MS&E 260

Introduction to Operations Management

Problem Session 7

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Outline

- Decision Analysis
- Final Review I

Decision Analysis

- What is a decision?
  - An allocation of resources that is somewhat irrevocable.
- What makes a decision hard?
  - Uncertainty
  - Lack of Alternatives
  - Unclear Preferences
  - Non Monetary Values
  - Time pressure
  - Etc…
- What is decision analysis?
  - A formal procedure for the analysis of decision problems[1].
  - It’s a *normative* science (vs. descriptive.)

[1] Howard, Ronald A., "Decision Analysis: Applied Decision Theory," Proceedings of the 4th International Conference on Operational Research (1966) 55-77

Decisions vs. Outcomes

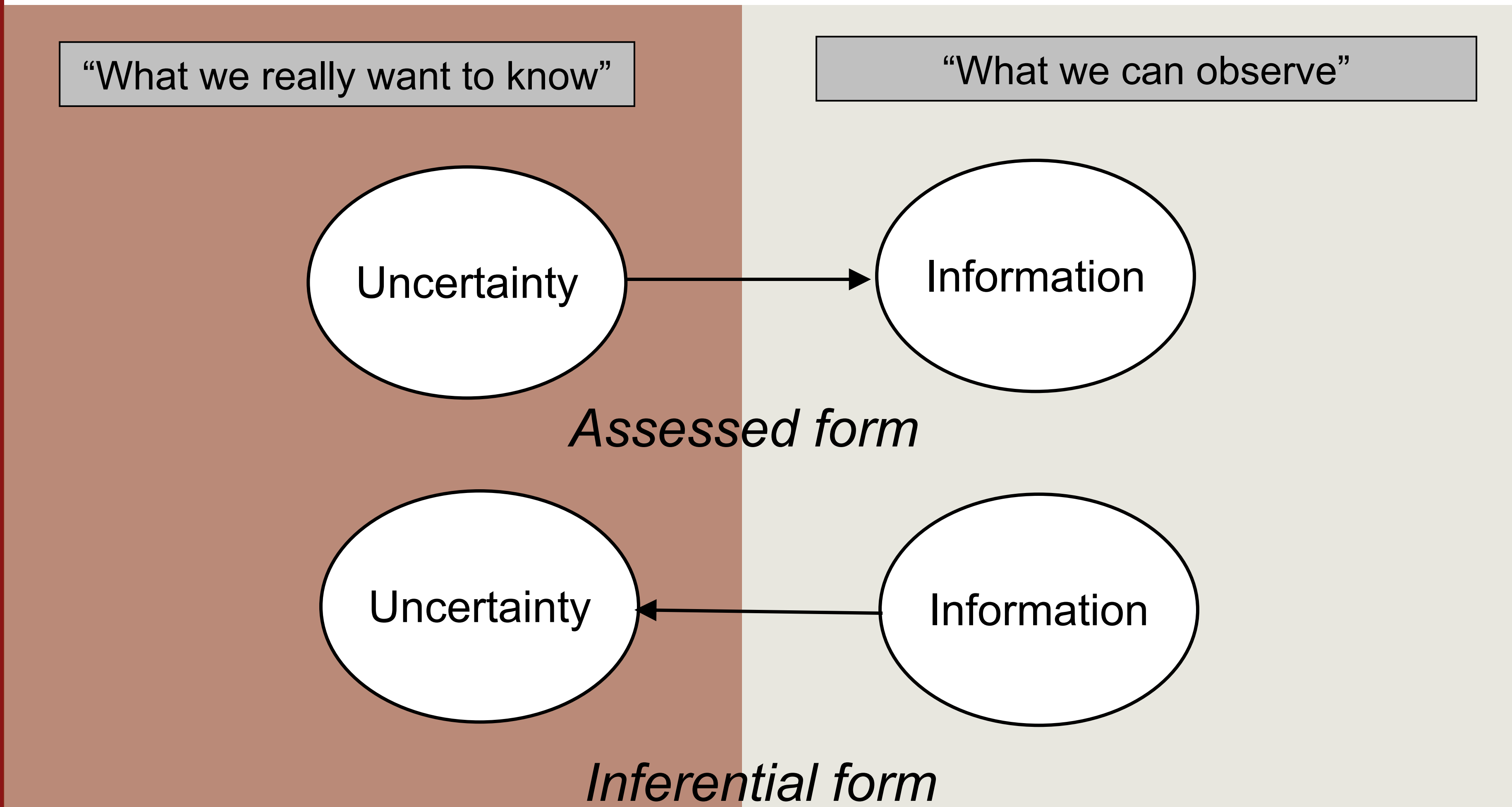
- One of the most important distinctions in DA
- You determine the quality of a decision ***before*** knowing the outcome
- Good decisions can have bad outcomes and bad decisions can have good outcomes.

Quality of Outcome

Quality of Decision	Bad	Good
Good	Driving sober and getting into an accident	Driving sober and arriving safely
Bad	Driving drunk and getting into an accident	Driving drunk and arriving safely

Information Gathering

- Involves using a test to illuminate an unobservable distinction



Bayesian Updating of Beliefs

- We use Bayes' Formula as a mechanism to update our prior beliefs into posterior beliefs

Prior

Likelihood

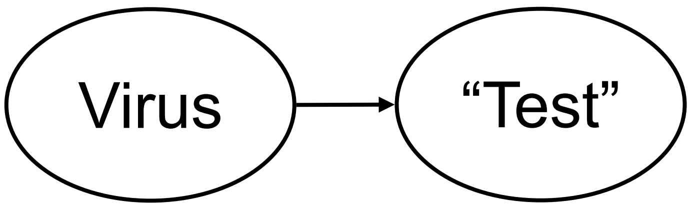
$$P(E|S) = \frac{P(E) \times P(S|E)}{P(S)}$$

Posterior

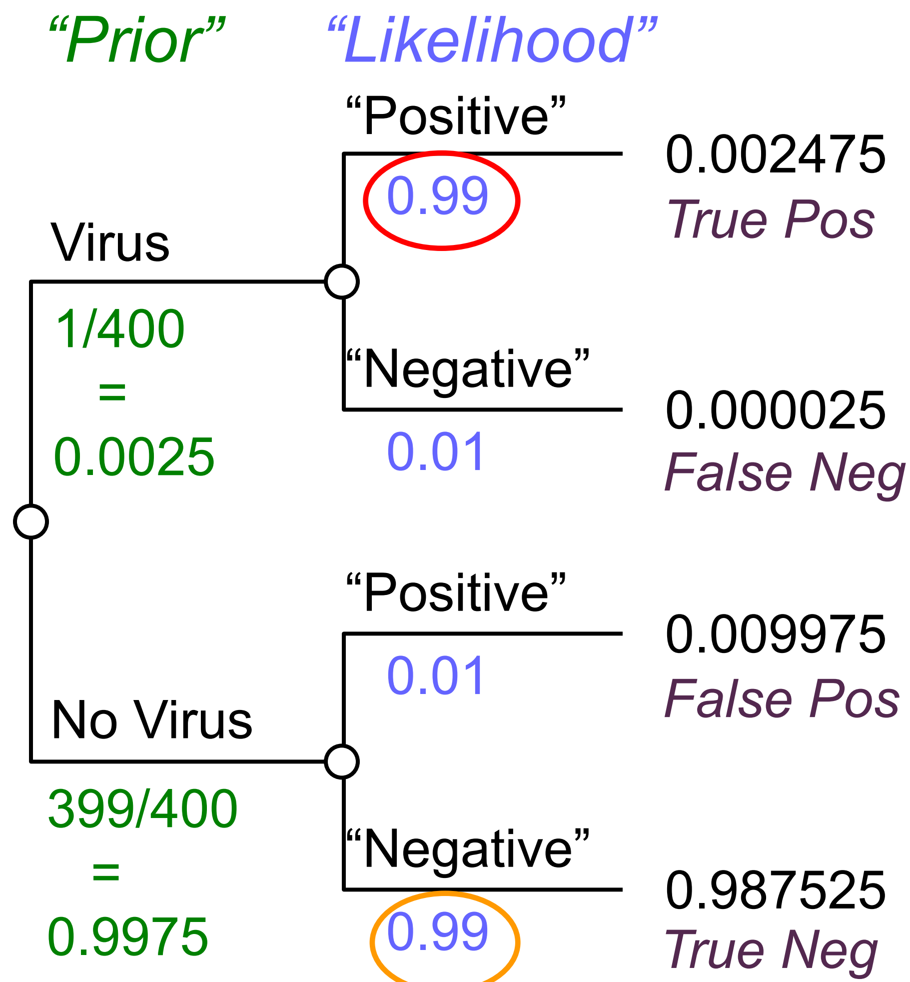
Pre-Posterior

“Assessed Tree”

- The “assessed” tree determines the joint probabilities from the prior and the likelihood.



Assessed Form

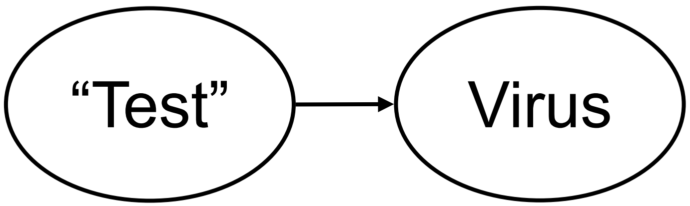


{“Pos” | Pos } = Test Sensitivity

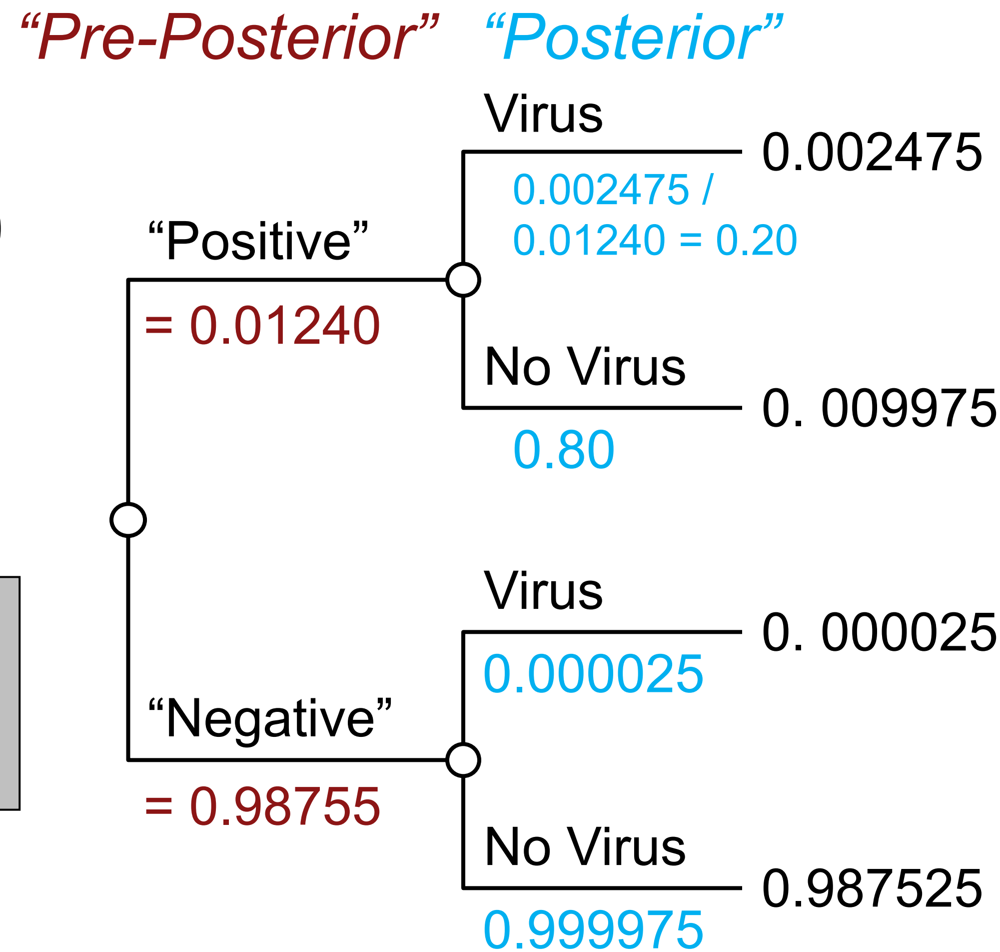
{“Neg” | Neg } = Test Specificity

“Inferential Tree”

- The “inferential” tree determines the posterior and pre-posterior probabilities from the joint.



Inferential Form



Let’s flip the previous tree!

Risk Preference In Decisions

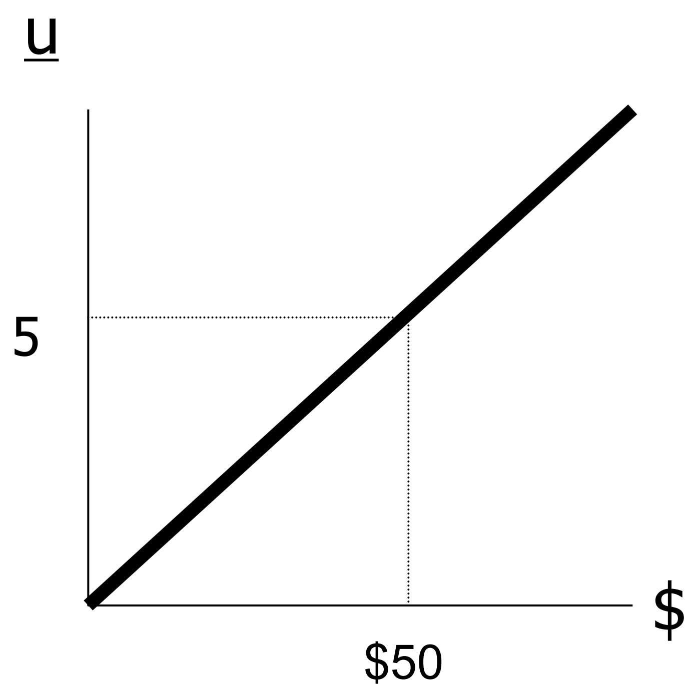
- Up to now, all our decisions have been in expected value
  - This implies that the decision-maker is risk neutral
  - i.e. $1 is worth $1 of utility
- Oftentimes, decision-makers have some degree of risk aversion
- We generally represent risk attitude with utility curves of the form

$$u(x) = a - b \times e^{-\gamma x}$$

- where
  - $\gamma =$ the decision−makers risk aversion coefficient
  - $x =$ the value in original units
  - $u(x) =$ utility associated with x
- Rules for γ:
  - $\gamma > 0$ indicates risk averse attitude
  - $\gamma < 0$ indicates risk seeking attitude
  - $\gamma = 0$ indicates risk neutral attitude

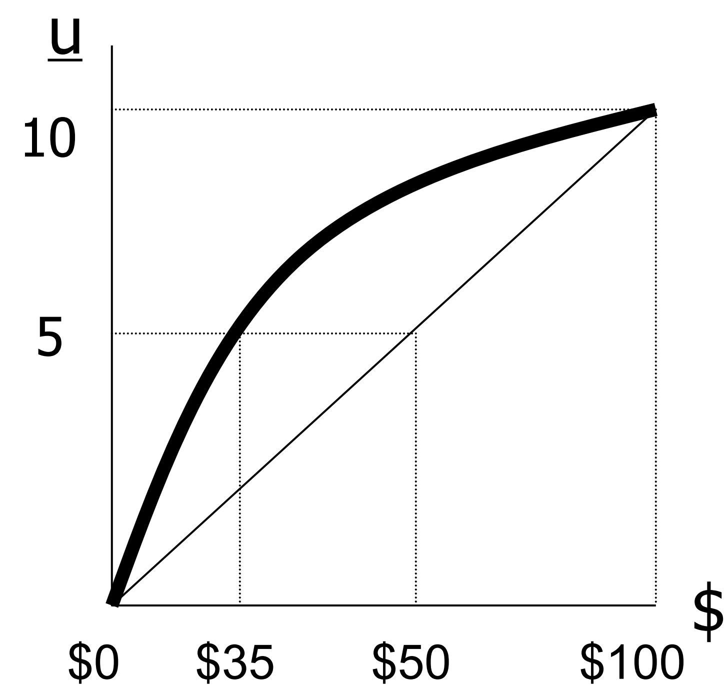
When dealing with risk attitude, we are solving for **expected utility**, not expected value

U-curves and U-values - Risk attitude



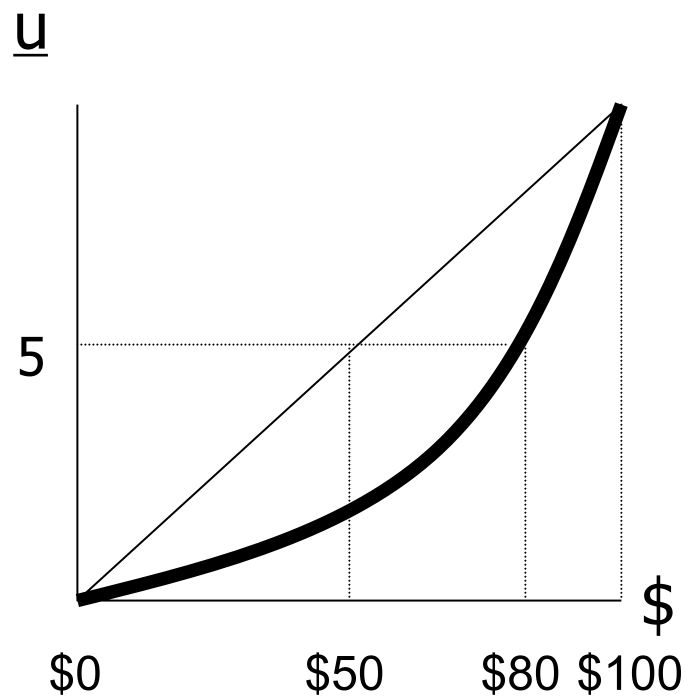
- Risk-neutral
- u-curve is a straight line
- e-value of dollar value of a deal is equal to $ certain equivalent of that deal

U-curves and U-values - Risk attitude



- Risk-averse
- u-curve lies above the straight line
- e-value of dollar value of a deal is bigger than $ certain equivalent of that deal
  - What's the e-value of the dollar value of the deal with equal chance of winning $100 or $0?
  - What's the certain equivalent of this deal?

U-curves and U-values - Risk attitude



- Risk-seeking
- u-curve lies below the straight line
- e-value of dollar value of a deal is smaller than $ certain equivalent of that deal
  - What's the e-value of the dollar value of the deal with equal chance of winning $100 or $0?

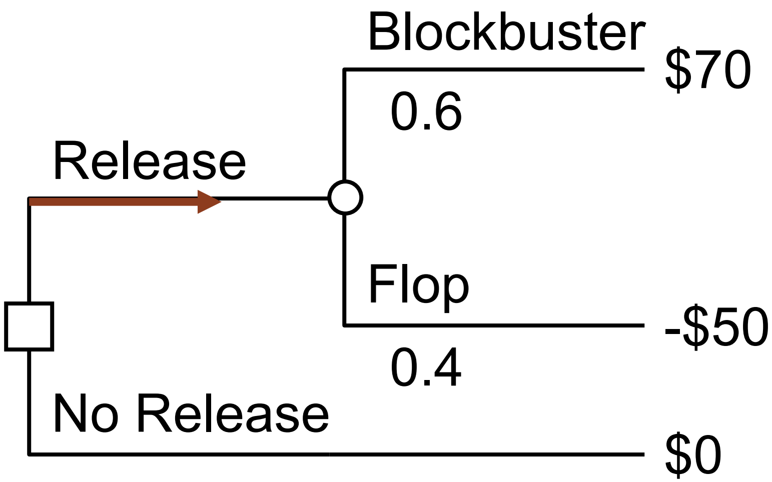
Case Study Example - DA Studio

- DA Studios, a major entertainment corporation, has just produced a motion picture called “Claire the Clairvoyant Goes to College”.
- Fanny, VP of Movie Marketing, needs to decide whether to release the film or not.
- The decision is difficult because Fanny does not know whether the movie will be a “Blockbuster” or a “Flop”.

Case Study Example - DA Studio

- Fanny says the probability the movie will be a "Blockbuster" *if released*, {Blockbuster | Released,&}, is 0.6.
- If it is a Blockbuster, DA Studios will earn $70M. If it is a Flop, DA Studios will lose $50M.
- Fanny says if they do not release the movie, DA Studios breaks even, i.e. does not earn or lose any money.
- Fanny also explains that this is a small deal compared to the company as a whole, so she is comfortable being risk-neutral.

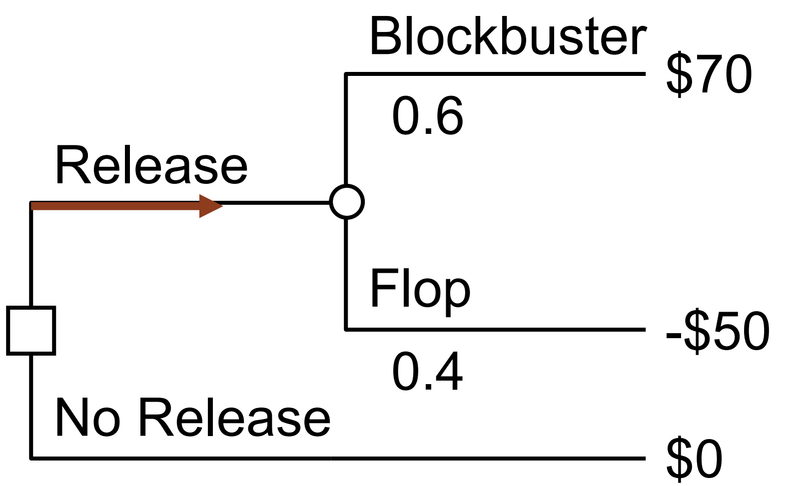
How do you draw the decision tree? What is the CE?



CE = max{$42 + (-$20)[2], $0} = $22M

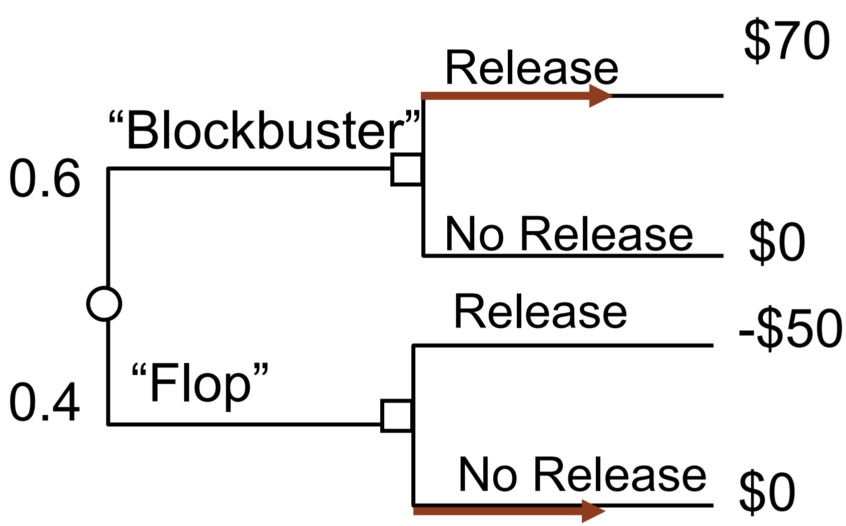
[2] True for risk-neutral decision maker

What is the VFC? And Value of Clairvoyance?



CE = max{$42 + (-$20), $0} = $22M

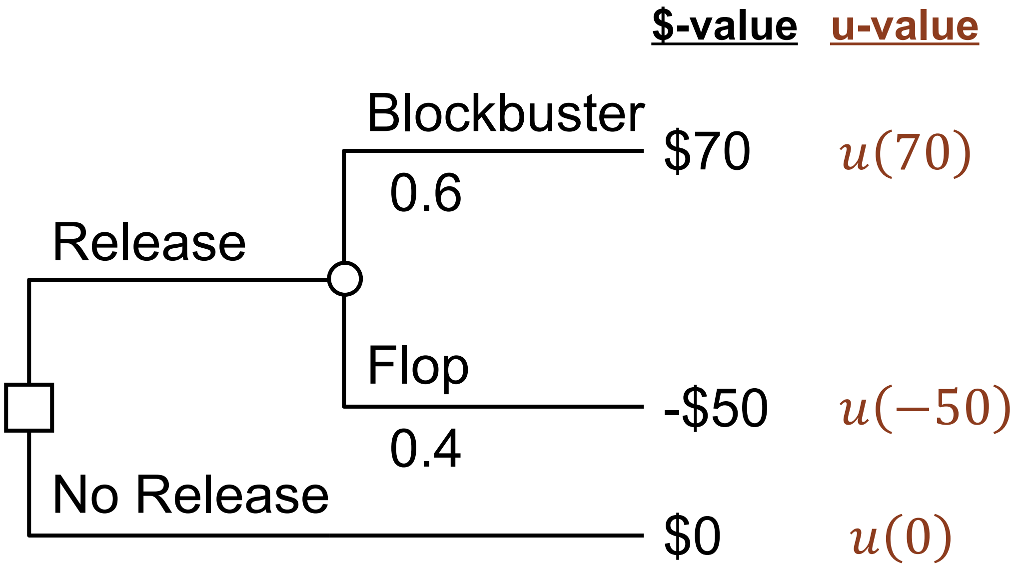
Value of Clairvoyance
= VFC - CE (original deal)[2]
= $42M - $22M
= $20M



Value with Free Clairvoyance (VFC)
= 0.6 ($70) + 0.4 ($0)
= $42M

[2] True for risk-neutral decision maker

What is the CE if DA Studio were risk-averse?



- Let's say

$$u(x) = -e^{-5x}$$

Deal Worth (in u-value) = max{0.6 x $u(70)$ + 0.4 x $u(-50)$, $u(0)$}

CE (in $-value) = $u^{-1}(Deal\ Worth\ in\ u - value)$

Case Study Example - DA Studio

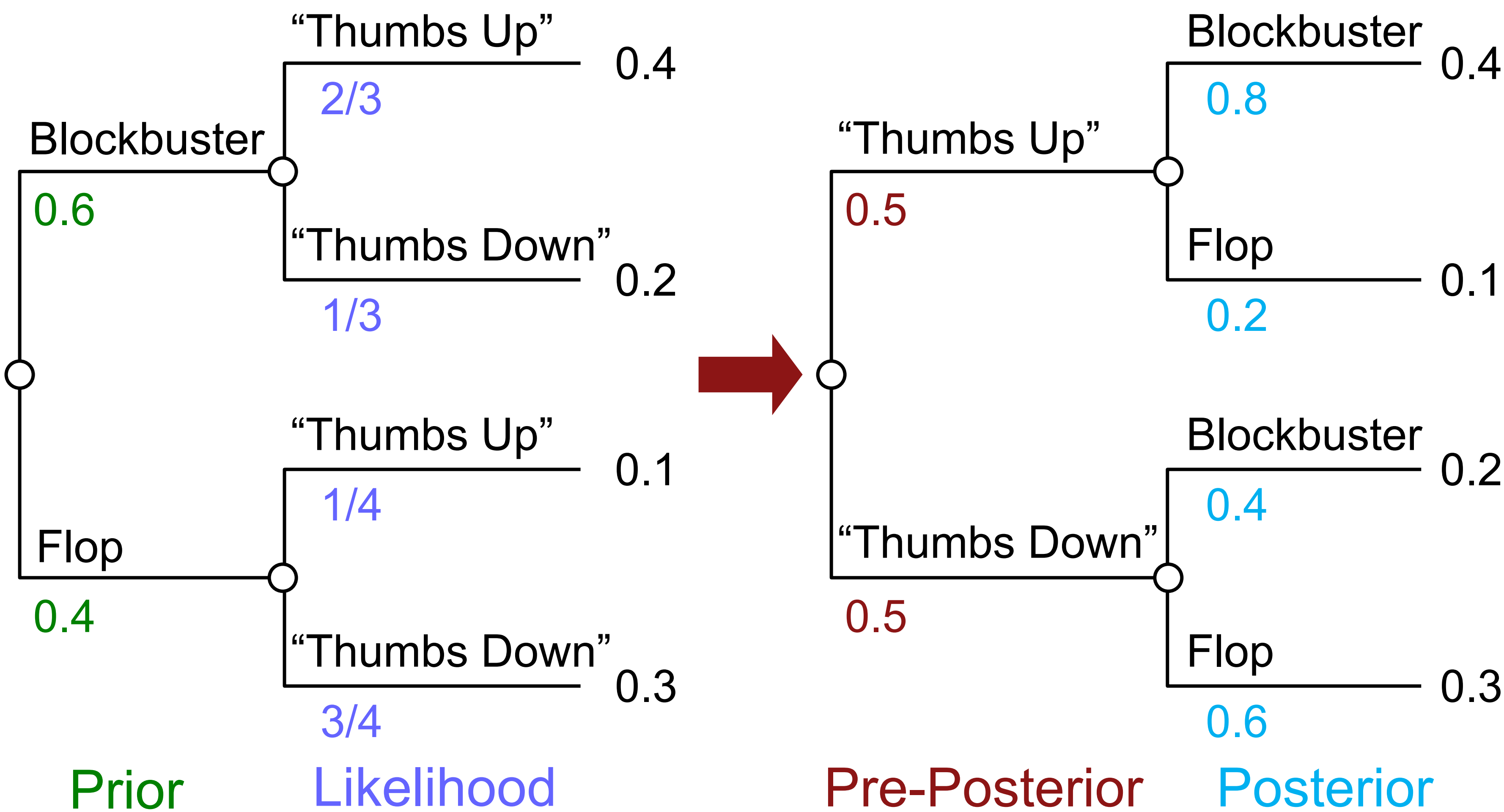
- Now, an imperfect sneak preview "test" is available. How do we think about this? We assume again that the decision maker is risk-neutral.

- Fanny explains that from time to time they do have "Sneak Previews" for selected films.
  - A sneak preview is a special showing, after which the audience fills out a questionnaire stating their opinion.
  - The questionnaire contains only two boxes: "Thumbs Up" or "Thumbs Down".
- Fanny provides you with the following sneak preview data from past films:
  - {"Thumbs Up" | Blockbuster} = 2/3
  - {"Thumbs Down" | Flop} = 3/4
  - Sneak Preview cost = $0.5 million

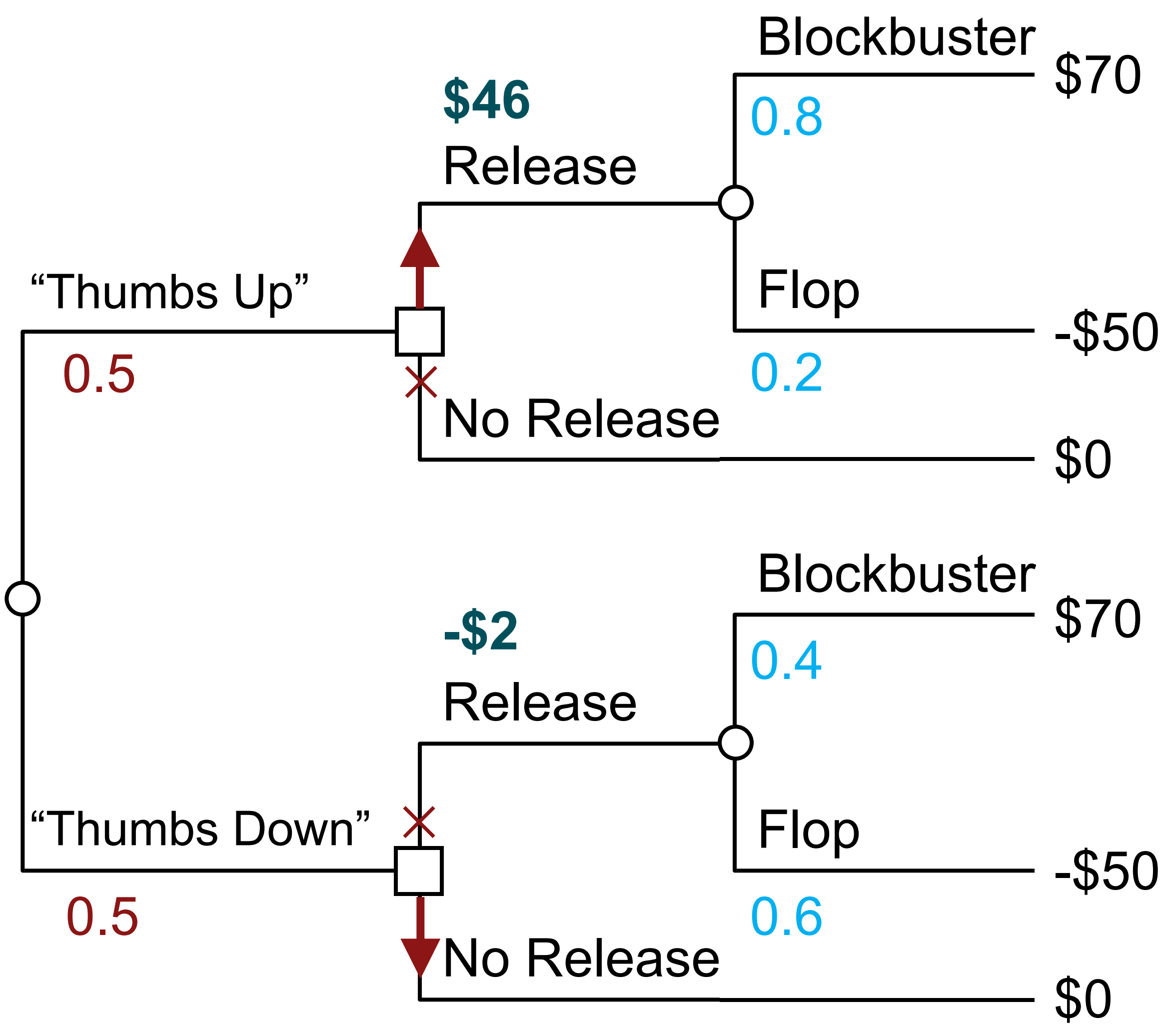
- What should Fanny do?

Sneak Preview "Test"

- We need to infer the posterior probabilities of a Blockbuster or Flop.



Sneak Preview "Test"



Alternative | "Thumbs Up"
→ Release

CE(with test | "Thumbs Up")
= max{$56 + (-$10), $0}
= max{$46, $0}
= $46M

CE(with test)
= 0.5 ($46M) + 0.5 ($0M)
= $23M

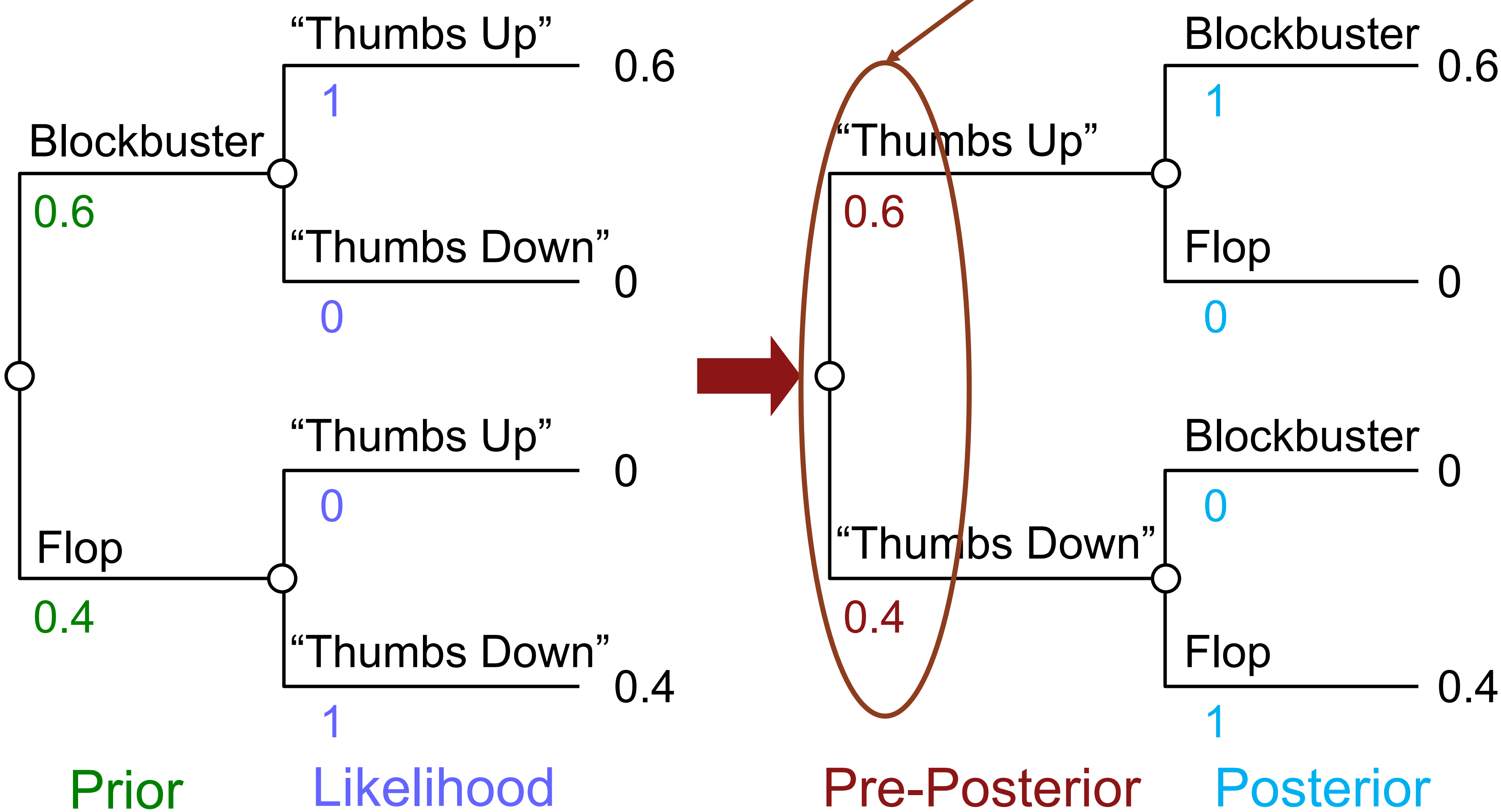
CE(with test | "Thumbs Down")
= max{$28 + (-$30), $0}
= max{$-2, $0}
= $0M

Alternative | "Thumbs Down"
→ No Release

Sneak Preview "Test"

- What if it were a **perfect test**? What changes?

The same as in the tree a few slides ago.



Final Review I

- Inventory Management
  - ❑ EOQ Model
  - ❑ Newsvendor Model
  - ❑ (Q, R) policy
- Capacity and Waiting Time
  - ❑ M/M/1 Queue
  - ❑ G/G/N
  - ❑ Load and Capacity
- Supply Chain Management
- Revenue Management
- Decision Analysis

Inventory Management with Certain Demand

Consider a single inventory item. Demand rate is fixed at λ units/time. Shortage is not allowed, and order quantity is fixed at Q per cycle. There is no lead time. Two common types of questions seen:

- Determine Q^* in EOQ
- Determine Q^* in EOQ with discount(s)
  - All-unit discount
  - Incremental discount

Newsvendor Model

Consider a single inventory item. There is only one period, and the demand follows a distribution (e.g. normal/uniform distribution, etc.) Common types of questions seen:

- Determine Q^*
- Determine expected profits/sales/leftover

(Q,R) Model, with Service Levels

Single product or no product interactions. Inventory levels are reviewed continuously. Expected demand is λ per unit time with lead time τ. Choose R to meet the demand during lead time.

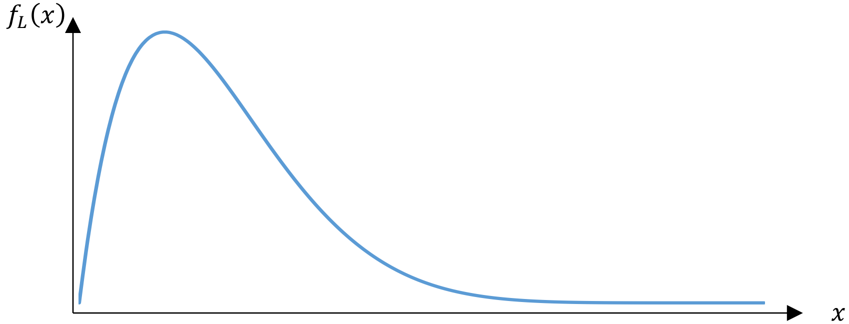
- Determine (Q^*, R^*)
- Type I service level (α)
  - The proportion of cycles in which no stockouts occur
- Type II service level (β)
  - Fraction of demand satisfied on time

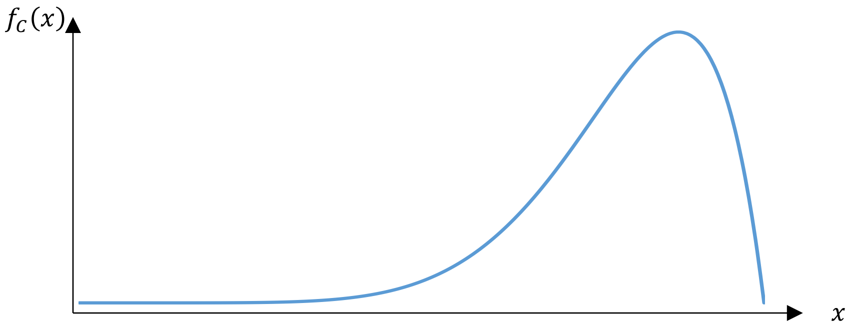
M/M/1 Queue and G/G/N

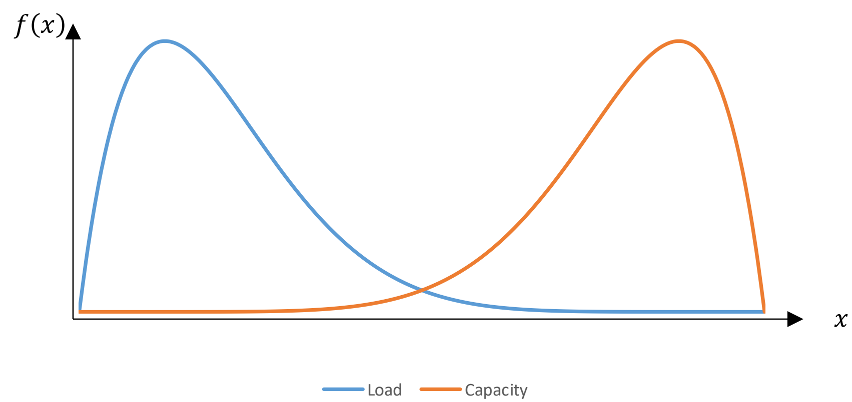
- M/M/1: Arrival rate λ follows a Poisson distribution. Service rate follows an exponential distribution.

- G/G/N: Not sure about the distributions of arrival rate and service rate.

Load and Capacity







Probability of failure (P_f) computation

$$P_f = \int_x f_L(x) F_C(x)\,dx$$
$$= P(L = x \text{ and } C < x)$$

$$P_f = \int_x f_C(x) G_L(x)\,dx$$
$$= P(C = x \text{ and } L > x)$$

where

$$G_L(x) = 1 - F_L(x)$$

Supply Chain Management

- Perfectly-coordinated supply chain
- Revenue-sharing Contract
- Buy-back Contract
- Know how to compute: expected sales, expected loss, retailer and supplier's expected profits, etc.

Revenue Management

- **Static Pricing:**
  - Demand is deterministic. How do you price the item?

$$\max_{p} \; d(p) \cdot p$$

- **2-Segmentation Pricing:**
  - Demand is deterministic, but we have two target groups. How do you price the item?

- **Dynamic Pricing:**
  - Demand is stochastic. How do you price the item?

Decision Analysis

- A normative science
- Bayesian update
- Influence diagrams
- Decision trees
- Risk attitudes
- Value of information

Example: Newsvendor Model (2016 Midterm)

John buys newspapers from the local kiosk at 10 cents a copy and sells them at 35 cents a copy. He buys all his papers at once early in the morning and he sells on each day until he is pretty sure that no one else will buy his papers, or he runs out of copies. John recycles the unsold papers at 5 cents.

a) Find the underage and overage cost.

b) John knows that the demand is normally distributed with mean 100 and variance 100. How many papers should John buy each day?

c) What is the expected fill rate with the quantity you found in (b)?

d) Now assume that the demand has a normal distribution with mean μ and variance 100. What is the minimum value of μ such that John's daily expected profit is at least \$40?