

MS&E 260 Homework 2

Summer 2019, Stanford University
Due: July 10th, 2019, at 10:30AM (PDT)

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Problem 1:

Jerry plans on selling burritos at Palo Alto Burrito Festival. From last year's sales history, Jerry estimates that the demand for burritos during the day would be normally distributed with mean 250 and standard deviation 30. Jerry's supplier charges \$5 per burrito and Jerry decides to charge \$13 per burrito for sale. Since Jerry does not know how to calculate the optimal order quantity that will maximize his expected profit, he asked his brother, Phil who took MS&E 260 last year, for help. After solving the problem using a newsvendor model, Phil told Jerry "You should order 280 burritos considering that you can sell every unsold burrito to the nearby grocery store after the game."

- (a) When solving the newsvendor problem, Phil assumed that the nearby grocery store would buy entire left over burritos at \$x per burrito, where $x < \$12$. What is the value of \$x? Please round your answer to the nearest tenth.

Answer:

$$\text{Overage cost} = c_o = 5 - x$$

$$\text{Underage cost} = c_u = 13 - 5 = 8$$

$$\text{Mean} = \mu = 250$$

$$\text{Standard deviation} = \sigma = 30$$

$$\text{Optimal quantity} = Q^* = 280$$

$$\text{Critical Ratio} = F(Q) = \frac{c_u}{c_u + c_o} = \frac{8}{8 + 5 - x} = \frac{8}{13 - x}$$

$$Q^* = \sigma(z) + \mu \Rightarrow 280 = 30z + 250 \Rightarrow 30z = 30 \Rightarrow z = 1$$

Looking the value of $z = 1$ in normal density table, critical ratio is 0.8413

$$\frac{8}{13 - x} = 0.8413 \Rightarrow x = \frac{2.9369}{0.8413} = 3.49$$

Rounding off to nearest tenth $x = 3.5$

- (b) Now assume that $x = \$2$. The weather forecast predicts that there is a high chance of heavy rain on the day of the festival. Based on this report, Jerry comes up with the following discrete demand distribution. What is the optimal order quantity, given this demand distribution?

Q	240	245	250	255	260	265	270
P(demand = Q)	0.07	0.12	0.23	0.17	0.16	0.20	0.05

Table 1: Estimated Discrete Demand Distribution

Answer:

$$\text{Overage cost} = c_o = 5 - 2 = 3$$

$$\text{Underage cost} = c_u = 13 - 5 = 8$$

$$\text{Critical Ratio} = F(Q) = \frac{c_u}{c_u + c_o} = \frac{8}{8 + 3} = 0.7272$$

From the table :

$$\text{Adding all the discrete demand distributions: } 0.07 + 0.12 + 0.23 + 0.17 + 0.16 = 0.75$$

Based on $P(D \leq Q^*)$, we can chose Q^* to be 260.

- (c) Since there is a high chance of rain, Jerry also plans to sell disposable umbrellas. He can buy these umbrellas for \$2 each and sell them for \$10 each if it rains that day. However, the salvage value of unsold umbrella is \$0. There is a 60% chance of rain and the demand will be 400 if it rains. But if it doesn't rain, the demand will be zero. How many umbrellas should be purchased?

Answer:

$$c_o = 2$$

$$c_u = 10 - 2 = 8$$

For $Q \leq 400$

$$\text{Profit equation1} = 0.6 (Q(10 - 2)) + 0.4 (10 * 0 - 2 Q) = 4.8 Q - 0.8 Q = 4Q$$

For $Q \geq 400$

$$\begin{aligned} \text{profit equation2} &= 0.6 (400 * (10 - 2) - (Q - 400) * 2) + 0.4 (0 * 10 - Q * 2) \\ &= 0.6 (3200 - 2 Q + 800) + 0.4 (- 2Q) \\ &= 2400 - 1.2Q - 0.8 Q \\ &= 2(1200 - Q) \end{aligned}$$

Using $Q = 400$ we get

$$\text{Equation 1} = 4 * 400 = 1600$$

$$\text{Equation 2} = 2400 - 2 * 400 = 1600$$

Thus $Q^* = 400$

Problem 2:

In the lecture, we discussed the derivation of the critical ratio:

$$F(Q) = \frac{c_u}{c_o + c_u}$$

Please briefly discuss the implication of this equation.

Answer:

The function gives us an important ratio called “Critical Ratio” that helps to analyze, costs associated with having too much vs. too less inventory.

This function has certain set of pre-requisites viz. we are trying to sell a single item during a single amount of time, it's a one-time decision, the items are perishable and demand is normally distributed.

As we can see this function does not depend on how much is the demand, we can get a ratio that can help us scale the cost in proper proportion with respect to the demand.

Basically it's a cumulative distribution function, that gives us the probability of demands we have satisfied given that we have ordered Q^* units. Just like in any CDF or according to the CDF definition it can evaluate the probability that our demand will take a value less or equal to Q^* units. Or in other words, it's a CDF of the demand given that we have ordered Q^* units.

The ideal value of critical ratio will be when overage cost is equals to underage cost. In which case, the ratio will be 0.5. But in other case or real world scenario, Q^* is the equilibrium between expected overage and underage costs.

Problem 3:

Stanford warehouse of the famous wine distributor WS&E stocks materials required for the cases of wines. One type of wine that Stanford warehouse distributes is the Burgundy Chardonnay. Each case of this wine is purchased by the warehouse for \$200. Since it is sent from Europe in intermodal containers it has a high lead time of 2 months (1/6 years) and the company uses an inventory carrying charge based on a 20% annual interest rate. The cost of order processing and receipt is \$1,000 per order. Annual demand for this wine follows a normal distribution with mean 240 cases and variance of 600 cases (standard deviation of ~24.5 cases). Assume that if a case of wine is demanded when the warehouse is out of stock, then the demand is backordered, and the cost associated with each backordered case is estimated to be at \$80.

$$k = 1000$$

$$i = 20 \%$$

$$c = 200$$

$$p = 80$$

$$T(LT) = 2 \text{ months} = 1/6 \text{ year}$$

$$\text{Mean} = \mu = 240$$

$$\text{variance} = 600$$

$$\text{Standard deviation} = \sigma = 24.5$$

(a) Compute the mean and standard deviation of demand during lead time.

Answer:

$$\mu_{LT} = \frac{240}{6} = 40$$

$$\sigma_{LT} = \sqrt{\frac{600}{6}} = 10$$

(b) The manager of the warehouse uses (Q,R) policy. Find the optimal values of the order quantity and the reorder level.

Answer:

----- Step 1 -----

$$EOQ = Q^* = \sqrt{\frac{2 k \lambda}{h}} = \sqrt{\frac{2 * 100 * 240}{0.2 * 200}} = 110$$

$$Q_0 = 110$$

$$F(R_1) = 1 - \frac{Q h}{p \lambda} = 1 - \frac{110 * 40}{80 * 240} = 1 - 0.23 = 0.77 \Rightarrow z_1 = 0.74$$

$$R_1 = \sigma z + \mu = 10 * 0.74 + 40 = 47$$

----- Step 2 -----

$$N(R_1) = \sigma L(z_1) = 10 * L(0.74) = 10 * 0.1334 = 1.334$$

$$Q_1 = \sqrt{\frac{2 \lambda [k + p n(R_1)]}{h}} = \sqrt{\frac{2 * 240 [1000 + 80 * 1.334]}{0.2 * 200}} = 115$$

$$F(R_2) = 1 - \frac{Q h}{p \lambda} = 1 - \frac{115 * 40}{80 * 240} = 1 - 0.24 = 0.76$$

$$\Rightarrow z_2 = 0.71$$

$$R_2 = \sigma z + \mu = 10 * 0.71 + 40 = 47$$

Since $R_2 = R_1$, our values have converge and we can say
Optimal (Q, R) = (115, 47)

(c) Determine the safety stock.

Answer:

$$\text{Safety stock} = \text{reorder level} - \text{expected demand during lead time} = R - \mu = 47 - 40 = 7$$

(d) What are the average annual holding, setup and stockout costs associated with this wine?

Answer:

$$\text{Holding cost} = h \left(s + \frac{Q}{2} \right) = 0.2 * 200 \left(7 + \frac{115}{2} \right) = \$2580$$

$$\text{fixed/setup cost} = \frac{k}{T} = \frac{k \lambda}{Q} = \frac{1000 * 240}{115} = \$2086.95$$

$$\begin{aligned} \text{shortage cost} &= p \left(\frac{n(R)}{T} \right) = p \left(\frac{\sigma L(z_2) \lambda}{Q} \right) = 80 \left(\frac{10 L(0.71) 240}{115} \right) = 80 \left(\frac{10 * 0.1405 * 240}{115} \right) \\ &= \$234.57 \end{aligned}$$

$$\begin{aligned} \text{Total Cost} &= \text{Holding cost} + \text{fixed cost} + \text{shortage cost} \\ &= 2580 + 2086.95 + 234.57 = 4901.52 \end{aligned}$$

(e) What is the cost of uncertainty? (You may compare to a case that there is no uncertainty, think about what this case refers to.)

Answer:

Cost of uncertainty (COU) = Total cost in QR model - total cost in EOQ model

$$\text{Total cost in EOQ model} = \sqrt{2 k \lambda h} = \sqrt{2 * 1000 * 240 * 0.2 * 200} = 4381.78$$

$$COU = 4901.52 - 4381.78 = 519.74$$

(f) What is the proportion of order cycles in which no stock-outs occur?

Answer:

This can also be interpreted as proportion of demand satisfied on time, which is the Type II service

$P(D \leq R)$,

$$F(R^*) = 0.76 = 76\%$$

(g) What is the expected proportion of demand that cannot be met at once?

Answer:

$$\text{demand that can not be met at once} = \frac{\text{loss function}}{\text{quantity}} = \frac{n(R^*)}{Q^*} = \frac{\sigma L(z_2)}{115} = \frac{1.405}{115} = 1.221 \%$$

Problem 4:

Read the following articles:

- <https://readwrite.com/2019/04/27/can-ai-save-the-supply-chain-from-its-own-destruction/>
- <https://hbr.org/2015/05/the-3-d-printing-revolution>

Select one of these two articles, and do the following:

- 1) Briefly summarize the article.
- 2) Provide a few interpretive thoughts on the article, using what you have learned from class.
- 3) Provide one recommendation on how the dilemma posed in the article could be resolved.

Some notes:

- Please limit your responses to one page, double spaced, 12 point font.
- There is no right answer to this question. We are evaluating your ability to apply what you learn in class to practical applications.
- This question is not intended to be free points. If you do not demonstrate a sufficient level of critical thinking, full credit will not be awarded.

Answer:

The article by Brian Wallace on 27th Apr 2019 about AI/Tech, describes importance of AI in supply chain to reduce cost and maximize profit. The write up is appropriately backed up by examples like use of VRS (voice response system) at call centers, shipping routes efficiency, optimizing empty truck spaces, UPS left turn analysis, etc. AI can be used to identify a pattern in the past data and optimize supply chain, shipping inefficiency, empty space in trucks/containers, routes, inventory, equipment maintenance/breakdowns and market demand predictions. Not only it reduces the cost and increases profit, it also provide customer satisfaction and most importantly helps environment.

Intelligence is something that separates us human beings apart from other living organisms. It is the quality of observing past data, identifying a pattern, learn from it, and make decisions to improve the process. We all use it every day either cautiously or sub-cautiously in most cases. Simplest example could be, if you get stuck in traffic for consecutive days on a street during commute, you eventually start avoiding that street. You decide in your mind, that it will save you time and gas. Similar analysis and suggestions provided by a computer program is nothing but "Artificial Intelligence". The best example that comes to mind is spam detection program. These programs constantly reads older spam emails/messages and generates a key table to analyze a newer email/text message for spam or phishing content.

Some people believe that AI is taking away jobs or reducing the number of jobs. Although this might be true in a very few cases, but the positive effects are way superseding. As the computational power is increasing, analyzing vast amount of data is becoming simpler and faster. But, computation alone does not derive meaningful results, it has to run on a meaningful clean set of data. Thus, whenever possible every human should contribute towards the learning data, so AI can help build a better society without destroying our beautiful planet.