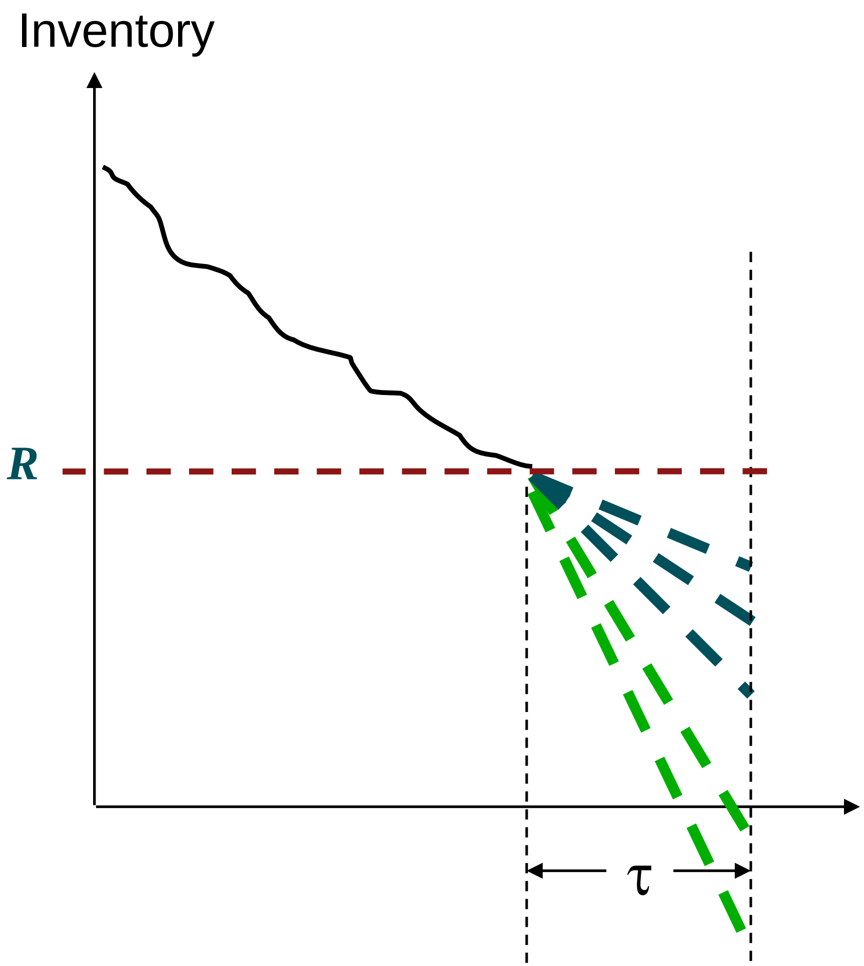


What will happen if demand follows one of these patterns?

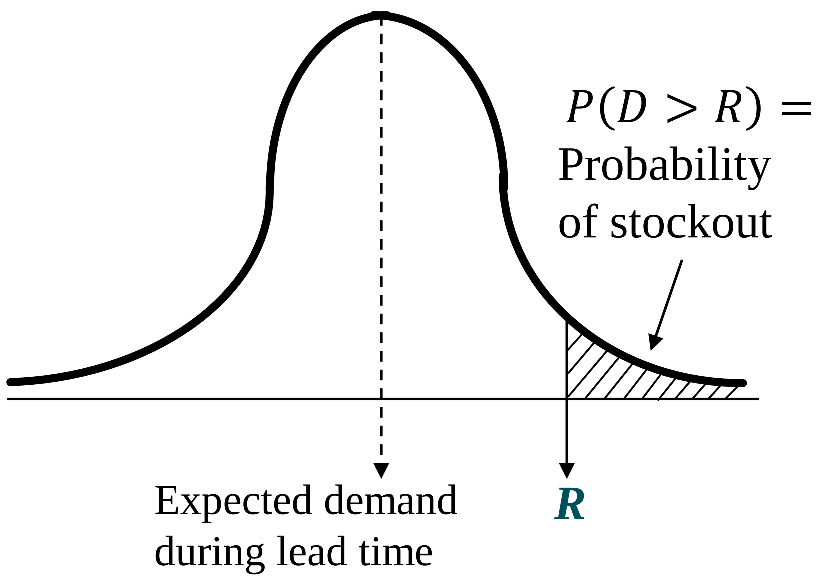
Describing Demand

- The response time of the system in this case is the time that elapses from the point an order is placed until it arrives
- Hence, the uncertainty that must be protected against is the uncertainty of demand during the lead time
- We assume that D represents the **demand during the lead time** and has probability distribution $f(x)$ with mean μ and standard deviation σ

Demand During Lead Time



Often the probability distribution of demand during lead time follows a Normal pattern



(Q,R) Model: Expected Total Cost per Unit Time

$$C(Q,R) = \overbrace{h\left(s + \frac{Q}{2}\right)}^{\text{holding cost}} + \underbrace{\frac{K}{T}}_{\text{fixed cost}} + \overbrace{p\left(\frac{n(R)}{T}\right)}^{\text{shortage cost}} \qquad T = \frac{Q}{\lambda}$$

s = average inventory level before an order arrives

$= (reorder\ level) - (expected\ demand\ during\ lead\ time) = R - \mu$

$n(R) =$ expected amount of shortage per cycle

$D > R \Rightarrow shortage = D - R$

$D < R \Rightarrow shortage = 0$

$$n(R) = \int_0^R 0 f(x)dx + \int_R^\infty (x-R)f(x)dx = \int_R^\infty (x-R)f(x)dx := \overbrace{\sigma L(z)}^{\substack{\text{Standard loss function} \\ (D\sim Normal)}}$$

(Q,R) Model: Expected Total Cost per Unit Time

$C(Q,R)$ = holding cost + fixed cost + shortage cost

$$= h\left(\frac{Q}{2} + R - \lambda\tau\right) + K\frac{\lambda}{Q} + p\left(\frac{\lambda n(R)}{Q}\right)$$

$$\boxed{Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}}}$$

$$\boxed{F(R) = 1 - \frac{Qh}{p\lambda}}$$

Questions: How do we pull Q and R from these equations?

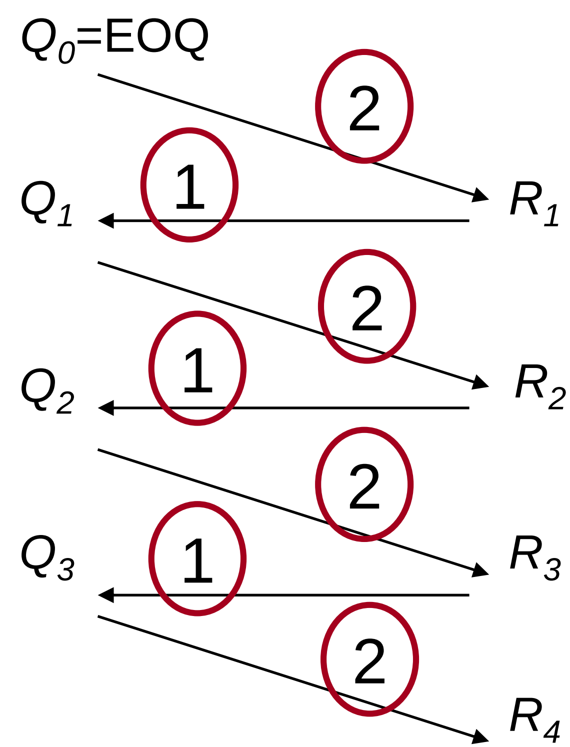
Answer: Solve iteratively!

Solving for Optimal Q and R

- Start with a Q_0 value and iterate until the **Q or R values converge**

1 $Q = \sqrt{\frac{2\lambda[K+pn(R)]}{h}}$

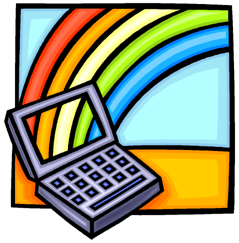
2 $F(R) = 1 - \frac{Qh}{p\lambda}$



Remember: To find Q, you need $n(R) = \sigma L(z)$. Lookup for z in the Normal Tables. Alternatively, you can calculate it using Normdist and Normsdist functions in Excel. Also:

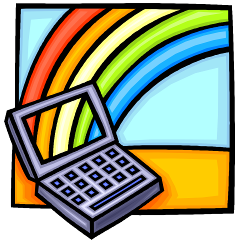
$$L(z) = \phi(z) - z\big(1 - \Phi(z)\big)$$

Remember $L(z)$ in Excel: '=NORMDIST(z,0,1,FALSE)-z*NORMSDIST(-z,0,1,TRUE)'



(Q,R) Model Example: Rainbow Colors

- Rainbow Colors paint store uses a (Q, R) inventory system to control its stock levels. For a popular eggshell latex paint, historical data show that the distribution of **monthly demand** is approximately **Normal**, with **mean 28** and **standard deviation 8**. Replenishment **lead time** for this paint is about **14 weeks**. Each can of paint **costs** the store **$6**. Although excess demands are backordered, each unit of **stockout** costs about **$10** due to bookkeeping and loss of goodwill. **Fixed cost** of replenishment is **$15** per order and holding costs are based on a **30% annual interest rate**.
- What is the optimal lot size (order quantity) and reorder level?
- What is the expected inventory level (safety stock) just before an order arrives?



(Q,R) Model Example: Rainbow Colors

- Given Input:
  - Monthly demand: Normal with mean 28 and standard deviation 8
  - $\tau = 14$ weeks
  - $c = \$6$, $p = \$10$, $K = \$15$
  - $h = ic = (0.3)(6) = \$1.8$/unit/year
- Computed input:
  - $\lambda = (28)(12) = 336$ units/year (expected annual demand)
  - Expected demand during lead time

$$\mu = \frac{(28)(12)\; units/year}{52\; weeks/year} \times (14\; weeks) = 90\; units$$

  - Variance of demand during lead time

annual variance $= (12)(8^2) = 768$

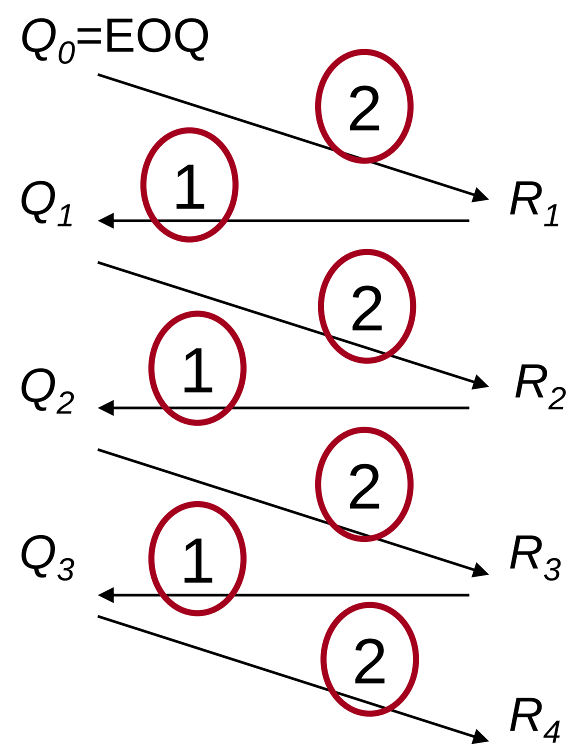
variance of lead time demand $= 768 \times \frac{14}{52} = 206.77 \Rightarrow \sigma = 14.38$

Solving for Optimal Q and R

- Start with a Q_0 value and iterate until the Q values converge

1 $Q = \sqrt{\frac{2\lambda[K+pn(R)]}{h}}$

2 $F(R) = 1 - \frac{Qh}{p\lambda}$



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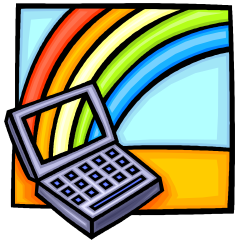
(Q,R) Model Example: Rainbow Colors



- Iteration 0: Computer EOQ

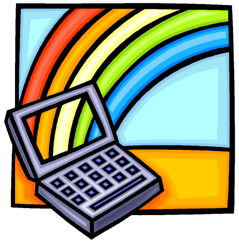
$$Q = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2(15)(336)}{1.8}} = 75$$

- Iteration 1: **Compute $\boldsymbol{R_1}$ (given $\boldsymbol{Q_0}$)** and then compute Q_1 (given R_1)

$$F(R) = 1 - \frac{Qh}{p\lambda} = 1 - \frac{(75)(1.8)}{(10)(336)} \approx 0.96 = \Phi(z) \Rightarrow z = 1.75$$

$$R = \sigma z + \mu \Rightarrow R_1 = (14.38)(1.75) + 90 \approx 115$$

(Q,R) Model Example: Rainbow Colors



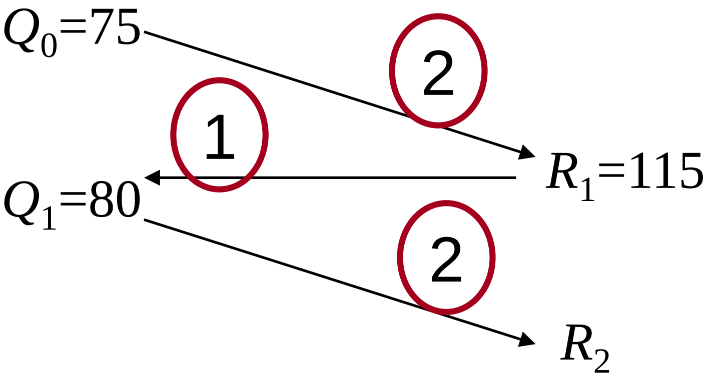
- Iteration 1 (continued): Compute Q_1 (given R_1)

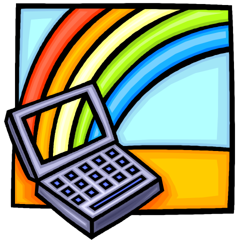
$$Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}}$$

$$n(R_1) = \sigma L(1.75) = (14.38)(0.0162) = 0.233$$

$$Q_1 = \sqrt{\frac{2(336)[15 + (10)(0.233)]}{1.8}} \approx 80$$

- Q_0 and Q_1 are not close, continue iterating





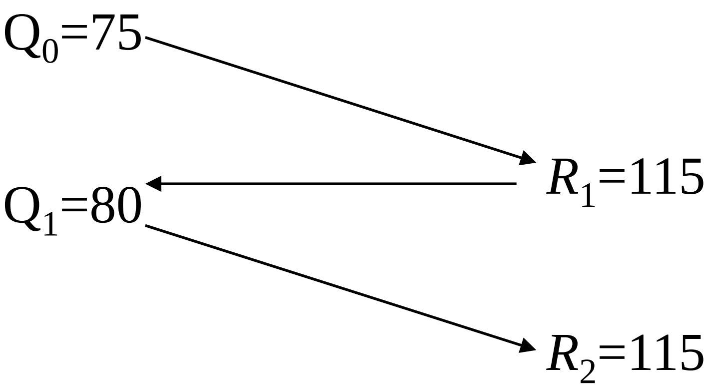
(Q,R) Model Example: Rainbow Colors

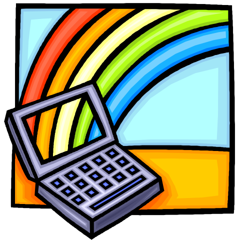
- Iteration 2: **Compute $\boldsymbol{R_2}$ (given $\boldsymbol{Q_1}$)** and then compute Q_2 (given R_2)

$$F(R_1) = 1 - \frac{Q_1 h}{p\lambda} = 1 - \frac{(80)(1.8)}{(10)(336)} \approx 0.957 = \Phi(z) \Rightarrow z = 1.72$$

$$R = \sigma z + \mu \Rightarrow R_2 = (14.38)(1.72) + 90 \approx 115$$

STOP! R values have converged,
optimal $(Q, R) = (80{,}115)$





(Q,R) Model Example: Rainbow Colors

- $(Q, R) = (80{,}115)$
  - Reorder level is larger than expected demand during the lead time. Why?
  - Optimal order quantity is larger than EOQ. Why?

Sensitivity Analysis with Respect to Q

$$Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}}$$

$$F(R) = 1 - \frac{Qh}{p\lambda}$$

- As the order quantity Q increases:
  - There are fewer order cycles per unit time
  - The impact of the shortage term $pn(R)$ decreases
  - Less safety stock is required
  - There are higher holding costs (for $Q > Q^*$)

Sensitivity Analysis with Respect to R

$$Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}}$$

$$F(R) = 1 - \frac{Qh}{p\lambda}$$

- As the reorder level R increases:
  - There are fewer expected shortages per cycle ($n(R)$ decreases)
  - This reduces the expected shortage cost incurred in each cycle
  - Therefore, the order quantity decreases

Summary: (Q,R) Models

- Balance between holding cost, setup/fixed cost, and shortage cost
  - To save on the **shortage cost**, we want **large $\boldsymbol{R}$**
  - To save on the **holding cost**, we want **small $\boldsymbol{Q}$** and **small $\boldsymbol{R}$**
  - To save on the **fixed cost**, we want **large $\boldsymbol{Q}$**
- Choose Q and R to strike a good balance among these three costs

Inventory Control Subject to Uncertain Demand

- Inventory Systems with Uncertain (Stochastic) Demand
  - Newsvendor Model (single period)
  - Continuous Review (Q,R) Model (multiple periods)
    - Per-Unit Backorder Penalty
    - **Service Level (Fill Rate)**

Service Levels

- In many circumstances, the penalty cost, p, is difficult to estimate
- For this reason, it is common business practice to set inventory levels to meet a specified service objective instead
- The two most common service objectives are:
  - Type I service level (α)
    - The proportion of cycles in which no stockouts occur
    - Example: 90% Type I service level $\Rightarrow$ There are no stockouts in 9 out of 10 cycles (on average)
  - Type II service level (fill rate, β)
    - Fraction of demand satisfied on time

*See Nahmias for additional explanation

Service Levels Example

Order cycle	Demand	Stock-outs
1	180	0
2	75	0
3	235	150
4	140	0
5	180	0
6	200	140
7	150	0
8	90	0
9	160	0
10	40	0
TOTAL:	1450	290

Fraction of periods with no stock-outs $= \frac{8}{10}$

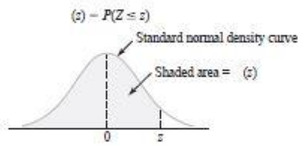
Type I service = 80% (α = 0.8)

Fraction of demand satisfied on time $= \frac{1450-290}{1450} = 0.8$

Type II service = 80% (β = 0.8)

Normal z-Table

Table A.3 Standard Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas *(cont.)* $\Phi(z) = P(Z \le z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Normal Loss Function

Standard Normal Loss Function Table, $L(z)$

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00
−4.0	4.0900	4.0800	4.0700	4.0600	4.0500	4.0400	4.0300	4.0200	4.0100	4.0000
−3.9	3.9900	3.9800	3.9700	3.9600	3.9500	3.9400	3.9300	3.9200	3.9100	3.9000
−3.8	3.8900	3.8800	3.8700	3.8600	3.8500	3.8400	3.8300	3.8200	3.8100	3.8000
−3.7	3.7900	3.7800	3.7700	3.7600	3.7500	3.7400	3.7300	3.7200	3.7100	3.7000
−3.6	3.6900	3.6800	3.6700	3.6600	3.6500	3.6400	3.6300	3.6200	3.6100	3.6000
−3.5	3.5900	3.5800	3.5700	3.5600	3.5500	3.5400	3.5301	3.5201	3.5101	3.5001
−3.4	3.4901	3.4801	3.4701	3.4601	3.4501	3.4401	3.4301	3.4201	3.4101	3.4001
−3.3	3.3901	3.3801	3.3701	3.3601	3.3501	3.3401	3.3301	3.3201	3.3101	3.3001
−3.2	3.2901	3.2801	3.2701	3.2601	3.2502	3.2402	3.2302	3.2202	3.2102	3.2002
−3.1	3.1902	3.1802	3.1702	3.1602	3.1502	3.1402	3.1302	3.1202	3.1103	3.1003
−3.0	3.0903	3.0803	3.0703	3.0603	3.0503	3.0403	3.0303	3.0204	3.0104	3.0004
−2.9	2.9904	2.9804	2.9704	2.9604	2.9505	2.9405	2.9305	2.9205	2.9105	2.9005
−2.8	2.8906	2.8806	2.8706	2.8606	2.8506	2.8407	2.8307	2.8207	2.8107	2.8008
−2.7	2.7908	2.7808	2.7708	2.7609	2.7509	2.7409	2.7310	2.7210	2.7110	2.7011
−2.6	2.6911	2.6811	2.6712	2.6612	2.6512	2.6413	2.6313	2.6214	2.6114	2.6015
−2.5	2.5915	2.5816	2.5716	2.5617	2.5517	2.5418	2.5318	2.5219	2.5119	2.5020
−2.4	2.4921	2.4821	2.4722	2.4623	2.4523	2.4424	2.4325	2.4226	2.4126	2.4027
−2.3	2.3928	2.3829	2.3730	2.3631	2.3532	2.3433	2.3334	2.3235	2.3136	2.3037
−2.2	2.2938	2.2839	2.2740	2.2641	2.2542	2.2444	2.2345	2.2246	2.2147	2.2049
−2.1	2.1950	2.1852	2.1753	2.1655	2.1556	2.1458	2.1360	2.1261	2.1163	2.1065
−2.0	2.0966	2.0868	2.0770	2.0672	2.0574	2.0476	2.0378	2.0280	2.0183	2.0085
−1.9	1.9987	1.9890	1.9792	1.9694	1.9597	1.9500	1.9402	1.9305	1.9208	1.9111
−1.8	1.9013	1.8916	1.8819	1.8723	1.8626	1.8529	1.8432	1.8336	1.8239	1.8143
−1.7	1.8046	1.7950	1.7854	1.7758	1.7662	1.7566	1.7470	1.7374	1.7278	1.7183
−1.6	1.7087	1.6992	1.6897	1.6801	1.6706	1.6611	1.6516	1.6422	1.6327	1.6232
−1.5	1.6138	1.6044	1.5949	1.5855	1.5761	1.5667	1.5574	1.5480	1.5386	1.5293
−1.4	1.5200	1.5107	1.5014	1.4921	1.4828	1.4736	1.4643	1.4551	1.4459	1.4367
−1.3	1.4275	1.4183	1.4092	1.4000	1.3909	1.3818	1.3727	1.3636	1.3546	1.3455
−1.2	1.3365	1.3275	1.3185	1.3095	1.3006	1.2917	1.2827	1.2738	1.2650	1.2561
−1.1	1.2473	1.2384	1.2296	1.2209	1.2121	1.2034	1.1946	1.1859	1.1773	1.1686
−1.0	1.1600	1.1514	1.1428	1.1342	1.1257	1.1172	1.1087	1.1002	1.0917	1.0833
−0.9	1.0749	1.0665	1.0582	1.0499	1.0416	1.0333	1.0250	1.0168	1.0086	1.0004
−0.8	0.9923	0.9842	0.9761	0.9680	0.9600	0.9520	0.9440	0.9360	0.9281	0.9202
−0.7	0.9123	0.9045	0.8967	0.8889	0.8812	0.8734	0.8658	0.8581	0.8505	0.8429
−0.6	0.8353	0.8278	0.8203	0.8128	0.8054	0.7980	0.7906	0.7833	0.7759	0.7687
−0.5	0.7614	0.7542	0.7471	0.7399	0.7328	0.7257	0.7187	0.7117	0.7047	0.6978
−0.4	0.6909	0.6840	0.6772	0.6704	0.6637	0.6569	0.6503	0.6436	0.6370	0.6304
−0.3	0.6239	0.6174	0.6109	0.6045	0.5981	0.5918	0.5855	0.5792	0.5730	0.5668
−0.2	0.5606	0.5545	0.5484	0.5424	0.5363	0.5304	0.5244	0.5186	0.5127	0.5069
−0.1	0.5011	0.4954	0.4897	0.4840	0.4784	0.4728	0.4673	0.4618	0.4564	0.4509
0.0	0.4456	0.4402	0.4349	0.4297	0.4244	0.4193	0.4141	0.4090	0.4040	0.3989

Standard Normal Loss Function Table, $L(z)$ (Concluded)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.3989	0.3940	0.3890	0.3841	0.3793	0.3744	0.3697	0.3649	0.3602	0.3556
0.1	0.3509	0.3464	0.3418	0.3373	0.3328	0.3284	0.3240	0.3197	0.3154	0.3111
0.2	0.3069	0.3027	0.2986	0.2944	0.2904	0.2863	0.2824	0.2784	0.2745	0.2706
0.3	0.2668	0.2630	0.2592	0.2555	0.2518	0.2481	0.2445	0.2409	0.2374	0.2339
0.4	0.2304	0.2270	0.2236	0.2203	0.2169	0.2137	0.2104	0.2072	0.2040	0.2009
0.5	0.1978	0.1947	0.1917	0.1887	0.1857	0.1828	0.1799	0.1771	0.1742	0.1714
0.6	0.1687	0.1659	0.1633	0.1606	0.1580	0.1554	0.1528	0.1503	0.1478	0.1453
0.7	0.1429	0.1405	0.1381	0.1358	0.1334	0.1312	0.1289	0.1267	0.1245	0.1223
0.8	0.1202	0.1181	0.1160	0.1140	0.1120	0.1100	0.1080	0.1061	0.1042	0.1023
0.9	0.1004	0.0986	0.0968	0.0950	0.0933	0.0916	0.0899	0.0882	0.0865	0.0849
1.0	0.0833	0.0817	0.0802	0.0787	0.0772	0.0757	0.0742	0.0728	0.0714	0.0700
1.1	0.0686	0.0673	0.0659	0.0646	0.0634	0.0621	0.0609	0.0596	0.0584	0.0573
1.2	0.0561	0.0550	0.0538	0.0527	0.0517	0.0506	0.0495	0.0485	0.0475	0.0465
1.3	0.0455	0.0446	0.0436	0.0427	0.0418	0.0409	0.0400	0.0392	0.0383	0.0375
1.4	0.0367	0.0359	0.0351	0.0343	0.0336	0.0328	0.0321	0.0314	0.0307	0.0300
1.5	0.0293	0.0286	0.0280	0.0274	0.0267	0.0261	0.0255	0.0249	0.0244	0.0238
1.6	0.0232	0.0227	0.0222	0.0216	0.0211	0.0206	0.0201	0.0197	0.0192	0.0187
1.7	0.0183	0.0178	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146
1.8	0.0143	0.0139	0.0136	0.0132	0.0129	0.0126	0.0123	0.0119	0.0116	0.0113
1.9	0.0111	0.0108	0.0105	0.0102	0.0100	0.0097	0.0094	0.0092	0.0090	0.0087
2.0	0.0085	0.0083	0.0080	0.0078	0.0076	0.0074	0.0072	0.0070	0.0068	0.0066
2.1	0.0065	0.0063	0.0061	0.0060	0.0058	0.0056	0.0055	0.0053	0.0052	0.0050
2.2	0.0049	0.0047	0.0046	0.0045	0.0044	0.0042	0.0041	0.0040	0.0039	0.0038
2.3	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
2.4	0.0027	0.0026	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021
2.5	0.0020	0.0019	0.0019	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015
2.6	0.0015	0.0014	0.0014	0.0013	0.0013	0.0012	0.0012	0.0012	0.0011	0.0011
2.7	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008
2.8	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
2.9	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004
3.0	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
3.1	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.2	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001
3.3	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.4	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.5	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Service Levels in Production: Krispy Kreme