EOQ Example — All-unit discount

$K = 100$

$C = 8$

$i = 0.20$

$\lambda = 5600$/year

$$TC(Q) = \left(\frac{\lambda}{Q}\right)K + \left(\frac{Q}{2}\right)(iC) + \lambda C$$

$$Q_1 = \sqrt{\frac{2K\lambda}{iC_1}} = \sqrt{\frac{2 \cdot 100 \cdot 5600}{0.20 \cdot 8}} = \sqrt{10(100)(700)} = \sqrt{700{,}000} = 836.66$$

not in range

$\therefore Q_1^* = 800$

$$TC(Q_1^* = 800) = \left(\frac{5600}{800}\right)(100) + \left(\frac{800}{2}\right)(0.20 \cdot 8) + 5600 \cdot 8$$

$= 700 + 640 + 44{,}800$

$= 46140$

$C_2 = 7$

$$Q_2 = \sqrt{\frac{2(100)(5600)}{0.20 \cdot 7}} = \sqrt{800{,}000} = 894.43$$

in range

$$TC(Q_2^* = 894.43) = \left(\frac{5600}{894.43}\right)(100) + \left(\frac{894.43}{2}\right)(0.2 \cdot 7) + 5600 \cdot 7$$

$= 40452.20$

$C_3 = 6$

$$Q_3 = \sqrt{\frac{2(100)(5600)}{0.20 \cdot 6}} = 966.09$$

not in range

$\therefore Q_3^* = 1000$

$$TC(Q_3^* = 1000) = \left(\frac{5600}{1000}\right)(100) + \frac{1000}{2} \cdot 0.2 \cdot 6 + 5600 \cdot 6$$

$= 560 + 600 + 33600$

$= 34760$

the lowest!

Incremental discount — Method 1

$$C_{AVG} = \frac{c(Q)}{Q}$$

$C_{AVG_1} = 8$ $\quad Q_1^* = 836.66$ $\quad C_{AVG_2} = 7.9 + \frac{100}{Q}$ $\quad C_{AVG_3} = 7.8 + \frac{300}{Q}$

$$TC(Q_1^* = 836.66) = \left(\frac{5600}{836.66}\right)(100) + \left(\frac{836.66}{2}\right)(0.2 \cdot 8) + 5600 \cdot 8$$

$$= 46138.66$$

$$TC(Q) = \left(\frac{5600}{Q}\right)(100) + \left(\frac{Q}{2}\right)0.2\left(7.9 + \frac{100}{Q}\right) + 5600\left(7.9 + \frac{100}{Q}\right)$$

$$= \frac{560060}{Q} + 0.79Q + 10 + 5600 \cdot 7.9 + \frac{560000}{Q}$$

$$= 0.79Q + 44250 + \frac{1120000}{Q}$$

$$TC'(Q) = 0.79 - \frac{1120000}{Q^2} = 0$$

$Q_2 = 1190.68$

in range

$TC(Q_2^* = 1190.68)$

$= 46131.28$

$$TC(Q) = \frac{5600}{Q}(100) + \left(\frac{Q}{2}\right)0.2\left(7.8 + \frac{300}{Q}\right) + 5600\left(7.8 + \frac{300}{Q}\right)$$

$$= \frac{560000}{Q} + 0.78Q + 30 + 5600 \cdot 7.8 + \frac{1680000}{Q}$$

$$TC'(Q) = 0.78 - \frac{2240000}{Q^2}$$

$Q_3 = 1694.64$

out of range, won't matter (for increm. discount)

but as sanity check:

$TC(Q_3^* = 2000) = 46390$

Method 2 — easier to get Q!

- $8
- $7.9 with additional cost to order $100
- $7.8 with additional cost to order $300

$C_1 = 8$

$Q_1 = \sqrt{\frac{2K\lambda}{iC_1}} = 836.66 = Q_1^*$

$TC(Q_1^*) = 46138.66$

$C_2 = 7.9$

$Q_2 = \sqrt{\frac{2(K+100)\lambda}{iC_2}}$

$= \sqrt{\frac{2(200)\cdot 5600}{0.2 \cdot 7.9}}$

$= 1190.68 = Q_2^*$

$TC(Q_2^*) = 46131.28$

$C_3 = 7.8$

$Q_3 = \sqrt{\frac{2(K+300)\lambda}{iC_3}}$

$= 1694.64$

not in range

$Q_3^* = 2000$

$TC(Q_3^*) = 46390$

EOQ - finite pdn rate

$K = 300$

$h = 5$ / unit per year

$\lambda = 1800$ units per year

(a) $EOQ = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2 \cdot 300 \cdot 1800}{5}} = 464.76$

(b) Set up cost (annual) $= \left(\frac{\lambda}{Q}\right) K = \left(\frac{1800}{464.76}\right)(300) = \$1161.90$

$\psi = 8000$

(c) $EOQ' = \sqrt{\frac{2K\lambda}{h\left(1 - \frac{\lambda}{\psi}\right)}} = \sqrt{\frac{2 \cdot 300 \cdot 1800}{5\left(1 - \frac{1800}{9000}\right)}} = \sqrt{\frac{2 \cdot 300 \cdot 1800}{4}}$

$= 519.62$

(d) holding cost (annual) $= \left(\frac{Q}{2}\right) h \left(1 - \frac{\lambda}{\psi}\right)$

$= \frac{519.62}{2} \cdot 5 \cdot \left(1 - \frac{1800}{9000}\right) = 519.62 \cdot 2$

$= \$1039.23$