MS&E 260 Homework 2 Solutions

Summer 2019, Stanford University

Due: July 10th, 2019, at 10:30AM (PDT)

Problem 1. (a)

$$c_o = 5 - x$$
$$c_u = 13 - 5 = 8$$
$$\text{critical ratio} = \frac{c_u}{c_0 + c_u} = \frac{8}{(5-x)+8}$$

Also, $Q^* = 280$. We have $z = \frac{280-250}{30} = 1$, and it follows that $\phi(1) = 0.8413$. So x needs to satisfy:

$$\frac{8}{(5-x)+8} = 0.8413$$

which gives us $x \approx 3.5$.

(b) First, we derive the cumulative probability of the demand distribution:

Q	240	245	250	255	260	265	270
P(demand = Q)	0.07	0.12	0.23	0.17	0.16	0.20	0.05
P(demand $\leq$ Q)	0.07	0.19	0.42	0.59	0.75	0.95	1

Table 1: Estimated Discrete Demand Distribution

Next, we compute the critical ratio:

$$c_o = 5 - 2 = 3$$
$$c_u = 13 - 5 = 8$$
$$\text{critical ratio} = \frac{c_u}{c_o + c_u} = \frac{8}{11} = 0.73$$

Since $F(260) = 0.75$, the optimal quantity $Q^* = 260$.

(c) Let q be the quantity that Jerry is going to purchase. There are two cases:

i. If $q \leq 400$, then we have expected profit $E(\pi)$:

$$E(\pi) = 0.6 \times [(10 - 2) \times q] + 0.4 \times [(0 - 2) \times q] = 4q$$

To maximize the expected profit, $q^* = 400$.

ii. If $q \geq 400$, then we have expected profit $E(\pi)$:

$$E(\pi) = 0.6 \times [(10 - 2) \times 400 + (0 - 2) \times (q - 400)] + 0.4 \times [(0 - 2) \times q] = 2{,}400 - 2q$$

To maximize the expected profit, $q^* = 400$.

Therefore, the optimal quantity $q^* = 400$.

Problem 2. The equation balances the expected overage costs and expected underage costs.

Problem 3. (a)

$$\mu = \frac{240}{6} = 40$$
$$\sigma = \frac{24.495}{\sqrt{6}} = 10$$

(b) Step 1:

$$Q_0 = \sqrt{\frac{2 \times 1{,}000 \times 240}{40}} = 110$$
$$1 - F(R_1) = \frac{110 \times 40}{80 \times 240} = 0.23$$

Thus, $F(R_1) = 0.77$. From the table $z_1 = 0.74$. Therefore,

$$R_1 = \mu + \sigma \times z_1 = 40 + 10 \times 0.74 = 47$$

Step 2:

$$n(R_1) = \sigma L(z_1) = 10 \times 0.1334 = 1.334$$
$$Q_1 = \sqrt{\frac{2 \times 240 \times (1{,}000 + 80 \times 1.334)}{40}} = 115$$
$$1 - F(R_2) = \frac{115 \times 40}{80 \times 240} = 0.24$$

Hence, $F(R_2) = 0.76$. From the table $z_2 = 0.79$. Therefore, $R_2 = \mu + \sigma \times z_2 = 40 + 10 \times 0.71 = 47$. Notice that R has converged. Thus $(Q^*, R^*) = (115, 47)$.

(c) $s = R^* - \mu = 47 - 40 = 7$ units

(d)

$$\text{Average annual holding cost} = h \times (\frac{Q^*}{2} + R - \mu) = 40 \times (\frac{115}{2} + 7) = 2{,}580$$
$$\text{Average annual setup cost} = \frac{K\lambda}{Q^*} = \frac{1{,}000 \times 240}{115} = 2{,}087$$
$$\text{Average annual stockout cost} = \frac{p\lambda n(R^*)}{Q^*} = \frac{80 \times 240 \times 1.334}{115} = 222.72$$
$$\text{Total Cost} = 4{,}889.72$$

(e) When there is no variability, the EOQ solution is optimal. Thus,

$$\text{total cost with EOQ} = \sqrt{2K\lambda h} = 4{,}381.78$$

$$\text{Cost of uncertainty} = 4,889.72 - 4,381.78 = 507.94$$

(f) $P(D \leq R^*) = F(R^*) = 0.76$, thus 76%

(g) $\frac{n(R^*)}{Q^*} = \frac{1.334}{115} = 1.16\%$

Problem 4. Solutions vary. Answers are evaluated on the basis of completeness on the required criteria. This includes summary of the article, critical thinking applied to interpretation of the article, and application of lectured concepts to the topics discussed in the article.