MS&E 260 Homework 5

Summer 2019, Stanford University

Due: August 12th, 2019, at 10:30AM (PDT)

Problem 1. You are selling coats and forecast that the demand function is $D(p) = 1{,}000 - 5p$.

(a) What is the optimal price? How many units do you sell at the optimal price and what is your revenue?

(b) From your past experience, you notice that you can actually segment the market perfectly. You observe that customers who are willing to pay at least \$50 will always be attracted to buy online and customers who are willing to pay less than \$50 will always go to the store. What prices would you choose for the store and for the website (assuming there is no delivery cost)? What is the total revenue?

(c) Suppose that you sold at one optimal price as in part (a), and you decide to sell at a lower price for the remaining inventory you have (customers who bought in the original price will no longer buy in the markdown price.) What will be the optimal markdown? What will be your total revenue (including the revenue in part (a))? Is your total revenue here larger or smaller than what you got in part (b)? Explain briefly why.

Problem 2. A firm is considering various pricing strategies for their new product. The firm estimates that customers have valuations $\theta \sim U(0,1)$. In general, the utility of a customer with valuation θ from buying a product at price p is equal to $\theta - p$. We assume that a buyer is willing to buy a product at a given price p if and only if his utility is at least 0.

The firm is planning to sell initially to high valuation customers. Next year, it will sell the product to customers with lower values. Low-value customers anticipate prices will fall and delay purchases until price falls. There is a discount factor δ for both the seller and the buyers and all other costs are zero.

Note: Since there is a discount factor for period 2, we have to be more careful with buyers' utilities in period 2. For example, if a buyer with valuation 0.3 buys the product in year 2 at a price of 0.2, then his utility is $\delta(0.3 - 0.2) = 0.1\delta$.

(a) How would you approximate the expected demand in period 2 given the price p of the product when you want to sell to customers of valuations $[0, \theta_1]$?

(b) Suppose that the firm sells to customers of valuations $[0, \theta_1]$ in year 2. Express the profit of the firm in year 2 as a function of θ_1 and the price of the product in year 2. What is the optimal price p_2^* maximizing the profit in year 2? Express p_2^* as a function of θ_1.

(c) Find a condition such that the customers of valuation θ_1 are indifferent between buying in years 1 and 2. Use this result to express the price of the product in year 1, as a function of θ_1 and δ.

(d) Use parts (b) and (c) to express the total profit from year 1 and 2 as a function of θ_1 and δ only.

(e) Use part (d) to formulate the problem of finding the valuation θ_1^* that maximizes the total profit as a maximization problem. Solve this maximization problem for the optimal type θ_1^*.

Problem 3. John is the chief operating officer of Alpha Corp, which produces optical imagers for mobile phone manufacturers. John is considering an expansion of Alpha's current business into a new business line: producing specialized optical imagers for medical devices. This will require capital investment to build new manufacturing infrastructure. If Alpha expands into this new business line, then there is one relevant uncertainty that John must consider: market demand. If market demand is high, then Alpha will receive a net income of \$50M in the next year. If market demand is low, then Alpha will incur a \$10M loss in the next year, due to the uncovered capital expenditures associated with the business line expansion. John's prior belief on the market demand is that it will be high with a probability of 0.4. Suppose the market research from Beta Corp is able to provide better information on the demand uncertainty for John, in the form of a market research report. Beta Corp is widely respected for the accuracy of their reports, and John estimates that if there will be high demand, Beta's reports will predict high demand 90% of the time. If there will be low demand, Beta's report will predict low demand 80% of the time.

(a) Assuming John is a risk neutral decision maker, what is his optimal alternative (to form the new business line, or not), without using Beta's market research report? What is John's expected value for his optimal alternative?

(b) What is the maximum price that John should be willing to pay for Beta's market research report?

(c) For the original decision (with no market research report), how much does John's prior belief on the market demand uncertainty need to change before his optimal alternative changes?

(d) Based upon this answer, would you consider John's optimal alternative (as given in Part (a)) to be a robust decision?

(e) If Beta's market research report was a perfect test of future market demand (no false positive or false negative signals), what is the value of this test?

Problem 4. Read the following articles:

- `https://hbr.org/2016/05/can-lean-manufacturing-put-an-end-to-sweatshops`
- `https://www.forbes.com/sites/douggollan/2019/04/05/united-airlines-move-to-stop-publishing-award-levels-is-unfriendly/#7c64c4a817fa`

Select one of these two articles, and do the following:

1) Briefly summarize the article.
2) Provide a few interpretive thoughts on the article, using what you have learned from class.
3) Provide one recommendation on how the dilemma posed in the article could be resolved.

Some notes:

- Please limit your responses to one page, double spaced, 12 point font.
- There is no right answer to this question. We are evaluating your ability to apply what you learn in class to practical applications.
- This question is not intended to be free points. If you do not demonstrate a sufficient level of critical thinking, full credit will not be awarded.