MS&E 260

Introduction to Operations Management

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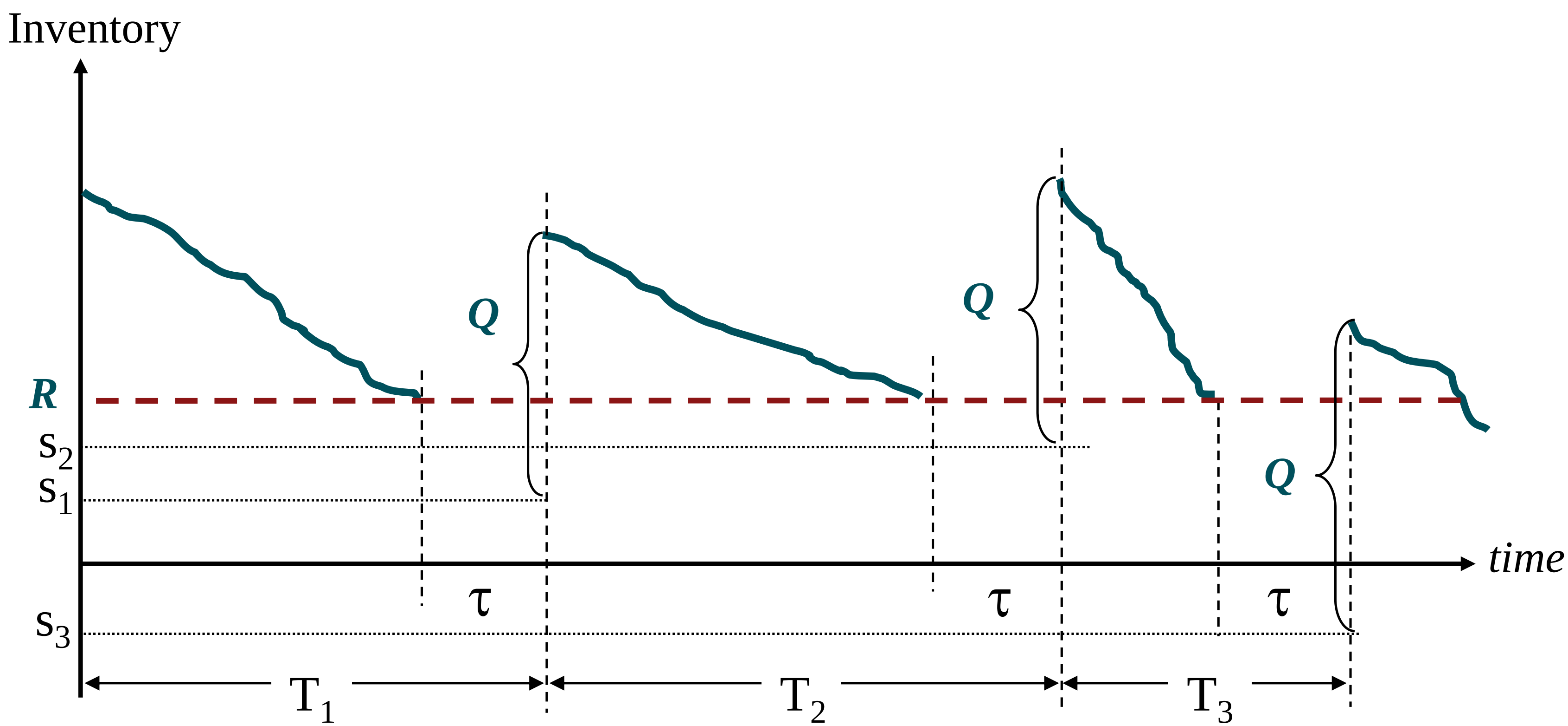
Inventory Management with Uncertain Demand

Inventory Control Subject to Uncertain Demand

- Inventory Systems with Uncertain (Stochastic) Demand
  - Newsvendor Model (single period)
  - **Continuous Review (Q,R) Model (multiple periods)**
    - **Per-Unit Backorder Penalty**
    - **Service Level (Fill Rate)**

Uncertain Demand



- Both Q and R are decision variables
- Cycle time is no longer constant

(Q,R) Decisions

- We choose R to **meet the demand during lead time**
  - Service levels: Protect against uncertainties in demand (or lead time)
  - Balance the costs: stock-outs and inventory
- Tradeoff in Q: Fixed cost versus holding cost
- Objective:
  - Minimize expected
    - fixed cost + holding cost + stockout (backorder) cost

Q tradeoffs R tradeoffs

(Q,R) Model Assumptions

- Inventory levels are reviewed continuously
- Single product or no product interactions
- **Demand** is **random** and **stationary**. Expected demand is λ per unit time
- Lead time is τ
  - Time elapsed is from the time an order is placed until it arrives
- The relevant costs are:
  - K Setup cost per order
  - h Holding cost per unit per unit time
  - c Purchase price (cost) per unit
  - p Penalty cost per unit of unsatisfied demand

(Q,R) Model: Expected Total Cost per Unit Time

$$C(Q) = \overbrace{h\left(s + \frac{Q}{2}\right)}^{\text{holding cost}} + \underbrace{\frac{K}{T}}_{\text{fixed cost}} + \overbrace{p\left(\frac{n(R)}{T}\right)}^{\text{shortage cost}} \qquad T = \frac{Q}{\lambda}$$

s = average inventory level before an order arrives

$= (reorder\ level) - (expected\ demand\ during\ lead\ time) = R - \mu$

$n(R) =$ expected amount of shortage per cycle

$D > R \Rightarrow shortage = D - R$

$D < R \Rightarrow shortage = 0$

$$n(R) = \int_0^R 0 f(x)dx + \int_R^\infty (x - R)f(x)dx = \int_R^\infty (x - R)f(x)dx := \overbrace{\sigma L(z)}^{\substack{\text{Standard loss function} \\ (D\sim Normal)}}$$

(Q,R) Model: Expected Total Cost per Unit Time

$C(Q)$ = holding cost + fixed cost + shortage cost

$$= h\left(\frac{Q}{2} + R - \lambda\tau\right) + K\frac{\lambda}{Q} + p\left(\frac{\lambda n(R)}{Q}\right)$$

$$\boxed{Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}}}$$

$$\boxed{F(R) = 1 - \frac{Qh}{p\lambda}}$$

Questions: How do we pull Q and R from these equations?

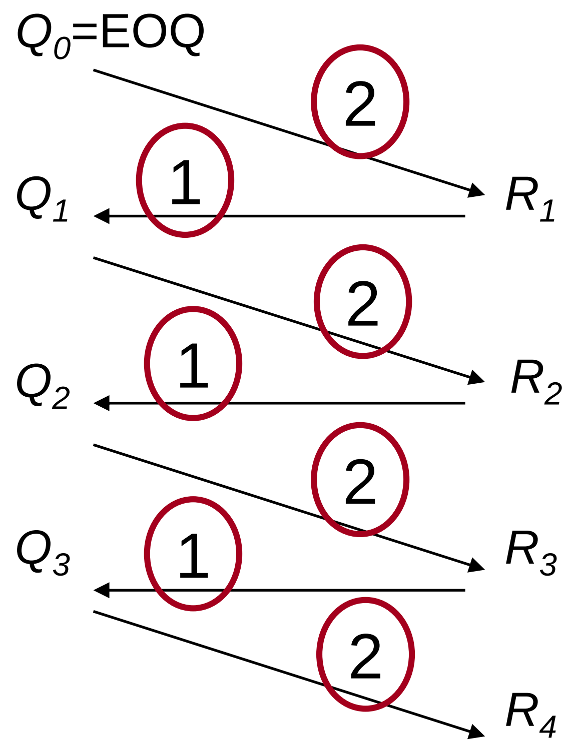
Answer: Solve iteratively!

Solving for Optimal Q and R

- Start with a Q_0 value and iterate until the **$\boldsymbol{Q}$ or $\boldsymbol{R}$ values converge**

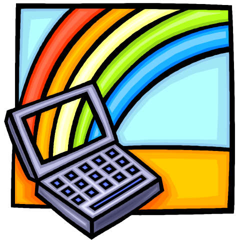
1 $Q = \sqrt{\frac{2\lambda[K+pn(R)]}{h}}$

2 $F(R) = 1 - \frac{Qh}{p\lambda}$



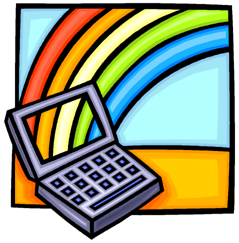
Remember: To find Q, you need $n(R) = \sigma L(z)$. Lookup for z in the Normal Tables. Alternatively, you can calculate it using Normdist and Normsdist functions in Excel: $L(z) = \phi(z) - z\big(1 - \Phi(z)\big)$

Remember $L(z)$ in Excel: '=NORMDIST(z,0,1,FALSE)-z*NORMSDIST(-z)'



(Q,R) Model Example: Rainbow Colors

- Rainbow Colors paint store uses a (Q, R) inventory system to control its stock levels. For a popular eggshell latex paint, historical data show that the distribution of **monthly demand** is approximately **Normal**, with **mean 28** and **standard deviation 8**. Replenishment **lead time** for this paint is about **14 weeks**. Each can of paint **costs** the store **$6**. Although excess demands are backordered, each unit of **stockout** costs about **$10** due to bookkeeping and loss of goodwill. **Fixed cost** of replenishment is **$15** per order and holding costs are based on a **30% annual interest rate**.
- What is the optimal lot size (order quantity) and reorder level?
- What is the expected inventory level (safety stock) just before an order arrives?



(Q,R) Model Example: Rainbow Colors

- Given Input:
  - Monthly demand: Normal with mean 28 and standard deviation 8
  - $\tau = 14$ weeks
  - $c = \$6,\ p = \$10,\ K = \$15$
  - $h = ic = (0.3)(6) = \$1.8$/unit/year
- Computed input:
  - $\lambda = (28)(12) = 336$ units/year (expected annual demand)
  - Expected demand during lead time

$$\mu = \frac{(28)(12)\ units/year}{52\ weeks/year} \times (14\ weeks) = 90\ units$$

  - Variance of demand during lead time

annual variance $= (12)(8^2) = 768$

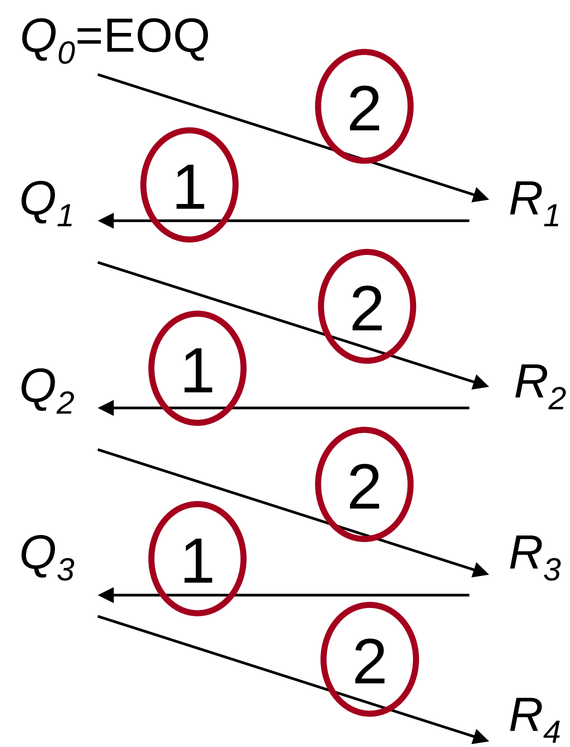
variance of lead time demand $= 768 \times \frac{14}{52} = 206.77 \Rightarrow \sigma = 14.38$

Solving for Optimal Q and R

- Start with a Q_0 value and iterate until the Q values converge

1 $Q = \sqrt{\frac{2\lambda[K+pn(R)]}{h}}$

2 $F(R) = 1 - \frac{Qh}{p\lambda}$



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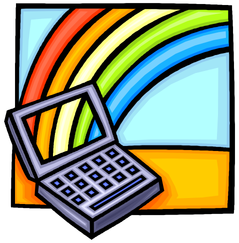


(Q,R) Model Example: Rainbow Colors

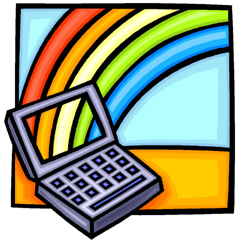
- Iteration 0: Computer EOQ

$$Q = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2(15)(336)}{1.8}} = 75$$

- Iteration 1: **Compute $\boldsymbol{R_1}$ (given $\boldsymbol{Q_0}$)** and then compute Q_1 (given R_1)

$$F(R) = 1 - \frac{Qh}{p\lambda} = 1 - \frac{(75)(1.8)}{(10)(336)} \approx 0.96 = \Phi(z) \Rightarrow z = 1.75$$

$$R = \sigma z + \mu \Rightarrow R_1 = (14.38)(1.75) + 90 \approx 115$$



(Q,R) Model Example: Rainbow Colors

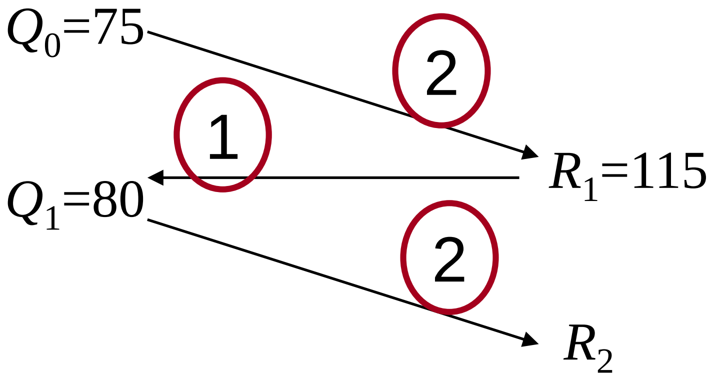
- Iteration 1 (continued): Compute Q_1 (given R_1)

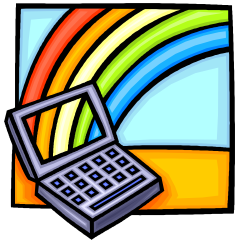
$$Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}}$$

$$n(R_1) = \sigma L(1.75) = (14.38)(0.0162) = 0.233$$

$$Q_1 = \sqrt{\frac{2(336)[15 + (10)(0.233)]}{1.8}} \approx 80$$

- Q_0 and Q_1 are not close, continue iterating





(Q,R) Model Example: Rainbow Colors

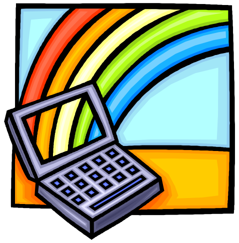
- Iteration 2: **Compute $\boldsymbol{R_2}$ (given $\boldsymbol{Q_1}$)** and then compute Q_2 (given R_2)

$$F(R_1) = 1 - \frac{Q_1 h}{p\lambda} = 1 - \frac{(80)(1.8)}{(10)(336)} \approx 0.957 \Rightarrow z = 1.72$$

$$R = \sigma z + \mu \Rightarrow R_2 = (14.38)(1.72) + 90 \approx 115$$

STOP! R values have converged, optimal $(Q, R) = (80{,}115)$

$$\begin{array}{ccc} Q_0=75 & \searrow & \\ & & R_1=115 \\ Q_1=80 & \leftarrow & \\ & \searrow & R_2=115 \end{array}$$



(Q,R) Model Example: Rainbow Colors

- $(Q, R) = (80{,}115)$
  - Reorder level is larger than expected demand during the lead time. Why?
    - Safety stock: $S = R - \mu = 115 - 90 = 25\ units$
  - Optimal order quantity is larger than EOQ. Why?

Sensitivity Analysis

$$F(R) = 1 - \frac{Qh}{p\lambda}$$

- As the order quantity Q increases:
  - There are fewer cycles per unit time
  - The impact of the shortage term $pn(R)$ decreases
  - Less safety stock is required
  - There are higher holding costs

Optimal Q as a Function of R

$$Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}}$$

- As the reorder level R increases:
  - There are fewer expected shortages per cycle ($n(R)$ decreases)
  - This reduces the expected shortage cost incurred in each cycle
  - Therefore, the order quantity decreases

Summary: (Q,R) Models

- Balance between holding cost, setup/fixed cost, and shortage cost
  - To save on the **shortage cost**, we want **large $\boldsymbol{R}$**
  - To save on the **holding cost**, we want **small $\boldsymbol{Q}$** and **small $\boldsymbol{R}$**
  - To save on the **fixed cost**, we want **large $\boldsymbol{Q}$**
- Choose Q and R to strike a good balance among these three costs

Inventory Control Subject to Uncertain Demand

- Inventory Systems with Uncertain (Stochastic) Demand
  - Newsvendor Model (single period)
  - **Continuous Review (Q,R) Model (multiple periods)**
    - **Per-Unit Backorder Penalty**
    - **Service Level (Fill Rate)**

Service Levels

- In many circumstances, the penalty cost, p, is difficult to estimate
- For this reason, it is common business practice to set inventory levels to meet a specified service objective instead
- The two most common service objectives are:
  - Type I service level (α)
    - The proportion of cycles in which no stockouts occur
    - Example: 90% Type I service level $\Rightarrow$ There are no stockouts in 9 out of 10 cycles (on average)
  - Type II service level (β)
    - Overall fill rate across all cycles
    - Fraction of demand satisfied on time

Service Levels Example 1

Order cycle	Demand	Stock-outs
1	180	0
2	75	0
3	235	45
4	140	0
5	180	0
6	200	10
7	150	0
8	90	0
9	160	0
10	40	0
TOTAL:	1450	55

Fraction of periods with no stock-outs $= \frac{8}{10}$

Type I service = 80% (α = 0.8)

Fraction of demand satisfied on time $= \frac{1450-55}{1450} = 0.96$

Type II service = 96% (β = 0.96)

Service Levels Example 2

Order cycle	Demand	Stock-outs
1	180	0
2	75	0
3	235	150
4	140	0
5	180	0
6	200	140
7	150	0
8	90	0
9	160	0
10	40	0
TOTAL:	1450	290

Fraction of periods with no stock-outs $= \frac{8}{10}$

Type I service = 80% (α = 0.8)

Fraction of demand satisfied on time $= \frac{1450-290}{1450} = 0.8$

Type II service = 80% (β = 0.8)

Service Levels Example 3

Order cycle	Demand	Stock-outs
1	180	0
2	75	0
3	235	235
4	140	0
5	180	0
6	200	200
7	150	0
8	90	0
9	160	0
10	40	0
TOTAL:	1450	435

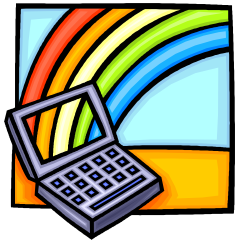
Fraction of periods with no stock-outs $= \frac{8}{10}$

Type I service = 80% (α = 0.8)

Fraction of demand satisfied on time $= \frac{1450-435}{1450} = 0.7$

Type II service = 70% (β = 0.7)

(Q,R) Model with Service Level: Example – Rainbow Colors



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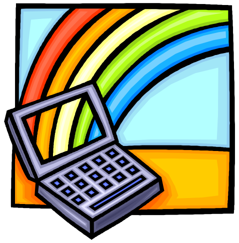
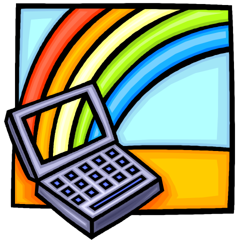
- Rainbow Colors is not sure whether the $10 estimate for the shortage cost is accurate. Hence, they decided to use a service level approach.



(Q,R) Model with Service Level: Example – Rainbow Colors

- What are the optimal (Q, R) values if we want to
  - Achieve no stockouts in 90% of the order cycles?
  - Satisfy 95% of the demand on time?

(Q,R) Model with Service Level: Example – Rainbow Colors



- Input:
  - Monthly demand: Normal with mean $=$ 28 and standard deviation $=$ 8
  - $\tau = 14$ weeks
  - $c = \$6,\ p = \$10,\ K = \$15$
  - $h = ic = (0.3)(6) = \$1.8/unit/year$
  - $\alpha = 0.9$ or $\beta = 0.9$
- Computed input:
  - $\lambda = (28)(12) = 336\ units/year$ (Expected annual demand)
  - Expected demand during lead time

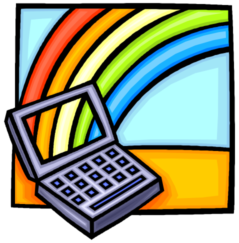
$$\mu = \frac{(28)(12)\ units/year}{52\ weeks/year} \times (14\ weeks) = 90\ units$$

  - Variance of demand during lead time

$$Annual\ variance = (12)(8^2) = 768$$

$$Variance\ of\ lead\ time\ demand = 768 \times \frac{14}{52} = 206.77 \Rightarrow \sigma = 14.38$$

Rainbow Colors: Type I Service Level

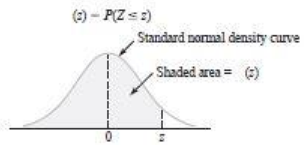


- Find (Q, R) to have 90% Type I service level
  - $Q = EOQ = 75$
  - $\Phi(z) = \alpha = 0.9 \Rightarrow z = 1.28$
  - $R = \sigma z + \mu \Rightarrow R = (14.38)(1.28) + 90 = 108$
  - For 90% Type I service level: $(Q, R) = (75,108)$

- Recall: With unit penalty cost of $\$10$, we found $(Q, R) = (80,115)$.
- What is the Type I service level that corresponds to $(Q, R) = (80,115)$?
  - $R = \sigma z + \mu \Rightarrow 115 = (14.38)z + 90 \Rightarrow z = 1.7385$
  - $\Phi(1.7385) = \alpha = 0.96 \Rightarrow 96\%$ Type I service level when $(Q, R) = (80,115)$

Normal z-Table

Table A.3 Standard Normal Curve Areas



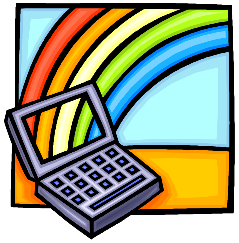
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas *(cont.)* $\Phi(z) = P(Z \le z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Rainbow Colors:
Type II Service Level



- β: Fraction of demand met on time
- $1-\beta$: Fraction of demand not met on time (stock-outs)
- Recall:

Expected number of stockouts per unit time

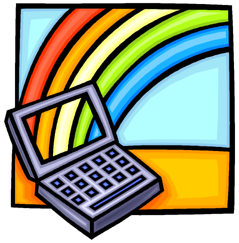
$$= \frac{n(R)}{T} = \frac{\lambda n(R)}{Q} \left(since\ T = \frac{Q}{\lambda} \right)$$

$$1-\beta = \frac{\text{Expected number of stockouts per unit time}}{\text{Expected demand per unit time}} = \frac{n(R)}{Q}$$

$$\Rightarrow 1-\beta = \frac{n(R)}{Q}$$

- With this information, for a given (Q, R), we can compute β

Rainbow Colors:
Type I vs. Type II Service Level



- For 90% Type I service level we found $(Q, R) = (75{,}108)$
- Question: What is the Type II service level which corresponds to this policy?

$$\frac{R-\mu}{\sigma} = \frac{108-90}{14.38} = 1.25 = z$$

$$n(R) = \sigma L(z) = \sigma L(1.25) = (14.38)(0.0506) = 0.7276$$

$$1-\beta = \frac{n(R)}{Q} = \frac{0.7276}{75} = 0.0097 \Rightarrow \beta \approx 0.99$$

- The same policy results in 90% Type I service and 99% TYPE II service!

Normal Loss Function

Standard Normal Loss Function Table, $L(z)$

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00
−4.0	4.0900	4.0800	4.0700	4.0600	4.0500	4.0400	4.0300	4.0200	4.0100	4.0000
−3.9	3.9900	3.9800	3.9700	3.9600	3.9500	3.9400	3.9300	3.9200	3.9100	3.9000
−3.8	3.8900	3.8800	3.8700	3.8600	3.8500	3.8400	3.8300	3.8200	3.8100	3.8000
−3.7	3.7900	3.7800	3.7700	3.7600	3.7500	3.7400	3.7300	3.7200	3.7100	3.7000
−3.6	3.6900	3.6800	3.6700	3.6600	3.6500	3.6400	3.6300	3.6200	3.6100	3.6000
−3.5	3.5900	3.5800	3.5700	3.5600	3.5500	3.5400	3.5301	3.5201	3.5101	3.5001
−3.4	3.4901	3.4801	3.4701	3.4601	3.4501	3.4401	3.4301	3.4201	3.4101	3.4001
−3.3	3.3901	3.3801	3.3701	3.3601	3.3501	3.3401	3.3301	3.3201	3.3101	3.3001
−3.2	3.2901	3.2801	3.2701	3.2601	3.2502	3.2402	3.2302	3.2202	3.2102	3.2002
−3.1	3.1902	3.1802	3.1702	3.1602	3.1502	3.1402	3.1302	3.1202	3.1103	3.1003
−3.0	3.0903	3.0803	3.0703	3.0603	3.0503	3.0403	3.0303	3.0204	3.0104	3.0004
−2.9	2.9904	2.9804	2.9704	2.9604	2.9505	2.9405	2.9305	2.9205	2.9105	2.9005
−2.8	2.8906	2.8806	2.8706	2.8606	2.8506	2.8407	2.8307	2.8207	2.8107	2.8008
−2.7	2.7908	2.7808	2.7708	2.7609	2.7509	2.7409	2.7310	2.7210	2.7110	2.7011
−2.6	2.6911	2.6811	2.6712	2.6612	2.6512	2.6413	2.6313	2.6214	2.6114	2.6015
−2.5	2.5915	2.5816	2.5716	2.5617	2.5517	2.5418	2.5318	2.5219	2.5119	2.5020
−2.4	2.4921	2.4821	2.4722	2.4623	2.4523	2.4424	2.4325	2.4226	2.4126	2.4027
−2.3	2.3928	2.3829	2.3730	2.3631	2.3532	2.3433	2.3334	2.3235	2.3136	2.3037
−2.2	2.2938	2.2839	2.2740	2.2641	2.2542	2.2444	2.2345	2.2246	2.2147	2.2049
−2.1	2.1950	2.1852	2.1753	2.1655	2.1556	2.1458	2.1360	2.1261	2.1163	2.1065
−2.0	2.0966	2.0868	2.0770	2.0672	2.0574	2.0476	2.0378	2.0280	2.0183	2.0085
−1.9	1.9987	1.9890	1.9792	1.9694	1.9597	1.9500	1.9402	1.9305	1.9208	1.9111
−1.8	1.9013	1.8916	1.8819	1.8723	1.8626	1.8529	1.8432	1.8336	1.8239	1.8143
−1.7	1.8046	1.7950	1.7854	1.7758	1.7662	1.7566	1.7470	1.7374	1.7278	1.7183
−1.6	1.7087	1.6992	1.6897	1.6801	1.6706	1.6611	1.6516	1.6422	1.6327	1.6232
−1.5	1.6138	1.6044	1.5949	1.5855	1.5761	1.5667	1.5574	1.5480	1.5386	1.5293
−1.4	1.5200	1.5107	1.5014	1.4921	1.4828	1.4736	1.4643	1.4551	1.4459	1.4367
−1.3	1.4275	1.4183	1.4092	1.4000	1.3909	1.3818	1.3727	1.3636	1.3546	1.3455
−1.2	1.3365	1.3275	1.3185	1.3095	1.3006	1.2917	1.2827	1.2738	1.2650	1.2561
−1.1	1.2473	1.2384	1.2296	1.2209	1.2121	1.2034	1.1946	1.1859	1.1773	1.1686
−1.0	1.1600	1.1514	1.1428	1.1342	1.1257	1.1172	1.1087	1.1002	1.0917	1.0833
−0.9	1.0749	1.0665	1.0582	1.0499	1.0416	1.0333	1.0250	1.0168	1.0086	1.0004
−0.8	0.9923	0.9842	0.9761	0.9680	0.9600	0.9520	0.9440	0.9360	0.9281	0.9202
−0.7	0.9123	0.9045	0.8967	0.8889	0.8812	0.8734	0.8658	0.8581	0.8505	0.8429
−0.6	0.8353	0.8278	0.8203	0.8128	0.8054	0.7980	0.7906	0.7833	0.7759	0.7687
−0.5	0.7614	0.7542	0.7471	0.7399	0.7328	0.7257	0.7187	0.7117	0.7047	0.6978
−0.4	0.6909	0.6840	0.6772	0.6704	0.6637	0.6569	0.6503	0.6436	0.6370	0.6304
−0.3	0.6239	0.6174	0.6109	0.6045	0.5981	0.5918	0.5855	0.5792	0.5730	0.5668
−0.2	0.5606	0.5545	0.5484	0.5424	0.5363	0.5304	0.5244	0.5186	0.5127	0.5069
−0.1	0.5011	0.4954	0.4897	0.4840	0.4784	0.4728	0.4673	0.4618	0.4564	0.4509
0.0	0.4456	0.4402	0.4349	0.4297	0.4244	0.4193	0.4141	0.4090	0.4040	0.3989

Standard Normal Loss Function Table, $L(z)$ (Concluded)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.3989	0.3940	0.3890	0.3841	0.3793	0.3744	0.3697	0.3649	0.3602	0.3556
0.1	0.3509	0.3464	0.3418	0.3373	0.3328	0.3284	0.3240	0.3197	0.3154	0.3111
0.2	0.3069	0.3027	0.2986	0.2944	0.2904	0.2863	0.2824	0.2784	0.2745	0.2706
0.3	0.2668	0.2630	0.2592	0.2555	0.2518	0.2481	0.2445	0.2409	0.2374	0.2339
0.4	0.2304	0.2270	0.2236	0.2203	0.2169	0.2137	0.2104	0.2072	0.2040	0.2009
0.5	0.1978	0.1947	0.1917	0.1887	0.1857	0.1828	0.1799	0.1771	0.1742	0.1714
0.6	0.1687	0.1659	0.1633	0.1606	0.1580	0.1554	0.1528	0.1503	0.1478	0.1453
0.7	0.1429	0.1405	0.1381	0.1358	0.1334	0.1312	0.1289	0.1267	0.1245	0.1223
0.8	0.1202	0.1181	0.1160	0.1140	0.1120	0.1100	0.1080	0.1061	0.1042	0.1023
0.9	0.1004	0.0986	0.0968	0.0950	0.0933	0.0916	0.0899	0.0882	0.0865	0.0849
1.0	0.0833	0.0817	0.0802	0.0787	0.0772	0.0757	0.0742	0.0728	0.0714	0.0700
1.1	0.0686	0.0673	0.0659	0.0646	0.0634	0.0621	0.0609	0.0596	0.0584	0.0573
1.2	0.0561	0.0550	0.0538	0.0527	0.0517	0.0506	0.0495	0.0485	0.0475	0.0465
1.3	0.0455	0.0446	0.0436	0.0427	0.0418	0.0409	0.0400	0.0392	0.0383	0.0375
1.4	0.0367	0.0359	0.0351	0.0343	0.0336	0.0328	0.0321	0.0314	0.0307	0.0300
1.5	0.0293	0.0286	0.0280	0.0274	0.0267	0.0261	0.0255	0.0249	0.0244	0.0238
1.6	0.0232	0.0227	0.0222	0.0216	0.0211	0.0206	0.0201	0.0197	0.0192	0.0187
1.7	0.0183	0.0178	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146
1.8	0.0143	0.0139	0.0136	0.0132	0.0129	0.0126	0.0123	0.0119	0.0116	0.0113
1.9	0.0111	0.0108	0.0105	0.0102	0.0100	0.0097	0.0094	0.0092	0.0090	0.0087
2.0	0.0085	0.0083	0.0080	0.0078	0.0076	0.0074	0.0072	0.0070	0.0068	0.0066
2.1	0.0065	0.0063	0.0061	0.0060	0.0058	0.0056	0.0055	0.0053	0.0052	0.0050
2.2	0.0049	0.0047	0.0046	0.0045	0.0044	0.0042	0.0041	0.0040	0.0039	0.0038
2.3	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
2.4	0.0027	0.0026	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021
2.5	0.0020	0.0019	0.0019	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015
2.6	0.0015	0.0014	0.0014	0.0013	0.0013	0.0012	0.0012	0.0012	0.0011	0.0011
2.7	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008
2.8	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
2.9	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004
3.0	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
3.1	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.2	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001
3.3	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.4	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.5	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Rainbow Colors:
Find (Q,R) for desired type II service level

- If the desired Type II service level is 0.95:
- Approximate Q with EOQ

$$Q = \sqrt{\frac{2\lambda K}{h}} = \sqrt{\frac{2 \times 15 \times 336}{1.8}} = 75$$

- Find R based on $\sigma L(z) = n(R) = (1 - \beta)Q$

$$\sigma L(z) = (1 - 0.95) \times 75 = 3.75$$

$$L(z) = \frac{3.75}{14.38} = 0.2608 \Rightarrow z = 0.3158$$

$$R = \sigma z + \mu \Rightarrow R = 14.28 \times 0.3158 + 90 \approx 95$$

Finding the Optimal (Q,R) for a Desired Type II Service Level

- Optimal solution when we have stockout cost p:

$$Q = \sqrt{\frac{2\lambda[K + pn(R)]}{h}} \quad (1)$$

$$F(R) = 1 - \frac{Qh}{p\lambda} \quad (2)$$

- From (2) :

$$p = \frac{Qh}{\lambda\big(1 - F(R)\big)} \quad (3)$$

- Substitute (3) into (1), solve for Q to get:

$$Q = \frac{n(R)}{1 - F(R)} + \sqrt{\frac{2K\lambda}{h} + \left(\frac{n(R)}{1 - F(R)}\right)^2} \quad (4)$$

- To be solved simultaneously with

$$n(R) = (1 - \beta)Q \quad (5)$$

Impact of Service Level β on R

- For a given Q:
  - As $\beta \uparrow$, $n(R) = (1 - \beta)Q \downarrow$
  - i.e. $\beta \uparrow$, $R \uparrow$
- As the service level increases, the reorder level increases as well

Summary: (Q,R) Models

- Balance between holding cost, setup/fixed cost, and shortage cost
  - To save on the **shortage cost**, we want **large $\boldsymbol{R}$**
  - To save on the **holding cost**, we want **small $\boldsymbol{Q}$** and **small $\boldsymbol{R}$**
  - To save on the **fixed cost**, we want **large $\boldsymbol{Q}$**
- Choose Q and R to strike a good balance among these three costs

Periodic Review Models

(s, S) Policy

- The (Q, R) policy is appropriate when inventory levels are reviewed continuously
- In the case of periodic review, a slight alteration of this policy is required:
  - Define two levels, $s < S$, and let u be the starting inventory at the beginning of a period. Then

If $u \leq s$, order $S - u$

If $u > s$, do not order

- In general, computing the optimal values of s and S is much more difficult than computing Q and R
- But, we can use a (Q, R) approximation:

$s = R$ and $S = R + Q$

(T, S) Policy

- Every T periods order up to S units
  - Let $T = EOQ/\lambda$
  - Then, the order-up-to level S should cover the demand during $T + \tau$ periods

$$S = (mean\ demand + safety\ stock)\ over\ (T + \tau)\ periods$$

$$= \mu_{T+\tau} + z\sigma_{T+\tau}$$

  - Determine z as in a (Q, R) system