MS&E 260

Introduction to Operations Management

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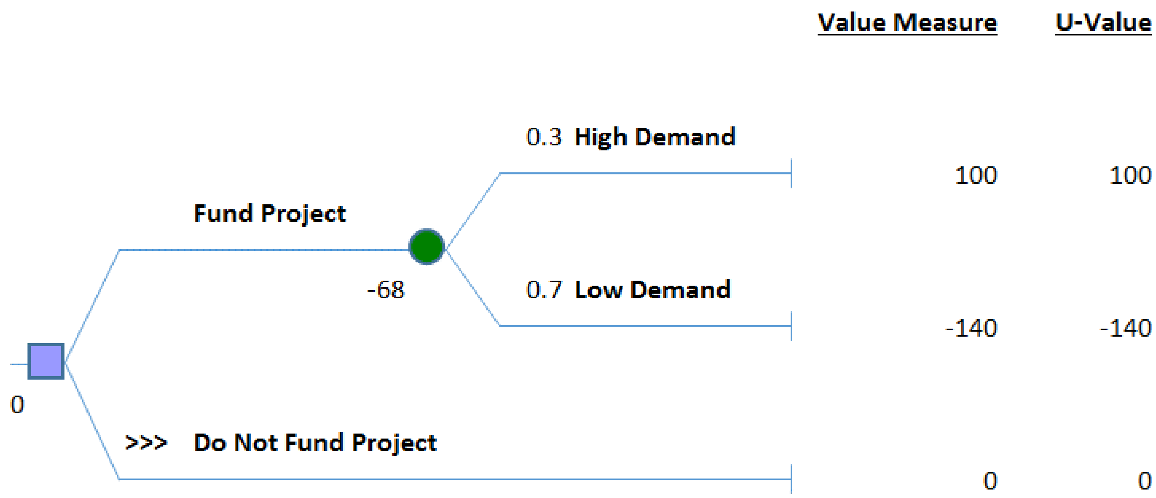
Decision Analysis

Some Key Decision Analysis Terms

- Deal/Lottery
- Five Rules
- U-value/U-curve
- E-value
- E-value of U-values
- Risk Neutral
- Delta Property/Constant Risk Aversion
- Value of Clairvoyance

Step Back: What Is Management Science?

- Management Science: the study of organizations and how they achieve their goals
  - Our approach is different from pedagogies in the other fields
  - Decision Analysis is a foundational element of management science: it helps us make decisions, in spite of uncertainty
  - Decision Tree is a tool that helps us make decisions



example decision tree

Decision Making In Two Forms

Descriptive: how people make decisions

- Methods
  - Pros and Cons
  - Rate and Weight
  - Flipping Coins
- Imprecise, results not easily repeatable

Tight coupling between decisions and outcomes

Normative: how people should make decisions

- Method
  - Decision Analysis
- Provides an objective framework for evaluating quantitative measures

Decouples decisions and outcomes

Why Do We Need Decision Analysis? (1/2)

- Humans tend to be very poor decision makers!
- Have you ever…
  - missed a great airfare because you were unsure what future prices would be?
  - been paralyzed by a to-do list with 20-items, with no clue on how and in what order to tackle the list?
  - worked in an organization which decided on projects on the basis of net present value?

These are all examples of decision-making in non-normative ways

Why Do We Need Decision Analysis? (2/2)

- Humans tend toward some well-known biases
  - Availability bias: tendency to think that if an event is more easily imaginable, then it is more probable
    - Is it more likely for an English word to start with 'R', or have 'R' in the third position?
  - Recency bias: tendency to base decisions on events that have occurred more recently
    - Bike riders wear helmets after seeing an accident
    - Intelligence analysts overly cautious of bad intelligence after Iraq
  - Anchoring bias: tendency to rely too heavily on the first piece of information offered when making decisions (example)

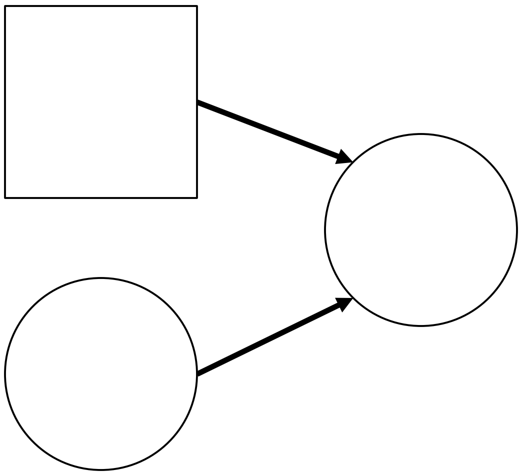
Decision Analysis can help us overcome these biases

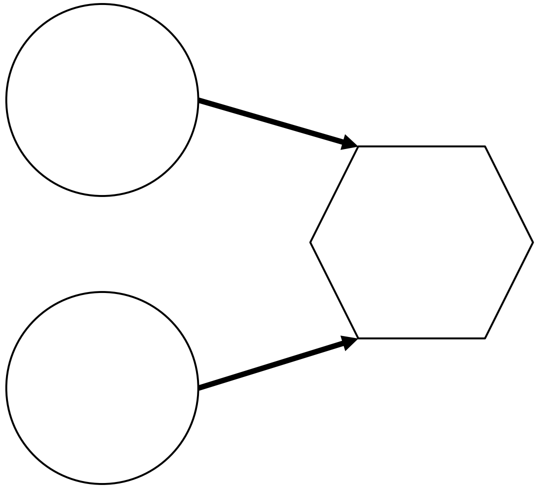
Components of a Decision

1. Decisions
   - A choice between two or more alternatives that involves an irrevocable allocation of resources
   - e.g. invest/do not invest in new advanced weapon system
2. Uncertainties
   - Events that can occur, probabilistically with different degrees, which affect the outcome
   - e.g. mission success, readiness of technology, adversary countermeasures
3. Deterministic Nodes
   - If all inputs to a deterministic node are known, then there is no longer any uncertainty about it
   - e.g. net income if revenue and expenses are known
4. Outcomes (symbol varies, sometimes or)
   - Quantitative representation of the value of each combination of decision and uncertainty
   - e.g. monetary value, expected casualties

Influence Diagrams

- Conditional dependence
  - Arrow between two uncertainties
  - Arrow from decision node to uncertainty
  - Absence of arrow between two nodes asserts independence
- Inputs to deterministic node
  - Arguments of a function
- Prior knowledge of an uncertainty or event
  - Arrow from any node into a decision indicates that the result of the node are known before decision is made
  - If the parent node of a decision is an uncertainty, that uncertainty has been resolved by the time of the decision
  - If the parent node is another decision, the order of the decisions is made explicit



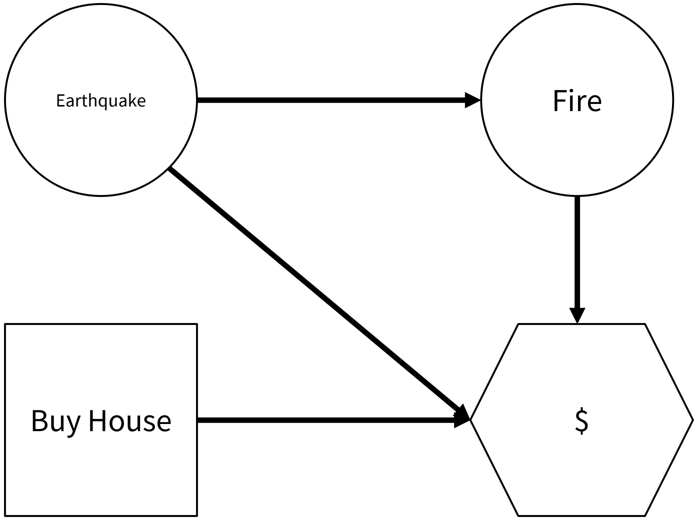


Influence Diagrams: Simple Example (1/2)

- Scenario:
  - There is an historic house for sale in San Francisco with an amazing view. The house has not been seismically retrofitted
  - The asking price is \$1M. Conditional on the house not getting destroyed by an ‘act of God,’ you would be willing to pay up to \$3M for the house
  - You believe there are two kinds of ‘acts of God’ that might destroy the house: earthquake and fire
  - If an earthquake occurs, even if it doesn’t destroy the house directly, it increases the probability of a fire
- Problem:
  - Draw an influence diagram representing your decision to buy the house

Influence Diagrams: Simple Example (2/2)

- Earthquake → Fire
  - Probability of fire depends on whether or not an earthquake has occurred.
- Buy House → $, Earthquake → $, Fire → $
  - Value that we “book” by making the decision depends on
    - (a) the result of our decision
    - (b) whether or not the house is subsequently destroyed
- Why no arrows from Buy House to Earthquake or Fire?
- Why no arrows from Earthquake or Fire into Buy House ?



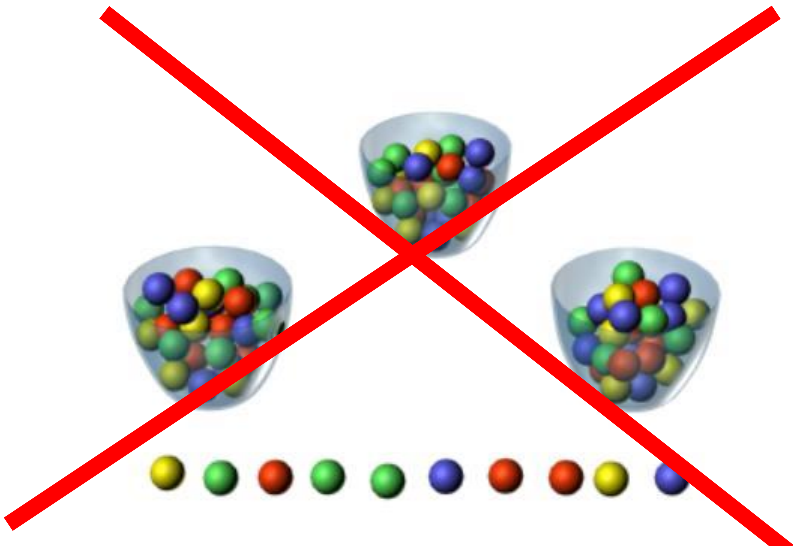
Some Philosophical Thoughts on Probability (1/3)

- We often conflate these terms: "Expectation," "Expected Value," and "Things We Expect to Happen"
- "Expectation" or "Expected Value" of a random variable is the weighted average of the outcomes
- Expectation of the decision is not an outcome you should expect
  - e.g. Rolling a die: expected numerical value = 3.5, an impossible outcome. We would never expect to roll a 3.5

Expected values of a decision are useful because they help us weigh alternatives while considering uncertainty

Some Philosophical Thoughts on Probability (2/3)

- Probabilities are a belief
  - Your probability may be different from my probability on the same uncertainty



Some Philosophical Thoughts on Probability (3/3)

- All probabilities are conditioned on the total sum of your life experiences: “&”
  - e.g. My belief on the probability of rain tomorrow =
$$P(rain \mid \&)$$
  & = my knowledge of seasons, red shoes, etc
- We should strive to become “Bayesian Thinkers”
  - We have the ability to observe a body of evidence, and update our beliefs based upon new evidence

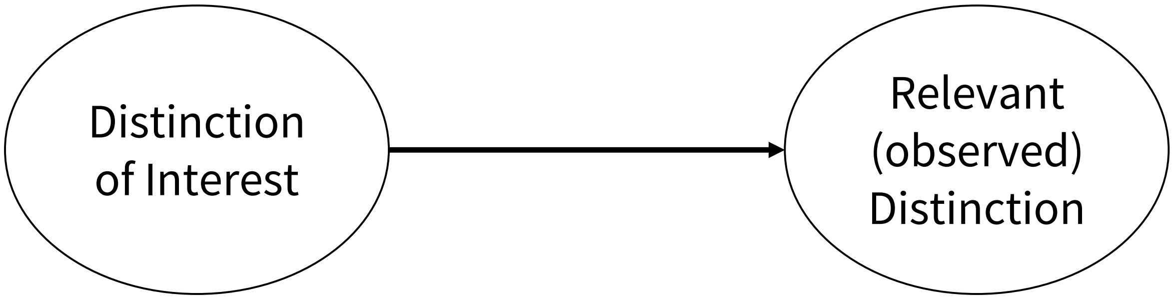
Thomas Bayes
1701-1761

Questions

- What is the gross tonnage of dog food sold in the United States in the calendar year 2012?
- What is the probability of the Los Angeles Dodgers scoring more than 4 runs when they play the Oakland Athletics on Tuesday, August 7, 2018?

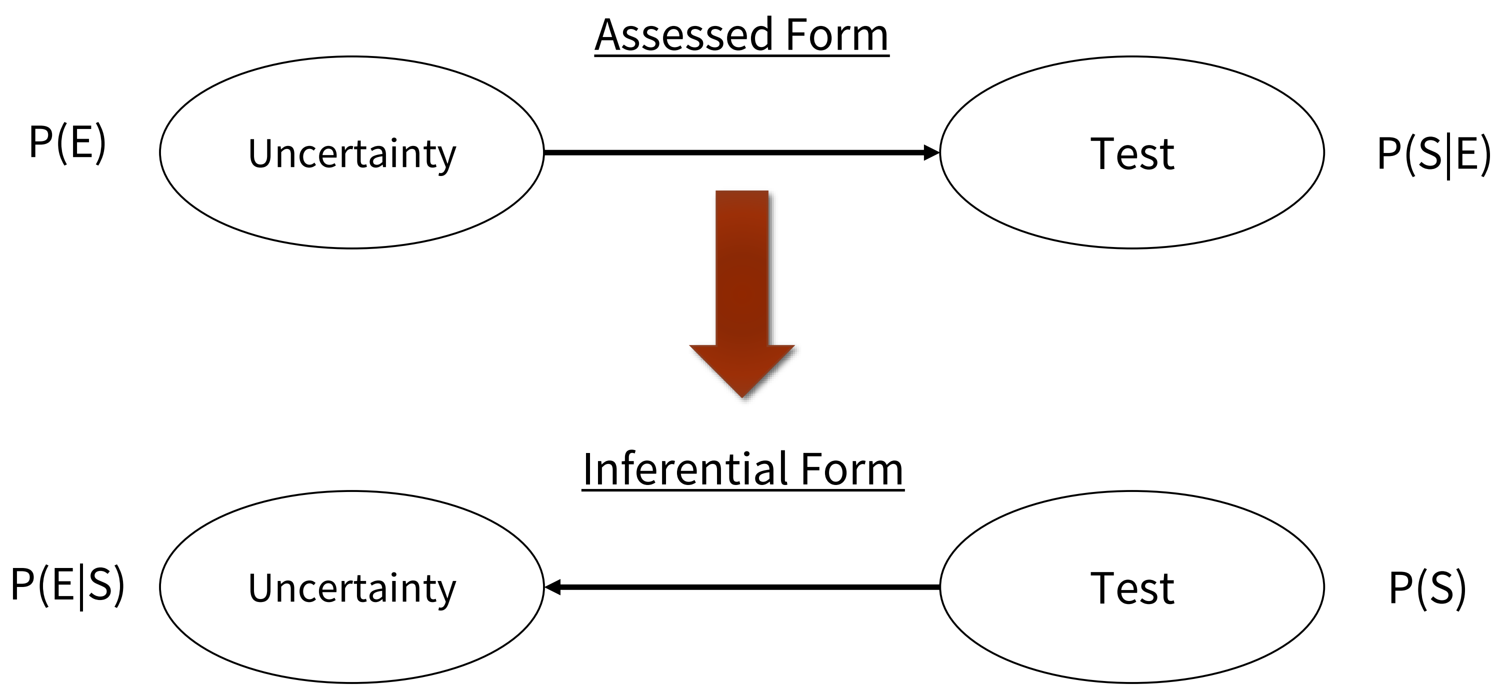
Bayesian Inference

- We observe a distinction that is relevant (probabilistically dependent) to our distinction of interest and use that information to update our probability distribution on the distinction of interest



Updating Beliefs

- We can gather information (e.g. from a test) to update our probability on an unobservable distinction
- Given:
  - E: event that we want to know more about
  - S: signal that we observe regarding the event E



Bayesian Updating of Beliefs

- We use Bayes' Formula as a mechanism to update our prior beliefs into posterior beliefs

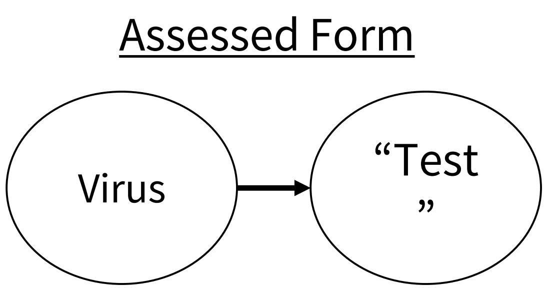
$$\underbrace{P(E|S)}_{\text{Posterior}} = \frac{\overbrace{P(E)}^{\text{Prior}} \times \overbrace{P(S|E)}^{\text{Likelihood}}}{\underbrace{P(S)}_{\text{Preposterior}}}$$

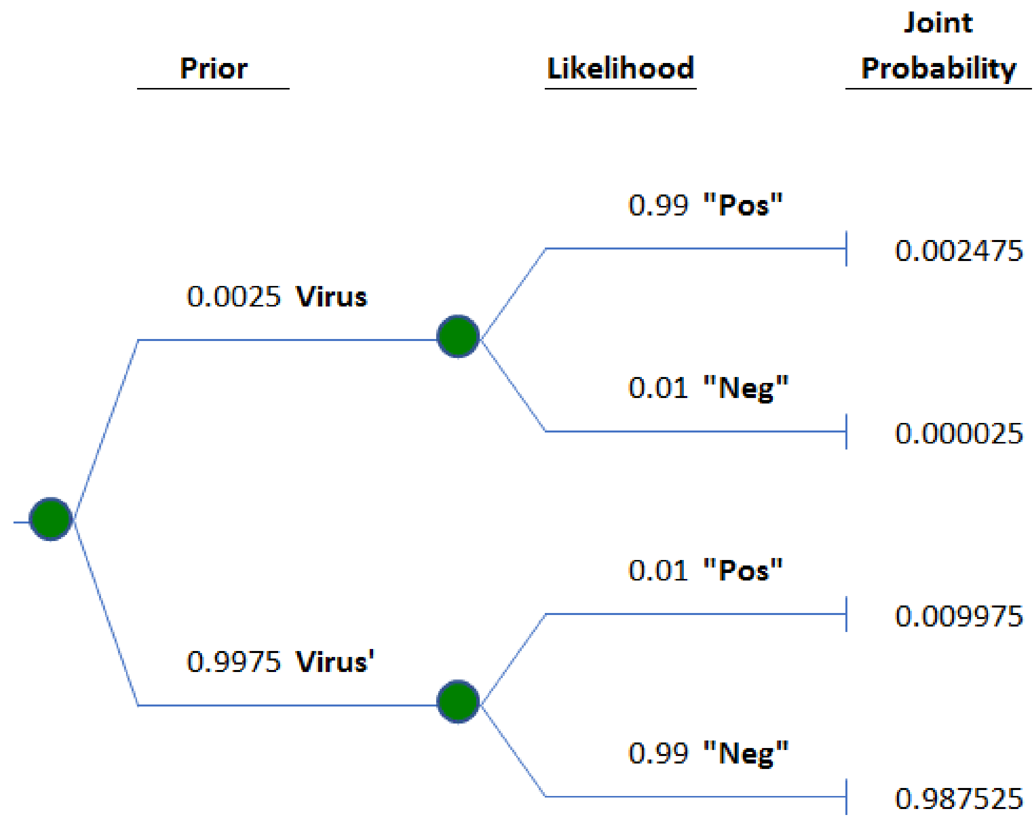
Example: Testing for a Virus (1/3)

- Suppose that you want to study a virus
- 1/400th of the population is infected
- A test is available:
  - If used on an infected person, the test is 99% likely to indicate that the person is indeed infected
  - If the person is not infected, the test is 99% likely to indicate that the person is not infected

Example: Testing for a Virus (2/3)

- First, we characterize the performance of the test
  - This is the Assessed form of the tree
- The Assessed form determines the joint probabilities of the prior and likelihood



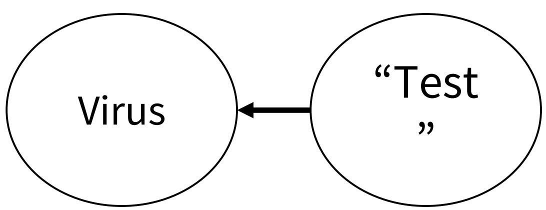


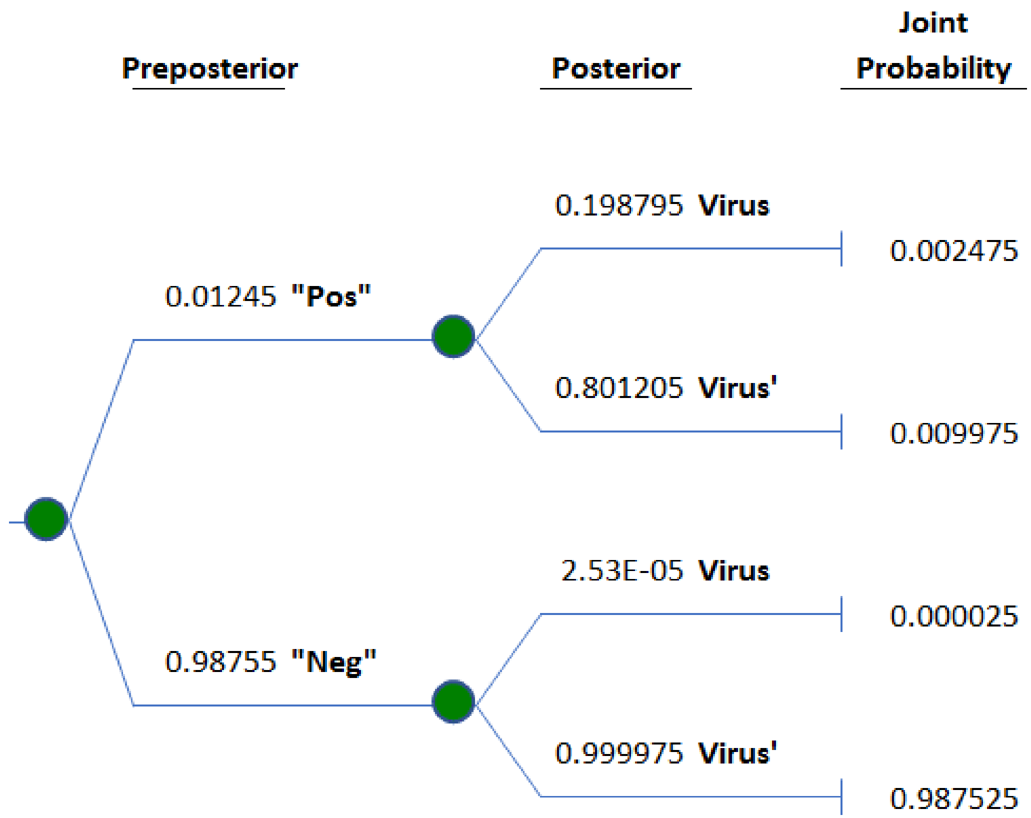
{“Pos” | Pos } = Test Sensitivity
{“Neg” | Neg } = Test Specificity

Example: Testing for a Virus (3/3)

- Next, we "flip the tree"
  - Reverse the order of the uncertainties
- This is the Inferential form of the tree: we can now make inferences on the virus that are not directly assessed

Inferential Form



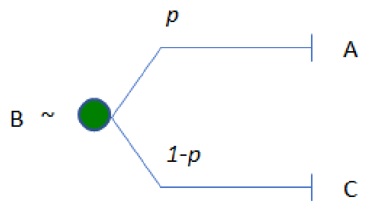


Decision Analysis Foundation: The Five Rules

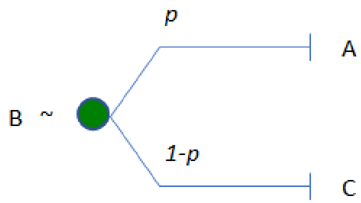
1. **Probability**
   - Consider all information relevant to a decision in terms of possible prospects and associated probabilities.
2. **Preference Ordering**
   - We can order our list of prospects according to our preference (ties allowed).

$$A > B > C$$

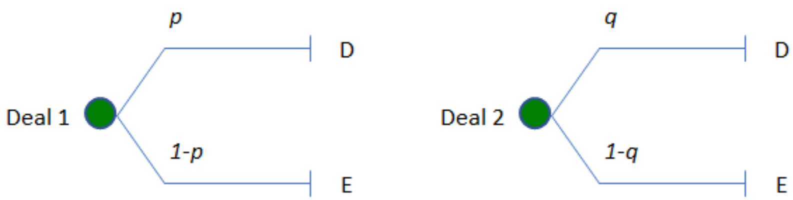
3. **Equivalence**
   - Given the preference ordering ($A > B > C$), we can state a probability p that would make us indifferent between B and the lottery of A and C shown right.



4. **Substitution**
   - If we faced the decision situation in (3), for that value of p, we would be indifferent between B and the lottery of A and C.



5. **Choice**
   - Given the following deals, if you prefer D over E, and $p > q$, then you must choose Deal 1 over Deal 2.



Decision Analysis Fundamentals (1/2)

- All alternatives, uncertainties, and values must pass the Clarity Test
  - e.g. What constitutes “hostile action” in space?
- Who is the decision maker?
  - Identify a single decision maker, and be consistent
  - Often, different decision makers in the same decision situation will have opposing objectives
    - e.g. military escalation: President and military leadership

Decision Analysis Fundamentals (2/2)

- Values should be quantifiable measures to maximize or minimize, as appropriate
  - Money, lives, etc
- Be as complete as appropriate in quantifying value
- e.g. What are implications of introducing a new drug to market?
  - Reputation of the pharmaceutical company
  - Value of unintended deaths versus money
  - Public support for new drug

Components of a Decision

1. Decisions
   - A choice between two or more alternatives that involves an irrevocable allocation of resources
   - e.g. invest/do not invest in new advanced weapon system
2. Uncertainties
   - Events that can occur, probabilistically with different degrees, which affect the outcome
   - e.g. mission success, readiness of technology, adversary countermeasures
3. Deterministic Nodes
   - If all inputs to a deterministic node are known, then there is no longer any uncertainty about it
   - e.g. net income if revenue and expenses are known
4. Outcomes (symbol varies, sometimes ◁ or ⬡)
   - Quantitative representation of the value of each combination of decision and uncertainty
   - e.g. monetary value, expected casualties

Decision Trees

- Scenarios depicted in influence diagrams can also be depicted in event trees
- If no information about any uncertainties is known at the time of a decision, the decision is usually drawn as the root node of the event tree
  - The branches of the node are the different alternatives that could be selected
  - Each subsequent node is an uncertainty (along with its possible realizations) or a subsequent decision
- If an uncertainty precedes a decision in an event tree, then the uncertainty is resolved and is known to the decision maker
- The leaves of the tree represent all possible realizations of the joint distribution over the uncertainties together with the decision(s)
  - Leaves may be labeled with value the decision maker places on that specific outcome

Probabilities in Decision Trees

- Probabilities are values between 0 and 1

$$0 \leq P(Success \mid New\ Technology) \leq 1$$

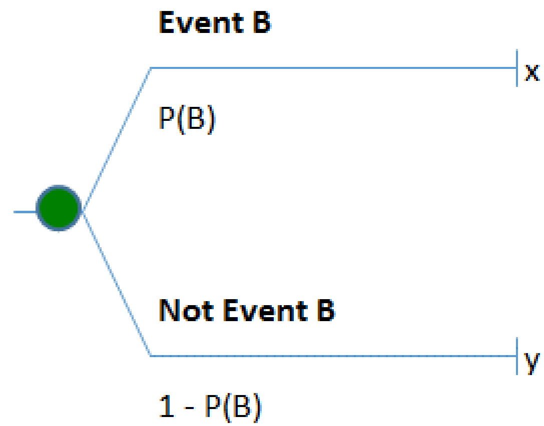
- All probabilities associated with each degree of an uncertainty must sum to 1 (example 1)

$$P(Success \mid New\ Technology) + P(Failure \mid New\ Technology) = 1$$

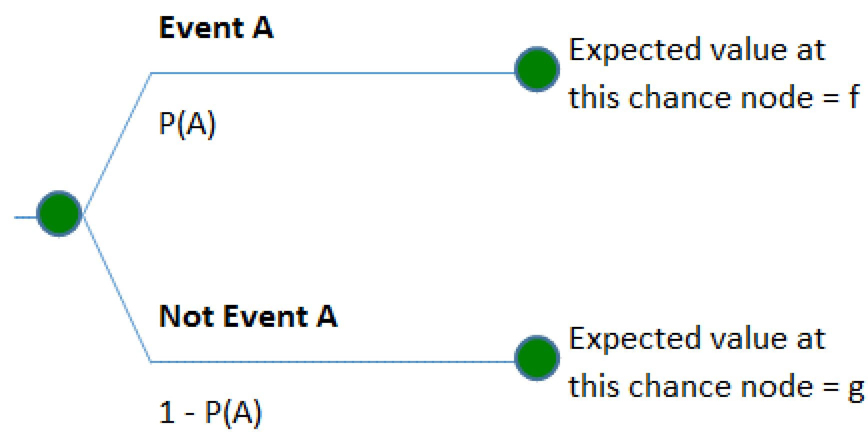
- Conditional Probabilities
  - Probability of an event conditional on knowing another piece of information
  - In example 1, probabilities are conditioned on what technology has been chosen

Calculating Expectation of Each Decision Alternative

$$\text{Expected Value} = xP(B) + y\big(1 - P(B)\big)$$



$$\text{Expected Value} = fP(A) + g\big(1 - P(A)\big)$$



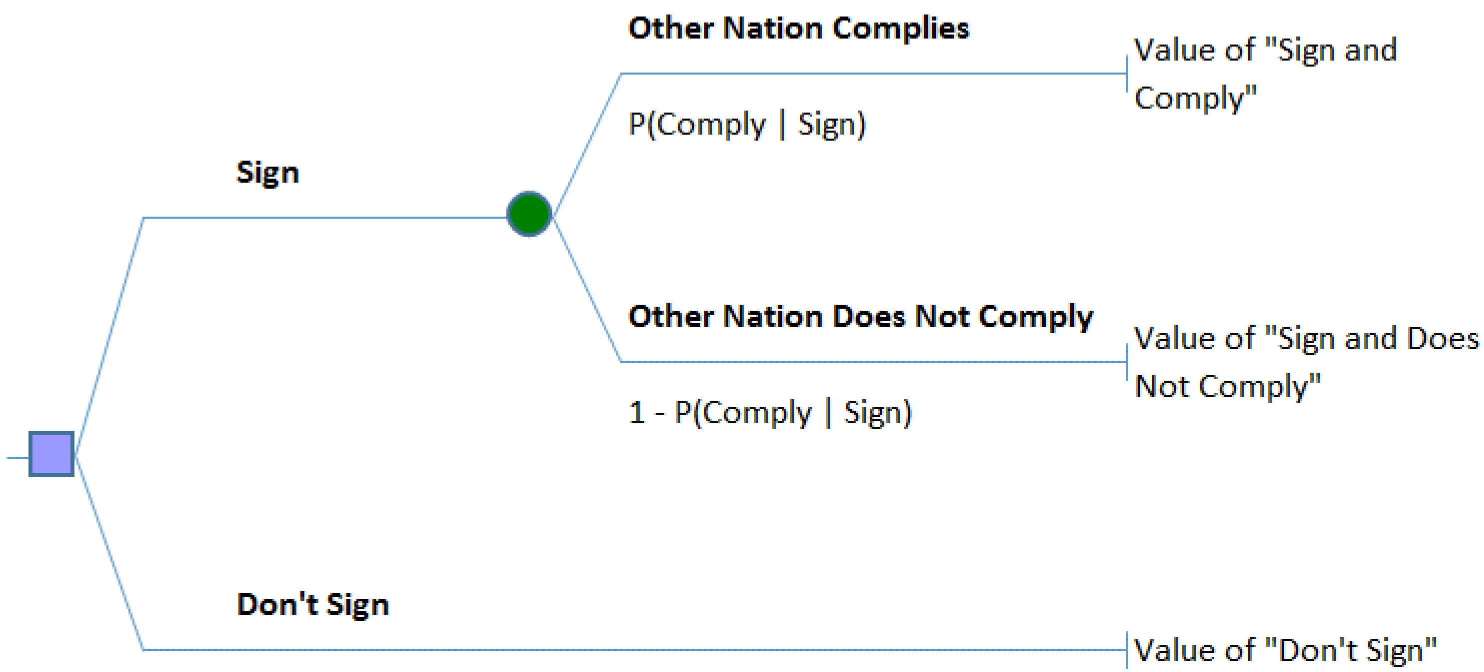
Expected values are calculated as sequential weighted averages, right-to-left on the tree (example)

Example Decision Tree: Arms Agreement Tree (1/4)

- As National Security Advisor, you have to decide whether to recommend signing a major bilateral arms elimination agreement
- If you don't sign, the opposing nation will definitely not relinquish their weapons and you expect ongoing violence to result in one million lost lives
- If you do sign, the opposing nation will also sign, but may not comply
  - If they comply, some lives will still be lost in residual violence (20,000 lives)
  - If they do not comply, the current situation will be aggravated and the expected number of lives lost is four million
  - Experts estimate that the opposing nation will not comply with the agreement with a probability of 0.2

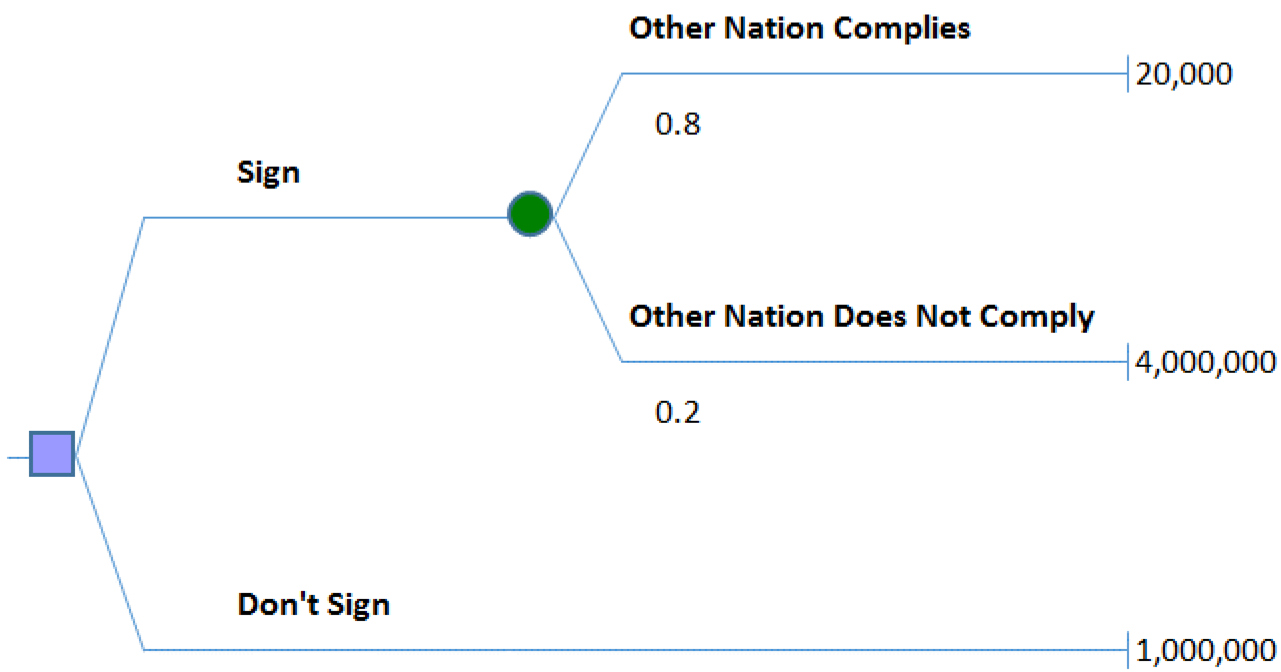
Example Decision Tree: Arms Agreement Tree (2/4)

Objective: Choose the alternative that minimizes the expected number of lives lost



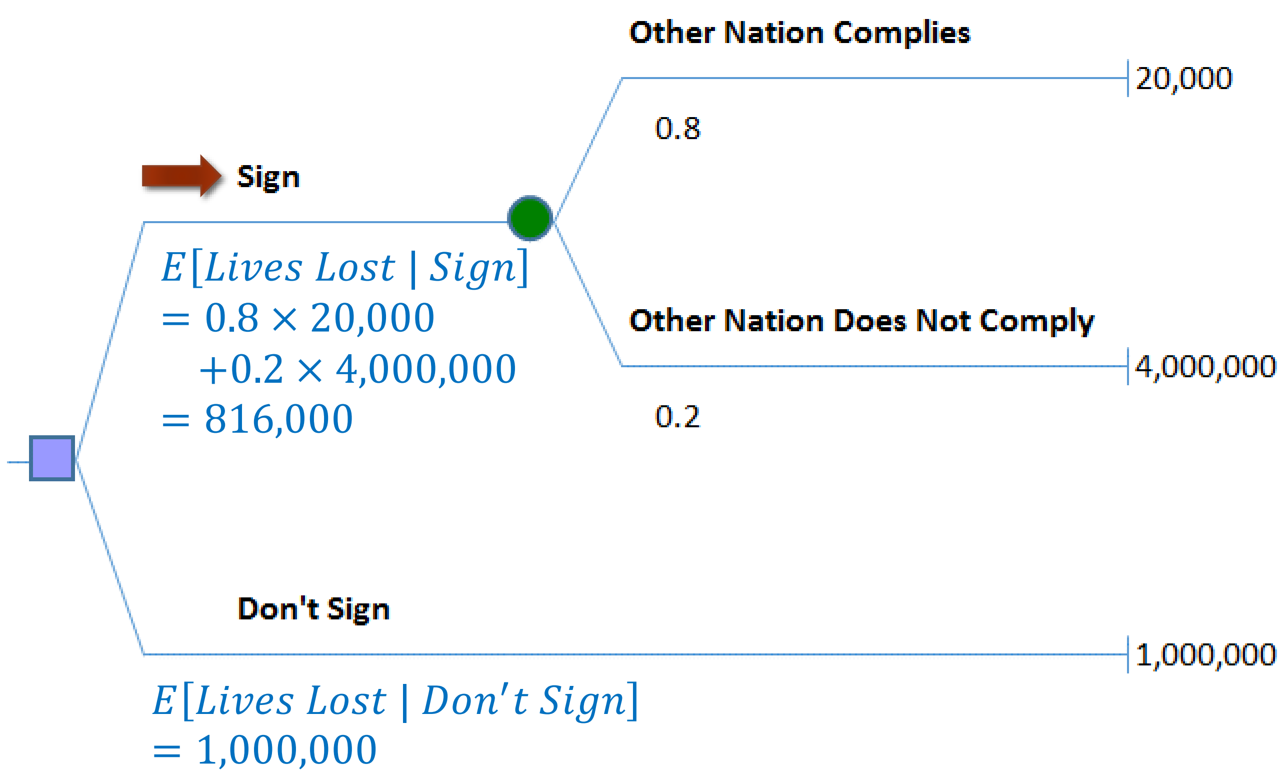
Example Decision Tree: Arms Agreement Tree (3/4)

Objective: Choose the alternative that minimizes the expected number of lives lost



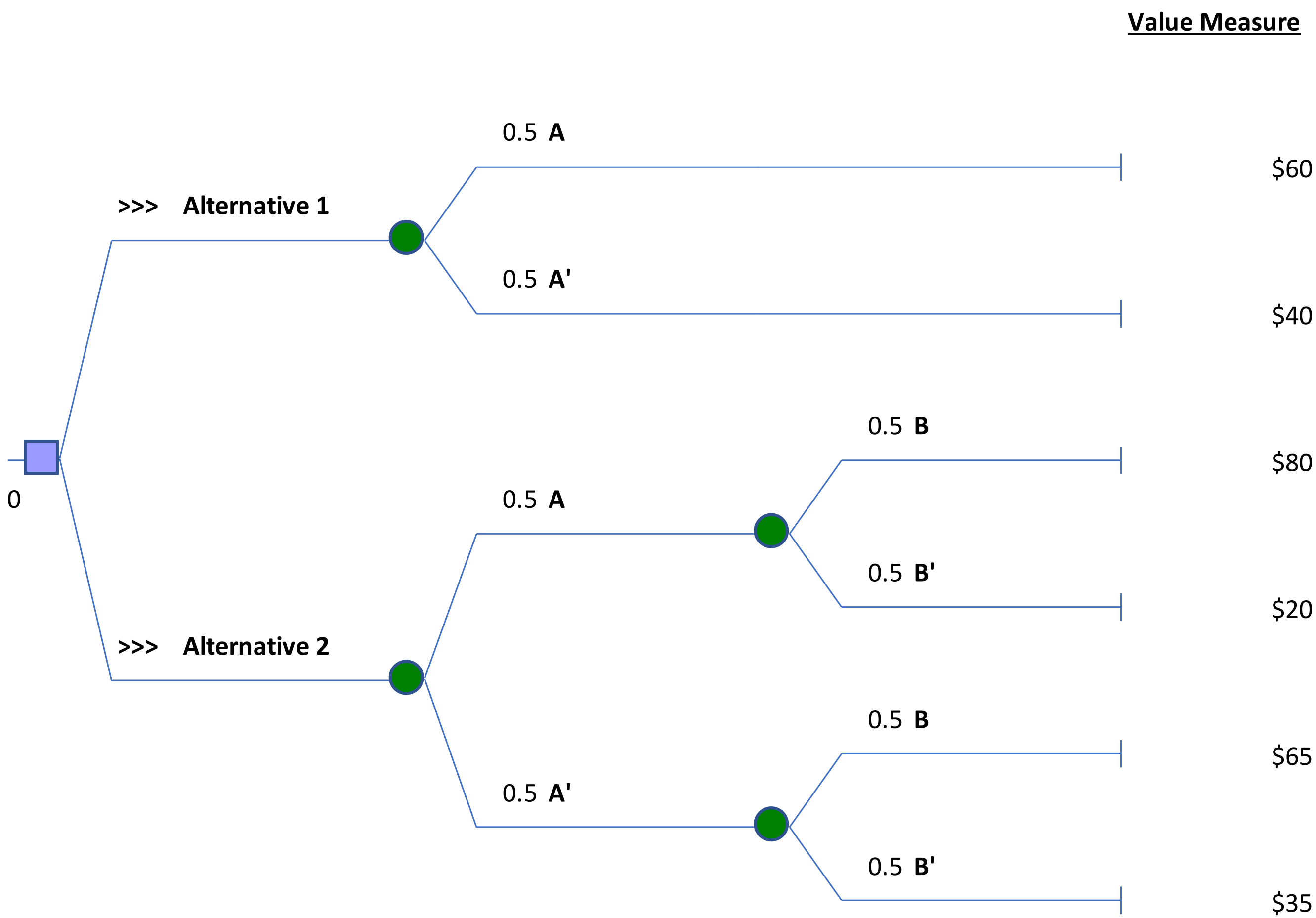
Example Decision Tree: Arms Agreement Tree (4/4)

Objective: Choose the alternative that minimizes the expected number of lives lost



Introduction into Risk Preference

- Which alternative will an infinitely risk-seeking person choose?
- Which alternative will an infinitely risk-averse person choose?



Risk Preference In Decisions

- Up to now, all our decisions have been in expected value
  - This implies that the decision-maker is risk neutral
  - i.e. $1 is worth $1 of utility
- Oftentimes, decision-makers have some degree of risk aversion
- We generally represent risk attitude with utility curves of the form

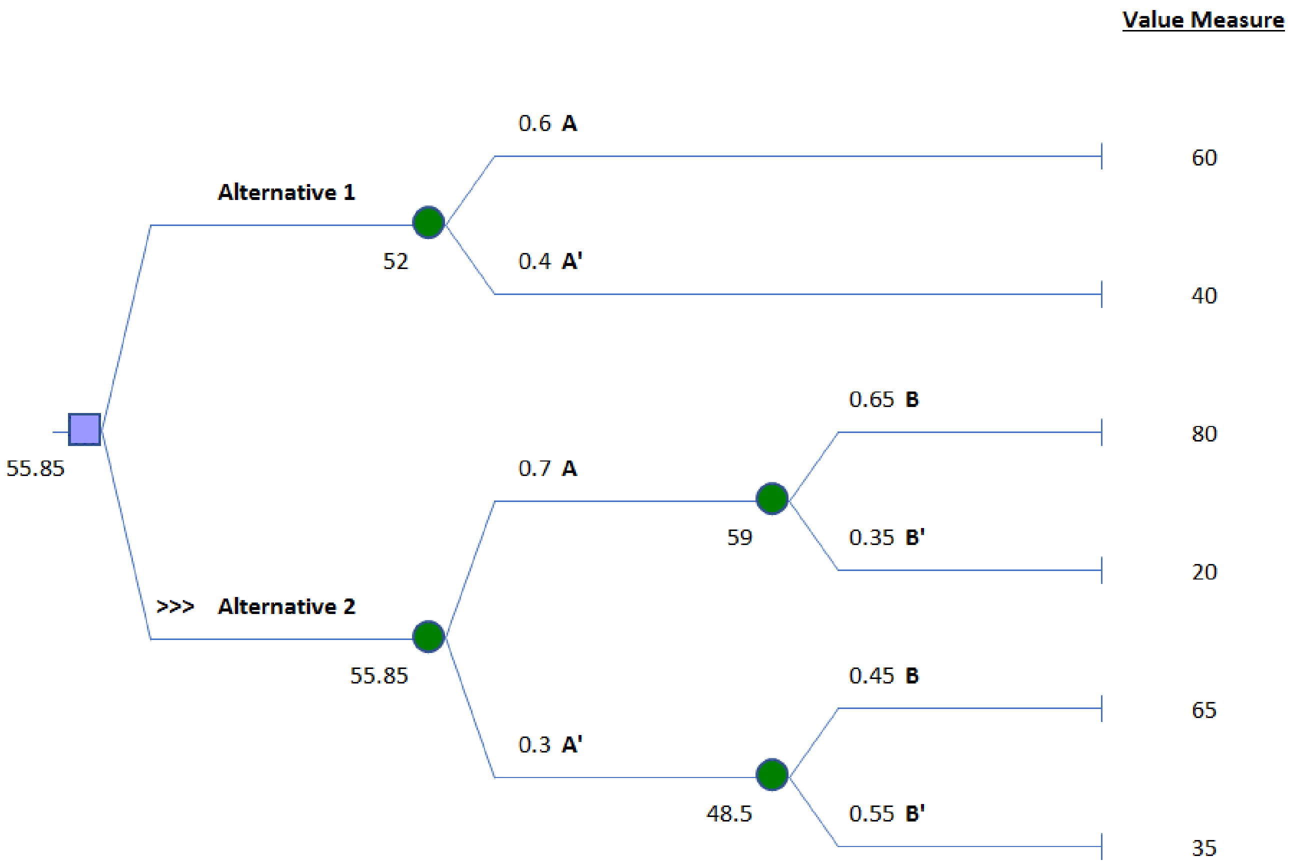
$$u(x) = a - b \times e^{-\gamma x}$$

- where
  - $\gamma =$ the decision−makers risk aversion coefficient
  - $x =$ the value in original units
  - $u(x) =$ utility associated with x
- Rules for γ:
  - $\gamma > 0$ indicates risk averse attitude
  - $\gamma < 0$ indicates risk seeking attitude
  - $\gamma = 0$ indicates risk neutral attitude

When dealing with risk attitude, we are solving for **expected utility**, not expected value

Example Decision Tree: Incorporating Risk Preference (1/3)

- Consider the following decision situation for a risk neutral decision-maker:

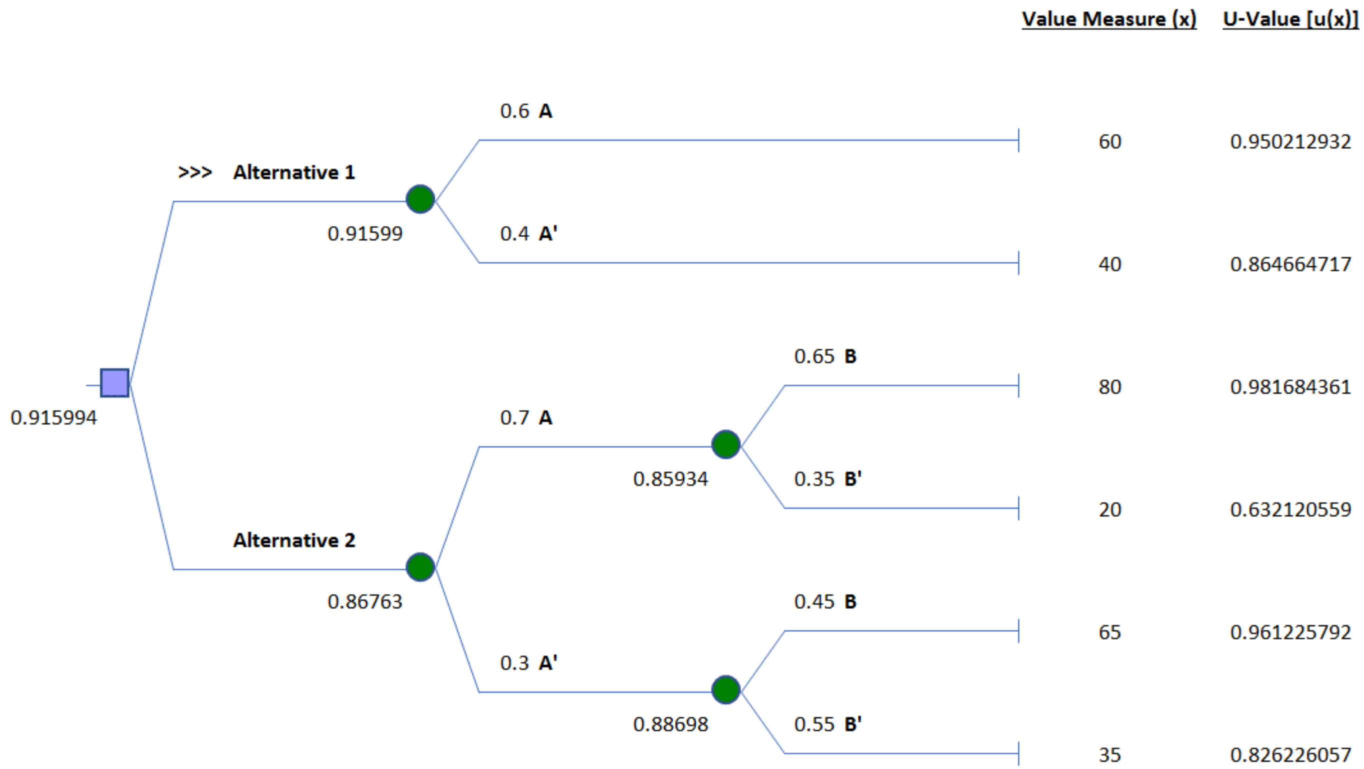


- The **expected value** of this decision is 55.85

Example Decision Tree: Incorporating Risk Preference (2/3)

- Suppose the decision-maker is risk averse with the following risk curve:

$$u(x) = 1 - e^{-0.05x}$$



- The **expected utility** of this decision is 0.915994

Example Decision Tree: Incorporating Risk Preference (3/3)

- The expected utility of the decision is the maximum expected utility of the two alternatives:

$$u(x) = max\big(u(Alternative\ 1), u(Alternative\ 2)\big) = max(0.91599, 0.86763)$$
$$= 0.91599$$

- Now, we can use the inverse of the utility curve to transform utility back to the original units:

$$x = \frac{-ln\big(1 - u(x)\big)}{0.05}$$

- Plugging in $u(x) = 0.91599$, we get

$$x = 49.53726$$

- This is the **certain equivalent of the expected utility** of this decision, given the decision-makers risk preference
  - Recall, the same tree for the risk neutral decision-maker was valued at 55.85

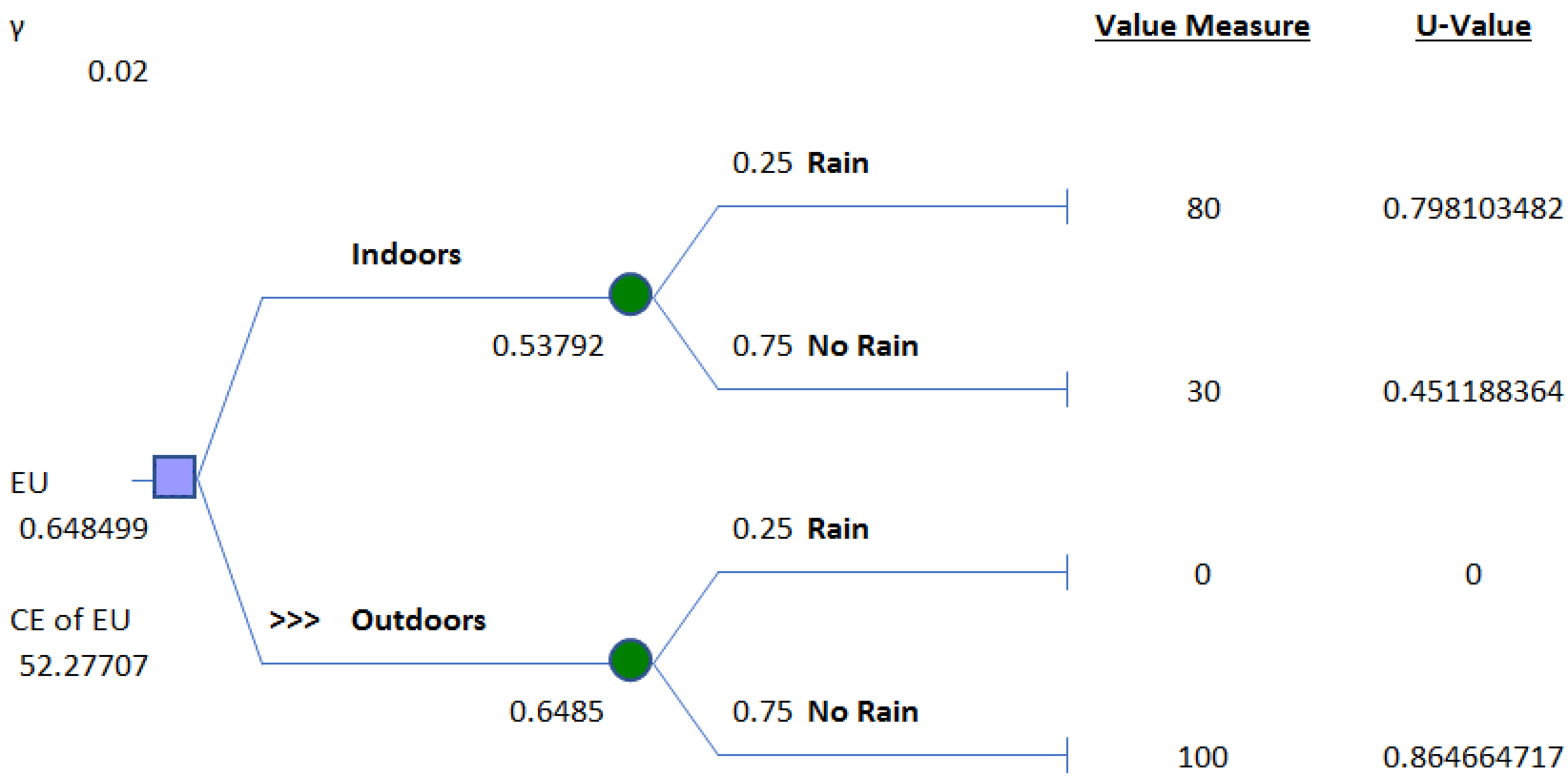
Putting It All Together: An Integrated Example

- Suppose you are interested in throwing a party tomorrow. You can either hold the party Indoors or Outdoors. You are concerned with one uncertainty: rain. You assign the following dollar values to each of the four possible outcomes:

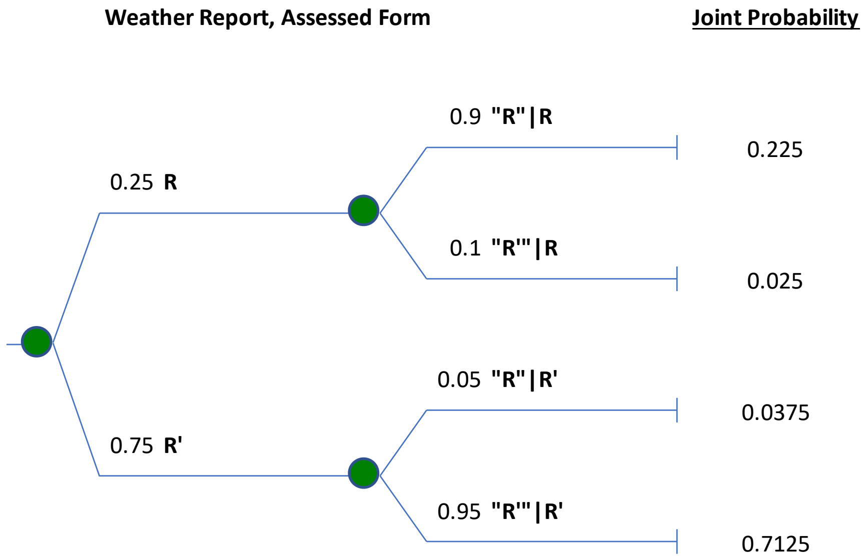
Outcome	Value
Indoors, No Rain	$30
Indoors, Rain	$80
Outdoors, No Rain	$100
Outdoors, Rain	$0

- You believe the probability of rain tomorrow is 0.25. There is a specialized weather report, which correctly predicts rain given rain 90% of the time, and correctly predicts no rain given no rain 95% of the time. What is the maximum price you should be willing to pay for this weather report?
- Note: You have a risk aversion coefficient of $\gamma = 0.02$ and you can use the utility curve $u(x) = 1 - e^{-\gamma x}$

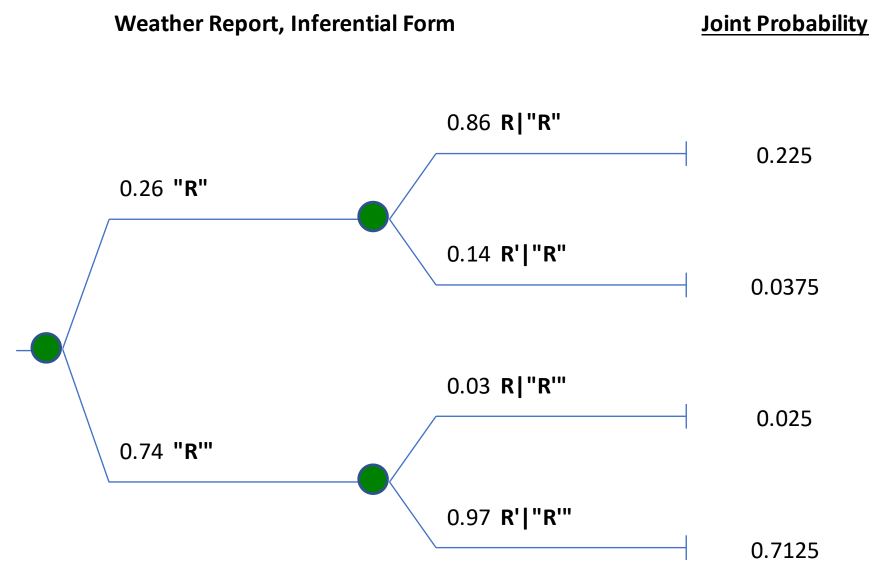
Integrated Example, Step 1: Solve the Baseline Decision Tree



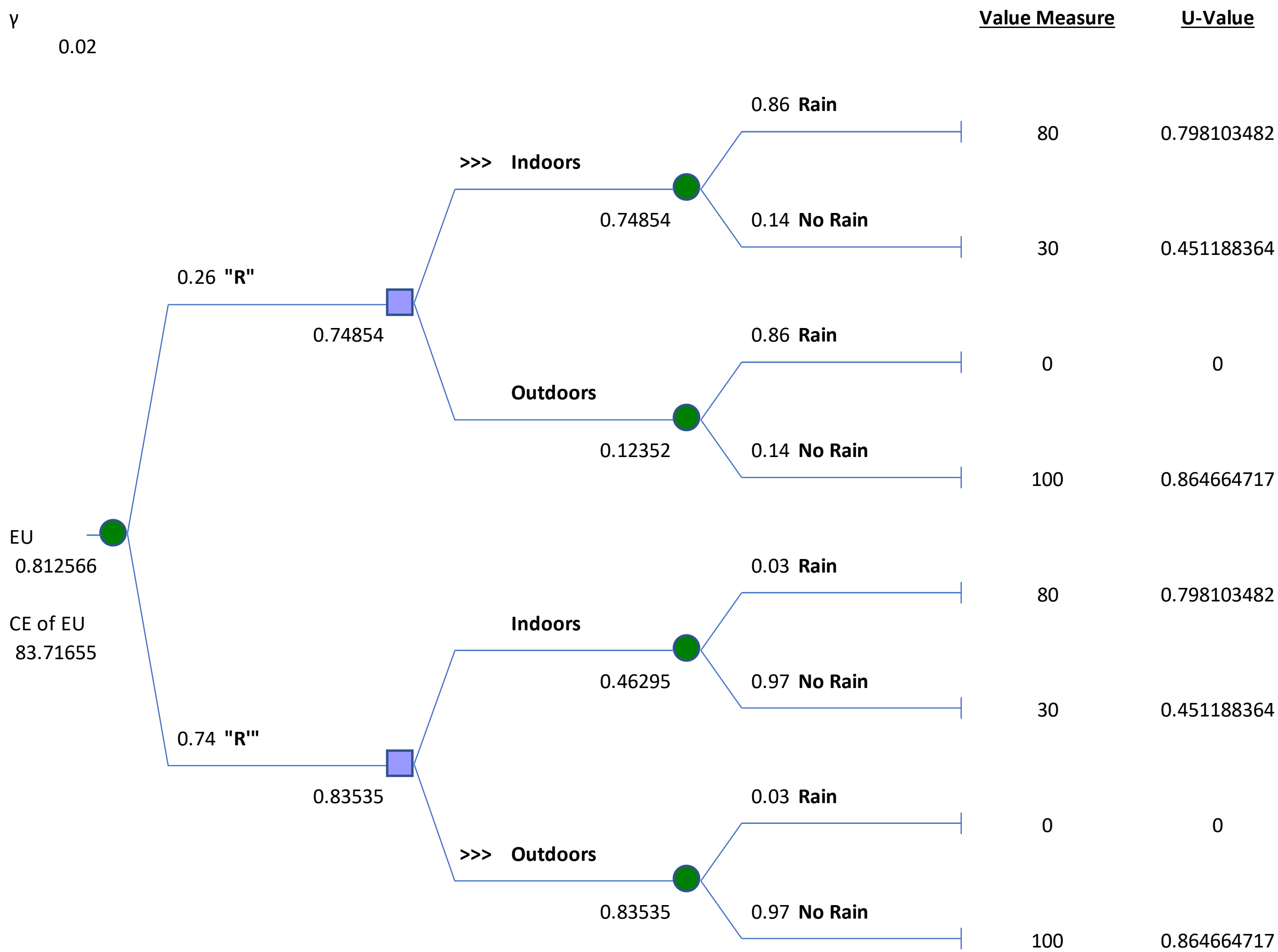
Integrated Example, Step 2: Characterize the Weather Report



flip the tree



Integrated Example, Step 3: Reformulate the Original Tree, But With New Information From Weather Report (Assume Report is Free)



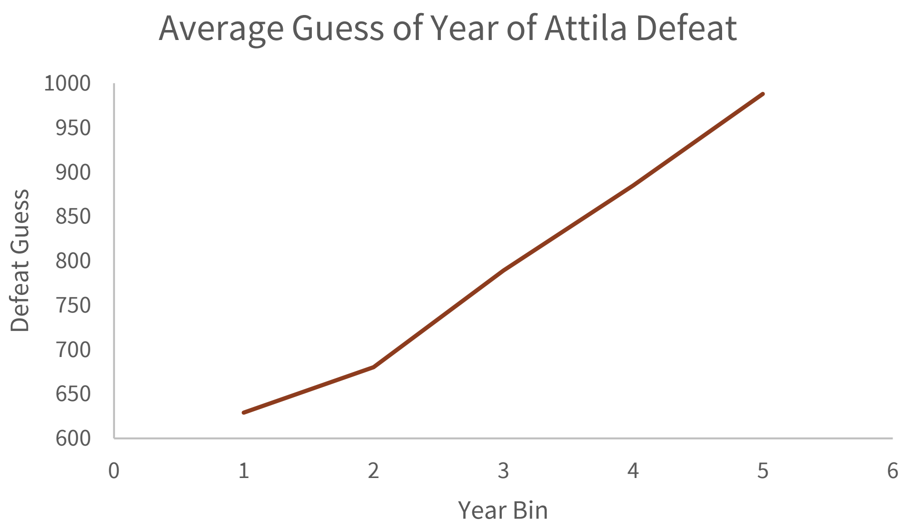
Integrated Example, Step 4: Compute Value of the Weather Report

- Certain equivalent of expected utility of decision tree without weather report: \$52.28
- Certain equivalent of expected utility of decision tree with weather report: \$78.50
- Value of weather report = value with report – value without report
  - Value = \$83.72 – \$52.28 = \$31.44
- You should be willing to pay **up to** \$31.44 for the weather report, but not any more!

Backup

Results of Anchoring Question

Range of Anchor (last 3 + 400)	Average Guess of Year of Attila Defeat
400 to 599	629
600 to 799	680
800 to 999	789
1000 to 1199	885
1200 to 1399	988



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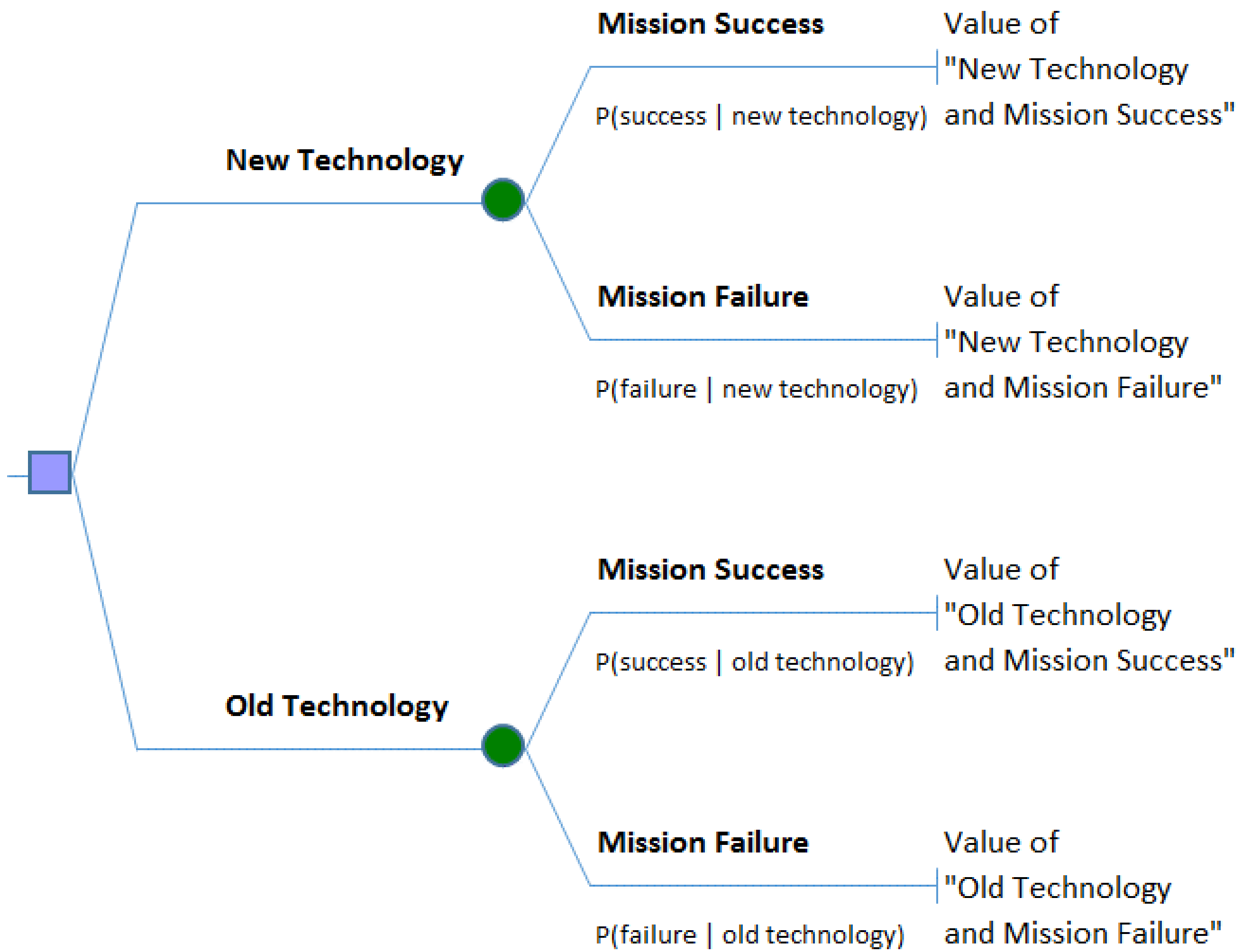
Example Decision Tree #1



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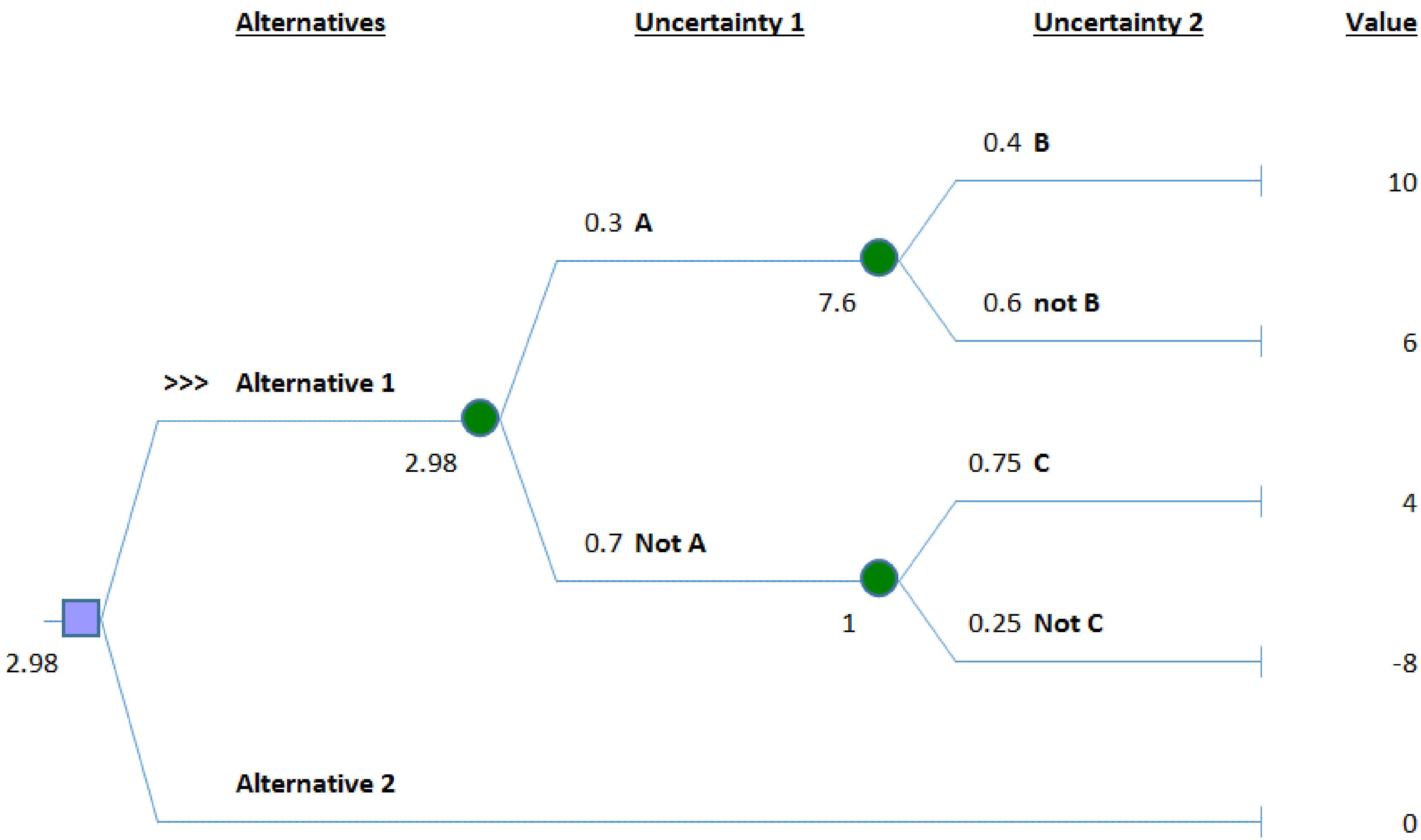
Generic Decision Tree



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