

MS&E 260 Homework 4

Summer 2019, Stanford University

Due: July 31st, 2019, at 10:30AM (PDT)

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Problem 1. Imagine a two-firm supply chain that consists of a supplier and a retailer. The supplier has a marginal cost $c = \$40$ and a wholesale price $w = \$60$. The retailer is looking to sell its product at $p = \$100$; at this price point, demand over the lifespan of the product is distributed normally with mean 200 and standard deviation of 10. The salvage value of the product is $s = \$5$ for the retailer.

Given:

$$c = \$40 \quad w = \$60 \quad p = \$100 \quad s = \$5 \quad \mu = 200 \quad \sigma = 10$$

(a) What is the expected profit for the supplier and the retailer?

$$c_o = \text{wholesale price} - \text{salvage value} = 60 - 5 = 55$$

$$c_u = \text{sell price} - \text{wholesale price} = 100 - 60 = 40$$

$$\text{critical ratio} = \frac{c_u}{c_u + c_o} = \frac{40}{40 + 55} = 0.421$$

$$z(0.421) = -0.1993, \quad L(z) = 0.5065$$

$$Q^* = \mu + \sigma z = 200 + 10 * (-0.1993) = 198.007 \cong 198$$

$$E[\text{Sales}_R] = \mu - \sigma L(z) = 200 - 10(0.5065) = 194.93 \cong 195$$

$$E[\text{LeftOver}_R] = Q^* - E[\text{Sales}_R] = 198 - 195 = 3$$

$$E[\text{Profit}_R] = E[\text{Sales}_R] p - Q^* w + E[\text{LeftOver}_R] s = 195 * 100 - 198 * 60 + 3 * 5 = 7635$$

$$E[\text{Profit}_S] = Q^* (w - c) = 198 * (60 - 40) = 3960$$

(b) Suppose the supply chain was integrated (or alternatively, the supply chain is perfectly coordinated.) What is the optimal order quantity of the retailer? What is the expected total profit for the supply chain?

$$c_o = 40 - 5 = 35$$

$$c_u = 100 - 40 = 60$$

$$\text{critical ratio} = \frac{c_u}{c_u + c_o} = \frac{60}{60 + 35} = 0.6315$$

$$z(0.6315) = 0.3358, \quad L(z) = 0.2533$$

$$Q^* = \mu + \sigma z = 200 + 10 * (0.3358) = 203.35 \cong 203$$

$$E[\text{Sales}] = \mu - \sigma L(z) = 200 - 10(0.2533) = 197.467 \cong 197$$

$$E[\text{LeftOver}] = Q^* - E[\text{Sales}_R] = 203 - 197 = 6$$

$$E[\text{Profit}_{SC}] = E[\text{Sales}] p - Q^* c + E[\text{LeftOver}] s = 197 * 100 - 203 * 40 + 6 * 5 = 11610$$

- (c) Now assume the firms seek to form a revenue sharing contract. Let f be the upfront fee and θ be the revenue share percentage. Suppose they decide on an upfront fee of \$30 per unit. What is the share θ so that the chain is perfectly coordinated? What is the expected profit for the retailer under this contract?

$$f = 30 \quad c = \$40 \quad p = \$100 \quad s = \$5$$

$$\frac{p - c}{p - s} = \frac{\theta p - f}{\theta p - s} \Rightarrow \frac{100 - 40}{100 - 5} = \frac{\theta 100 - 30}{\theta 100 - 5} \Rightarrow \theta = 0.7286$$

$$\text{critical ratio} = \frac{\theta p - f}{\theta p - s} = \frac{0.7286 * 100 - 30}{0.7286 * 100 - 5} = 0.6316$$

$$z = 0.336$$

$$L(z) = 0.2532$$

$$Q^* = \mu + \sigma z = 200 + 10 (0.336) = 203.36 \cong 203$$

$$E[\text{Sales}_R] = \mu - \sigma L(z) = 200 - 10(0.2523) = 197.477 \cong 197$$

$$E[\text{Profit}_R] = \theta_R p E[\text{Sales}_R] - f Q^* = 0.7286 * 100 * 197 - 30 * 203 = 8263.42 \cong 8263$$

- (d) Suppose the salvage value for the retailer is \$0, but the salvage value for the supplier is $s_s = \$5$. Assume now that the firms are looking to form a buyback contract. What would be the buy-back price that perfectly coordinates the supply chain assuming the wholesale price remains \$60 per unit? What is the expected profit for the supplier and the retailer?

$$c = \$40 \quad w = \$60 \quad p = \$100 \quad s_s = \$5 \quad \mu = 200 \quad \sigma = 10$$

$$\Rightarrow b = p - (p - w) \times \frac{p - s}{p - c} = 100 - (100 - 60) \times \frac{100 - 5}{100 - 40} = 36.667$$

For a perfect coordination = critical ratio = 0.6351 (as calculated in Part b)

$$z(0.6315) = 0.3358, \quad L(z) = 0.2533$$

$$Q^* = \mu + \sigma z = 200 + 10 * (0.3358) = 203.35 \cong 203$$

$$E[\text{Sales}_R] = \mu - \sigma L(z) = 200 - 10 (0.2533) = 197.467 \cong 197$$

$$E[\text{LeftOver}_R] = Q^* - E[\text{Sales}_R] = 203 - 197 = 6$$

$$E[\text{Profit}_R] = E[\text{Sales}_R] p - Q^* w + E[\text{LeftOver}_R] s = 197 * 100 - 203 * 60 + 6 * 36.667 = 7740$$

$$E[\text{Profit}_S] = Q^* (w - c) - E[\text{LeftOver}_R] b + E[\text{LeftOver}_R] s = 203 * (60 - 40) - 6 * 36.667 + 6 * 5 = 3869.998 \cong 3870$$

Problem 2. Company A produces two types of items, product 1 and product 2. The marginal cost of producing product 1 is \$8, and the annual total expected demand is normally distributed with $\mu = 300$ and standard deviation 80. The marginal cost of producing product 2 is \$10, and its annual total expected demand is normally distributed with $\mu = 200$ and standard deviation 50. The production lead time is six months.

Company B is the retailer of these two products, and the retail prices for product 1 and 2 are \$20 and \$24, respectively. The products not sold at the end of the sales period will be sold to another company at a price of \$3 for product 1 and \$4 for product 2.

(a) Company A charges a wholesale price \$13 for product 1 and a wholesale price \$15 for product 2.

What are the optimal quantities q^*_1 and q^*_2 for products 1 and 2 respectively such that Company B maximizes its profits?

	c	μ	σ	p	s	w	c_o	c_u	critical ratio $\frac{c_u}{c_u + c_o}$	z	$Q^* = \mu + \sigma z$
Product 1	8	300	80	20	3	13	$13-3 = 10$	$20-13 = 7$	0.4117	-0.2231	$= 300 + 80(-0.2231)$ $= 282.152$ $\cong 282$
Product 2	10	200	50	24	4	15	$15-4 = 11$	$24-15 = 9$	0.45	-0.1256	$= 200 + 50(-0.1256)$ $= 193.72$ $\cong 194$

(b) Assume now that the management team of Company A is not satisfied with this agreement for product 1 and wants to develop a new supply chain contract. The following two types of contracts are available. Which of them should Company B accept?

- Contract 1: Buy-back contract: $w = 13$ and $b = 5$
- Contract 2: Revenue-sharing contract: $f = 8$ and share = 85%

	Contract Params	critical ratio	Q^* $= \mu + \sigma z$	$E[\text{Sales}_R]$ $= \mu - \sigma L(z)$	$E[\text{Profit}_R]$
Contract 1 Buy back	$b = 5$	$= \frac{p - w}{p - b}$ $= \frac{20 - 13}{20 - 5}$ $= 0.4667$	$= 300 + 80$ (-0.0835) $= 293.32$ $\cong 293$	$= 300 - 80 * (0.4421)$ $= 264.63$ $\cong 265$	$= E[\text{Sales}_R]p - Q^* w + (Q^* - E[\text{Sales}_R]) b$ $= 265 * 20 - 293 * 13 + (293 - 265) 5$ $= 1631$
Contract 2 Revenue Sharing	$f = 8$ share = 85%	$= \frac{\theta p - f}{\theta p - s}$ $= \frac{(0.85)20 - 8}{(0.85)20 - 3}$ $= 0.6428$	$= 300 + 80$ (0.3659) $= 329.272$ $\cong 329$	$= 300 - 80 * (0.2423)$ $= 280.616$ $\cong 281$	$= \theta_R p E[\text{Sales}_R] - f Q^*$ $= 0.85 * 20 * 281 - 8 * 329$ $= 2145$

Thus the Company B (retailer) should go with Contract 2 Revenue sharing as the profit numbers are higher in that case.

Problem 3. Watch the following video: <https://youtu.be/wPkhyKGJLvs>

In the video, Joe references a problem regarding excessive demand for Disney restaurants. What are some ways in which Joe can use operations management in order to solve this problem? In your response, discuss three ways to solve this problem, using concepts such as revenue management, capacity optimization, and supply chain management, among others. Make sure to provide a qualitative discussion on how these proposed solutions will influence the underlying mathematical models associated with the concepts (such as revenue management, capacity optimization, and supply chain management).

The video is presented by Joe who is the director of pricing and revenue management at Disney land and Disney world. In his talk, there are multiple problems that he focuses on e.g. problem of getting a table at the restaurant as they are full, finding what visitors want and get in the right restaurant, how to put right pricing and packaging, etc. Joe and his team at Disney uses operations management and quantitative analysis for all the solutions and apply them globally at multiple locations. I think it's a genuine problem to have for a park that receives so many visitors for a whole day. Some suggestions that can help resolve part of the problem:

1. wait times during popular lunch and dinner hours
just like showing average ride wait times via Disney's digital FastPass app, average wait time at restaurants can also be shown on the app. That will redistribute the load and not everyone will gather in restaurants next to the popular rides. This will make sure that all the space available throughout Disney will be optimally utilized during popular lunch and dinner hours and remaining time as well. Also the food inventory at each restaurant will be evenly utilized resulting in less losses.
2. Revenue sharing food court
Food courts are really popular and get a lot of visitors as everyone in a party has so many options to grab food from. Disney should extend more food courts and have suppliers run them. In return Disney can ask for revenue share.
3. grab and go food trucks and kiosks
Disney can create more grab and go food trucks/kiosks in the parks. A lot of food like sushi, cold sandwiches, fruit salads, pre-packages salads, hot dogs, etc. can be served on the go by a couple of servers. This will take some revenue from dine-in restaurants but initial setup costs is also very less here. The revenue can be kept unaffected by pricing the item right.

Problem 4. Discuss the differences between the revenue sharing contract model and the buyback contract model. In particular, what types of situations would one be advantageous over the other?

In revenue sharing contract suppliers pay a certain upfront fee and then shares a percent of revenue each item sold. While in buyback model, retailer and supplier decides on a price at which supplier will buy back the unsold units from retailer. Both the models are popular and are widely used. There can be multiple factors that supplier and retailers can look into before deciding which model is best suited for them.

If the shipping cost is huge or the items are highly perishable buyback model might not be suitable. Neither retailer nor supplier would want to take the cost of shipping back or perishing products. When the items are nonperishable (like furniture) and can be sold in other market, buy back model would work out great for both the parties.

Demand variation also plays a good role in deciding the contract. For items in regular constant use e.g. toothpaste, hand soap, beverages, etc. usually have a definite demand. These products will be sold eventually if not right now. Bearing a cost of shipping back might not be a good idea here and revenue sharing should work out great. Just like an example discussed in the class about how Costco utilizes revenue sharing for wine and other products on racks.

A lot of time it might be personal financial preference as well. Retailer with higher capital to risk can go with buyback contract to maximize the profits., while retailers with lower risk money might want to explore revenue sharing model.

Throughout the supply chain logistics there can be multiple factors like shipping, weather, perishability, capital, holding costs, etc. Each factor should be considered in contract selection process as overall profit will be affected by them.

Problem 5. Read the following articles:

- <https://www.forbes.com/sites/paulmartyn/2018/04/27/the-nfl-draft-a-data-driven-supply-chain-if-ever-there-was/#6644401443cf>
- <https://www.sunherald.com/news/local/military/article161039534.html>

Select one of these two articles, and do the following:

- 1) Briefly summarize the article.
- 2) Provide a few interpretive thoughts on the article, using what you have learned from class.
- 3) Provide one recommendation on how the dilemma posed in the article could be resolved.

Some notes:

- Please limit your responses to one page, double spaced, 12 point font.
- There is no right answer to this question. We are evaluating your ability to apply what you learn in class to practical applications.
- This question is not intended to be free points. If you do not demonstrate a sufficient level of critical thinking, full credit will not be awarded.

The article “The NFL Draft – A Data – Driven Supply Chain If Ever There Was” by Paul Martyn on Apr 27, 2018 tries to cover the aspects and details involved in NFL player selection process. As mentioned in the article all the coaches from different teams re-strategize during this event based on past data, what has worked what has not, and their finances to select next set of players for the team. Multiple factors are discussed such as supply management, sourcing management, finances and the profit impact of each decision taken. Not just budget allocation amongst players and resources affect the outcome of the teams, but factors like medical history of the players, depth of the injuries, injury resolution time, team coordination, personal performance ego and many more personal historic data also plays a big role in the picks.

In my childhood during a game of basketball, football or cricket, all the players used to make a circle facing inwards, put our palms facing down on top of each other and then at the count of 3 we all have an option to flip our palms. Whosoever is palm faced up goes to one team and the rest goes to other team. That was a pretty random process keeping coordination and games interesting. Some games used to be better, while some games used to be just one sided based on the random selection. This was all feasible as no money was involved there and goal was to have fun and workout. But that’s the risk coaches cannot afford to take. Coaches need to make sure they are executing the most efficient process to minimize resources on non-critical items and maximize on critical ones.

On a higher level one might think that more the money, better the team and so will be the outcome. But this is just not true at many levels. NFL is a team sports league and each team needs to work together with players possessing different skills blending in together. Programming model methods such as linear programming, dynamic programming, convex optimization, etc. should help a great deal here. Each factor considered should be converted to a probability e.g. out of last 50 games if the player got injured in 10 of them, injury probability is 10/50, or if the player touch down in 6 games, the touch down probability is 6/50. Using these probabilities modeling equations can be formed for offence and defense teams separately and hence optimal solution can be found. This might give a good enough guestimate but for sure real life non-mathematical factors might take over precedence and hence should be prioritized as such.