MS&E 260 Homework 3
Summer 2019, Stanford University
Due: July 24th, 2019, at 10:30AM (PDT)

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Problem 1.

Think Geek sells cans of Unicorn Meat, an excellent source of sparkles and with magic in every bite. (If you're curious, check it out on Amazon.) Weekly demand of the good is normally distributed with mean 310 and variance of 1,280. Think Geek purchases a can of Unicorn Meat at a cost of $7 each. The processing cost for each order is $120 and the lead time is 5 weeks. Think Geek uses a 25% annual inventory holding cost rate. Under the current inventory policy, Think Geek purchases 1,240 cans of Unicorn Meat whenever the inventory falls below 1,650. Assume that there are 50 weeks in a year.

demand = 310 / week = 310 * 50 per year = 15500 per year
Expected yearly demand λ = 15500 per year
Expected demand during lead time μ = 15500 * 5 / 50 = 1550

Variance Weekly = 1280
Variance yearly = 1280 * 50 = 64000
Variance during lead time = 64000 * 5 / 50 = 6400
Standard deviation during lead time = $\sigma = \sqrt{6400} = 80$

c = $7
k = 120
T = 5 weeks
i = 25% annual
Annual inventory holding cost rate h = .25 * 7 = 1.75

Q = 1240
R = 1650
1 year = 50 weeks

a) What is the fill rate under the current inventory policy?

$R = \sigma z + \mu \Rightarrow z = (R - \mu) / \sigma = (1650 - 1550) / 80 \Rightarrow z = 1.25$

$$1 - \beta = \frac{n(R)}{Q} = \frac{\sigma L(z)}{Q} = \frac{80 * 0.0506}{1240} \Rightarrow \beta = 1 - 0.0032 = 0.9967 = 99.67\,\%$$

b) What order quantity and reorder point would minimize the total annual setup, holding and shortage costs subject to having a fill rate at least equal to 98%?

$$EOQ = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2 * 120 * 15500}{0.25 * 7}} = 1457.98 \cong 1458$$

$$1 - \beta = \frac{n(R)}{Q} = \frac{\sigma L(z)}{Q} \Rightarrow L(z) = \frac{(1-\beta)\,Q}{\sigma} = \frac{(1-0.98)\,1458}{80} = 0.3645 \Rightarrow z = 0.07$$

$R = \sigma\, z + \mu \Rightarrow R$ = 80 * 0.07 + 1550 = 1555.6 ≅ 1556

For 98% Type II service level: Q, R = 1458, 1556

c) Would the solution in part (b) change if Think Geek has a minimum Type I service level constraint of 80%? Explain why.

$$EOQ = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2\, *\, 120\, *\, 15500}{0.25\, *\, 7}} = 1457.98 \cong 1458$$

$\Phi(z)$ = 0.8 => z = 0.8416

$R = \sigma z + \mu \Rightarrow R$ = 80 * 0.8416 + 1550 = 1617.33 ≅ 1617

For 80% Type I service level: Q, R = 1458, 1617

The EOQ value does not change but the reorder level value R does changes from 98 % Service Level Type II to 80% service level Type I.

Problem 2.

Consider a queueing system where customers arrive according to a Poisson process with a mean rate of 30 customers per hour. Customers wait in a single line and are served by the next available server when they reach the front of the line. The service times are exponentially distributed.

Suppose for (a) and (b) ONLY that the time it takes for a server to serve one customer is 5 minutes on average:

mean arrival λ = 30 customers per hour
service rate μ = 60 / 5 services per hour = 12 services per hour

a) If four servers are used, what is the fraction of time that some server is not busy?

no work time = 1 – utilization = 1- ρ = 1 - λ / N μ = 1 – (30 / 4 * 12) = 0.375 = 37.5%

b) What is the minimum number of servers that could be used in order to avoid infinite queue size?

Utilization rate $\rho = \lambda$ / N μ
N=1 => $\rho = \lambda$ / N μ = 30/ (1*12) = 2.5 =250%
N=2 => $\rho = \lambda$ / N μ = 30/ (2*12) = 1.25 =125%
N=3 => $\rho = \lambda$ / N μ = 30/ (3*12) = 0.8333 = 83.33%

Since 100% utilization rate tends to make the queue infinite, total number of 3 servers or adding 2 more servers should keep the utilization rate below 100% and avoid infinite queue.

Suppose for the rest of the question that only one server is used:

c) The server has a service cost of $16 per hour per service. Assume that a significant cost is incurred by making a customer wait because of potential lost future business. We estimate the cost to be $0.50 for each minute a customer spends in the system, counting both waiting time and service time. What is the service rate that minimizes the long run average cost per unit time? What is the optimal cost in this case?

service cost c = \$16 per hour per service
waiting cost h = \$0.50 each minute customer is in the system = \$30 per hour per customer is in the system
mean arrival λ = 30 customers per hour

$$Optimal\ Service\ rate\ = \mu^* = \lambda + \sqrt{\frac{\lambda\, h}{c}} = 30 + \sqrt{\frac{30 * 30}{16}} = 30 + (30/4) = 37.4$$

$$Optimal\ cost\ = C(\mu^*) = c\,\lambda + 2\sqrt{\lambda\, h\, c} = 16 * 30 + 2\sqrt{30 * 30 * 16} = 720$$

d) For the service rate you found in part (c), what is the average length of the waiting line? (Please round your answer to the closest integer.)

Since assuming only one server is used
$\rho = \lambda / \mu$ = 30 / 37.4 = 0.802

Average length of the waiting line =

$$\text{No. of people in queue} = \mathrm{L_q} = \frac{\rho^2}{1 - \rho} = \frac{(0.802)^2}{1 - 0.802} = 3.248 \cong 3$$

e) Consider now a similar system which is empty at time 0. There is one arrival every two time slots, i.e. at time slots 1, 3, 5... The service time per customer is 1. What is the average length of the waiting line?

Since one arrival every alternate time slot so the arrival rate is reduced to half.
or $\lambda = 15$ customers / hour
$\mu = 1/1$ services per min $= 60$ services / hour

$\rho = \lambda / \mu$ = 15 / 60 = 0.25

Average length of the waiting line =

$$No. of\ people\ in\ queue\ = L_q = \frac{\rho^2}{1 - \rho} = \frac{(0.25)^2}{1 - 0.25} = 0.083 \cong 0$$

Since the service rate is faster than the arrival rate, the line will always be empty

Problem 3:
A drive-through carwash has 5 machines. The time between cars arriving at the carwash has unknown distribution with mean 4 minutes and a variance of 3 minutes. Cars are washed on a first come first served basis. The time it takes to wash a car per machine has a uniform distribution with a maximum of 8 minutes and a minimum of 6 minutes. Please answer the following questions:

Number of servers N = 5

arrival mean E[A] = 4 mins
arrival standard deviation σ[A] = $\sqrt{3}$ mins
average arrival rate λ = 1/4 customers per min = 60/4 = 15 customers per hour
Inter-arrival time distribution C_A = σ[A] / E[A] = $\sqrt{3}$ /4
$C_A^2 = 3 / 4^2 = 3/16 = 0.188$

service uniform distribution a = 6, b= 8
service mean time E[S] = (a + b) / 2 = 7
$$service\ standard\ deviation\ \sigma\ [S] = \frac{b - a}{\sqrt{12}} = \frac{8 - 6}{\sqrt{12}} = 0.577$$
service rate μ = 1/7 per min = 60 / 7 services per hour = 8.571 per hour
Inter-service time distribution C_s = σ [S] / E[S] = 0.577 / 7 = 0.0825
$$C_S^2 = (0.0825)^2 = 0.0068$$

N servers average capacity utilization $\rho = \lambda / (N\ \mu) = = \frac{1}{4} * \frac{7}{5} = 0.35$
$$\rho' = \rho^{\sqrt{2(N+1)} - 1} = 0.35^{\sqrt{2(5+1)} - 1} = 0.03075$$

a) How long does a car need to wait before getting served on average?

$$W_q = \frac{1}{\mu\, N} \times \frac{\rho^{\sqrt{2(N+1)} - 1}}{1 - \rho} \times \frac{C_A^2 + C_S^2}{2} = \frac{7}{5} \times \frac{0.03075}{0.65} \times \frac{0.188 + 0.0068}{2} = 0.00645\ mins \cong 0$$

b) How many people are waiting to be served on average?

$L_q = \lambda\, W_q$ = 0.00645 / 4 = 0.0016125

c) What is the expected time a car will spend in the carwash?

$$W = W_q + \frac{1}{\mu} = 0.00645 + 7 = 7.00645\ mins$$

d) How many people are in the carwash on average?

$L = \lambda\, W$ = *1/4 customers per min * 7.00645 mins = 1.752 customers*

e) What is the fraction of time there is no car in the carwash?

Idle time = no work time = 1 – utilization = 1- ρ = 1. 0.35 = 0.65 = 65.0 %

Problem 4.

Isaac runs a data science business and revenue depends on his website being accessible. He is concerned about access to the site being blocked by basic distributed denial of service at- tacks. To investigate this, you will do a "back of the envelope" risk calculation.

You are given the following information: The attackers utilize very basic attack tools, such as a simple open-source application called Praetox Technologies. Once the application is downloaded - either voluntarily or in a variant form via a malicious link - the application recruits computers into a network that floods a designated website with traffic until it slows or collapses under the load. If an attack occurs, we believe that the number of connections per millisecond to Isaac's server will be uniformly distributed between 200 and 300, which we denote as $F_L(x)$.

Assume that the number of connections is constant during an attack. If the number of connection attempts per time unit exceeds the server's capability, the site will become inaccessible to real users.

a) If Isaac's server has an effective server capacity of C = 250 connections per millisecond, what is the probability of a failure, given that an attack occurs (failure occurs when the capacity exceeds 250 connections per millisecond)?

Probability of failure = probability that load exceeds 250

$$P(Failure) = P(Load > 250) = 1 - P(Load \leq 250) = 1 - \frac{250 - 200}{300 - 200} = 0.5$$

b) Now suppose that the server's capacity is also uncertain, and has the following probability density function (again in units of connections/millisecond):

$$g_c(x) = \begin{cases} 0.05 - 0.0002x & \text{if } 150 \leq x \leq 250 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability of a failure now?

$$P(failure) = \int_{-\infty}^{\infty} f(x)\,[1 - F_L(x)]\,dx$$

$$F_L(x) = \begin{cases} 0, & x < 200 \\ \dfrac{x - 200}{300 - 200}, & x \in [200,300] \\ 1 & x > 300 \end{cases}$$

$$P(failure) = \int_{150}^{200} (0.05 - 0.0002\,x) \times 1\,dx + \int_{200}^{250} (0.05 - 0.0002\,x) \times \frac{300 - x}{100}\,dx \cong 0.95$$

c) Isaac has calculated that a denial of service attack will result in $100K in loss of revenue (one time only.) In other words, attackers will not attack the server again after an attack has been successful. Google has also offered to increase Isaac's server capacity in order to reduce the probability of success for a denial of service attack. The Google server upgrade costs $50K (one time only.) This improvement results in the following probability density function (again in units of connections/millisecond):

$$g_c(x) = \begin{cases} 0.1 - 0.0004x & \text{if } 179.29 \leq x \leq 250 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability of a failure with the upgraded server?

$$P(failure) = \int_{179.29}^{200} (0.1 - 0.0004\,x) \times 1\,dx + \int_{200}^{250} (0.1 - 0.0004\,x) \times \frac{300 - x}{100}\,dx$$

$$P(failure) = [0.1\,x - 0.0002\,x^2]_{179.29}^{200} + \left[\frac{x\,(x^2 - 825\,x + 225000)}{750000}\right]_{200}^{250}$$

$$P(failure) = 0.49998 + 0.4167 = 0.91668 \cong 0.93$$

d) Assuming Isaac is a rational decision maker, should he purchase the Google server up- grade?
probable loss in case of b = 0.95 * 100 K = 95 K
Pr0bable loss in case of c = 0.93 * 150 K = 139.5 K
Since probable losses after upgrading is more, Isaac should not purchase server up-grade. The reduction in failure probability is not significant.

Problem 5.

Read the following articles:

- https://www.latimes.com/business/la-fi-disneyland-wait-times-20170712-htmlstory.html
- http://fortune.com/2019/04/30/artificial-intelligence-walmart-stores

Select one of these two articles, and do the following:

1. Briefly summarize the article.
2. Provide a few interpretive thoughts on the article, using what you have learned from class.
3. Provide one recommendation on how the dilemma posed in the article could be resolved.

   Some notes:

- Please limit your responses to one page, double spaced, 12 point font.
- There is no right answer to this question. We are evaluating your ability to apply what you learn in class to practical applications.
- This question is not intended to be free points. If you do not demonstrate a sufficient level of critical thinking, full credit will not be awarded.

The article "Disneyland raised prices to shorten waits. Here are the results" by Hugo Martin and Ben Poston primarily describes how big of a deal is queueing for Disney. The park has seen long queues due to its popularity since the day it has opened and has made officials pioneer in dealing with effective queueing mechanisms. The popularity just keeps on increasing and no solution alone is sufficient enough to facilitate decrease in wait times. Disney is very well aware of all the increment in already long wait times and have taken measures such as: adding average wait time displays, adding Fastpass'es, adding separate one person queues, changing pricing for different days, adding new rides to distribute load, etc. In spite of these measures, the average wait time is increasing every year and worst cases waits can be up to 5 hours in some cases on peak days.

The wait time problem at Disney is so big that there are AI and ML companies that have stated analyzing past data and provide effective ways to reduce the wait times. Just like mentioned in the article, what day of the week, what month of the year and what hour of the day to take which specific ride makes a big difference in the wait times. While long queue times adds frustration and spoils the fun of it, no wait time also does not help. Some small wait time gives the visitor the opportunity to savor the decor and build up enthusiasm. Some laughs and time with friends and family makes the attraction more important and adds to the overall experience of the park.

With today's tech available mobile phones are the best friend, when it comes to solving any such issues. As mentioned in the article there are digital versions of Fastpass available now, I think that app version can be improved to solve the bigger problem. There can be an app each member login's as themselves and join a group. The group plans to visit the park on the day between certain hours. Machine learning and AI can be used to study the past data for average time of n people for a particular ride. Using that data suggestions, routing and booking windows can be suggested and booked for the whole group. The booking window may be about 20 mins at the ride. This will ensure some wait time and no frustrations for the whole group. This will not solve the whole problem as there might be subgroups, no shows, etc. But it will help a majority of the visitors make the most of their buck.