MS&E 260 Homework 3 Solutions

Summer 2019, Stanford University

Due: July 24th, 2019, at 10:30AM (PDT)

Problem 1. (a) The current fill rate is

$$\begin{aligned}
\sigma_L^2 &= 5 \times 1{,}280 = 6{,}400 \\
\sigma_L &= 80 \\
z &= \frac{ss}{\sigma_L} = \frac{1{,}650 - 310 \times 5}{80} = 1.25 \\
L(z) &= 0.053 \\
\beta &= 1 - \frac{\sigma_L L(z)}{Q} = 1 - \frac{80 \times 0.053}{1{,}240} = 99.96\%
\end{aligned}$$

(b) The order quantity and reorder point would be

$$\begin{aligned}
K &= 120 \\
\lambda &= 310 \times 50 = 1{,}550 \\
h &= 7 \times 0.25 = 1.75 \\
Q &= EOQ = \sqrt{\frac{2K\lambda}{h}} = 1{,}458 \\
n(R) &= (1-\beta)Q = (1 - 0.98) \times 1{,}458 = 29.16 = \sigma_L L(z) \\
L(z) &= \frac{29.16}{80} = 0.3645 \\
\Rightarrow z &= 0.08 \\
R &= \sigma_L \cdot z + \mu \cdot L = 0.08 \times 80 + 310 \times 5 = 1{,}557
\end{aligned}$$

(c) Type II service level = 98% $\Rightarrow (Q, R) = (1{,}458,\ 1{,}557)$. We need to solve what α is for that (Q, R).

$$\begin{aligned}
R &= \sigma_L \cdot z + \mu \cdot L \\
1{,}557 &= 80z + 310 \times 5 \\
\Rightarrow z &= 0.08 \\
\Phi(0.0875) &= 0.53 \\
\alpha &= 53\%
\end{aligned}$$

The solution would change since it currently only has a Type I service level of 53% which is less than 80%.

Problem 2. (a) $\lambda = 30$ customers/hr, and $\mu = \frac{60}{5} = 12$ customers/hr. Hence fraction is

$$1 - \frac{\lambda}{N \times \mu} = 1 - \frac{30}{4 \times 12} = \frac{3}{8}$$

(b) Let N be the number of machines used. N has to satisfy:

$$\frac{\lambda}{N\mu} < 1 \Rightarrow N > \frac{\lambda}{\mu} = \frac{30}{12} = 2.5$$

Hence $N_{min} = 3$.

(c) For this question, we know $c = 16$ and $h = 0.5 \times 60 = 30$.

$$\mu^* = \lambda + \sqrt{\frac{\lambda h}{c}} = 30 + \sqrt{\frac{30 \times 30}{16}} = 37.5 \text{ customers/hr}$$
$$c(\mu^*) = c\lambda + 2\sqrt{\lambda h c} = 16 \times 30 + 2\sqrt{30 \times 30 \times 16} = 720$$

(d)

$$\rho = \frac{\lambda}{\mu} = \frac{30}{37.5} = 0.8$$
$$l_q = \frac{\rho^2}{1-\rho} = \frac{0.8^2}{1-0.8} = 3.2 \approx 3$$

(e) The key idea is to observe that we have deterministic arrival and service times. When a customer enters at time slot i, she finds the system empty. She leaves the system at time $i + 1$. The next customer arrives at time slot $i + 2$. Therefore, any time we can have at most one customer in the system and no customer waits to be served. Hence, the length of the queue is 0.

Problem 3. First, we compute the following parameters:

$$\lambda = \frac{1}{4}$$
$$C_A^2 = \frac{\sigma_A^2}{E(A)^2} = \frac{3}{4^2} \approx 0.188$$
$$\mu = \frac{1}{7}$$
$$C_S^2 = \frac{\sigma_S^2}{E(S)^2} = \frac{\frac{(8-6)^2}{12}}{7^2} \approx 0.0068$$
$$\rho = \frac{\frac{1}{4}}{5 \times \frac{1}{7}} = \frac{7}{20} \approx 0.35$$

(a)

$$\begin{aligned} W_q &= \frac{1}{5\mu} \times \frac{\rho^{\sqrt{2(5+1)}-1}}{1-\rho} \times \frac{C_A^2 + C_S^2}{2} \\ &= \frac{7}{5} \times \frac{0.35^{\sqrt{12}-1}}{1-0.35} \times \frac{0.188 + 0.0068}{2} \\ &\approx 0.0158 \end{aligned}$$

(b) $L_q = \lambda \times W_q \approx 0.00395$

(c) $W = W_q + \frac{1}{\mu} \approx 7.0158$

(d) $L = \lambda \times W \approx 1.76$

(e) The fraction is $1 - \rho \approx 0.65$

Problem 4. (a)

$$
\begin{aligned}
P(\text{failure}) &= P(\text{load} > 250) \\
&= 1 - P(\text{load} \le 250) \\
&= 1 - \frac{250 - 200}{300 - 200} \\
&= 0.5
\end{aligned}
$$

(b)

$$
P(\text{failure}) = \int_{-\infty}^{\infty} g_C(x)[1 - F_L(x)]\ dx
$$

$$
F_L(x) = \begin{cases} 0 & x < 200 \\ \frac{x-200}{300-200} & x \in [200, 300] \\ 1 & x > 300 \end{cases}
$$

Therefore,

$$
\begin{aligned}
P(\text{failure} &= \int_{150}^{200} (0.05 - 0.0002x)(1)\ dx + \int_{200}^{250} (0.05 - 0.0002x)\left(\frac{300 - x}{300 - 200}\right)\ dx \\
&= 0.9583333
\end{aligned}
$$

(c)

$$
\begin{aligned}
P(\text{failure}) &= \int_{179.29}^{200} (0.1 - 0.0004x)(1)\ dx + \int_{200}^{250} (0.1 - 0.0004x)\left(\frac{300 - x}{300 - 200}\right)\ dx \\
&= 0.9166475
\end{aligned}
$$

(d)

$$
\begin{aligned}
\text{Expected Loss} &= P(\text{Failure}) \times \text{Lost Revenue} \\
\text{Old Expected loss} &= (0.9583333) \times (\$100{,}000) \\
&= \$95{,}833.33 \\
\text{New Expected loss} &= (0.9166475) \times (\$100{,}000) \\
&= \$91{,}664.75
\end{aligned}
$$

\$95,833.33 - \$91,664.75 = \$4,168.58 is less than \$50,000. Isaac should not purchase the server upgrade from Google because it results in less savings than the cost of the upgrade.

Problem 5. Solutions vary. Answers are evaluated on the basis of completeness on the required criteria. This includes summary of the article, critical thinking applied to interpretation of the article, and application of lectured concepts to the topics discussed in the article.