MS&E 260 Homework 1 Solutions

Summer 2019, Stanford University

Due: July 3rd, 2019, at 10:30AM (PDT)

Problem 1. (a) Let x_i be the amount of alloy i used in production. The problem is equivalent to solving the following optimization problem

$$
\begin{aligned}
\min_{x_1, x_2, x_3, x_4} \quad & x_1 + 3x_2 + 2x_3 + 4x_4 \\
\text{s.t.} \quad & 0.2x_1 + 0.4x_2 + 0.1x_3 + 0.35x_4 = 0.6, \\
& 0.8x_1 + 0.6x_2 + 0.9x_3 + 0.65x_4 = 0.4, \\
& x_i \geq 0 \quad \forall i \in \{1, 2, 3, 4\}
\end{aligned}
$$

(b) The following formulation solves the desired optimization problem:

$$
\begin{aligned}
\min \quad & \sum_{ij} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n
\end{aligned}
$$

(c) Let x_{A_i} be the amount transported from source A to disposal site i. Let x_{B_i} be the amount transported from source B to disposal site i. Note that $i \in \{1, 2, 3\}$.

$$
\begin{aligned}
\min_{x_{A_i}, x_{B_i}} \quad & 35x_{A_1} + 15x_{A_2} + 20x_{A_3} + 20x_{B_1} + 5x_{B_2} + 15x_{B_3} \\
\text{s.t.} \quad & \sum_i x_{A_i} = 200, \\
& \sum_i x_{B_i} = 400, \\
& x_{A_1} + x_{B_1} \leq 250, \\
& x_{A_2} + x_{B_2} \leq 150, \\
& x_{A_3} + x_{B_3} \leq 100, \\
& x_{A_1}, x_{A_2}, x_{A_3}, x_{B_1}, x_{B_2}, x_{B_3} \geq 0
\end{aligned}
$$

Problem 2. (a)

$$Q^* = \sqrt{\frac{2 \times 250 \times 24{,}000}{3 \times 0.15}} = 5{,}164$$

(b) i.

$$Q^*_{\$3} = \sqrt{\frac{2 \times 250 \times 24{,}000}{3 \times 0.15}} = 5{,}164$$

Since $Q \le 5{,}000$, we use $Q^* = 5{,}000$ to calculate the total cost:

$$TC(Q = 5{,}000, \$3) = \frac{24{,}000}{5{,}000} \times 250 + \frac{5{,}000}{2} \times 3 \times 0.15 + 24{,}000 \times 3 = 74{,}325$$

ii.

$$Q^*_{\$2} = \sqrt{\frac{2 \times 250 \times 24{,}000}{2 \times 0.15}} = 6{,}325$$

Since $Q < 6{,}250$, we use $Q^* = 6{,}249$ to calculate the total cost:

$$TC(Q = 6{,}249, \$2) = \frac{24{,}000}{6{,}249} \times 250 + \frac{6,249}{2} \times 2 \times 0.15 + 24{,}000 \times 2 = 49{,}897.5$$

iii.

$$Q^*_{\$1} = \sqrt{\frac{2 \times 250 \times 24{,}000}{1 \times 0.15}} = 8{,}944$$

Since $Q \ge 6{,}250$, we use $Q^* = 8{,}944$ to calculate the total cost:

$$TC(Q = 8{,}944, \$1) = \frac{24{,}000}{8{,}944} \times 250 + \frac{8{,}944}{2} \times 1 \times 0.15 + 24{,}000 \times 1 = 25{,}341.64$$

The optimal order quantity Q^* is 8,944 donuts with a total cost of \$25,341.64.

(c) We solve $\frac{C(Q)}{Q}$:

i. \$3 for $Q \le 6{,}000$
ii. $\frac{600}{Q} + \$2.90$ for $6{,}000 < Q < 10{,}000$
iii. $\frac{2{,}000}{Q} + \$2.80$ for $Q \ge 10{,}000$

We have:

i. First increment

$$TC(Q_3) = 250(\frac{24{,}000}{Q}) + (3 \times 0.15)\frac{Q}{2} + 24{,}000 \times 3 = \frac{6{,}000{,}000}{Q} + 0.225Q + 72{,}000$$

$$TC'(Q_3) = -\frac{6{,}000{,}000}{Q^2} + 0.225 = 0$$

$$Q_3^* = \sqrt{\frac{6{,}000{,}000}{0.225}} = 5{,}164$$

$$TC(Q_3^*) = \frac{6{,}000{,}000}{5{,}164} + 0.225 \times 5{,}164 + 72{,}000 = \$74{,}323.79$$

ii.

$$
\begin{aligned}
TC(Q_{2.90}) &= 250(\frac{24{,}000}{Q}) + 0.15 \times (\frac{600}{Q} + 2.90)\frac{Q}{2} + 24{,}000(\frac{600}{Q} + 2.90) \\
&= \frac{6{,}000{,}000}{Q} + 45 + 0.2175Q + \frac{14{,}400{,}000}{Q} + 69{,}600 \\
&= \frac{20{,}400{,}000}{Q} + 0.2175Q + 69{,}645
\end{aligned}
$$

$$TC'(Q_{2.90}) = -\frac{20{,}400{,}000}{Q^2} + 0.2175 = 0$$

$$Q^*_{2.90} = \sqrt{\frac{20{,}400{,}000}{0.2175}} = 9{,}685$$

$$TC(Q^*_{2.90}) = \frac{20{,}400{,}000}{9{,}685} + 0.2175 \times 9{,}685 + 69{,}645 = \$73{,}857.84$$

iii. Third increment

$$
\begin{aligned}
TC(Q_{2.80}) &= 250(\frac{24{,}000}{Q}) + 0.15 \times (\frac{2{,}000}{Q} + 2.80)\frac{Q}{2} + 24{,}000(\frac{2{,}000}{Q} + 2.80) \\
&= \frac{6,000,000}{Q} + 150 + 0.21Q + \frac{48{,}000{,}000}{Q} + 67{,}200 \\
&= \frac{54{,}000{,}000}{Q} + 0.21Q + 67{,}350
\end{aligned}
$$

$$TC'(Q_{2.80}) = -\frac{54{,}000{,}000}{Q^2} + 0.21 = 0$$

$$Q^*_{2.80} = \sqrt{\frac{54{,}000{,}000}{0.21}} = 16{,}036$$

$$TC(Q^*_{2.80}) = \frac{54{,}000{,}000}{16{,}036} + 0.21 \times 16{,}036 + 67{,}350 = \$74{,}084.98$$

$TC(Q^*_{2.90})$ is the lowest, so we order 9,685 donuts for a total cost of \$73,857.84.

Another method:

For \$2.90 and \$2.80 increments, we must first find the additional cost to order.

- **\$2.90:** $\$18{,}000 + \$2.90(Q - 6{,}000) = 18{,}000 + 2.90Q - 17{,}400 = 600 + 2.90Q$
  The additional cost to order is now \$600.
- **\$2.80:** $\$30{,}000 + \$2.80(Q - 10{,}000) = 30{,}000 + 2.80Q - 28{,}000 = 2{,}000 + 2.80Q$
  The additional cost to order is now \$2,000.

We continue with the same logic to find Q^* and total costs.

$$Q^*_{\$3} = \sqrt{\frac{2 \times 250 \times 24{,}000}{3 \times 0.15}} = 5{,}164$$

$$TC(Q = 5{,}164, \$3) = \frac{24{,}000}{5{,}164} \times 250 + \frac{5{,}164}{2} \times 3 \times 0.15 + 24{,}000 \times 3 = \$74{,}323.79$$

$$Q^*_{\$2.90} = \sqrt{\frac{2 \times (250 + 600) \times 24{,}000}{2.90 \times 0.15}} = 9{,}685$$

$$TC(Q = 9{,}685, \$2.90) = \frac{24{,}000}{9{,}685} \times (250+600) + \frac{9{,}685}{2} \times 2.90 \times 0.15 + 24{,}000 \times 2.90 = \$73{,}812.84$$

$$Q^*_{\$2.80} = \sqrt{\frac{2 \times (250 + 2{,}000) \times 24{,}000}{2.80 \times 0.15}} = 16{,}036$$

Since $Q < 6{,}250$, we use $Q^* = 6{,}249$ to calculate the total cost:

$$TC(Q = 16{,}036, \$2.80) = \frac{24{,}000}{16,036} \times (250+2{,}000) + \frac{16{,}036}{2} \times 2.80 \times 0.15 + 24{,}000 \times 2.80 = \$73{,}934.98$$

The optimal order quantity is 9,685 donuts with a total cost of \$73,812.84.

Problem 3. (a)

$$Q^* = \sqrt{\frac{2 \times 200 \times 1{,}600}{4}} = 400 \text{ units}$$

(b)

$$\text{Reorder point} = \frac{1}{52} \times 1{,}600 = 30.8 \sim 31 \text{ units}$$

(c)

$$\text{Setup cost} = \frac{200 \times 1{,}600}{400} = \$800$$

(d)

$$Q^* = \sqrt{\frac{2 \times 200 \times 1{,}600}{4 \times (1 - \frac{1{,}600}{8{,}000})}} = 447.2 \sim 447 \text{ units}$$

(e)

$$\text{The maximum inventory level} = (1 - \frac{1{,}600}{8{,}000}) \times Q^* = 358 \text{ units}$$

(f)

$$\text{The total annual holding cost} = 4 \times \frac{447}{2} \times (1 - \frac{1{,}600}{8{,}000}) = \$715.20$$

Problem 4. Solutions vary. Answers are evaluated on the basis of completeness on the required criteria. This includes summary of the article, critical thinking applied to interpretation of the article, and application of lectured concepts to the topics discussed in the article.