MS&E 260 Homework 4 Solutions

Summer 2019, Stanford University

Due: July 31st, 2019, at 10:30AM (PDT)

Problem 1. (a) We first calculate the

$$\text{Underage cost: } c_u = 100 - 60 = 40$$
$$\text{Overage cost: } c_o = 60 - 5 = 55$$

The retailer's critical ratio is $\frac{c_u}{c_u+c_o} = \frac{40}{55+40} = 0.42$. So $z = \Phi^{-1}(0.42) = -0.1992$.

We can evaluate the retailer's expected profit. Let π_r denote the profit:

$$E[\pi_r] = pE[\text{sales}] - wQ + s(Q - E[\text{sales}]) = p[\mu - \sigma \cdot L(z)] - cQ + s(Q - \mu + \sigma \cdot L(z))$$

We have that:

$$E[\text{sales}] = \mu - \sigma \cdot L(z) = 200 - 10 \times 0.5064 = 194.94$$
$$E[\text{leftover}] = Q - E[\text{sales}] = 198 - 194.94 = 3.06$$

so we finally get that $E[\mu_r] \approx \$7{,}629$.

The supplier's expected profit is $E[\pi_s] = (w - c) \times Q = \$3{,}960$

(b) $c_u = 100 - 40 = 60$ and $c_o = 40 - 5 = 35$

The supply chain's critical ratio is $\frac{c_u}{c_u+c_o} = \frac{60}{60+35} = 0.632$. So $z = \Phi^{-1}(0.632) = 0.336$.

Hence the retailer's optimal order quantity is $Q = \mu + \sigma \cdot z = 200 + 0.336 \times 10 \approx 203$.

We can evaluate the integrated chain's expected profit:

$$E[\text{sales}] = \mu - \sigma \cdot L(z) = 200 - 10 \times 0.245 \approx 197.5$$
$$E[\text{leftover}] = Q - E[\text{sales}] = 203 - 197.5 = 5.5$$
$$E[\pi] = \$11{,}657.50$$

(c) Critical ratio:

$$\frac{p-c}{p-s} = \frac{\theta p - f}{\theta p - s} \Rightarrow \frac{60}{95} = \frac{100\theta - 30}{100\theta - 5}$$

Hence, $\theta = 0.729$. The expected profit is the same as in the integrated firm case, which is \$11,657.50.

(d) Optimal buy back price:

$$\begin{aligned}
\frac{p-c}{p-s} &= \frac{p-w}{(w-b)+p-w} \\
b &= p-(p-w)\cdot\frac{p-s}{p-c} \\
&= 100-(100-60)\times\frac{100-5}{100-40} \\
&= \$36.67
\end{aligned}$$

We have $\frac{c_u}{c_u+c_o} = \frac{40}{63.33} = 0.632$ and $z = \Phi^{-1}(0.632) = 0.336$. So the retailer's optimal order quantity is $Q = \mu + \sigma \cdot z = 200 + 0.336 \times 10 \approx 203$.

Retailer's expected profit:

$$\begin{aligned}
E[\text{sales}] &= \mu - \sigma \cdot L(z) = 200 - 10 \times 0.265 \approx 197.5 \\
E[\text{leftover}] &= Q - E[\text{sales}] = 203 - 197.5 = 5.5 \\
E[\pi_r] &\approx \$7{,}771
\end{aligned}$$

Supplier's profit:

- Case 1: assuming that the buy-back units will be salvaged by the supplier,

$$E[\pi_s] = 203 \times (60-40) - 5.5 \times (36.67-5) \approx \$3{,}886$$

- Case 2: assuming that the buy-back units will be kept by the supplier for resale or inventory and not salvaged,

$$E[\pi_s] = 203 \times (60-40) - 5.5 \times 36.67 \approx \$3{,}858$$

Problem 2. (a) Product 1: $c_u = 20 - 13 = 7$, $c_o = 13 - 3 = 10$. So the critical ratio $= \frac{7}{7+10} \approx 0.412 = F(q_1^*)$. Since the demand is normally distributed with mean 300 and standard deviation 80, we have $q_1^* \approx 282$.

Product 2: $c_u = 24 - 15 = 9$, $c_o = 15 - 4 = 11$. So the critical ratio $= \frac{9}{11+9} \approx 0.45 = F(q_2^*)$. Since the demand is normally distributed with mean 200 and standard deviation 50, we have $q_2^* \approx 194$.

(b) Contract 1: $c_u = 20 - 13 = 7$, $c_o = 13 - 5 = 8$. So the critical ratio $= \frac{7}{7+8} \approx 0.467 = F(Q^*)$. Since the demand is normally distributed with mean 300 and standard deviation 80, we have $z = -0.0837$ and $Q^* \approx 293$. Now, we compute the profit for company B.

$$\begin{aligned}
E[\text{sales}] &= \mu - \sigma \cdot L(z) = 300 - 80 \times L(-0.0837) = 300 - 80 \times 0.44 \approx 264.8 \\
E[\text{leftover}] &= 293 - 264.8 = 28.2 \\
E[\pi_B] &= E[\text{sales}] \times p + b(Q^* - E[\text{sales}]) - Q^* \cdot w \\
&= 264.8 \times 20 + 28.2 \times 5 - 293 \times 13 \\
&\approx 1{,}628
\end{aligned}$$

Contract 2: $c_u = 20 \times 0.85 - 8 = 9$, $c_o = 8 - 3 = 5$. So the critical ratio $= \frac{9}{9+5} \approx 0.643 = F(Q^*)$. Since the demand is normally distributed with mean 300 and standard deviation 80, we have $z = 0.366$ and $Q^* = 329$. Therefore:

$$\begin{aligned}
E[\text{sales}] &= \mu - \sigma \cdot L(z) = 300 - 80 \times L(0.366) = 300 - 80 \times 0.242 \approx 280.6 \\
E[\text{leftover}] &= 329 - 280.6 = 48.4 \\
E[\pi_B] &= E[\text{sales}] \times p + b(Q^* - E[\text{sales}]) - Q^* \cdot w \\
&= 20 \times 0.85 \times 280.6 + 48.4 \times 3 - 329 \times 8 \\
&\approx 2{,}283
\end{aligned}$$

Since 1,628 < 2,283, company B should choose the second contract.

Problem 3. Various answers are acceptable.

Problem 4. 1. Revenue sharing contract

   (a) Requires a high level of coordination between the two firms.
   (b) Supplier becomes highly dependent on retailer's marketing strategies, etc.
   (c) Works best when future success of both companies are dependent on each other, and companies are not competing.

2. Buy-back contract

   (a) Requires less coordination between the two firms.
   (b) Supplier is still more dependent on retailers behavior than the uncoordinated case.
   (c) Works best when supplier has alternative retail channels to focal retailer, and logistics do not play a major issue (not that expensive to transport, goods don't get damaged).

Problem 5. Solutions vary. Answers are evaluated on the basis of completeness on the required criteria. This includes summary of the article, critical thinking applied to interpretation of the article, and application of lectured concepts to the topics discussed in the article.