MS&E 260 Midterm Examination

Stanford University

July 19th, 2019

SOLUTIONS

Name: ____________________________________

SUNet ID: ______________________@stanford.edu

Question	Points Available	Points Earned
1	25	
2	25	
3	25	
4	25	
Total	100	

Instructions:

1. This examination contains **9 pages**, including this page and the table pages.
2. You have **80 minutes** to complete the examination. As a courtesy to your classmates, we ask that you not leave during the last fifteen minutes.
3. You may use one single-sided 8.5" $\times$ 11" page with notes that you have prepared. You may not use any other resources, including lecture notes, books, other students or other engineers.
4. You may use a calculator. You may not share a calculator with anyone. If you didn't bring a calculator, you may use your phone, **but** you must put it on **flight mode** and clear all visible notifications **before** the examination starts, and you must not open any applications other than the calculator and a timer.
5. Please sign the below Honor Code statement.

In recognition of and in the spirit of the Stanford University Honor Code, I certify that I will neither give nor receive unpermitted aid on this examination.

Signature: ____________________________________

Question 1: Artisan Raw Vegan Chocolate!

[25 pts] Stanford student, Abby, found an amazing vegan chocolate supplier at the Los Angeles Mar Vista farmers' market last July. It is called Hopf Chocolate and the owner uses no refined sugar but sweetens with low glycemic, mineral-rich coconut nectar! Abby is not a chocolate connoisseur but was blown away by how smooth and tasty the chocolate was. Hopf Chocolate does not have a physical store nor a distribution channel set up serving Northern California but does deliver with USPS with ice packs. Abby, being an enterprising MS&E 260 student, decides to devise an ordering schedule/plan to satisfy her cravings as well as to share with friends and family the wonderful creations optimizing based on cost.

The chocolate bars cost \$9 each and she estimates that on average 4 chocolate bars per week will be consumed by others and herself. The cost of placing an order, mostly shipping with packaged ice packs, is \$20. The holding cost of the chocolates is 50% (think of this as a partial penalty for the semi-perishable chocolate bars and the freezer space it takes up in Abby's tiny graduate efficiency housing.)

(a) [3 pts] How many chocolate bars should Abby place per order?

Solution: $K = \$20$, $\lambda = 4$, $h = 50\% \times \$9 = \$4.5$

$Q^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2\times20\times4}{4.5}} = \sqrt{35.56} = 5.96$

Round this to 6 chocolate bars per week.

(b) [3 pts] How frequently should Abby order in number of weeks?

Solution: Reorder $\frac{4}{6} = 0.67$ time per week, i.e. once every 10.5 days.

(c) [3 pts] What is the total annual cost for Abby from these Hopf vegan raw chocolates? (Assume 52 weeks a year.)

Solution: There is a total weekly cost of $\$9 \times 4 + \$20 \times \frac{4}{6} + \frac{6}{2} \times \$4.5 = \$62.83$. Hence the annual cost is \$3,267.16.

(d) [8 pts] Abby would DM (direct message) the owner of Hopf, Andrea, on Instagram to tell her how much she loves the chocolates. Andrea appreciates Abby's enthusiasm and business hence decides to offer Abby an **all-units** discount in the following schedule:

- \$9/chocolate bar if $Q \leq 5$
- \$8/chocolate bar if $6 \leq Q \leq 10$
- \$6/chocolate bar if $Q \geq 11$

How many chocolate bars should Abby place per order now? What is the total annual cost for Abby from these chocolate bar purchases?

Solution:

$Q_1^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2\times20\times4}{4.5}} = \sqrt{35.56} = 5.96 > 5$ hence $Q_1^* = 5$

$TC(Q_1^*) = \$9 \times 4 + \$20 \times \frac{4}{5} + \frac{5}{2} \times \$4.5 = \$63.25$ weekly

$Q_2^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2\times20\times4}{4}} = \sqrt{40} = 6.32$ hence $Q_2^* = 6$

$TC(Q_2^*) = \$8 \times 4 + \$20 \times \frac{4}{6} + \frac{6}{2} \times \$4 = \$57.33$ weekly

$Q_3^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2\times20\times4}{3}} = \sqrt{53.33} = 7.30 < 10$ hence $Q_3^* = 11$

$TC(Q_3^*) = \$6 \times 4 + \$20 \times \frac{4}{11} + \frac{11}{2} \times \$3 = \$47.77$ weekly

Since $TC(Q_3^*)$ is the lowest, Abby would order 11 chocolate bars at a time per week for a total annual cost of $\$47.77 \times 52 = \$2{,}484.04$.

(e) [8 pts] Andrea wonders if a more modest **incremental** discount schedule is more profitable for her, since she's operating a one-woman business:

- \$9Q for $Q \leq 5$
- $\$45 + \$8.5(Q-5)$ for $6 \leq Q \leq 10$
- $\$87.5 + \$8(Q-10)$ for $Q \geq 11$

How many chocolate bars should Abby place per order now? What is the total annual cost for Abby from these chocolate bar purchases?

Solution:

We have $C(Q)$:

- 9Q for $Q \leq 5$
- $8.5Q + 2.5$ for $6 \leq Q \leq 10$
- $8Q + 7.5$ for $Q \geq 10$

We have found the additional cost to order. Hence, we can use the formula to find Q^*.

$Q_1 = \sqrt{\frac{2\times20\times4}{0.5\times9}} = 5.96$ as before, which is infeasible.

$Q_2 = \sqrt{\frac{2\times(20+2.5)\times4}{0.5\times8.5}} = 6.50$ which is feasible. $Q_2^* = 7$.

As a result, $TC(Q_2^*) = \frac{4}{7}\times 20 + \frac{7}{2}\times 0.5 \times (8.5 + \frac{2.5}{7}) + 4\times(8.5+\frac{2.5}{7}) = \$62.36$ weekly

$Q_3 = \sqrt{\frac{2\times(20+7.5)\times4}{0.5\times8}} = 7.42$ which is infeasible.

Hence, $TC(Q_2^*)$ is the lowest, so Abby orders 7 chocolate bars per week for a total annual cost of $\$62.36 \times 52 = \$3{,}242.72$.

Question 2: Tulip Mania!

[25 pts] You are the owner of a tulip store in San Francisco selling tulips to tourists. Purchasing the tulips from your Dutch supplier costs \$6 per bulb but can be sold at a price of \$20. As you are ordering the tulips from the Netherlands, there is a lead time between when the order is placed and when the order is received. Demand during this lead time is uncertain. The demand for tulips is normally distributed with a mean of 1,500 and a standard deviation of 400. You keep half of the un-sold tulips for your own garden and the other half you sell to a discount garden store for \$4.

(a) [5 pts] What is the probability that the product sells less than $\frac{3}{4}$ of the forecast?

Solution: Less than $\frac{3}{4}$ of the tulips will be sold if the demand is less than $\frac{3}{4} \times 1{,}500 = 1{,}125$ tulips. Normalizing that we get $\frac{1125-1500}{400} = -0.9375$. We find that $z(-0.9375) = 0.1743 = 17.43\%$.

(b) [5 pts] How many tulips should you buy from your Dutch supplier to maximize expected profit?

Solution: Since you sell only half of the unsold tulips to the discount store, there is an overage cost, $c_o = 6 - \frac{4}{2} = 4$ and an underage cost, $c_u = 20 - 6 = 14$. So the critical ratio is $\frac{14}{18} = 0.7778$. Then we find that $z(0.7778) = 0.7647$. So, $Q^* = 0.7647 \times 400 + 1{,}500 = 1{,}806$.

(c) [5 pts] If you want to ensure a 95% fill rate for your customers, how many tulips should you order?

Solution: If f is the fill rate, then $L(z) = (1 - f)(\frac{\mu}{\sigma}) = (1 - 0.95) \times \frac{1{,}500}{400} = 0.1875$. From the table we can see that $L(0.1875)$ is the closest to $L(0.192)$ which makes $z = 0.534$. So, $Q^* = 0.534 \times 400 + 1{,}500 = 1{,}714$.

(d) [5 pts] If you want to ensure a 98% in-stock probability, how many tulips should you order?

Solution: From the table we see that $z(2.06) = 0.9803$ so $z = 2.06$, which makes $Q^* = 2.05 \times 400 + 1{,}500 = 2{,}324$.

(e) [5 pts] Assuming you decide to buy 1,800 tulips, what is your company's expected profit?

Solution: If 1,800 tulips are ordered, then $z = \frac{Q-\mu}{\sigma} = \frac{1{,}800-1{,}500}{400} = 0.75$. We see that $L(0.75) = 0.1311$. The expected sales are $\mu - L(z)\sigma = 1{,}500 - 0.1311 \times 400 = 1{,}447.56$ tulips and the expected left-over inventory is $1{,}800 - 1{,}447.56 = 352.44$ tulips. So, the expected profit is $(20 - 6) \times 1{,}447.56 - (6 - \frac{4}{2}) \times 352.44 = \$18{,}856.08$.

Question 3: Lost on Maui?

[25 pts] Lost on Maui is a clothing and surf boutique located in the town of Paia on the Hawaiian island of Maui. The owner is thinking about expanding to women's apparel more by including sturdy and comfortable flip flops with feminine designs. However, he is a little lost regarding how much to order and enlists your help (in exchange for free surfboard rentals three days a week!)

You know Lost on Maui sells 150 flip flops per year on average, and that each pair of flip flops costs $15 to order from an outside supplier. An additional administrative cost of $5 is incurred per order. The annual holding cost is calculated with an annual interest rate of 10%. Assume that if Lost on Maui is out of stock when a pair of flip flops is demanded, demand is backordered at the cost of $4 each pair. The lead time is one month (4 weeks) and the demand during the lead time follows a uniform distribution from 0 to 100.

(a) [5 pts] Design a (Q, R) policy given the specific parameter values provided and the demand distribution during the lead time. The model should include expressions for the total cost function, Q, $F(R)$, and $n(R)$.

Solution: The total cost is $C(Q,R) = h\left(\frac{Q}{2} + R - \lambda\tau\right) + K\frac{\lambda}{Q} + p\lambda\frac{n(R)}{Q}$ where $s = R - \lambda\tau$ is the safety stock. The setup cost is $K = 5$ per order. We also have $h = 1.5$, $p = 4$, and $\lambda = 150$.

$$Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}} = \sqrt{\frac{2 \times 150 \times [5 + 4 \times n(R)]}{1.5}} = \sqrt{1000 + 800n(R)}$$

and

$$F(R) = 1 - \frac{Qh}{p\lambda} = 1 - \frac{Q}{400}$$

Since we have a uniform distribution $U[0, 50]$ for lead time demand,

$$f(x) = \frac{1}{b-a} = \frac{1}{100}$$

Therefore,

$$n(R) = \int_R^\infty (x - R)f(x)\, dx = \int_R^\infty \frac{(x-R)}{100}\, dx = \frac{R^2}{200} + 50 - R$$

(b) [10 pts] Show the steps for computing the reorder point via Q-R iteration. *Hint:* You should find $R^* = 91$.

Solution: We have the following iterations:

1. $EOQ = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2\times5\times150}{1.5}} = \sqrt{1{,}000} = 31.62$; $F(R_0) = 1 - \frac{31.62}{400} = 0.92$; $R_0 = 0.92 \times 100 = 92$
2. $n(R_0) = 0.32$; $Q_1 = \sqrt{1256} = 35.44$; $F(R_1) = 0.91$; $R_1 = 0.91 \times 100 = 91$
3. $n(R_1) = 0.405$; $Q_2 = \sqrt{1324} = 36.39$; $F(R_2) = 0.91$; $R_2 = 91$

The procedure has converged and $R^* = 91$.

(c) [5 pts] Determine the level of safety stock.

Solution: Since the lead time is 4 weeks and sales rate is 150 units per year, the safety stock is $s = R - \mu = 91 - 50 = 41$

(d) [5 pts] What is the proportion of order cycles in which no stock-outs occur?

Solution: $P(D \leq R^*) = P(D \leq 91) = \frac{91}{100} = 0.91$

Question 4: You've Got Mail

[25 pts] You are considering keeping a mailbox at the post office on Stanford campus after graduation in order to have a reason to visit your alma mater frequently. You are most concerned about the amount of wait time when getting a big package, since there is only one window that offers such a service.

You estimate that the inter-arrival time of customers like you is exponentially distributed with a mean of 12 minutes (unfortunately, the employee manning that particular window is not always there) and the servicing time is exponentially distributed with a mean of 5 minutes (she tends to take a long time to find a particular package.)

(a) [10 pts] Compute the expected number of customers in queue, the customers' expected wait time, and their expected total time in system (waiting and servicing time.)

Solution: We have $\lambda = \frac{1}{12}$ and $\mu = \frac{1}{5}$, so $\rho = \frac{\lambda}{\mu} = \frac{5}{12}$.

The expected number of customers in queue is

$$L_q = \frac{\rho^2}{1-\rho} = \frac{25 \times 12}{144 \times 7} = 0.30$$

The customers' expected wait time is

$$W_q = \frac{L_q}{\lambda} = \frac{0.30}{\frac{1}{12}} = 3.6 \text{ mins}$$

The customers' total wait time in system is

$$W = W_q + \frac{1}{\mu} = 3.6 + 5 = 8.6 \text{ mins}$$

(b) Read the following statements carefully and identify whether they are *True* or *False*.

(i) [5 pts] Queue size would remain stable even if the mean servicing time rose to 25 minutes.
Solution: *False*

(ii) [5 pts] The marginal benefit of adding servers/windows (in terms of average wait time) diminishes as more servers/windows are added.
Solution: *True*

(iii) [5 pts] Suppose a customer has been completing the service at the window (not waiting in line) for 5 minutes already. The expected time remaining until the completion of the process is 7 minutes.
Solution: *False*

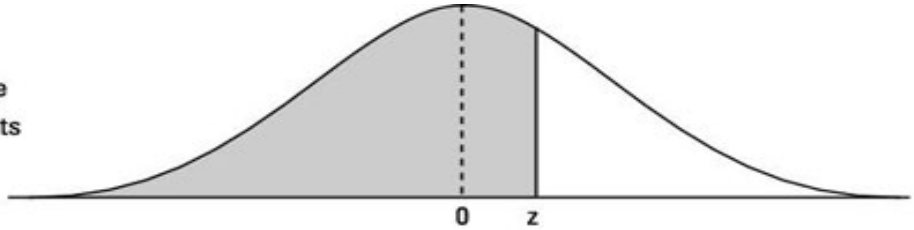
Standard Normal Loss Function Table, $L(z)$

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00
−4.0	4.0900	4.0800	4.0700	4.0600	4.0500	4.0400	4.0300	4.0200	4.0100	4.0000
−3.9	3.9900	3.9800	3.9700	3.9600	3.9500	3.9400	3.9300	3.9200	3.9100	3.9000
−3.8	3.8900	3.8800	3.8700	3.8600	3.8500	3.8400	3.8300	3.8200	3.8100	3.8000
−3.7	3.7900	3.7800	3.7700	3.7600	3.7500	3.7400	3.7300	3.7200	3.7100	3.7000
−3.6	3.6900	3.6800	3.6700	3.6600	3.6500	3.6400	3.6300	3.6200	3.6100	3.6000
−3.5	3.5900	3.5800	3.5700	3.5600	3.5500	3.5400	3.5301	3.5201	3.5101	3.5001
−3.4	3.4901	3.4801	3.4701	3.4601	3.4501	3.4401	3.4301	3.4201	3.4101	3.4001
−3.3	3.3901	3.3801	3.3701	3.3601	3.3501	3.3401	3.3301	3.3201	3.3101	3.3001
−3.2	3.2901	3.2801	3.2701	3.2601	3.2502	3.2402	3.2302	3.2202	3.2102	3.2002
−3.1	3.1902	3.1802	3.1702	3.1602	3.1502	3.1402	3.1302	3.1202	3.1103	3.1003
−3.0	3.0903	3.0803	3.0703	3.0603	3.0503	3.0403	3.0303	3.0204	3.0104	3.0004
−2.9	2.9904	2.9804	2.9704	2.9604	2.9505	2.9405	2.9305	2.9205	2.9105	2.9005
−2.8	2.8906	2.8806	2.8706	2.8606	2.8506	2.8407	2.8307	2.8207	2.8107	2.8008
−2.7	2.7908	2.7808	2.7708	2.7609	2.7509	2.7409	2.7310	2.7210	2.7110	2.7011
−2.6	2.6911	2.6811	2.6712	2.6612	2.6512	2.6413	2.6313	2.6214	2.6114	2.6015
−2.5	2.5915	2.5816	2.5716	2.5617	2.5517	2.5418	2.5318	2.5219	2.5119	2.5020
−2.4	2.4921	2.4821	2.4722	2.4623	2.4523	2.4424	2.4325	2.4226	2.4126	2.4027
−2.3	2.3928	2.3829	2.3730	2.3631	2.3532	2.3433	2.3334	2.3235	2.3136	2.3037
−2.2	2.2938	2.2839	2.2740	2.2641	2.2542	2.2444	2.2345	2.2246	2.2147	2.2049
−2.1	2.1950	2.1852	2.1753	2.1655	2.1556	2.1458	2.1360	2.1261	2.1163	2.1065
−2.0	2.0966	2.0868	2.0770	2.0672	2.0574	2.0476	2.0378	2.0280	2.0183	2.0085
−1.9	1.9987	1.9890	1.9792	1.9694	1.9597	1.9500	1.9402	1.9305	1.9208	1.9111
−1.8	1.9013	1.8916	1.8819	1.8723	1.8626	1.8529	1.8432	1.8336	1.8239	1.8143
−1.7	1.8046	1.7950	1.7854	1.7758	1.7662	1.7566	1.7470	1.7374	1.7278	1.7183
−1.6	1.7087	1.6992	1.6897	1.6801	1.6706	1.6611	1.6516	1.6422	1.6327	1.6232
−1.5	1.6138	1.6044	1.5949	1.5855	1.5761	1.5667	1.5574	1.5480	1.5386	1.5293
−1.4	1.5200	1.5107	1.5014	1.4921	1.4828	1.4736	1.4643	1.4551	1.4459	1.4367
−1.3	1.4275	1.4183	1.4092	1.4000	1.3909	1.3818	1.3727	1.3636	1.3546	1.3455
−1.2	1.3365	1.3275	1.3185	1.3095	1.3006	1.2917	1.2827	1.2738	1.2650	1.2561
−1.1	1.2473	1.2384	1.2296	1.2209	1.2121	1.2034	1.1946	1.1859	1.1773	1.1686
−1.0	1.1600	1.1514	1.1428	1.1342	1.1257	1.1172	1.1087	1.1002	1.0917	1.0833
−0.9	1.0749	1.0665	1.0582	1.0499	1.0416	1.0333	1.0250	1.0168	1.0086	1.0004
−0.8	0.9923	0.9842	0.9761	0.9680	0.9600	0.9520	0.9440	0.9360	0.9281	0.9202
−0.7	0.9123	0.9045	0.8967	0.8889	0.8812	0.8734	0.8658	0.8581	0.8505	0.8429
−0.6	0.8353	0.8278	0.8203	0.8128	0.8054	0.7980	0.7906	0.7833	0.7759	0.7687
−0.5	0.7614	0.7542	0.7471	0.7399	0.7328	0.7257	0.7187	0.7117	0.7047	0.6978
−0.4	0.6909	0.6840	0.6772	0.6704	0.6637	0.6569	0.6503	0.6436	0.6370	0.6304
−0.3	0.6239	0.6174	0.6109	0.6045	0.5981	0.5918	0.5855	0.5792	0.5730	0.5668
−0.2	0.5606	0.5545	0.5484	0.5424	0.5363	0.5304	0.5244	0.5186	0.5127	0.5069
−0.1	0.5011	0.4954	0.4897	0.4840	0.4784	0.4728	0.4673	0.4618	0.4564	0.4509
0.0	0.4456	0.4402	0.4349	0.4297	0.4244	0.4193	0.4141	0.4090	0.4040	0.3989

Standard Normal Loss Function Table, $L(z)$ (Concluded)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.3989	0.3940	0.3890	0.3841	0.3793	0.3744	0.3697	0.3649	0.3602	0.3556
0.1	0.3509	0.3464	0.3418	0.3373	0.3328	0.3284	0.3240	0.3197	0.3154	0.3111
0.2	0.3069	0.3027	0.2986	0.2944	0.2904	0.2863	0.2824	0.2784	0.2745	0.2706
0.3	0.2668	0.2630	0.2592	0.2555	0.2518	0.2481	0.2445	0.2409	0.2374	0.2339
0.4	0.2304	0.2270	0.2236	0.2203	0.2169	0.2137	0.2104	0.2072	0.2040	0.2009
0.5	0.1978	0.1947	0.1917	0.1887	0.1857	0.1828	0.1799	0.1771	0.1742	0.1714
0.6	0.1687	0.1659	0.1633	0.1606	0.1580	0.1554	0.1528	0.1503	0.1478	0.1453
0.7	0.1429	0.1405	0.1381	0.1358	0.1334	0.1312	0.1289	0.1267	0.1245	0.1223
0.8	0.1202	0.1181	0.1160	0.1140	0.1120	0.1100	0.1080	0.1061	0.1042	0.1023
0.9	0.1004	0.0986	0.0968	0.0950	0.0933	0.0916	0.0899	0.0882	0.0865	0.0849
1.0	0.0833	0.0817	0.0802	0.0787	0.0772	0.0757	0.0742	0.0728	0.0714	0.0700
1.1	0.0686	0.0673	0.0659	0.0646	0.0634	0.0621	0.0609	0.0596	0.0584	0.0573
1.2	0.0561	0.0550	0.0538	0.0527	0.0517	0.0506	0.0495	0.0485	0.0475	0.0465
1.3	0.0455	0.0446	0.0436	0.0427	0.0418	0.0409	0.0400	0.0392	0.0383	0.0375
1.4	0.0367	0.0359	0.0351	0.0343	0.0336	0.0328	0.0321	0.0314	0.0307	0.0300
1.5	0.0293	0.0286	0.0280	0.0274	0.0267	0.0261	0.0255	0.0249	0.0244	0.0238
1.6	0.0232	0.0227	0.0222	0.0216	0.0211	0.0206	0.0201	0.0197	0.0192	0.0187
1.7	0.0183	0.0178	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146
1.8	0.0143	0.0139	0.0136	0.0132	0.0129	0.0126	0.0123	0.0119	0.0116	0.0113
1.9	0.0111	0.0108	0.0105	0.0102	0.0100	0.0097	0.0094	0.0092	0.0090	0.0087
2.0	0.0085	0.0083	0.0080	0.0078	0.0076	0.0074	0.0072	0.0070	0.0068	0.0066
2.1	0.0065	0.0063	0.0061	0.0060	0.0058	0.0056	0.0055	0.0053	0.0052	0.0050
2.2	0.0049	0.0047	0.0046	0.0045	0.0044	0.0042	0.0041	0.0040	0.0039	0.0038
2.3	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
2.4	0.0027	0.0026	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021
2.5	0.0020	0.0019	0.0019	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015
2.6	0.0015	0.0014	0.0014	0.0013	0.0013	0.0012	0.0012	0.0012	0.0011	0.0011
2.7	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008
2.8	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
2.9	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004
3.0	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
3.1	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.2	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001
3.3	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.4	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.5	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Number in the table represents $P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999