MS&E 260

INTRODUCTION TO OPERATIONS MANAGEMENT

Problem Session 6
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Announcements

- Homework 5 due Monday, August 12th (instead of the usual Wednesday due date)

- Will cover DA and Final Review next Friday, August 9th

Today’s Outline

- Revenue Management
  - Static Pricing
  - 2-Segment Pricing
  - Dynamic Pricing

Revenue Management

- **Static Pricing:**
  - Demand is deterministic. How do you price the item?

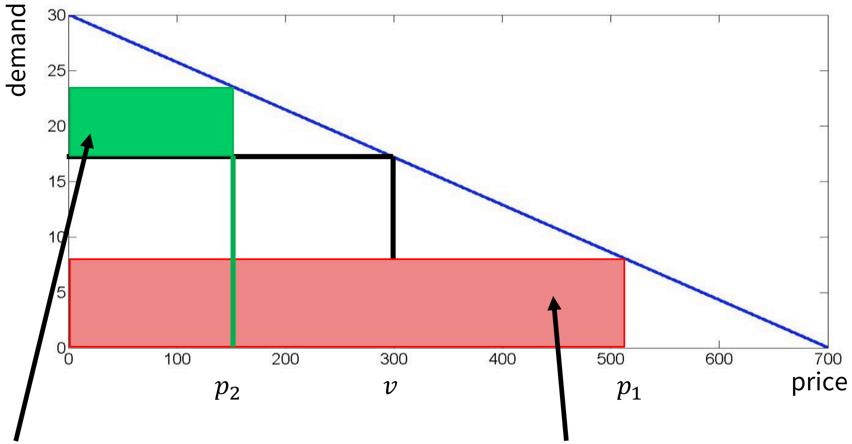
$$\max_{p} \; d(p) \cdot p$$

- **2-Segmentation Pricing:**
  - Demand is deterministic, but we have two target groups. How do you price the item?
- **Dynamic Pricing:**
  - Demand is stochastic. How do you price the item?

2-Segment Price Differentiation

- Two segments:
  - v: segmentation threshold
  - p_1: price for high-valuation segment
  - p_2: price for low-valuation segment
- Question: How to optimize for p_1 and p_2?

Remark: We assumed here that we perfectly segment the market so NO customer with value $> v$ buys the product with the lower price.



Revenue of low value segment

Revenue of high value segment

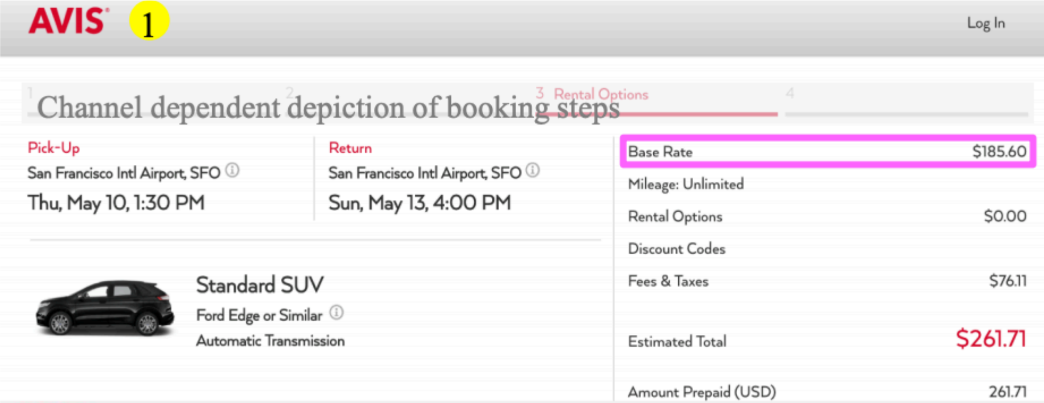
2-Segment Price Differentiation

- (More traditional) examples

- Examples in the digital age

2-Segment Price Differentiation – Example*

- Mac vs. Windows
  - Same pick-up and drop-off time
  - 40 hours booking in advance



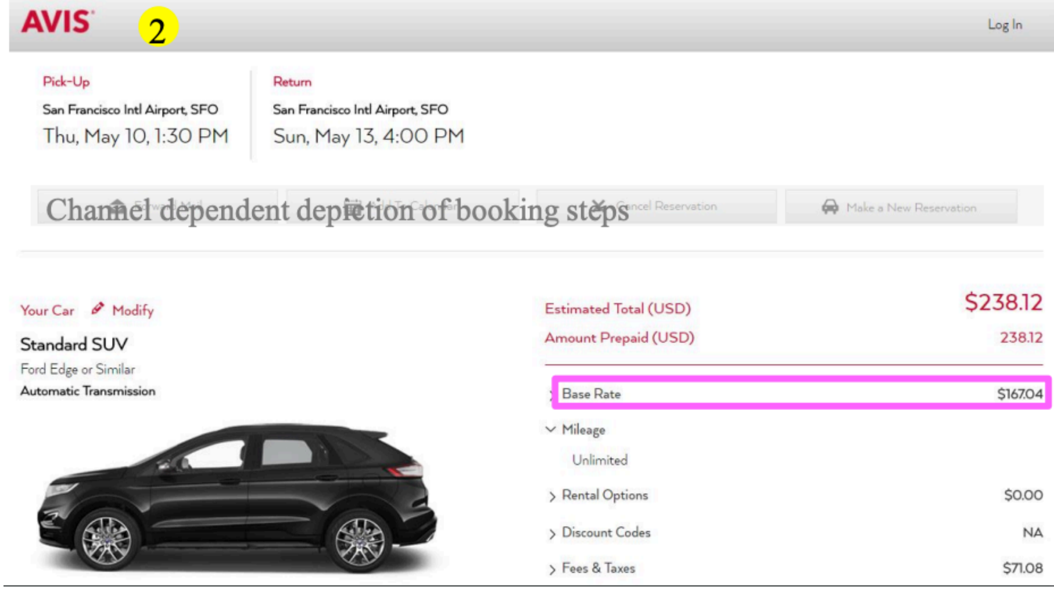
Channel dependent depiction of booking steps

Pick-Up
San Francisco Intl Airport, SFO
Thu, May 10, 1:30 PM

Return
San Francisco Intl Airport, SFO
Sun, May 13, 4:00 PM

Standard SUV
Ford Edge or Similar
Automatic Transmission

Base Rate	$185.60
Mileage: Unlimited	
Rental Options	$0.00
Discount Codes	
Fees & Taxes	$76.11
Estimated Total	$261.71
Amount Prepaid (USD)	261.71



Pick-Up
San Francisco Intl Airport, SFO
Thu, May 10, 1:30 PM

Return
San Francisco Intl Airport, SFO
Sun, May 13, 4:00 PM

Channel dependent depiction of booking steps

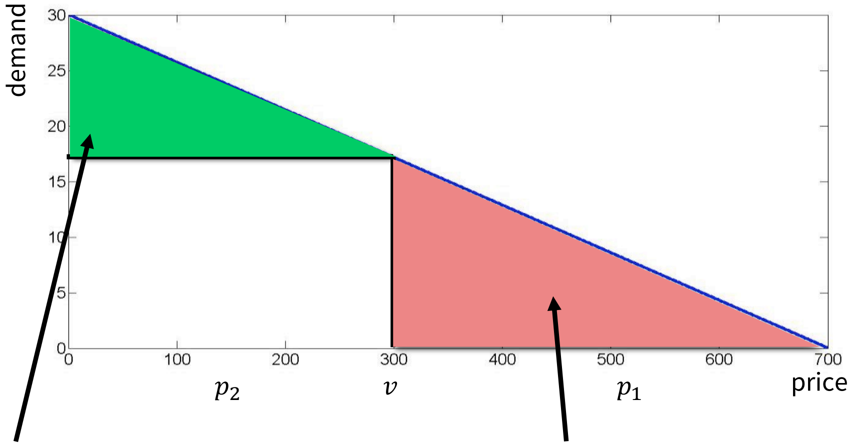
Your Car Modify
Standard SUV
Ford Edge or Similar
Automatic Transmission

Estimated Total (USD)	$238.12
Amount Prepaid (USD)	238.12
Base Rate	$167.04
Mileage	
Unlimited	
Rental Options	$0.00
Discount Codes	NA
Fees & Taxes	$71.08

*utdallas.edu/~metin

2-Segment Price Differentiation

- Question: How to optimize for p_1 and p_2?
  - v: segmentation threshold
  - p_1: price for high-valuation segment
  - p_2: price for low-valuation segment
- Find optimal price in each segment separately!



Possible revenue of low value segment

Possible revenue of high value segment

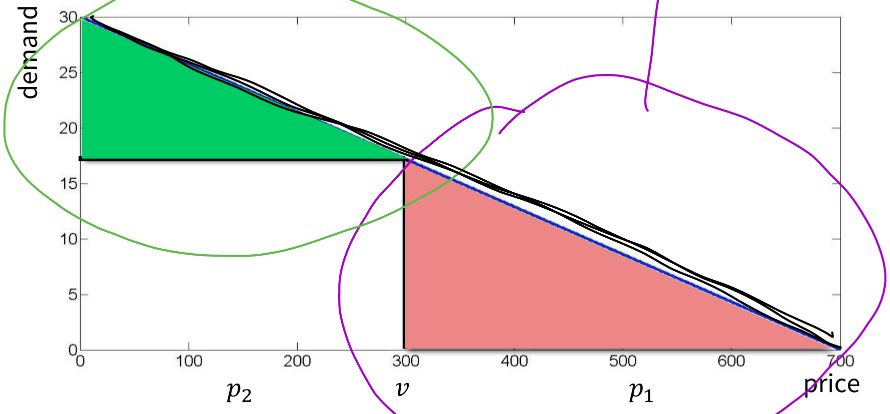
2-Segment Price Differentiation

optimization

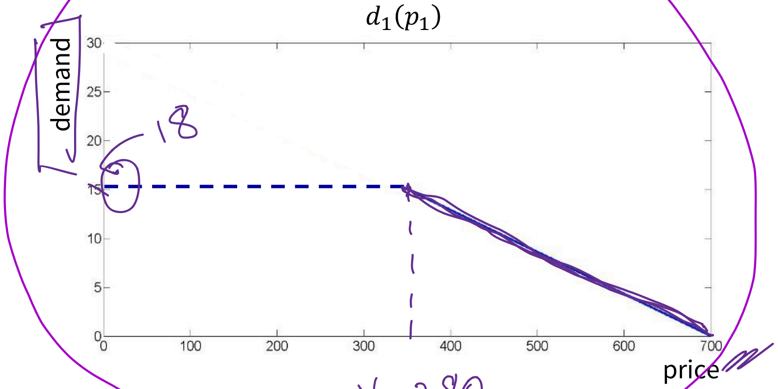
$\max \quad p_1 \cdot d_1(p_1)$

$d(p)$

$= 30 - 0.0429 p$



$d_1(p_1)$: price-response function of high-valuation segment



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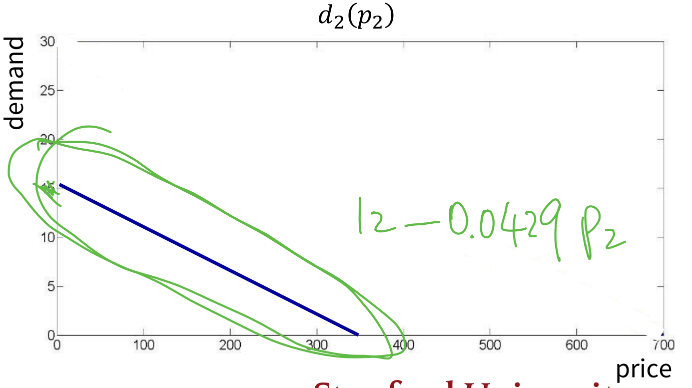
$v = 280$

$d_2(p_2)$: price-response function of high-valuation segment



$12 - 0.0429 p_2$

segment 1

$$\max_{p_1} \quad p_1 \cdot \underline{d_1(p_1)}$$

$v = 280$

↓

$30 - 0.0429\,v$

$= 18$

$$\Rightarrow \max_{p_1} \quad p_1 \cdot \min\{30 - 0.0429\,p_1 \,, 18\}$$

$$\text{s.t.} \quad p_1 \geq 280$$

$$p_1^* \approx 350$$

segment 2

$$\max_{p_2} \quad p_2 \cdot \underline{d_2(p_2)}$$

$Y =$

$$\Rightarrow \max_{p_2} \quad p_2 \cdot \left(\underline{12 - 0.0429\,p_2}\right)$$

$$\text{s.t.} \quad 0 \leq p_2 \leq 280$$

$$p_2^* \approx 140$$

Lecture Example – Electric Stand Mixer

- Segmentation threshold $= \$280$
- High-valuation segment revenue maximization:

$$\begin{array}{ll} \text{max} & p_1 min\{18, (30 - 0.0429 p_1)\} \\ \text{s.t.} & p_1 \geq 280 \end{array} \longrightarrow \begin{array}{l} p_1^* = \$349.65 \\ d(p_1^*) = 15 \\ revenue_{p_1^*} = \$5{,}244.76 \end{array}$$

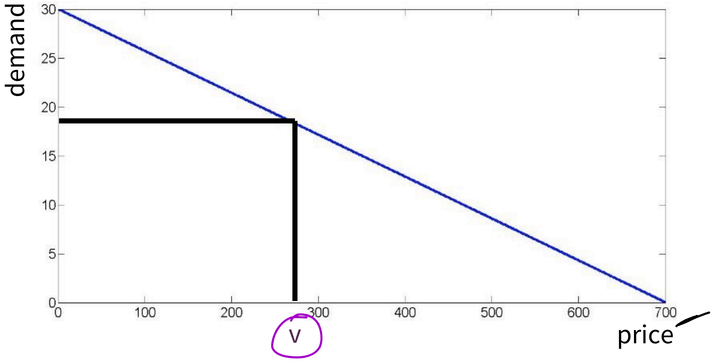
- Low-valuation segment revenue maximization:

$$\begin{array}{ll} \text{max} & p_2(12 - 0.0429 p_2) \\ \text{s.t.} & p_2 \leq 280 \\ & p_2 \geq 0 \end{array} \longrightarrow \begin{array}{l} p_2^* = \$139.86 \\ d(p_2^*) = 6 \\ revenue_{p_2^*} = \$839.16 \end{array}$$

Revenue increased by $840!

Lecture Example – Electric Stand Mixer

- Question: is $v = \$280$ the best price to segment the market?
- i.e. Total revenue = $\$5{,}244.76 + \$839.16 \approx \$6085$. Can we increase that?

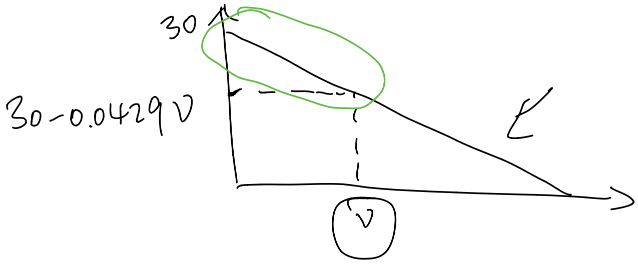


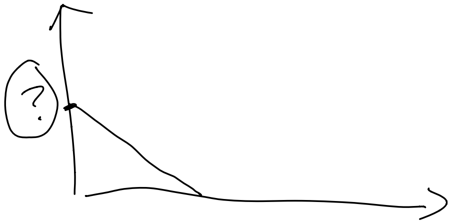
for $V \leq P_1 \leq 700$,

$$d_1(P_1) = 30 - 0.0429\, P_1$$

for $0 \le P_2 \le v$,

$$d_2(P_2) = 0.0429v - 0.0429P_2$$





want to

$$\max_{v} \left\{ \underset{\downarrow}{\text{high-val. seg. rev}} + \underset{\downarrow}{\text{low-val. seg. rev}} \right\}$$

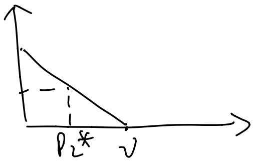
$$\max_{P_1} \; P_1 \cdot (30 - 0.0429P_1) \qquad \max_{P_2} \; P_2 \cdot (0.0429v - 0.0429P_2)$$

for a fixed v,

$$30 - 2 \cdot 0.0429 P_1 = 0$$
$$P_1^* = 350$$

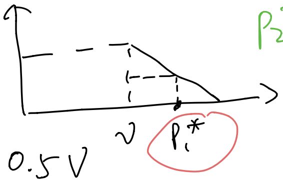
$$\left\{ \max_{P_1, P_2} \underline{\underline{(30p_1 - 0.0429p_1^2)}} + \underline{0.0429vP_2 - 0.0429P_2^2} \right\}$$

$$P_2^* = 0.5v$$



$$P_1 = \max\{v, 350\}$$

$$P_2 = \min\{0.5v, v\} = 0.5v$$



$$\max_{v}\Big\{ \max\{v,350\}\cdot\left(30-0.0429\cdot\max\{v,350\}\right) + \left[0.0429v-0.0429(0.5v)\right]\cdot 0.5v \Big\}$$

$$\max\{v,\ 350\} = \begin{cases} v & [\text{case } 1] \\ 350 & [\text{case } 2] \end{cases}$$

$$\begin{cases} \boxed{v^* = 466} \\ v^* = 0 \qquad \text{No} \end{cases}$$

total revenue $= 4660 + 2330$

$= 6990$

$> \quad 6085$

Revenue Management

- **Static Pricing:**
  - Demand is deterministic. How do you price the item?

$$\max_{p} \; d(p) \cdot p$$

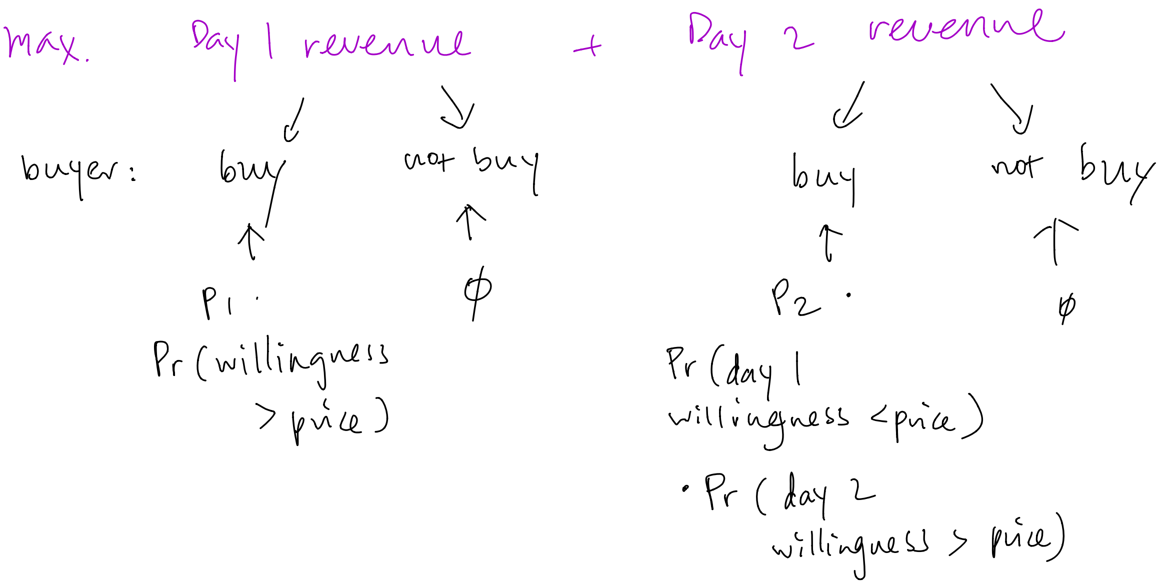
- **2-Segmentation Pricing:**
  - Demand is deterministic, but we have two target groups. How do you price the item?
- **Dynamic Pricing:**
  - Demand is stochastic. How do you price the item?

Example: Dynamic Pricing

You allocate 2 days to sell a single item. On day 1, buyer 1 arrives at the store and their willingness to pay, v_1, is drawn independently from $U[0, 100]$. On day 2, buyer 2 arrives and $v_2 \sim U[0, x]$. Using dynamic pricing optimization, the optimal price level is computed to be $p_1 = 62.5$ and $p_2 = 40$.

a) What is x?

b) Suppose that now you have three days to sell the item. On day 3, buyer 3 arrives with $v_3 \sim U[0,40]$. How will you price the item?



$$\max_{P_1, P_2} R = P_1 \cdot Pr(V_1 \geq P_1) + Pr(V_1 < P_1) \cdot P_2 \cdot Pr(V_2 \geq P_2)$$

$$= P_1 \cdot \left(1 - \frac{P_1}{100}\right) + \left(\frac{P_1}{100}\right) P_2 \left(1 - \frac{P_2}{X}\right)$$

$$\frac{\partial R}{\partial P_1} = 0 \Rightarrow 1 - \frac{2}{100} P_1 + \frac{P_1 P_2}{100} - \frac{P_1 P_2^2}{100 X} = 0$$

$$\frac{\partial R}{\partial P_2} = 0 \Rightarrow \frac{P_1}{100} - \frac{2 P_1}{100 X} P_2 = 0$$

$V_2 \sim U[0, X]$

$X \sim U[a, b]$ cdf, $Pr(X \leq x) = \frac{x - a}{b - a}$

$V_1 \sim U[0, 100]$

$$Pr(V_1 \geq P_1) = 1 - Pr(V_1 < P_1)$$
$$= 1 - \frac{P_1 - a}{b - a}$$
$$= 1 - \frac{P_1}{100}$$

Plug in P_1^*, P_2^*

$$\Rightarrow \frac{62.5}{100} - \frac{2 \cdot 62.5}{100 \cdot X} \cdot 40 = 0$$

$x = 80$

(b) $\max_{P_1, P_2, P_3} \{ \text{day } 1 + 2 + 3 \quad \text{rev's} \}$

$$\Rightarrow \max_{P_1, P_2, P_3} \; P_1 \cdot \left(1 - \frac{P_1}{100}\right) + \left(\frac{P_1}{100}\right) P_2 \left(1 - \frac{P_2}{80}\right)$$

$$+ \left(\frac{P_1}{100}\right)\left(\frac{P_2}{80}\right) P_3 \cdot \left(1 - \frac{P_3}{40}\right)$$