MS&E 260 Homework 5

Summer 2019, Stanford University

Due: August 12th , 2019, at 10:30AM (PDT)

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Problem 1. You are selling coats and forecast that the demand function is D(p) = 1,000 – 5p.

(a) What is the optimal price? How many units do you sell at the optimal price and what is your revenue?

revenue = f(p) = p(1000 – 5p) = 1000p – 5p²

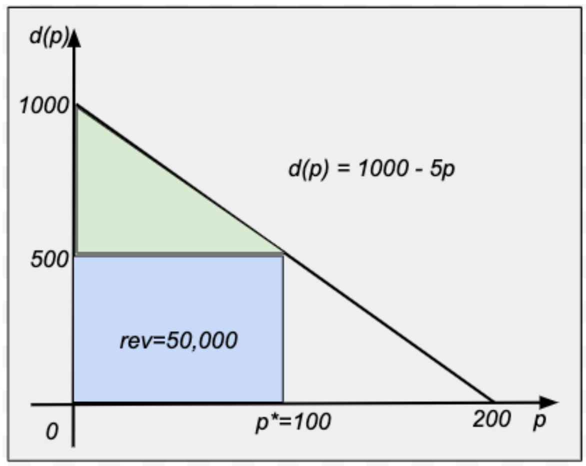
$$\frac{df}{dp} = -10\,p + 1000 = 0 => p^* = 100$$

units sold = demand function

= 1000 – 5p = 1000 – 5(100) = 500

revenue = *optimal price * units sold*

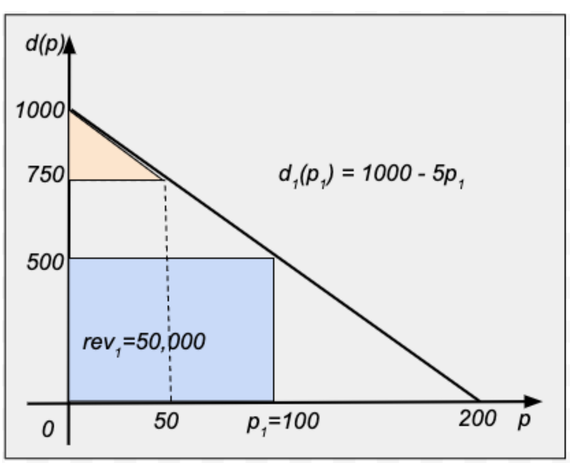
*= 100 * 500 = 50,000*

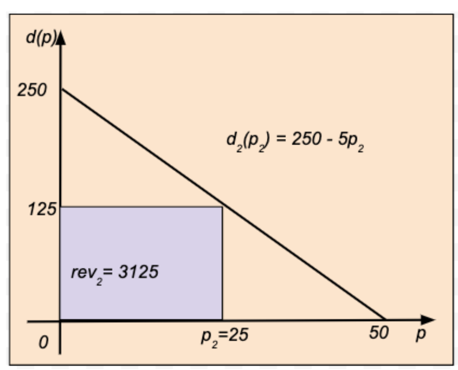


(b) From your past experience, you notice that you can actually segment the market perfectly. You observe that customers who are willing to pay at least $50 will always be attracted to buy online and customers who are willing to pay less than $50 will always go to the store. What prices would you choose for the store and for the website (assuming there is no delivery cost)? What is the total revenue?

segmentation threshold price = v = 50

*d(p) = 1000 – 5p = 1000 – 5*50 = 750*

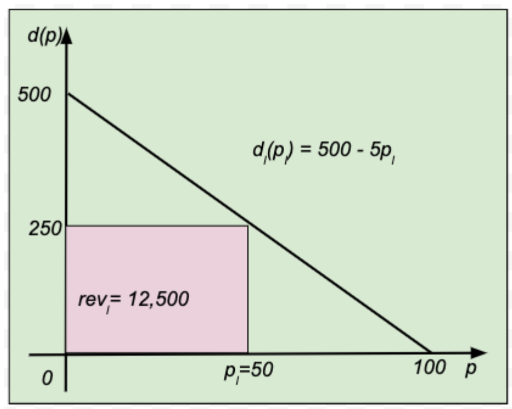




High (online) valuation segment:	Low (store) valuation segment:
demand: $d_1(p_1) = \min(750, (1000 - 5\ p_1))$ revenue max $p_1 * \min(750, (1000 - 5\ p_1))$ s.t $p_1 \geq 750$ $p_1^* = D / 2m = 1000 / 2*5 = 100$ $d_1(p_1^*) = 1000 - 5p = 500$ $rev^* = p_1^* * d_1(p_1^*) = 100 * 500 = 50{,}000$	y coordinate = 1000-750 = 250 revenue max $p_2 * (250 - 5\ p_2)$ s.t $p_2 \geq 0,\ p_2 \leq 750$ $p_2^* = D / 2m = 250 / 2*5 = 25$ $d_2(p_2^*) = 250 - 5(25) = 125$ $rev^* = p_2^* * d_2(p_2^*) = 25 * 125 = 3{,}125$

Total revenue = 50,000 + 3,125 = 53,125

(c) Suppose that you sold at one optimal price as in part (a), and you decide to sell at a lower price for the remaining inventory you have (customers who bought in the original price will no longer buy in the markdown price.) What will be the optimal markdown? What will be your total revenue (including the revenue in part (a))? Is your total revenue here larger or smaller than what you got in part (b)? Explain briefly why.



$p^* = D / 2m = 500 / 2*5 = 50$
$d(p^*) = 500 - 5p = 500 - 5(50) = 250$
$rev_l = 50 * 250 = 12{,}500$

Total revenue = 50000 + 12500 = 62,500

Total revenue here is higher than the one in part b. The higher price segment demand and revenue remains the constant in both the cases, but the lower price segment changes. In b the base price is set to 50 because of which the optimal price comes out to be 25 and thus the overall revenue is lower. while in case of c the optimal price is 50 which is equal to the segment threshold in b. Thus higher revenue overall in c.

Problem 2. A firm is considering various pricing strategies for their new product. The firm estimates that customers have valuations $\theta \sim U(0, 1)$. In general, the utility of a customer with valuation θ from buying a product at price p is equal to $\theta - p$. We assume that a buyer is willing to buy a product at a given price p if and only if his utility is at least 0.

The firm is planning to sell initially to high valuation customers. Next period, it will sell the product to customers with lower values. Low-value customers anticipate prices to fall and delay purchases until price falls. There is a discount factor δ for both the seller and the buyers and all other costs are zero.

Note: Since there is a discount factor for period 2, we have to be more careful with buyers' utilities in period 2. For example, if a buyer with valuation 0.3 buys the product in period 2 at a price of 0.2, then his utility is $\delta(0.3 - 0.2) = 0.1\delta$.

(a) How would you approximate the expected demand in period 2 given the price p of the product when you want to sell to customers of valuations $[0, \theta_1]$?

$p_1 = p$; $p_2 = \delta p_1 = \delta p$

$v_1 \sim U[0, \theta_1]$; $v_2 \sim U[0, \theta_2] \sim U[0, \delta\theta_1]$

$$\max_{p_1 p_2} \quad \text{Revnue} = p_1 . \text{Prob}(v_1 \geq p_1) + \text{Prob}(v_1 < p_1) . p_2 . \text{Prob}(v_2 \geq p_2)$$

$$\max_{p_1 p_2} \quad \text{Revnue} = p_1 \left(1\text{-} \frac{p_1}{\theta_1}\right) + \left(\frac{p_1}{\theta_1}\right)(p_2)\left(1\text{-} \frac{p_2}{\theta_2}\right)$$

$$\textit{Demand in period } 2 = \left(\frac{p_1}{\theta_1}\right)\left(1\text{-} \frac{p_2}{\theta_2}\right) =$$

$$\text{using the discount factor on price and valuation } = \left(\frac{p}{\theta_1}\right)\left(1\text{-} \frac{\delta p}{\delta\theta_1}\right) = \left(\frac{p}{\theta_1}\right)\left(1\text{-} \frac{p}{\theta_1}\right)$$

(b) Suppose that the firm sells to customers of valuations $[0, \theta_1]$ in period 2. Express the revenue of the firm in period 2 as a function of θ_1 and the price of the product in period 2. What is the optimal price p_2^* maximizing the revenue in period 2? Express p_2^* as a function of θ_1.

$v_1 \sim U[0, \theta_1]$

$v_2 \sim U[0, \theta_2] \sim U[0, \delta\theta_1]$

$$\max_{p_2} \quad \left(\frac{p_1}{\theta_1}\right)(p_2)\left(1\text{-} \frac{p_2}{\theta_2}\right) \Longrightarrow \textit{assuming} \left(\frac{p_1}{\theta_1}\right) \textit{iscontant 'c' for period} 2$$

$$f(p_2) = c(p_2)\left(1\text{-} \frac{p_2}{\delta\theta_1}\right) = cp_2 - \frac{c\,p_2^2}{\delta\theta_1} \Longrightarrow \textit{take differential}$$

$$c \text{ - } \frac{2\,c\,p_2}{\delta\theta_1} = 0 \text{ ==> } p_2^* = \frac{\delta\theta_1}{2}$$

(c) Find a condition such that the customers of valuation θ_1 are indifferent between buying in period 1 and 2. Use this result to express the price of the product in period 1, as a function of θ_1 and δ.

as the customers are indifferent between buying in period 1 and 2 thus revenue in both periods will be equal. Also p_2 value can be used $p_2^* = \frac{\delta\theta_1}{2}$

$$p_1\left(1\text{-} \frac{p_1}{\theta_1}\right) = \left(\frac{p_1}{\theta_1}\right)(p_2)\left(1\text{-} \frac{p_2}{\theta_2}\right) \Longrightarrow p_1\left(1\text{-} \frac{p_1}{\theta_1}\right) = \left(\frac{p_1}{\theta_1}\right)\left(\frac{\delta\theta_1}{2}\right)\left(1\text{-} \frac{\delta\theta_1}{2\,\delta\theta_1}\right)$$

$$1\text{-} \frac{p_1}{\theta_1} = \frac{\delta}{4} \text{ ==> } p_1^* = \theta_1\left(1\text{-} \frac{\delta}{4}\right)$$

(d) Use parts (b) and (c) to express the total revenue from period 1 and 2 as a function of θ_1 and δ only.

$$Revenue = p_1\left(1-\frac{p_1}{\theta_1}\right)+\left(\frac{p_1}{\theta_1}\right)(p_2)\left(1-\frac{p_2}{\theta_2}\right)$$

$$= \theta_1\left(1-\frac{\delta}{4}\right)\left(1-\left(1-\frac{\delta}{4}\right)\right)+\left(1-\frac{\delta}{4}\right)\frac{\delta\theta_1}{2}\left(1-\frac{\delta\theta_1}{2\,\delta\theta_1}\right)$$

$$= \theta_1\left(1-\frac{\delta}{4}\right)\frac{\delta}{4}+\left(1-\frac{\delta}{4}\right)\frac{\delta\theta_1}{4} = \frac{\delta\theta_1}{2}\left(1-\frac{\delta}{4}\right)$$

(e) Use part (d) to formulate the problem of finding the valuation θ_1^* that maximizes the total revenue as a maximization problem. Solve this maximization problem for the optimal type θ_1^*.

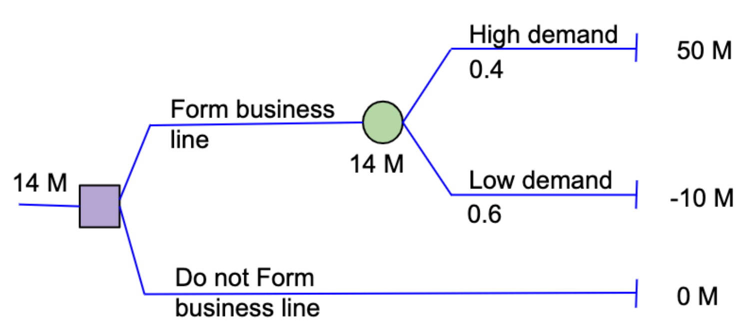
$$\begin{array}{ll} \max & \dfrac{\delta\theta_1}{2}\left(1-\dfrac{\delta}{4}\right) \\ \text{s. t.} & \theta_1 \sim U\,(0,\,1) \end{array}$$

Problem 3. John is the chief operating officer of Alpha Corp, which produces optical imagers for mobile phone manufacturers. John is considering an expansion of Alpha's current business into a new business line: producing specialized optical imagers for medical devices. This will require capital investment to build new manufacturing infrastructure. If Alpha expands into this new business line, then there is one relevant uncertainty that John must consider: market demand. If market demand is high, then Alpha will receive a net income of $50M in the next year. If market demand is low, then Alpha will incur a $10M loss in the next year, due to the uncovered capital expenditures associated with the business line expansion. John's prior belief on the market demand is that it will be high with a probability of 0.4. Suppose the market research from Beta Corp is able to provide better information on the demand uncertainty for John, in the form of a market research report. Beta Corp is widely respected for the accuracy of their reports, and John estimates that if there will be high demand, Beta's reports will predict high demand 90% of the time. If there will be low demand, Beta's report will predict low demand 80% of the time.

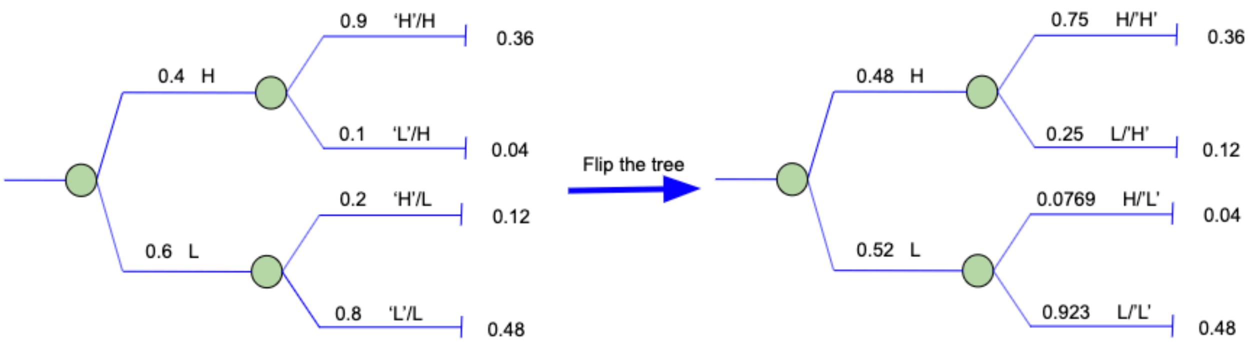
(a) Assuming John is a risk neutral decision maker, what is his optimal alternative (to form the new business line, or not), without using Beta's market research report? What is John's expected value for his optimal alternative?

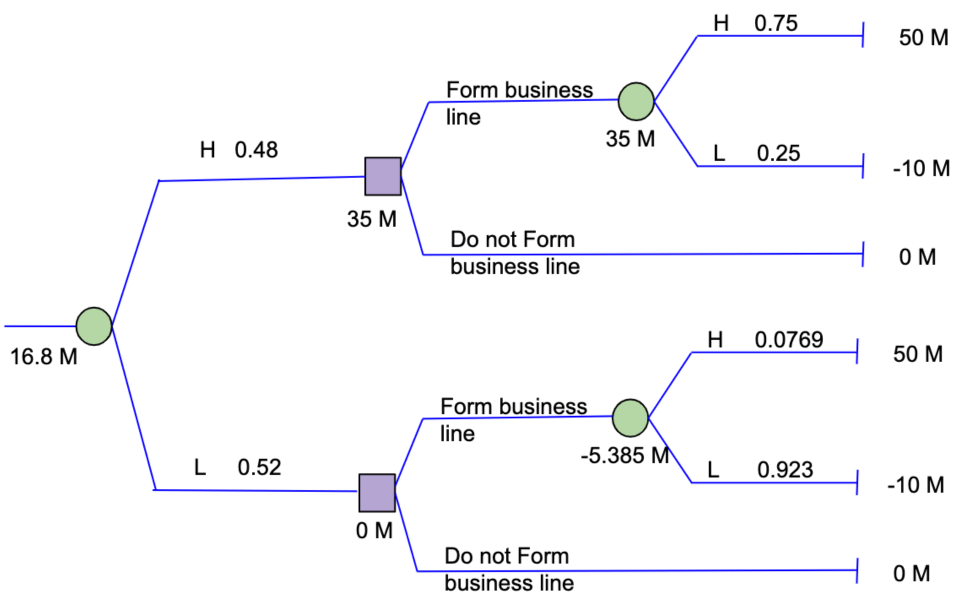
John should form the new business line.

John's expected value for forming business line is 14M.



(b) What is the maximum price that John should be willing to pay for Beta's market research report?





Cost of Beta's Market report = 16.8 – 14 = 2.8 M

(c) For the original decision (with no market research report), how much does John's prior belief on the market demand uncertainty need to change before his optimal alternative changes?

High demand probablility	Low demand probablility	Total value
40	60	14
39	61	13.4
38	62	12.8
37	63	12.2
36	64	11.6
35	65	11
34	66	10.4
33	67	9.8
32	68	9.2
31	69	8.6
30	70	8
29	71	7.4
28	72	6.8
27	73	6.2
26	74	5.6
25	75	5
24	76	4.4
23	77	3.8
22	78	3.2
21	79	2.6
20	80	2
19	81	1.4
18	82	0.8
17	83	0.2
16	84	-0.4
15	85	-1
14	86	-1.6

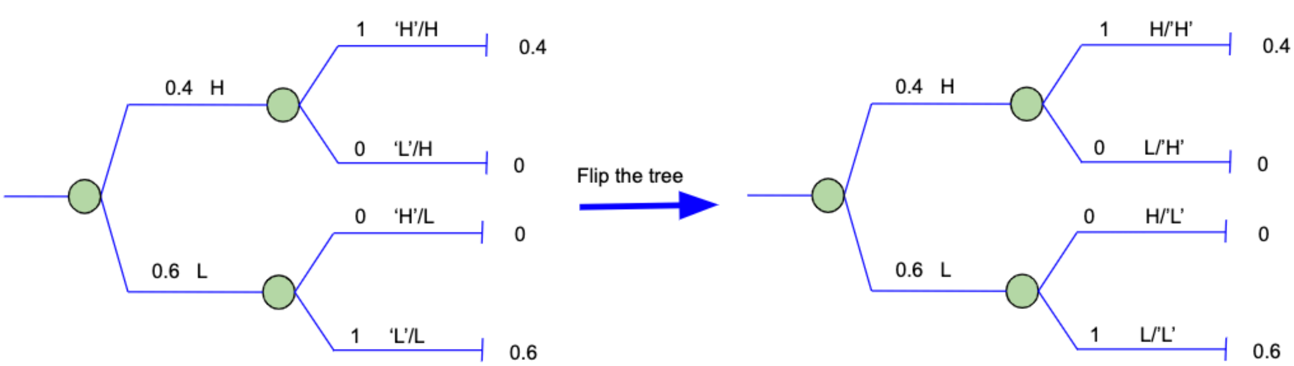
As seen in the table, the optimal alternative should change for high demand probability of 0.16 or lower.

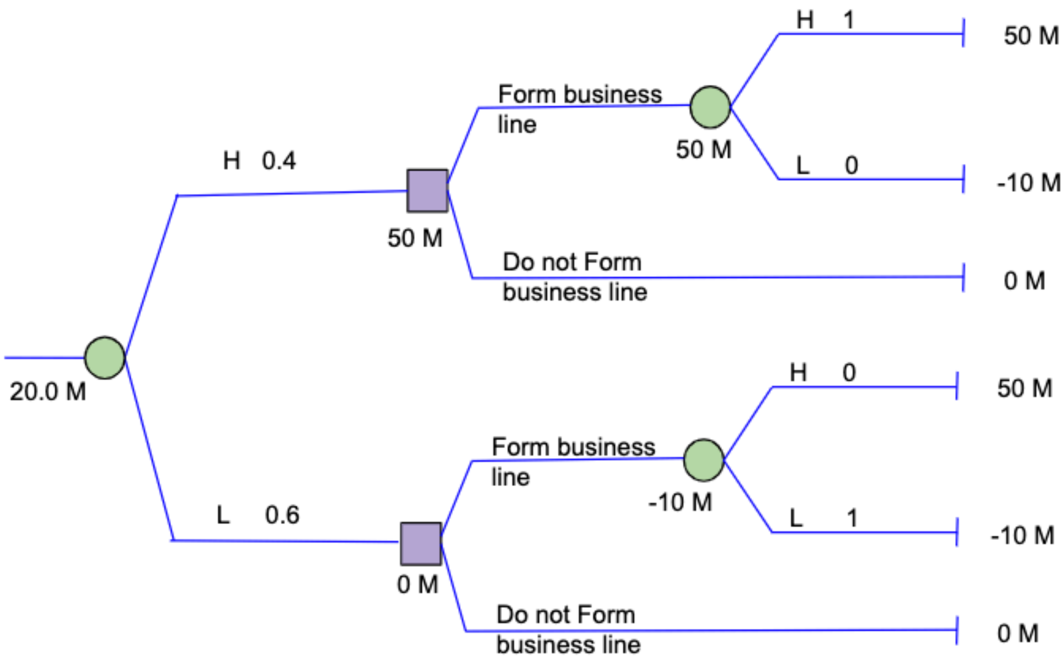
or in other words low demand probability of 0.84 or higher.

(d) Based upon this answer, would you consider John's optimal alternative (as given in Part (a)) to be a robust decision?

There is a gap of 0.23 between John's belief of 0.4 high demand probability and 0.17 where the optimal alternative changes. Thus John's optimal alternative should be a robust decision.

(e) If Beta's market research report was a perfect test of future market demand (no false positive or false negative signals), what is the value of this test?





Cost of Beta's Perfect Market report = 20 – 14 = 6 M

Problem 4. Read the following articles:

- https://hbr.org/2016/05/can-lean-manufacturing-put-an-end-to-sweatshops
- https://www.forbes.com/sites/douggollan/2019/04/05/united-airlines-move-to-stop-publishing-award-levels-is-unfriendly/#61a5c52917fa

Select one of these two articles, and do the following:

1) Briefly summarize the article.
2) Provide a few interpretive thoughts on the article, using what you have learned from class.
3) Provide one recommendation on how the dilemma posed in the article could be resolved.

Some notes:

- Please limit your responses to one page, double spaced, 12 point font.
- There is no right answer to this question. We are evaluating your ability to apply what you learn in class to practical applications.
- This question is not intended to be free points. If you do not demonstrate a sufficient level of critical thinking, full credit will not be awarded.

The article "United Airlines' move to stop publishing MileagePlus Award Levels Is unfriendly but not surprising" by Doug Gollan primarily highlights the shift of major American airlines from a static predefined pricing to a dynamic pricing. Delta airlines started dynamic pricing in 2015 and since then American airlines and recently united airlines has followed it. Airlines have been always trying to get the booking from customers irrespective if their pricing is the lowest or not. For this they started with fly miles program in 60’s and the more you stick to an airline, you get free flights using those points to vacation destinations. These flights were bought by fixed number of points gathered, irrespective of the day or season. In dynamic pricing the price varies based on day, availability, season, etc., and thus customer have to have so many points to be able to book that flight.

A couple of years back, one of the United Airlines flight was overbooked and a passenger was hurt while being escorted out of the plane. This caused a big backlash on the airlines and practices used by the airlines to maximize utilization of each flight. Later couple of similar incidents were also observed. People thought that they would never fly United again, and it’s going to bankrupt, but customers forgot all that and did buy the cheapest possible tickets suitable to their needs and it worked. Later, United became smarter on how not to be aggressive in booking and not lose money by using dynamic pricing for ticket’s booking and fly miles utilization.

Dynamic pricing is great for flexible customers, as while some flights with more demand costs more points, there are other’s where flights are cheaper due to various reasons. Using the points towards those would be a great benefit. Recently Google flights started generic search where I can say give me a flight to anywhere in US for $50 during next weekend. It will send you a reminder if something opens up and it’s also a great way to explore new places on budget. Customers should also consider signing up for Alliances (like start alliance, etc.) where you gain point via flying member airline, and can use those points with any other member airlines. It just gives more options to use the points and still reward you for loyalty.