

MS&E 260 Homework 3

Summer 2019, Stanford University

Due: July 24th, 2019, at 10:30AM (PDT)

Problem 1. Think Geek sells cans of Unicorn Meat, an excellent source of sparkles and with magic in every bite. (If you're curious, check it out on Amazon.) Weekly demand of the good is normally distributed with mean 310 and variance of 1,280. Think Geek purchases a can of Unicorn Meat at a cost of \$7 each. The processing cost for each order is \$120 and the leadtime is 5 weeks. Think Geek uses a 25% annual inventory holding cost rate. Under the current inventory policy, Think Geek purchases 1,240 cans of Unicorn Meat whenever the inventory falls below 1,650. Assume that there are 50 weeks in a year.

- (a) What is the fill rate under the current inventory policy?
- (b) What order quantity and reorder point would minimize the total annual setup, holding and shortage costs subject to having a fill rate at least equal to 98%?
- (c) Would the solution in part (b) change if Think Geek has a minimum Type I service level constraint of 80%? Explain why.

Problem 2. Consider a queueing system where customers arrive according to a Poisson process with a mean rate of 30 customers per hour. Customers wait in a single line and are served by the next available server when they reach the front of the line. The service times are exponentially distributed.

Suppose for **(a) and (b) ONLY** that the time it takes for a server to serve one customer is 5 minutes on average:

- (a) If four servers are used, what is the fraction of time that some server is not busy?
- (b) What is the minimum number of servers that could be used in order to avoid infinite queue size?

Suppose **for the rest of the question** that only one server is used:

- (c) The server has a service cost of \$16 per hour per service. Assume that a significant cost is incurred by making a customer wait because of potential lost future business. We estimate the cost to be \$0.50 for each minute a customer spends in the system, counting both waiting time and service time. What is the service rate that minimizes the long run average cost per unit time? What is the optimal cost in this case?
- (d) For the service rate you found in part (c), what is the average length of the waiting line? (Please round your answer to the closest integer.)
- (e) Consider now a similar system which is empty at time 0. There is one arrival every two time slots, i.e. at time slots 1, 3, 5... The service time per customer is 1. What is the average length of the waiting line?

Problem 3. A drive-through carwash has 5 machines. The time between cars arriving at the carwash has an unknown distribution with mean 4 minutes and a variance of 3 minutes. Cars are washed on a first come first served basis. The time it takes to wash a car per machine has a uniform distribution with a maximum of 8 minutes and a minimum of 6 minutes. Please answer the following questions:

- (a) How long does a car need to wait before getting served on average?
- (b) How many people are waiting to be served on average?
- (c) What is the expected time a car will spend in the carwash?
- (d) How many people are in the carwash on average?
- (e) What is the fraction of time there is no car in the carwash?

Problem 4. Isaac runs a data science business and revenue depends on his website being accessible. He is concerned about access to the site being blocked by basic distributed denial of service attacks. To investigate this, you will do a “back of the envelope” risk calculation. You are given the following information: The attackers utilize very basic attack tools, such as a simple open-source application called Praetox Technologies. Once the application is downloaded - either voluntarily or in a variant form via a malicious link - the application recruits computers into a network that floods a designated website with traffic until it slows or collapses under the load. If an attack occurs, we believe that the number of connections per millisecond to Isaac’s server will be uniformly distributed between 200 and 300, which we denote as $F_L(x)$.

Assume that the number of connections is constant during an attack. If the number of connection attempts per time unit exceeds the server’s capability, the site will become inaccessible to real users.

- (a) If Isaac’s server has an effective server capacity of $C = 250$ connections per millisecond, what is the probability of a failure, given that an attack occurs (failure occurs when the capacity exceeds 250 connections per millisecond)?
- (b) Now suppose that the server’s capacity is also uncertain, and has the following probability density function (again in units of connections/millisecond):

$$g_c(x) = \begin{cases} 0.05 - 0.0002x & \text{if } 150 \leq x \leq 250 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability of a failure now?

- (c) Isaac has calculated that a denial of service attack will result in \$100K in loss of revenue (one time only.) In other words, attackers will not attack the server again after an attack has been successful. Google has also offered to increase Isaac’s server capacity in order to reduce the probability of success for a denial of service attack. The Google server upgrade costs \$50K (one time only.) This improvement results in the following probability density function (again in units of connections/millisecond):

$$g_c(x) = \begin{cases} 0.1 - 0.0004x & \text{if } 179.29 \leq x \leq 250 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability of a failure with the upgraded server?

- (d) Assuming Isaac is a rational decision maker, should he purchase the Google server upgrade?

Problem 5. Read the following articles:

- <https://www.latimes.com/business/la-fi-disneyland-wait-times-20170712-htmlstory.html>
- <http://fortune.com/2019/04/30/artificial-intelligence-walmart-stores/>

Select one of these two articles, and do the following:

- 1) Briefly summarize the article.
- 2) Provide a few interpretive thoughts on the article, using what you have learned from class.
- 3) Provide one recommendation on how the dilemma posed in the article could be resolved.

Some notes:

- Please limit your responses to one page, double spaced, 12 point font.
- There is no right answer to this question. We are evaluating your ability to apply what you learn in class to practical applications.
- This question is not intended to be free points. If you do not demonstrate a sufficient level of critical thinking, full credit will not be awarded.