# MS&E 260

## Introduction to Operations Management

Problem Session 1

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# What We Learned This Week

- Linear programming
- Types of Inventory
- Economic Order Quantity (EOQ): Cycle stock for deterministic demand
  - with all-unit and incremental quantity discounts
  - with finite product rate

# Linear Programming - Lecture Example

- Imagine that you manage a factory that produces four different types of wood paneling. Each type of paneling is made by gluing and pressing together a different mixture of pine and oak chips. The following table summarizes the required amount of gluing, pressing, and mixture of wood chips required to produce a pallet of 50 units of each type of paneling:

| | **Resources Required per Pallet of Paneling Type** | | | |
|---|---|---|---|---|
| | Tahoe | Pacific | Savannah | Aspen |
| Glue (quarts) | 50 | 50 | 100 | 50 |
| Pressing (hours) | 5 | 15 | 10 | 5 |
| Pine chips (pounds) | 500 | 400 | 300 | 200 |
| Oak chips (pounds) | 500 | 750 | 250 | 500 |

- In the next production cycle, you have 5,800 quarts of glue; 730 hours of pressing capacity; 29,200 pounds of pine chips; and 60,500 pounds of oak chips available. Further assume that each pallet of Tahoe, Pacific, Savannah, and Aspen panels can be sold for profits of $450, $1,150, $800, and $400, respectively.
- What is the optimal mix of paneling type to produce?

https://www.solver.com/product-mix-example

# Linear Program Example, Problem Formulation

- Let:
  - $x_1$ = number of Tahoe pallets produced
  - $x_2$ = number of Pacific pallets produced
  - $x_3$ = number of Savannah pallets produced
  - $x_4$ = number of Aspen pallets produced

- Problem formulation:

$$
\begin{array}{ll}
\max & 450x_1 + 1150x_2 + 800x_3 + 400x_4 \\
\text{s.t.} & 50x_1 + 50x_2 + 100x_3 + 50x_4 \leq 5800 \\
& 5x_1 + 15x_2 + 10x_3 + 5x_4 \leq 730 \\
& 500x_1 + 400x_2 + 300x_3 + 200x_4 \leq 29200 \\
& 500x_1 + 750x_2 + 250x_3 + 500x_4 \leq 60500 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{array}
$$

- This problem can be solved in many different ways, with an algorithm like simplex

# Free Linear Optimization Solver

- Available at: https://online-optimizer.appspot.com
- Code:

```
var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;

maximize z:     450*x1+1150*x2+800*x3+400*x4;

subject to c11: 50*x1+50*x2+100*x3+50*x4<=5800;
subject to c12: 5*x1+15*x2+10*x3+5*x4<=730;
subject to c13: 500*x1+400*x2+300*x3+200*x4<=29200;
subject to c14: 500*x1+750*x2+250*x3+500*x4<=60500;

end;
```

# Motivation for Holding Inventory

- Economies of scale: **cycle stock**
  - Average cycle inventory = $\frac{Q}{2}$
- Uncertainties: **safety stock**
  - Hedging against demand, supply, or lead time
- Speculation & smoothing: **anticipation stock**
  - Resources with increasing value & seasonal demand
- Lead times (supply chain): **pipeline stock**
  - Average pipeline inventory = demand (d) x lead time (L)

# Economic Order Quantity (EOQ)

- Assumptions:
  - Consider a single inventory item
  - Demand is fixed (deterministic) at $\lambda$ units/time
  - Shortages are not allowed
  - Order quantity is fixed at $Q$ per cycle
  - Orders are received instantaneously (no lead time)
- Cost structure:
  - Fixed and marginal order costs per cycle ($K + cQ$)
  - Holding cost at $h$ per unit held per unit time
- Objective:
  - Determine order quantity $Q^*$ to minimize sum of ordering cost and inventory holding cost

# EOQ Derivation

- Cost function
  - Total Cost = Setup (ordering) cost + Holding cost + Purchase cost

$$TC(Q) = \left(\frac{\lambda}{Q}\right)K + \left(\frac{Q}{2}\right)h + \lambda c$$

$Q$ = order (production) quantity: the **decision to be made**

$\lambda$ = demand rate

$K$ = fixed ordering (or setup) cost

c = cost per unit in inventory

$i$ = annual interest rate

$\left.\begin{array}{l} c \\ i \end{array}\right\} h$ = holding cost per unit per year $= ic$

# EOQ Derivation

- Cost function
  - Total Cost = Setup (ordering) cost + Holding cost + Purchase cost

$$TC(Q) = \left(\frac{\lambda}{Q}\right)K + \left(\frac{Q}{2}\right)h + \lambda c$$

  - $Q^*$ minimizes the total cost function

$$\frac{d[TC(Q)]}{dQ} = \left(-\frac{\lambda}{Q^2}\right)K + \left(\frac{h}{2}\right) = 0$$

$$Q^* = \sqrt{\frac{2K\lambda}{h}}$$

# EOQ Example with all-unit and incremental discounts

W company sells Baby Mops. There is a fairly steady demand of 5,600 Baby Mops per year. Traditionally, W company purchases these outfits from a producer in Germany at the price of $8/unit. It costs W company $100 to place an order. Inventory holding costs are based on an annual interest rate of 20%. Suppose that the Baby Mop supplier is offering a quantity discount applied to all units with the following schedule (<u>total</u> unit cost):

- $\$8Q$ for $Q \leq 800$
- $\$7Q$ for $800 < Q < 1{,}000$
- $\$6Q$ for $Q \geq 1{,}000$

a) What is the optimal order quantity in this case? Using your optimal order quantity, what is the total cost?

# EOQ Example – cont'd

b) Assume now the supplier is offering an incremental discount with the following schedule:

- $\$8Q$ for $Q \leq 1{,}000$
- $\$8{,}000 + \$7.90(Q - 1{,}000)$ for $1{,}000 < Q < 2{,}000$
- $\$15{,}900 + \$7.80(Q - 2{,}000)$ for $Q \geq 2{,}000$

What is the optimal order quantity in this case? Using your optimal quantity, what is the total cost?

# Finite Production Rate Derivation

- Suppose replenishment is not instantaneous, but production rate $\Psi$ is greater than demand rate $\lambda$

$$H = \text{maximum inventory level} = Q(1 - \frac{\lambda}{\Psi})$$

$$\frac{H}{2} = \text{average inventory level} = \frac{Q}{2}(1 - \frac{\lambda}{\Psi})$$

$$TC(Q) = \left(\frac{\lambda}{Q}\right)K + \left(\frac{Q}{2}\right)\left(1 - \frac{\lambda}{\Psi}\right)h + \lambda c$$

$$Q^* = \sqrt{\frac{2K\lambda}{h(1-\frac{\lambda}{\Psi})}}$$

# EOQ Example with finite production rate

The same W company also sells these ostrich pillows. They have a known and constant demand of 1,800 units per year. The fixed cost of the setup for each production run is $300 and the inventory holding cost is $5 per unit per year. Assuming there is infinite production capacity, compute:

a) The EOQ
b) The resulting annual setup cost

Now assume there is a finite production rate of 9,000 units per year. Compute:

c) The EOQ
d) The total annual holding cost