MS&E 260 Homework 5 Solutions

Summer 2019, Stanford University

Due: August 12th, 2019, at 10:30AM (PDT)

Problem 1. (a) We want to maximize the revenue

$$R(p) = p \cdot D(p) = p(1{,}000 - 5p)$$

Taking the derivative we get $R'(p) = 1{,}000 - 10p$ so the equation $R'(p) = 0$ has solution $p = 100$. Therefore the demand is $D(p) = 1{,}000 - 5 \times 100 = 500$. The optimal revenue would be $R(p) = 100 \times 500 = \$50{,}000$.

(b) We formulate this problem as two separate linear programs:

$$\begin{aligned} \max \quad & p_1 \min\{750, 1{,}000 - 5p_1\} \\ \text{s.t.} \quad & p_1 \geq 50 \end{aligned}$$

We find that $p_1 = 100, D_1 = 500, R_1 = \$50{,}000$.

$$\begin{aligned} \max \quad & p_2(250 - 5p_2) \\ \text{s.t.} \quad & 0 \leq p_2 \leq 50 \end{aligned}$$

We find that $p_2 = 25, D_2 = 125, R_2 = \$3{,}125$. Hence, the total revenue becomes \$53,125.

(c) We found that $D_a(p_a^*) = 500$. The new demand after the markdown will be $D_m(p) = 1{,}000 - 5p - D_a(p_a^*) = 500 - 5p$.

$$\max p(500 - 5p)$$

We get that $R'(p) = 500 - 10p = 0 \Rightarrow p = 50$ and $D(p) = 500 - 5 \times 50 = 250$. The revenue is \$62,500.

In both cases, we sell at the global optimal price $p_1 = 100$ from part (a). In part (b), we choose a second optimal price for the segment of customers with $p_2 \leq 50$. In part (c), we choose a second optimal price for the segment of customers who did not buy at the initial optimal price. The segment in part (c) is larger than in part (b), so optimizing for this larger segment allows us to have a higher revenue. In other words, in (b) we miss out in potentially selling to the customers who would buy online but did not buy at the optimal price.

Problem 2. (a) We use the concept of utility. Since a buyer with valuation θ is willing to buy at a price p if and only if $\theta - p \geq 0 \leftrightarrow \theta \geq p$, θ follows the uniform distribution and we are selling only to customers of valuations $[0, \theta_1]$, where $0 \leq p \leq \theta_1 \leq 1$. Therefore, we get that the expected demand is $\int_p^{\theta_1} d\theta = \theta_1 - p$.

(b) Revenue in period 2 is $\pi_2(p_2) = \delta(\theta_1 - p_2)p_2$ (revenue in period 2 is discounted by δ). Since $\pi_2'(p_2) = \delta(\theta_1 - 2p_2) = 0$ we get that $p_2^* = \frac{\theta_1}{2}$.

(c) His valuation in period 1 and 2 is $\theta_1 - p_1$ and $\delta(\theta_1 - p_2)$, respectively. The customer is indifferent if $\theta_1 - p_1 = \delta(\theta_1 - p_2)$ so $p_1 = (1 - \frac{\delta}{2})\theta_1$.

(d) As in part (a), we find that the demand in period 1 is $1 - \theta_1$. The revenue from period 1 is $\pi_1 = (1 - \theta_1)(1 - \frac{\delta}{2})\theta_1$ where we used the result from part (c). The revenue from period 2 becomes $\pi_2 = \delta(\theta_1 - p_2^*)p_2^* = \frac{\delta\theta^2}{4}$.

Total revenue: $\pi = (1 - \theta_1)(1 - \frac{\delta}{2})\theta_1 + \frac{\delta\theta^2}{4}$.

(e) The maximization problem is

$$\max_{0 \leq \theta_1 \leq 1} \quad (1 - \theta_1)(1 - \frac{\delta}{2})\theta_1 + \frac{\delta\theta_1^2}{4}$$

We find that $\theta_1^* = \frac{1 - \frac{\delta}{2}}{2 - \frac{3\delta}{2}}$.

Problem 3. (a) John's optimal alternative, without Beta's market research report, is to pursue the new business line as shown in Figure 1 below. His expected value is $14M.

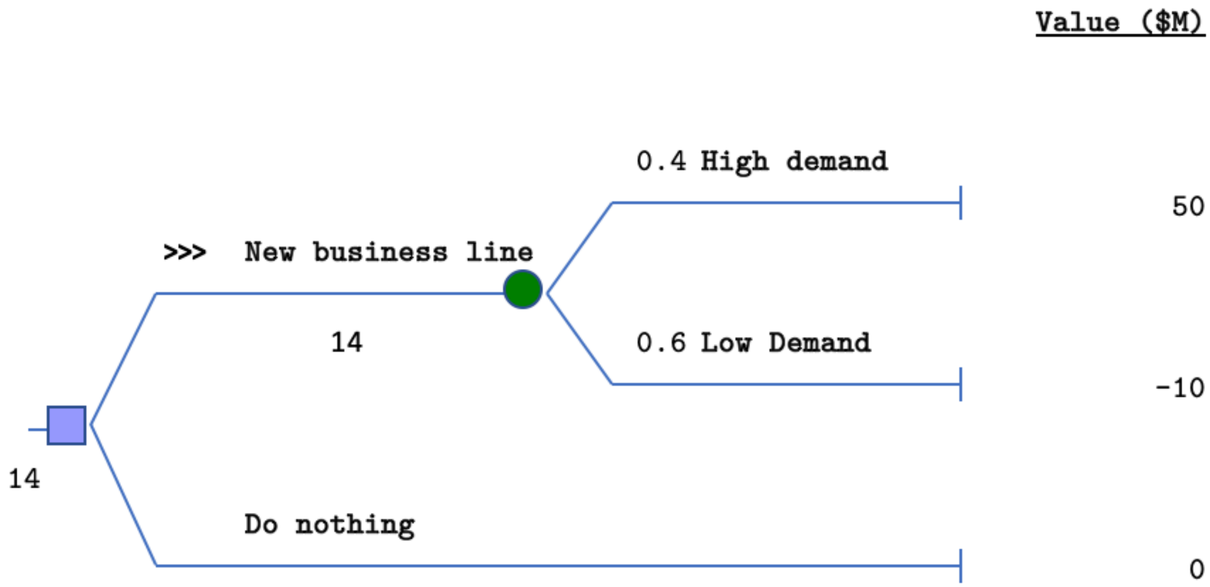
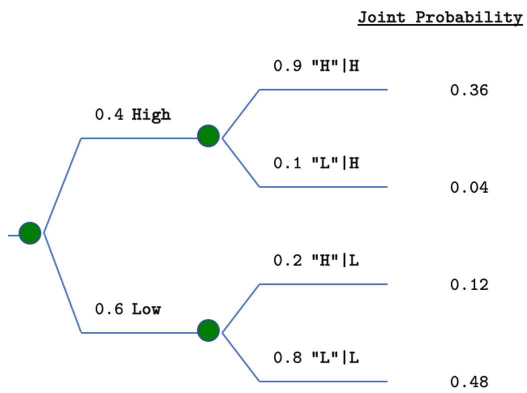
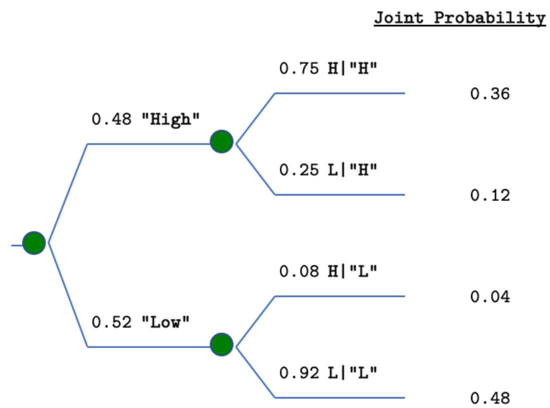


Figure 1: John's Original Decision Tree

(b) We first calculate the probabilities of the test giving "High" and "Low" results as well as of the inferred probabilities of High and Low in Figure 2.



(a) Assessed Form



(b) Inferential Form

Figure 2: The Market Research Report and its Probabilities

Then we draw a new decision tree as seen in Figure 3 with the new certain equivalent of \$16.8M. Hence,

$$VoI = \$16.8 - \$14 = \$2.8$$

This is the maximum price that John should be willing to pay for any information, such as Beta's market research report.

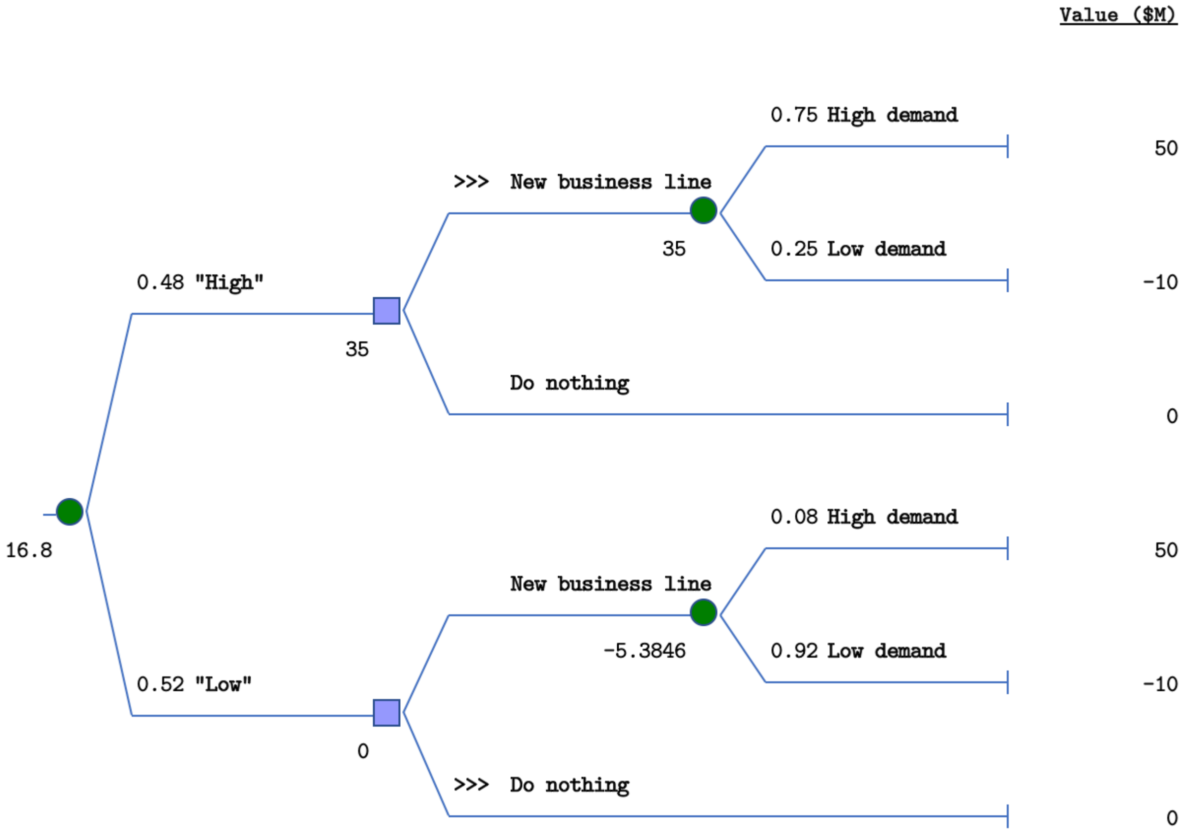


Figure 3: John's Modified Decision Tree with Market Report

(c) John is indifferent between the two alternatives if his prior is 0.17, as given by the decision tree below in Figure 4.

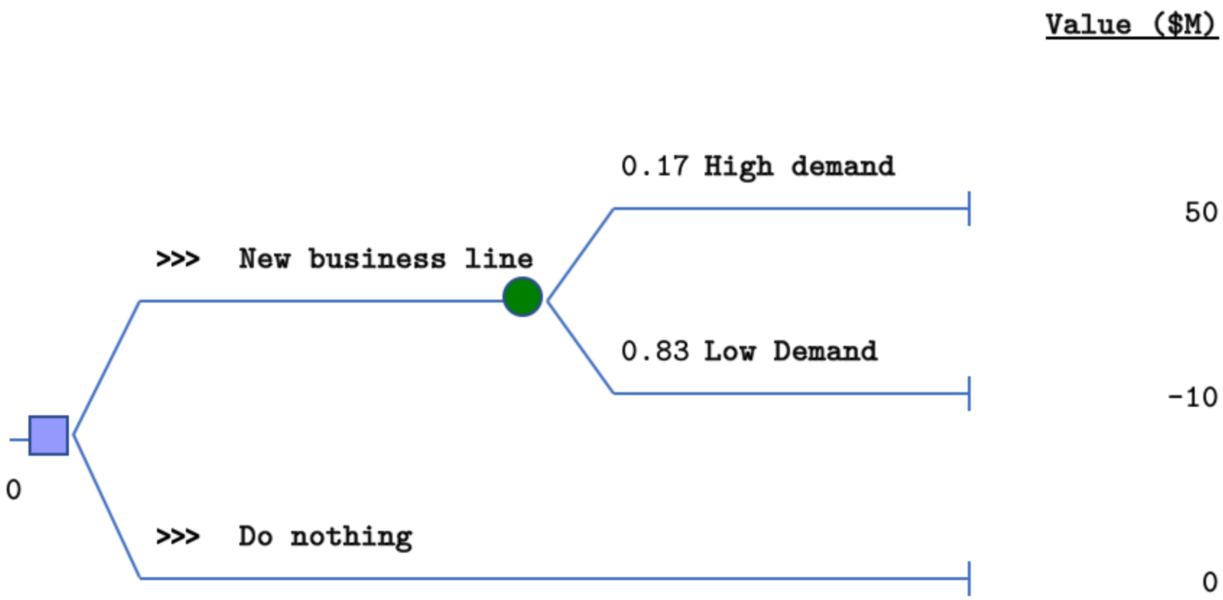


Figure 4: John's Indifferent with New Prior

(d) Any answer is acceptable, as long as it is well-justified. A reasonable answer is: yes, John's optimal alternative is robust because his prior (0.4) is much higher than the indifference point of the uncertainty. In other words, even if John's prior is cut in half, the optimal alternative (to pursue the new business line) still does not change.

(e) By re-doing the analysis, but with a perfect report (always providing correct signals), the maximum value of the report is \$6M.

Problem 4. Solutions vary. Answers are evaluated on the basis of completeness on the required criteria. This includes summary of the article, critical thinking applied to interpretation of the article, and application of lectured concepts to the topics discussed in the article.