MS&E 260

INTRODUCTION TO OPERATIONS MANAGEMENT

Problem Session 3

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Course Logistics Announcements

- Homework 3 due on July 24th (not this coming Wednesday)

- Midterm next Friday, July 19th
  - 24-hour take-home exam (5pm Thursday, July 18th through 4:59pm Friday, July 19th)
  - Detailed instructions will be provided on Canvas and Piazza
  - Course materials covered up till next Monday's lecture
  - Midterm review lecture next Wednesday, July 17th
  - No problem session next Friday, July 19th

- SCPD office hours next Monday (July 15th) will be held by Eline
  - Different Zoom link will be sent out

- Office hours next Tuesday (July 16th) will be held by Tina

What We Learned This Week

- Inventory control for uncertain demand (Q, R)

- Introduction to capacity and waiting times

Inventory Control Subject to Uncertain Demand

- Two types of inventory control models
  - **Fixed time period** – periodic review
    - One period (Newsvendor model)
    - Multiple periods
  - **Fixed order quantity** – Continuous review
    - (Q, R) models

(Q, R) Model Assumptions

- Inventory levels are reviewed continuously
- Single product or no product interactions
- Demand is random and stationary. Expected demand is λ per unit time
- Lead time is τ
  - Time elapsed is from the time an order is placed until it arrives
- The relevant costs are:
  - K Setup cost per order
  - h Holding cost per unit per unit time
  - c Purchase price (cost) per unit
  - p Penalty cost per unit of unsatisfied demand

(Q, R) Model: Expected Total Cost per Unit Time

$C(Q, R)$ = holding cost + fixed cost + shortage cost

$$= h\left(\frac{Q}{2} + R - \lambda\tau\right) + K\frac{\lambda}{Q} + p\left(\frac{\lambda \cdot n(R)}{Q}\right)$$

$$Q = \sqrt{\frac{2\lambda[K + p \cdot n(R)]}{h}}$$

$$F(R) = 1 - \frac{Qh}{p\lambda}$$

Questions: How do we pull Q and R from these equations?

Answer: Solve iteratively!

Optimal Q as a Function of R

$$\downarrow Q = \sqrt{\frac{2\lambda[K + p \cdot n(R)]}{h}}$$

expected shortage

- As the reorder level R increases: ↑
  - There are fewer expected shortages per cycle ($n(R)$ decreases) ↓
  - This reduces the expected shortage cost incurred in each cycle
  - Therefore, the order quantity decreases

Service Levels

- In many circumstances, the penalty cost, p, is difficult to estimate
- For this reason, it is common business practice to set inventory levels to meet a specified service objective instead
- The two most common service objectives are:
  - Type I service level (α)
    - The proportion of cycles in which no stock-outs occur
    - Example: 90% Type I service level $\rightarrow$ There are no stock-outs in 9 out of 10 cycles (on average)
  - Type II service level (fill rate, β)
    - Fraction of demand satisfied on time

Service Levels Example (inventory < demand)

Order Cycle	Demand	Stock-outs
1	180	0
2	75	0
3	235	150
4	140	0
5	180	0
6	200	140
7	150	0
8	90	0
9	160	0
10	40	0
Total	**1,450**	**290**

Fraction of periods with no stock-outs

$$= \frac{8}{10}$$

Type I service = 80% ($\alpha = 0.8$)

Fraction of demand satisfied on time

$$= \frac{1{,}450 - 290}{1{,}450} = 0.8$$

Type II service = 80% ($\beta = 0.8$)

(Q, R) With Service Level Example

Problem 3. Stanford warehouse of the famous wine distributor WS&E stocks materials required for the cases of wines. One type of wine that Stanford warehouse distributes is the Burgundy Chardonnay. Each case of this wine is purchased by the warehouse for \$200. Since it is sent from Europe in intermodal containers it has a high lead time of 2 months (1/6 years) and the company uses an inventory carrying charge based on a 20% annual interest rate. The cost of order processing and receipt is \$1,000 per order. Annual demand for this wine follows a normal distribution with mean 240 cases and variance of 600 cases (standard deviation of ∼24.5 cases). Assume that if a case of wine is demanded when the warehouse is out of stock, then the demand is backordered, and the cost associated with each backordered case is estimated to be at \$80.

$\mu = \frac{240}{6} = 40$ $\sigma = \frac{24.5}{\sqrt{6}} = 10$

(a) Compute the mean and standard deviation of demand during lead time.

(b) The manager of the warehouse uses (Q, R) policy. Find the optimal values of the order quantity and the reorder level. $(Q^*, R^*) = (115, 47)$

(c) Determine the safety stock.

(d) What are the average annual holding, setup and stockout costs associated with this wine?

(e) What is the cost of uncertainty? (You may compare to a case that there is no uncertainty, think about what this case refers to.)

Type I service level (f) What is the proportion of order cycles in which no stock-outs occur? 76%

1- Type II svc level (g) What is the expected proportion of demand that cannot be met at once? 1.16%

h) If WS&E actually aims for 95% Type II service level, does it over or underestimate the penalty cost?

i) Now assume that the penalty cost information is replaced by a Type II service level of 95%. Find the optimal (Q,R) values after 2 iterations.

(h) Type II service level $= 1 - 1.16\% = 0.9884$ > 0.95

$\Rightarrow$ overestimating the penalty cost

(i) $Q^* = 115$

$n(R_1) = Q^* \cdot (1 - 0.95) = 5.75$ $n(R_1) = \sigma \cdot L(z_1)$

$L(z_1) = \dfrac{n(R_1)}{\sigma} = \dfrac{5.75}{10} = 0.575$

$\Rightarrow z_1 = -0.31$

$R_1 = (-0.31) \cdot 10 + 40 = 36.9$

in Excel:

Normdist( )

$\vdots$

Q_1

Next Topic: Capacity + Wait Times

Little's Law ($L = \lambda W$)

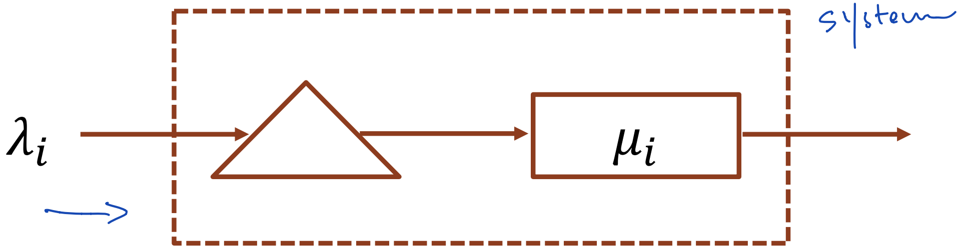
- Relationship between average inventory, average flow rate (throughput), and average flow time of a production system:

$$Average\ Inventory = Average\ Flow\ Rate \times Average\ Flow\ Time$$

"Q"

$L = \lambda \cdot W$

Demand/Capacity Analysis

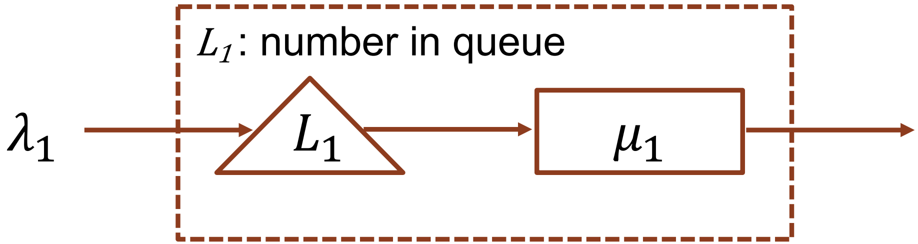


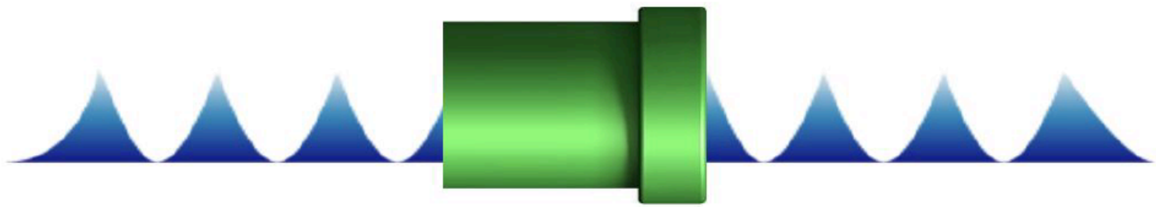
- For each process step i, determine:
  - λ_i: demand or input or arrival rate (in units of work per unit of time)
  - μ_i: realistic maximum service rate, assuming no idle time (in units of work per unit of time)

$\rho_i = \frac{\lambda_i}{\mu_i}$: capacity utilization

$\lambda_i - \mu_i$: buildup rate

Buildup Diagrams



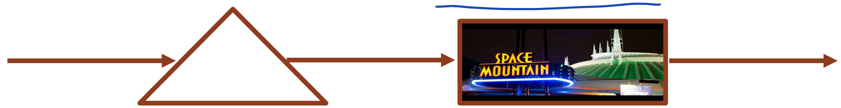


- Predictable variability
- $\lambda(t) > \mu(t)$ is okay
- Short run analysis
- Variable rates are okay

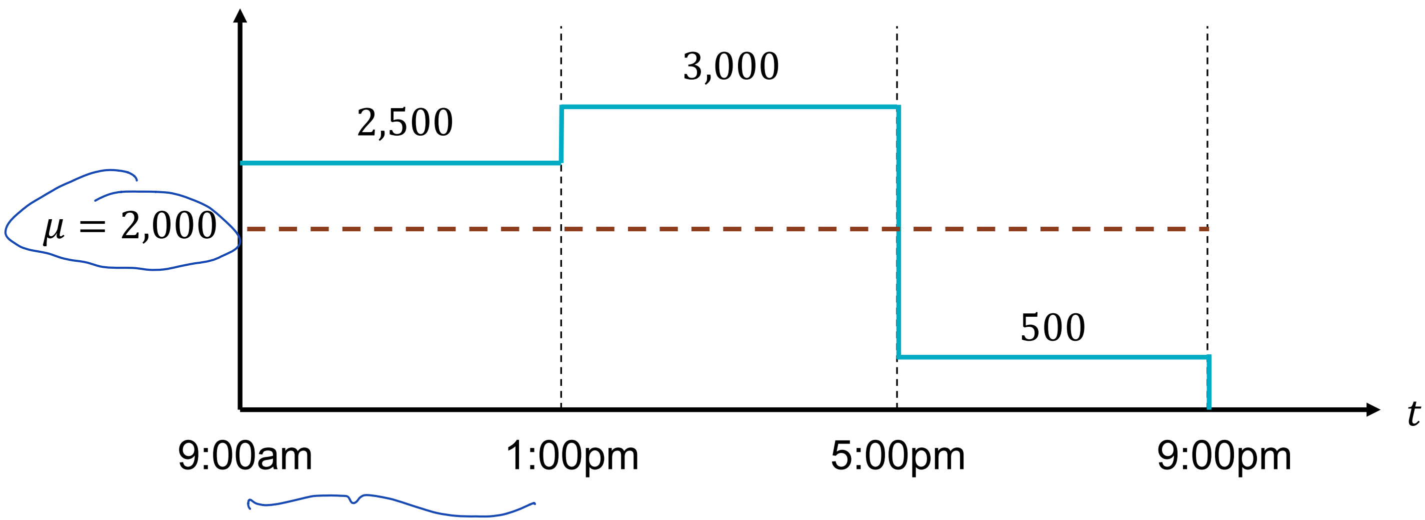
Buildup Diagram Example

Service rate $\mu = 2{,}000$ (People/Hour)

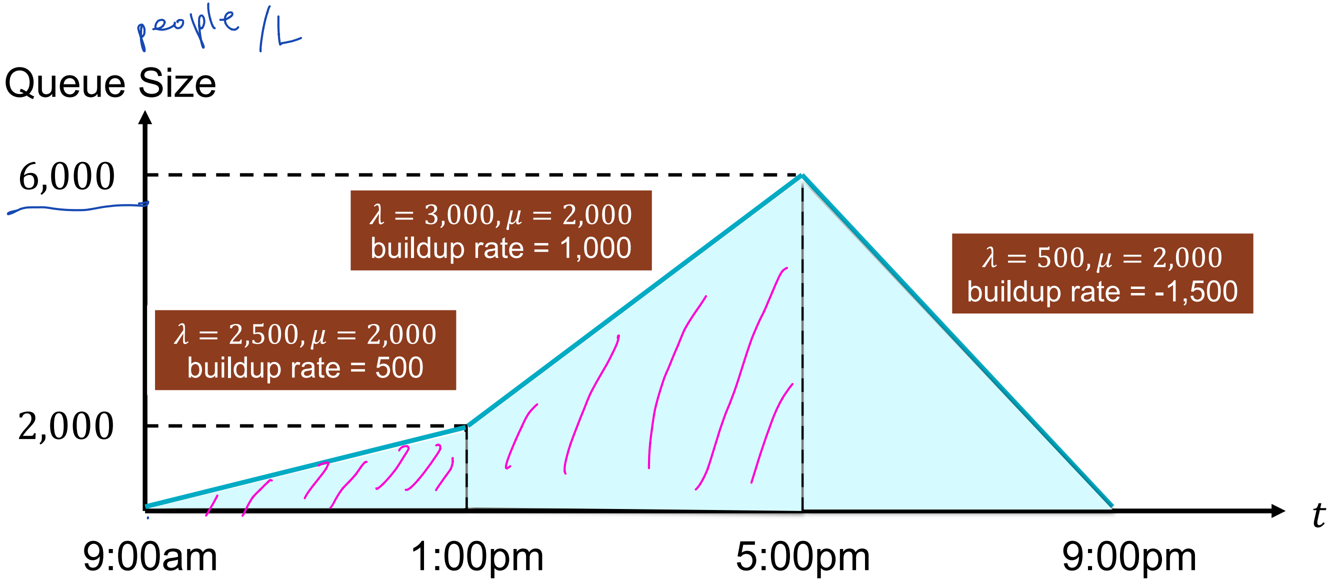
People arrive demand rate $\lambda(t)$



Demand rate $\lambda(t)$ (People/Hour)



Queue Buildup Example



- Average throughput = 2,000 people / hour
- Average queue size (inventory) = $2667 = \dfrac{2000\cdot 4/2 + \frac{(2000+6000)\cdot 4}{2} + \frac{6000\cdot 4}{2}}{12}$ $\dfrac{\text{"people}\cdot\text{hour"}}{\text{"hour"}}$
  unit: people
- Average wait = $\dfrac{2667}{2000}$

$L = \lambda W$

$= 1.33$ hours ≈ 80 mins

Variable Definitions

- $\lambda =$ arrival rate into system
- $\mu =$ service rate per server
- $N =$ number of servers
- $\rho =$ utilization rate $= \lambda/(N\mu)$
- $W_q =$ expected time customer spends in the queue in steady state
  - Wait time in queue
- $W =$ expected time customer spends in the system in steady state
  - Wait time plus service time
- $L_q =$ expected number of customers in the queue in steady state
  - Number of customers waiting
- $L =$ expected number of customers in the system in steady state
  - Number of customers in the system

memoryless memoryless

M/M/1 Queue

- Arrival rate λ follows a Poisson distribution (inter arrival follows exponential distribution)
- Exponential service rate with μ
- $N = 1$ (single server)

M/M/1 Queue: Number of People in the Queue

- Average time in queue (= average customers in system $\times$ average service time):

$$W_q = L \times \frac{1}{\mu}$$

- Average time in the system (= average time in queue + average service time):

$$W = L \times \frac{1}{\mu} + \frac{1}{\mu} = (L+1)\frac{1}{\mu}$$

- ⇒ By Little's Law ($L = \lambda W$):

$$W = \frac{1}{\mu} \times \frac{1}{1-\rho}$$

- Average number of people in the system:

$$L = \frac{\rho}{1-\rho}$$

- ⇒ Also given Little's Law ($L_q = \lambda W_q$):

$$L_q = \frac{\rho^2}{1-\rho}$$

" TV/ hour "

$\mu = 2$

Queueing Example

service time

- A repair man fixes broken televisions. The repair time is exponentially distributed with a mean of 30 minutes. Broken televisions arrive at his repair shop according to a Poisson stream, on average 10 broken televisions per day. Assume that the working hours of the repair man are 8 hours per day.

dd

$\lambda = \frac{10}{8} = 1.25$

"TV/hour"

fraction of idle time ? = 1 - utilization rate

a) What is the fraction of time that the repair man has no work to do?
b) How many televisions are, on average, at his repair shop?
c) What is the mean throughput time (waiting time plus repair time) of a television?
d) What is the probability that there are no televisions at his shop?

(a) $\underline{1-\rho} = 1 - \frac{\lambda}{\mu} = 1 - \frac{1.25}{2} = 1 - 0.625 = \underline{0.375}$

(b) $L = \frac{\rho}{1-\rho} = \frac{0.625}{0.375} = 1.667$ TVs

(c) $W = \frac{L}{\lambda} = \frac{1.667}{1.25} = 1.33$ hours

(d) same as part (a) : 37.5%