MS&E 260 Midterm Examination

Stanford University

July 19th, 2019

Name: Vishal Mittal

SUNet ID: vpmittal@stanford.edu

Question	Points Available	Points Earned
1	25	
2	25	
3	25	
4	25	
Total	100	

Instructions:

1. This examination contains **12 pages**, including this page and the table pages.
2. You have **80 minutes** to complete the examination. As a courtesy to your classmates, we ask that you not leave during the last fifteen minutes.
3. You may use one single-sided 8.5" × 11" page with notes that you have prepared. You may not use any other resources, including lecture notes, books, other students or other engineers.
4. You may use a calculator. You may not share a calculator with anyone. If you didn't bring a calculator, you may use your phone, **but** you must put it on **flight mode** and clear all visible notifications **before** the examination starts, and you must not open any applications other than the calculator and a timer.
5. Please sign the below Honor Code statement.

In recognition of and in the spirit of the Stanford University Honor Code, I certify that I will neither give nor receive unpermitted aid on this examination.

Signature: Vishal Mittal

Question 1: Artisan Raw Vegan Chocolate!

[25 pts] Stanford student, Abby, found an amazing vegan chocolate supplier at the Los Angeles Mar Vista farmers' market last July. It is called Hopf Chocolate and the owner uses no refined sugar but sweetens with low glycemic, mineral-rich coconut nectar! Abby is not a chocolate connoisseur but was blown away by how smooth and tasty the chocolate was. Hopf Chocolate does not have a physical store nor a distribution channel set up serving Northern California but does deliver with USPS with ice packs. Abby, being an enterprising MS&E 260 student, decides to devise an ordering schedule/plan to satisfy her cravings as well as to share with friends and family the wonderful creations optimizing based on cost.

The chocolate bars cost $9 each and she estimates that on average 4 chocolate bars per week will be consumed by others and herself. The cost of placing an order, mostly shipping with packaged ice packs, is $20. The holding cost of the chocolates is 50% (think of this as a partial penalty for the semi-perishable chocolate bars and the freezer space it takes up in Abby's tiny graduate efficiency housing.)

Given:
c = $9
λ = 4 * 52 chocolates / year = 208 chocolates / year
K = $20
i = 50%
h = i c = 50/100 * 9 = 4.5

(a) [3 pts] How many chocolate bars should Abby place per order?

$$Q^* = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2 * 20 * 208}{0.5 * 9}} = 42.998 \cong 43\ chocolates$$

(b) [3 pts] How frequently should Abby order in number of weeks?

$$L = \lambda W \;=>\; 208 = 43\,W \;=>\; W = 208/43 = 4.83\ weeks$$

Chocolates should be ordered every 4 weeks and 4 days or once every month approximately.

(c) [3 pts] What is the total annual cost for Abby from these Hopf vegan raw chocolates? (Assume 52 weeks a year.)

$$TC\ (Q^*) = \left(\frac{\lambda}{Q^*}\right)K + \left(\frac{Q^*}{2}\right)ic + \lambda\, c$$

$$TC\ (Q^*) = \left(\frac{208}{43}\right)20 + \left(\frac{43}{2}\right)0.5 * 9 + 208 * 9$$

$$TC\ (Q^*) = 96.744 + 96.75 + 1872 = 2065.494 \cong \$2065.5$$

(d) [8 pts] Abby would DM (direct message) the owner of Hopf, Andrea, on Instagram to tell her how much she loves the chocolates. Andrea appreciates Abby's enthusiasm and business hence decides to offer Abby an all-units discount in the following schedule:

i. \$9/chocolate bar if $Q \leq 5$
ii. \$8/chocolate bar if $6 \leq Q \leq 10$
iii. \$6/chocolate bar if $Q \geq 11$

How many chocolate bars should Abby place per order now? What is the total annual cost for Abby from these chocolate bar purchases?

---

for case (i)

$$Q^* = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2 * 20 * 208}{0.5 * 9}} = 42.998 \cong 43\,chocolates$$

43 is out of range so we will take boundary value of Q* = 5

$$TC\,(Q^*) = \left(\frac{\lambda}{Q^*}\right)K + \left(\frac{Q^*}{2}\right)ic + \lambda\,c = \left(\frac{208}{5}\right)20 + \left(\frac{5}{2}\right)0.5 * 9 + 208 * 9$$

$$TC\,(Q^*) = 832 + 11.25 + 1872 = 2715.25$$

---

for case (ii)

$$Q^* = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2 * 20 * 208}{0.5 * 8}} = 45.607 \cong 47\,chocolates$$

47 is out of range so we will take boundary value of Q* = 10

$$TC\,(Q^*) = \left(\frac{\lambda}{Q^*}\right)K + \left(\frac{Q^*}{2}\right)ic + \lambda\,c = \left(\frac{208}{10}\right)20 + \left(\frac{10}{2}\right)0.5 * 8 + 208 * 8$$

$$TC\,(Q^*) = 416 + 20 + 1664 = 2100$$

---

for case (iii)

$$Q^* = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2 * 20 * 208}{0.5 * 6}} = 52.66 \cong 53\,chocolates$$

Since 53 is in range so we will take Q* = 53

$$TC\,(Q^*) = \left(\frac{\lambda}{Q^*}\right)K + \left(\frac{Q^*}{2}\right)ic + \lambda\,c = \left(\frac{208}{53}\right)20 + \left(\frac{53}{2}\right)0.5 * 6 + 208 * 6$$

$$TC\,(Q^*) = 78.49 + 79.5 + 1248 = 1405.99$$

Since Total Cost for case (iii) is lowest, Abby should order 53 chocolates with total yearly cost of \$1405.99

(e) [8 pts] Andrea wonders if a more modest incremental discount schedule is more profitable for her, since she's operating a one-woman business:

i. \$9Q for $Q \leq 5$
ii. \$45 + \$8.5(Q - 5) for $6 \leq Q \leq 10$
iii. \$87.5 + \$8(Q - 10) for $Q \geq 11$

How many chocolate bars should Abby place per order now? What is the total annual cost for Abby from these chocolate bar purchases?

---

for case (i)

$$Q^* = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2 * 20 * 208}{0.5 * 9}} = 42.998 \cong 43\ chocolates$$

43 is out of range so we will take boundary value of Q* = 5

$$TC\ (Q^*) = \left(\frac{\lambda}{Q^*}\right)K + \left(\frac{Q^*}{2}\right)ic + \lambda\,c = \left(\frac{208}{5}\right)20 + \left(\frac{5}{2}\right)0.5 * 9 + 208 * 9$$

$$TC\ (Q^*) = 832 + 11.25 + 1872 = 2715.25$$

---

for case (ii)

$$c_{avg} = \frac{c\ (Q)}{Q} = \frac{45 + 8.5\ (Q-5)}{Q} = 8.5 + \frac{2.5}{Q}$$

$$TC\ (Q) = \left(\frac{208}{Q}\right)20 + \left(\frac{Q}{2}\right)0.5\left(8.5 + \frac{2.5}{Q}\right) + 208 * \left(8.5 + \frac{2.5}{Q}\right)$$

$$TC\ (Q) = \left(\frac{4160}{Q}\right) + 2.125\,Q + 0.625 + 1768 + \left(\frac{520}{Q}\right)$$
$$= \frac{4680}{Q} + 2.125\,Q + 1768.625$$

$$\frac{d\,[TC\ (Q)]}{dQ} = -\frac{4680}{Q^2} + 2.125 = \emptyset \Rightarrow Q* = \sqrt{\frac{4680}{2.125}} = 46.929 \cong 47$$

Since this is out of range we will use Q* = 10

$$TC\ (Q*) = \frac{4680}{10} + 2.125 * 10 + 1768.625 = 468 + 21.25 + 1768.625 = 2257.875$$
$$\cong \$2258$$

---

for case (iii)

$$c_{avg} = \frac{c\ (Q)}{Q} = \frac{87.5\ +\ 8\ (Q-10)}{Q} = 8 + \frac{7.5}{Q}$$

$$TC\ (Q) = \left(\frac{208}{Q}\right)20\ +\ \left(\frac{Q}{2}\right)0.5\ \left(8 + \frac{7.5}{Q}\right) + 208 * \left(8 + \frac{7.5}{Q}\right)$$

$$TC\ (Q) = \left(\frac{4160}{Q}\right) + 2\,Q\ +\ 1.875 + 1664\ + \left(\frac{1560}{Q}\right) = \frac{5720}{Q}\ +\ 2Q\ +\ 1665.875$$

$$\frac{d\ [TC\ (Q)]}{dQ} = -\frac{5720}{Q^2}\ +\ 2 = \emptyset \Rightarrow Q * = \sqrt{\frac{5720}{2}} = 53.47\ \cong\ 54$$

Since this is in range we will use Q* = 54

$$TC\ (Q\ *) = \frac{5720}{54}\ +\ 2\ *\ 54\ +\ 1665.875\ =\ 105.925 + 108 + 1665.875\ =\ 1879.8$$
$$\cong\ \$1880$$

Since Total Cost for case (iii) is lowest Abby should order 54 chocolates with total yearly cost of $1880

Question 2: Tulip Mania!

[25 pts] You are the owner of a tulip store in San Francisco selling tulips to tourists. Purchasing the tulips from your Dutch supplier costs $6 per bulb but can be sold at a price of $20. As you are ordering the tulips from the Netherlands, there is a lead time between when the order is placed and when the order is received. Demand during this lead time is uncertain. The demand for tulips is normally distributed with a mean of 1,500 and a standard deviation of 400. You keep half of the un-sold tulips for your own garden and the other half you sell to a discount garden store for $4.

Given:

Since probability un-sold tulip to be sold to garden store is 1/2 so while calculating overage cost we will consider 4 1/2 as salvage cost*

Overage cost = $c_o = 6 - (4/2) = 4$
Underage cost = $c_u = 20 - 6 = 14$
Mean = $\mu = 1500$
Standard deviation = $\sigma = 400$

(a) [5 pts] What is the probability that the product sells less than 3/4 of the forecast?

$$Critical\ Ratio\ = F(Q)\ = \frac{c_u}{c_u\ +\ c_o} = \frac{14}{14+4} = 0.778$$

F(Q) is the probability that demand does NOT exceed Q for Optimal Q (Q*)

$$\text{probability that product sells less than } 3/4 \text{ of the forecast} = \frac{3}{4} * \frac{14}{14+4}\ =\ 0.583$$

(b) [5 pts] How many tulips should you buy from your Dutch supplier to maximize expected profit?

Using the critical ratio 0.778 for lookup in z table, z = 0.77
Q* = $\sigma\ z + \mu$ = 400*(0.77) + 1500 = 1808

(c) [5 pts] If you want to ensure a 95% fill rate for your customers, how many tulips should you order?
expected total lost sales = L (z) σ
Expected sales = ($\mu - L\ (z)\ \sigma$)
Expected Fill Rate = Expected Sales/Expected Demand
= ($\mu - L\ (z)\ \sigma$) / μ

$$0.95\ = \frac{1500 - \mathrm{L(z)}\ *\ 400}{1500} \Rightarrow\ 0.95\ =\ 1\ -\ \frac{L(z)\ 4}{15} \Rightarrow\ L(z) = \frac{0.05\ *\ 15}{4}\ =\ 0.1875$$

from Standard Normal Loss Function Table z = 0.53
Q = $\sigma\ z + \mu$ = 400*(0.53) + 1500 = 1712

(d) [5 pts] If you want to ensure a 98% in-stock probability, how many tulips should you order?
F(Q*) = probability that demand does not exceed Q*
= in-stock probability for Q* items

F(Q) = 0.98 => From z table => z = 2.06
Q = $\sigma\ z + \mu$ = 400*(2.06) + 1500 = 2324

(e) [5 pts] Assuming you decide to buy 1,800 tulips, what is your company's expected profit?
Q = $\sigma\ z + \mu$ => 1800 = 400 z + 1500 => z = 0.75
Expected sales = $\mu - L\ (z)\ \sigma$ = 1500 – L(0.75) 400 = 1500 – (0.1312 * 400) = 1447.52 ≈ 1448

Buying cost = 1800 * 6 = 10800
Sales = 1448 *20 = 28960
Salvage money = (1800 - 1448)/2 * 4 = 704
Profit = sales - buying cost + salvage money = 28960 -10800 + 704 = 18864

Question 3: Lost on Maui?

[25 pts] Lost on Maui is a clothing and surf boutique located in the town of Paia on the Hawaiian island of Maui. The owner is thinking about expanding to women's apparel more by including sturdy and comfortable flip flops with feminine designs. However, he is a little lost regarding how much to order and enlists your help (in exchange for free surfboard rentals three days a week!)

You know Lost on Maui sells 150 flip flops per year on average, and that each pair of flip flops costs $15 to order from an outside supplier. An additional administrative cost of $5 is incurred per order. The annual holding cost is calculated with an annual interest rate of 10%. Assume that if Lost on Maui is out of stock when a pair of flip flops is demanded, demand is backordered at the cost of $4 each pair. The lead time is one month (4 weeks) and the demand during the lead time follows a uniform distribution from 0 to 100.

Given:

λ = 150

c = 15

K = 5

i = 10%

h = i c = 1.5

p = 4

T = 1 month = 1/12 year

μ = (0 + 100) / 2 = 50

$$\sigma = \frac{b - a}{\sqrt{12}} = \frac{100 - 0}{\sqrt{12}} = 28.87$$

(a) [5 pts] Design a (Q;R) policy given the specific parameter values provided and the demand distribution during the lead time. The model should include expressions for the total cost function, Q, F(R), and n(R).

$$EOQ = Q_0 = \sqrt{\frac{2\,k\,\lambda}{h}} = \sqrt{\frac{2 * 5 * 150}{1.5}} \cong 32$$

$$F(R_1) = 1 - \frac{Q\,h}{p\,\lambda} = 1 - \frac{32 * 1.5}{4 * 150} = 0.92 \Longrightarrow z_1 = 1.4051$$

$R_1 = \sigma z + \mu = 28.87 * 1.41 + 50 \approx 91$

$n(R_1) = \sigma L(z_1) = 28.87 * L(1.41) = 28.87 * 0.0363 = 1.05$

$$Q_1 = \sqrt{\frac{2\,\lambda\,[\,k + p\,n(R_1)\,]}{h}} = \sqrt{\frac{2 * 150\,[5 + 4 * 1.0364]}{1.5}} \cong 43$$

s = Safety stock = $reorder\ level - expected\ demand\ during\ lead\ time$ = $R - \mu$

Total Cost per unit time = holding cost + fixed cost + shortage cost

$$C\ (Q, R) = h\left(s\ +\ \frac{Q}{2}\right)\ +\ \frac{K}{T}\ +\ p\ \left(\frac{n(R)}{T}\right) \Rightarrow\ h\left((R-\mu)\ +\ \frac{Q}{2}\right)\ +\ \frac{K}{T}\ +\ p\ \left(\frac{n(R)}{T}\right)$$

(b) [10 pts] Show the steps for computing the reorder point via Q-R iteration. Hint: You should find R* = 91

$$F(R_2) = 1 - \frac{Q_1\ h}{p\ \lambda} = 1 - \frac{43\ *\ 1.5}{4\ *150} = 0.89 \Longrightarrow z_2 = 1.2265$$

*$R_2 = \sigma z + \mu$ = 28.87 * 1.2265 + 50 ≈ 85*

*$n(R_2) = \sigma L(z_2)$ = 28.87 * L (1.2265) = 28.87 * 0.0531 = 1.53*

$$Q_2 = \sqrt{\frac{2\ \lambda\ [\ k + p\ n(R_2)\ ]}{h}} = \sqrt{\frac{2\ *\ 150\ [5\ +\ 4\ *\ 1.53]}{1.5}}\ \cong\ 47$$

$$F(R_3) = 1 - \frac{Q_2\ h}{p\ \lambda} = 1 - \frac{47\ *\ 1.5}{4\ *150} = 0.88 \Longrightarrow z_3 = 1.175$$

*$R_3 = \sigma z + \mu$ = 28.87 * 1.175 + 50 ≈ 84*

*$n(R_3) = \sigma L(z_3)$ = 28.87 * L (1.175) = 28.87 * 0.059 = 1.7*

$$Q_3 = \sqrt{\frac{2\ \lambda\ [\ k + p\ n(R_2)\ ]}{h}} = \sqrt{\frac{2\ *\ 150\ [5\ +\ 4\ *\ 1.7]}{1.5}}\ \cong\ 49$$

$$F(R_4) = 1 - \frac{Q_3\ h}{p\ \lambda} = 1 - \frac{49\ *\ 1.5}{4\ *150} = 0.88 \Longrightarrow z_3 = 1.175$$

*$R_4 = \sigma z + \mu$ = 28.87 * 1.175 + 50 ≈ 84*

Since the value for R converges here we can say Q = 49 and R = 84

(c) [5 pts] Determine the level of safety stock.

s = Safety stock

= average inventory level before an order arrives

= $reorder\ level - expected\ demand\ during\ lead\ time$ = $R - \mu$ = 84 – 50 = 34

(d) [5 pts] What is the proportion of order cycles in which no stock-outs occur?

F(R) = the proportion of order cycles in which no stock-outs occur

$$F(R) = 1 - \frac{Q\,h}{p\,\lambda} = 1 - \frac{49 * 1.5}{4 * 150} = 0.88$$

Question 4: You've Got Mail

[25 pts] You are considering keeping a mailbox at the post office on Stanford campus after graduation in order to have a reason to visit your alma mater frequently. You are most concerned about the amount of wait time when getting a big package, since there is only one window that offers such a service.

You estimate that the inter-arrival time of customers like you is exponentially distributed with a mean of 12 minutes (unfortunately, the employee manning that particular window is not always there) and the servicing time is exponentially distributed with a mean of 5 minutes (she tends to take a long time to find a particular package.)

Given:

λ = arrival rate. = 1/12 customers per min = 5 customers per hour

μ = rate of service = 1/5 services per min = 12 services per hour

(a) [10 pts] Compute the expected number of customers in queue, the customers' expected wait time, and their expected total time in system (waiting and servicing time.)

ρ = utilization rate = λ / μ = 5/12 = 0.4167

$$No.\,of\ people\ in\ queue = L_q = \frac{\rho^2}{1 - \rho} = \frac{12 * (5/12)^2}{12 - 5} = \frac{25}{12 * 7} = 0.2976 \text{ people}$$

$$Average\ no.\,of\ people\ in\ the\ system = L = \frac{\rho}{1 - \rho} = \frac{12 * 5}{12(12 - 5)} = \frac{60}{84}$$
$$= 0.7142 \text{ people}$$

$$\text{Total time in system} = W = \frac{\text{L}}{\lambda} = 0.7142 / 5 = 0.1428 \text{ Hours} = 8.57 \text{ mins}$$

$$\text{Wait time in queue} = W_q = L \times \frac{1}{\mu} = 0.7142/12 = 0.0595 \text{ hours} = 3.57 \text{ mins}$$

(b) Read the following statements carefully and identify whether they are True or False.

(i) [5 pts] Queue size would remain stable even if the mean servicing time rose to 25 minutes.

μ = rate of service = 1/25 services per min = 2.4 services per hour
ρ = utilization rate = λ / μ = 5/2.4 = 2.083

$$No. of\ people\ in\ queue = L_q = \frac{\rho^2}{1-\rho} = \frac{(2.083)^2}{1-2.083} = \frac{4.34}{-1.083} = -4.007 \text{ people}$$

Since the service time rose to 25 mins, there will always be people in the queue right after the first arrival. Thus the utilization of 208 % or a negative number of people in queue which means there are always people there from the previous cycle. Like 4.007 people in this case.
Hence the statement is false.

(ii) [5 pts] The marginal benefit of adding servers/windows (in terms of average wait time) diminishes as more servers/windows are added.
Let N be the number of servers/windows then utilization rate is ρ = λ / Nμ. This means higher the number of servers lower will be utilization rate.

Let's consider above example and make a sheet :

λ	μ	N	ρ	L_q	L	W_q	W
5	2.4	49	0.042517007	0.001887967	0.04440497	0.01850207	0.00888099
5	2.4	50	0.041666667	0.001811594	0.04347826	0.01811594	0.00869565
5	2.4	51	0.040849673	0.001739765	0.04258944	0.0177456	0.00851789
5	2.4	52	0.040064103	0.001672124	0.04173623	0.01739009	0.00834725
5	2.4	53	0.039308176	0.001608354	0.04091653	0.01704855	0.00818331

Thus looking at the sheet:
For low utilization rates, adding a server to current servers will not make much of a difference to the wait time. Thus the marginal benefit in terms of wait time diminishes. The statement is True.

(iii) [5 pts] Suppose a customer has been completing the service at the window (not waiting in line) for 5 minutes already. The expected time remaining until the completion of the process is 7 minutes.
False. Since the customer has already been getting the service done, and the service time is 5 mins per service, customer should not have any further wait time.