MS&E 260

Introduction to Operations Management

Problem Session 2

Eline van den Haak

What We Learned This Week

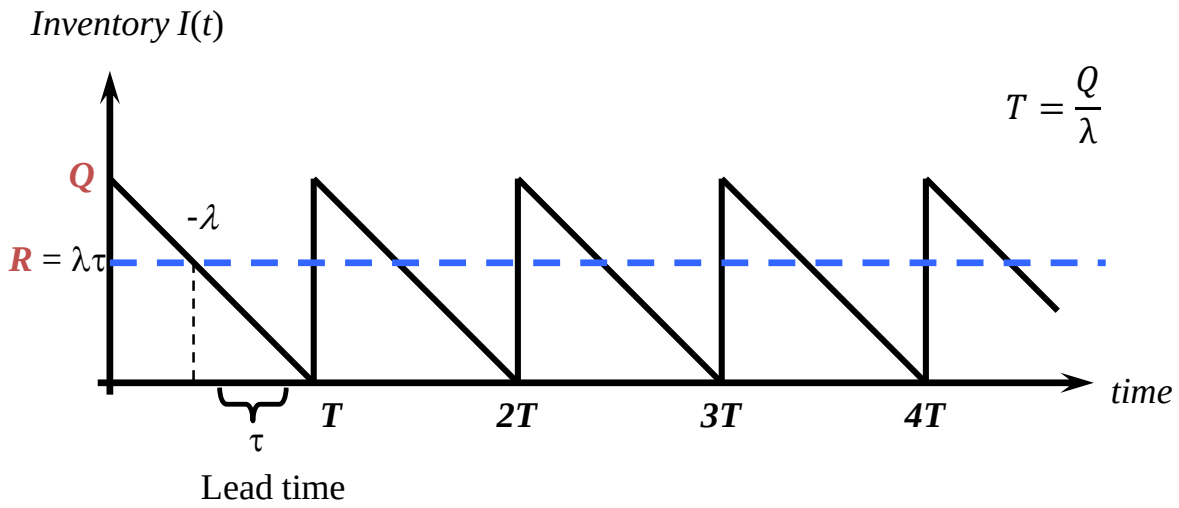
- Newsvendor
- Q,R Systems

Compare EOQ vs. Newsvendor? Continuous vs Periodic?

EOQ	Newsvendor
Deterministic = Demand is known	Stochastic = Demand is uncertain

Periodic	Continuous
(T,S) - Every T periods, order up to S units - Short shelf-life / backorder = costly	(Q,R) - When there are R units left, order exactly Q units - Costly to constantly be checking inventory

Recap: Basic EOQ



- Place an order when the inventory level is R. The order arrives after τ time periods
  - Q was the only decision variable
  - R could be computed easily because demand was deterministic

Economic Order Quantity (EOQ)

- Assumptions:
  - Consider a single inventory item
  - Demand is fixed (deterministic) at λ units/time
  - Shortages are not allowed
  - Order quantity is fixed at Q per cycle
  - Orders are received instantaneously (no lead time)
- Cost structure:
  - Fixed and marginal order costs per cycle $K + cQ$
  - Holding cost at h per unit held per unit time
- Objective: Determine order quantity Q^* to minimize sum of ordering cost and inventory holding cost

EOQ Derivation

- Cost function
  - Total Cost = Setup (Ordering) + Holding + Purchase

$$TC(Q) = \left(\frac{\lambda}{Q}\right)K + \left(\frac{Q}{2}\right)h + \lambda c$$

  - Q^* minimizes the total cost function

$$\frac{d[TC(Q)]}{dQ} = \left(-\frac{\lambda}{Q^2}\right)K + \left(\frac{h}{2}\right) = 0$$

$$Q^* = \sqrt{\frac{2K\lambda}{h}}$$

Newsvendor Model

- Motivation:
  - At the start of each day, a newspaper vendor must decide on the number of papers to purchase and sell
  - Daily sales cannot be predicted exactly, and are represented by the random variable, D
- Relevant costs:
  - c_o = unit cost of overage (not enough demand)
    Cost of having positive inventory left over at the end of period
  - c_u = unit cost of underage (too much demand)
    Cost of unsatisfied demand
- Example: Fashion
  - Retail: Stanford Shopping Center
  - Overage: Gilroy Outlet Mall

Newsvendor Model: Finding the Optimal Order Quantity

$G(Q, D)$: total overage + underage cost (if Q is ordered and demand is D)

$$G(Q, D) := \begin{cases} c_u(D - Q) & if\ D \geq Q \\ c_o(Q - D) & if\ D \leq Q \end{cases}$$

$G(Q) =$ expected overage + underage cost if Q is ordered

$$G(Q) = E[G(Q, D)] = \int_0^\infty G(Q, x) f(x) dx$$

$$= \int_0^Q c_o(Q - x) f(x) dx + \int_Q^\infty c_u(x - Q) f(x) dx \Rightarrow$$

$$F(Q) = \frac{c_u}{c_u + c_o}$$

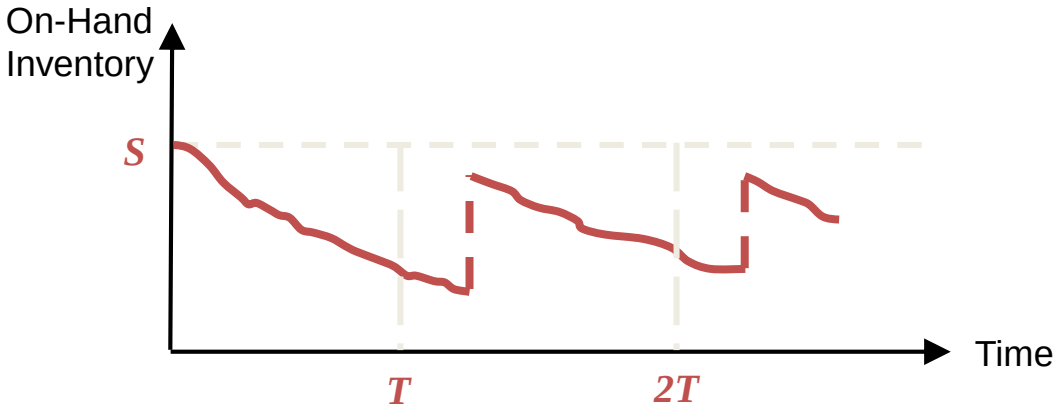
Critical Ratio!

Summary of Newsvendor Model

- Single period
- Depending on the relationship between the cost of shortage or excess inventory, we may order more or less than expected demand
- Optimal order quantity
  - Increases as shortage cost increases
  - Decreases as holding cost increases
- Higher variability may cause an increase or a decrease in the optimal order quantity
- As σ increases, Q^* will deviate more from the mean

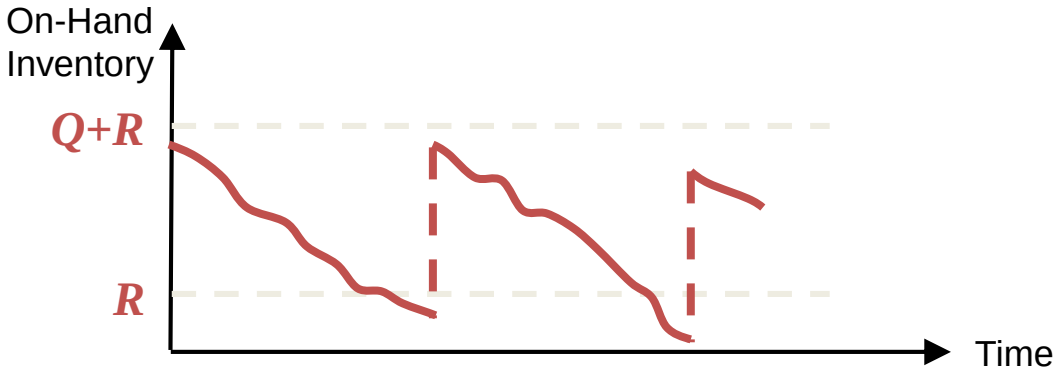
Periodic Review

- (T, S) System
  - Every T periods order up to S units



Continuous Review

- (Q, R) System
  - When the inventory level reaches R (order point), order exactly Q units (order quantity)



(Q,R) Model Assumptions

- Inventory levels are reviewed continuously
- Single product or no product interactions
- **Demand** is **random** and **stationary**. Expected demand is λ per unit time
- Lead time is τ
  - Time elapsed is from the time an order is placed until it arrives
- The relevant costs are:
  - K Setup cost per order
  - h Holding cost per unit per unit time
  - c Purchase price (cost) per unit
  - p Penalty cost per unit of unsatisfied demand

Exercises

EOQ Question

A Turkish racing bike manufacturer called XMB is using a special pillion seat in its brand new bike model. The bike sales for this model show a fairly steady demand of 5,600 bikes per year. Traditionally, XMB purchases these seats from a producer in Germany at a price of $8/unit. It costs XMB $100 to place an order. Inventory holding costs are based on an annual interest rate of 20%.
Suppose that the seat supplier is offering a quantity discount applied to all units with the following schedule (total unit cost):

- $8Q for $Q \leq 800$
- $7Q for $800 < Q < 1,000$
- $6Q for $1,000 \leq Q$

(A) What is the optimal order quantity in this case?
(B) Using your optimal order quantity, what is the total cost?

Newsvendor Example (w/ discrete distribution)

Source:
https://static1.squarespace.com/static/541ede81e4b02b592c3cc859/541ee543e4b000f16800f2c4/56ea742122482e1611ccc53a/1458562115204/SELFRIDGES.Rainbow+Bagels+%284%29.jpg?format=1500w

Newsvendor

Nancy's High-Five Bagel sells fresh baked bagels every morning. The daily demand for bagels is a random variable with a distribution estimated from prior experience given by the following table.

Number of bagels sold in one day	*Cumulative probability*
0	0.05
5	0.13
10	0.22
15	0.32
20	0.47
25	0.72
30	0.87
35	1.00

Each bagel costs Nancy's 9 cents to make, and they are sold for 50 cents each. All leftover units should be discarded at the end of the day at no net value.

(a) Based on the given discrete distribution, what are the overage and underage costs? And how many bagels should Nancy bake at the start of each day?

(b) Now suppose that the bagels unsold at the end of the day are purchased by a nearby charity soup kitchen for 3 cents each. What are the overage and underage costs in this case? And how many bagels should Nancy bake at the start of each day?

Note: For all parts, you can either interpolate from the empirical cumulative distribution function (and round consistently) to find a quantity, or simply pick one of the multiples of five as if there is no possibility of demand realizations in between.

Newsvendor Cont.

(c) Now instead of baking the bagels in-house, Nancy sells the equipment to buy the bagels from Kelly's store every morning at 15 cents each, to sell them for 50 cents. In addition, to promote the sales of bagels, Kelly's store buys back the left-over bagels at the end of the day from the Nancy's at 5 cents. What are the overage and underage costs in this case? And how many bagels should Nancy buy at the start of each day?

(d) Nancy still buys the bagels from Kelly's store every morning at 15 cents each and sells them for 50 cents. Now, instead of buying back the unsold units, Kelly's store agrees 1 to pay Nancy a rebate of 1 cent for every unit sold to end customers at Nancy's. What are the overage and underage costs in this case? And how many bagels should Nancy buy at the start of each day?

(Q,R) question

Stanford warehouse of the famous wine distributor WS&E stocks materials required for the cases of wines. One type of wine that Stanford warehouse distributes is the Burgundy Chardonnay. Each case of this wine is purchased by the warehouse for $200. Since it is sent from Europe in intermodal containers it has a high lead time of 2 months (1/6 years) and the company uses an inventory carrying charge based on a 20% annual interest rate. The cost of order processing and receipt is $1000 per order. Annual demand for this wine follows a normal distribution with mean 240 cases and variance of 600 cases2 (standard deviation of ∼24.5 cases). Assume that if a case of wine is demanded when the warehouse is out of stock, then the demand is backordered, and the cost associated with each backordered case is estimated to be as $80.

(a) Compute the mean and standard deviation of demand during lead time.
(b) The manager of the warehouse uses (Q, R) policy. Find the optimal values of the order quantity and the reorder level.