MS&E 260

Introduction to Operations Management

Problem Session 2- Solutions

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EOQ Question

A Turkish racing bike manufacturer called XMB is using a special pillion seat in its brand new bike model. The bike sales for this model show a fairly steady demand of 5,600 bikes per year. Traditionally, XMB purchases these seats from a producer in Germany at a price of $8/unit. It costs XMB $100 to place an order. Inventory holding costs are based on an annual interest rate of 20%.

Suppose that the seat supplier is offering a quantity discount applied to all units with the following schedule (total unit cost):

- $8Q for $Q \leq 800$
- $7Q for $800 < Q < 1,000$
- $6Q for $1,000 \leq Q$

(A) What is the optimal order quantity in this case?

(B) Using your optimal order quantity, what is the total cost?

EOQ Solution

First, we calculate our optimal Economic Order Quantities:

- $Q{*}\ \$8 = ((2 * 100 * 5,600)/(.2 * 8))0.5 = 836.7 \sim 837$
- $Q{*}\ \$7 = ((2 * 100 * 5,600)/(.2 * 7))0.5 = 894.4 \sim 894$
- $Q{*}\ \$6 = ((2 * 100 * 5,600)/(.2 * 6))0.5 = 966.1 \sim 966$

For an \$8 unit cost, $Q{*}\ \$8 = 837$ is infeasible, so we use $Q = 800$ to calculate our Total Cost. For a \$7 unit cost, our calculated $Q{*}\ \$7$ is feasible (within our bounds). For a \$6 unit cost, $Q{*}\ \$6 = 966$ is infeasible, so we use $Q = 1000$.

- $TC(Q = 800, \$8) = (100 * 5,600)/800 + (800/2) * .2 * 8 + 8 * 5,600 = \$46,140$
- $TC(Q = 894, \$7) = (100 * 5,600)/894 + (894/2) * .2 * 7 + 7 * 5,600 = \$40,452$
- $TC(Q = 1,000, \$6) = (100 * 5,600)/1,000 + (1,000/2) * .2 * 6 + 6 * 5,600 = \$34,760$

The optimal order quantity is $Q = 1,000$ (at a price of \$6 per unit), for a Total Cost = \$34,760.

Newsvendor

Nancy's High-Five Bagel sells fresh baked bagels every morning. The daily demand for bagels is a random variable with a distribution estimated from prior experience given by the following table.

Number of bagels sold in one day	*Cumulative probability*
0	0.05
5	0.13
10	0.22
15	0.32
20	0.47
25	0.72
30	0.87
35	1.00

Each bagel costs Nancy's 9 cents to make, and they are sold for 50 cents each. All leftover units should be discarded at the end of the day at no net value.

(a) Based on the given discrete distribution, what are the overage and underage costs? And how many bagels should Nancy bake at the start of each day?

(b) Now suppose that the bagels unsold at the end of the day are purchased by a nearby charity soup kitchen for 3 cents each. What are the overage and underage costs in this case? And how many bagels should Nancy bake at the start of each day?

Note: For all parts, you can either interpolate from the empirical cumulative distribution function (and round consistently) to find a quantity, or simply pick one of the multiples of five as if there is no possibility of demand realizations in between.

Newsvendor Cont.

(c) Now instead of baking the bagels in-house, Nancy sells the equipment to buy the bagels from Kelly's store every morning at 15 cents each, to sell them for 50 cents. In addition, to promote the sales of bagels, Kelly's store buys back the left-over bagels at the end of the day from the Nancy's at 5 cents. What are the overage and underage costs in this case? And how many bagels should Nancy buy at the start of each day?

(d) Nancy still buys the bagels from Kelly's store every morning at 15 cents each and sells them for 50 cents. Now, instead of buying back the unsold units, Kelly's store agrees 1 to pay Nancy a rebate of 1 cent for every unit sold to end customers at Nancy's. What are the overage and underage costs in this case? And how many bagels should Nancy buy at the start of each day?

Newsvendor Solutions

(a)
Overage cost: co = 9
Underage cost: cu = 50 − 9 = 41
Critical ratio = cu/(cu + co) = 41/(41 + 9) = 0.82
Then we need to choose smallest $Q*$ so that $P(D \leq Q*)$ is greater than or equal to 0.82. Thus $Q* = 30$ (linearly interpolated cumulative density, 27.67 → 28).

(b)
Overage cost: co = 9 − 3 = 6
Underage cost: cu = 50 − 9 = 41
Critical ratio = cu/(cu + co) = 41/(41 + 6) = 0.872
Then we need to choose smallest $Q*$ so that $P(D \leq Q*)$ is greater than or equal to 0.872 Thus $Q* = 35$ (linearly interpolated cumulative density, 30.08 → 31).
(c) Overage cost: co = 15 − 5 = 10 Underage cost: cu = 50 − 15 = 35 Critical ratio = cu/(cu + co) = 35/(35 + 10) = 0.777 Then we need to choose smallest $Q*$ so that $P(D \leq Q*)$ is greater than or equal to 0.777. Thus $Q* = 30$ (linearly interpolated cumulative density, 26.9 → 27). (d) Overage cost: co = 15 Underage cost: cu = 50 − 15 + 1 = 36 Critical ratio = cu/(cu + co) = 36/(36 + 15) = 0.705 Then we need to choose smallest $Q*$ so that $P(D \leq Q*)$ is greater than or equal to 0.705. Thus $Q* = 25$(linearly interpolated cumulative density, 24.7 → 25)

Newsvendor Solutions

(c)
Overage cost: $co = 15 - 5 = 10$
Underage cost: $cu = 50 - 15 = 35$
Critical ratio $= cu/(cu + co) = 35/(35 + 10) = 0.777$
Then we need to choose smallest $Q*$ so that $P(D \leq Q*)$ is greater than or equal to 0.777. Thus $Q* = 30$ (linearly interpolated cumulative density, $26.9 \rightarrow 27$).

(d)
Overage cost: $co = 15$
Underage cost: $cu = 50 - 15 + 1 = 36$
Critical ratio $= cu/(cu + co) = 36/(36 + 15) = 0.705$
Then we need to choose smallest $Q*$ so that $P(D \leq Q*)$ is greater than or equal to 0.705. Thus $Q* = 25$(linearly interpolated cumulative density, $24.7 \rightarrow 25$)

(Q,R) question

Stanford warehouse of the famous wine distributor WS&E stocks materials required for the cases of wines. One type of wine that Stanford warehouse distributes is the Burgundy Chardonnay. Each case of this wine is purchased by the warehouse for $200. Since it is sent from Europe in intermodal containers it has a high lead time of 2 months (1/6 years) and the company uses an inventory carrying charge based on a 20% annual interest rate. The cost of order processing and receipt is $1000 per order. Annual demand for this wine follows a normal distribution with mean 240 cases and variance of 600 cases2 (standard deviation of ~24.5 cases). Assume that if a case of wine is demanded when the warehouse is out of stock, then the demand is backordered, and the cost associated with each backordered case is estimated to be as $80.

(a) Compute the mean and standard deviation of demand during lead time.
(b) The manager of the warehouse uses (Q, R) policy. Find the optimal values of the order quantity and the reorder level.

(Q,R) Solution

(a) $\mu = 240/6 = 40$, $\sigma = 24.495/\sqrt{6} = 10$

(b)

Step 1 :

$Q0 = q\ 2*1000*240\ 40 = 110$

$1 - F(R1) = 110*40\ 80*240 = 0.23$. Thus, $F(R1) = 0.77$.

From the table $z1 = 0.74$.

Therefore, $R1 = \mu + \sigma * z1 = 40 + 10 * 0.74 = 47$

Step 2 :

$n(R1) = \sigma L(z1) = 10 * 0.1334 = 1.334$

$Q1 = q2*240*(1000+80*1.334)\ 40 = 115$

$1 - F(R2) = 115*40\ 80*240 = 0.24$. Thus, $F(R2) = 0.76$. From the table $z2 = 0.79$.

Therefore, $R2 = \mu + \sigma * z2 = 40 + 10 * 0.71 - 47$

Notice that R has converged. Thus, $(Q*, R*) = (115, 47)$.