MS&E 260 Homework 1

Summer 2019, Stanford University

Due: July 3rd, 2019, at 10:30AM (PDT)

Submitted by: Vishal Mittal (X250025)

Problem 1:

Formulate each of the following problems as a linear programming problem:

(a) A manufacturer wishes to produce an alloy, that is, by weight 60% of metal X and 40% of metal Y . There are four alloys available for the manufacturer to extract metal X and Y :

Table 1: Alloys Composition and Price

Alloys	1	2	3	4
% X	20	40	10	35
% Y	80	60	90	65
Price/ton	1	3	2	4

The manufacturer wishes to find the amounts of the various alloys needed to produce one ton of desired alloy and to determine the least expensive combination.

Answer 1(a):

- Formulation

  Let:

  a_1 = amount of alloy 1 needed

  a_2 = amount of alloy 2 needed

  a_3 = amount of alloy 3 needed

  a_4 = amount of alloy 4 needed

  $$\begin{aligned} \text{min} \quad & 1a_1 + 3a_2 + 2a_3 + 4a_4 \\ \text{s. t.} \quad & 20a_1 + 40a_2 + 10a_3 + 35a_4 >= 60 \\ & 80a_1 + 60a_2 + 90a_3 + 65a_4 >= 40 \\ & a_1, a_2, a_3, a_4 \geq 0 \end{aligned}$$

- Linear Optimization Code:

```
var a1 >= 0;
var a2 >= 0;
var a3 >= 0;
var a4 >= 0;

minimize z:  1*a1 + 3*a2 + 2*a3 + 4*a4;
subject to c1: 20*a1 + 40*a2 + 10*a3 + 35*a4 >= 60;
subject to c2: 80*a1 + 60*a2 + 90*a3 + 65*a4 >= 40;

end;
```

(b) Quantities a1, a2, ..., am, respectively, of a certain product are to be shipped from each of m locations and received in amounts b1, b2, ..., bn, respectively, at each of n destinations. We assume that the total amount shipped is equal to the total amount received. Associated with the shipping of a unit of product from origin i to destination j is a shipping cost c_{ij} . It is desired to determine the amounts x_{ij} to be shipped between each origin-destination pair i ∈ 1, 2, ...,m and j ∈ 1, 2, ..., n so as to satisfy the shipping requirements and minimize the total cost of transportation.

Answer 1(b):

Given:

a_i = quantity of product shipped from i^{th} location
b_j = quantity of product to receive at j^{th} location
m = number of total shipping locations
n = number of total receiving locations
c_{ij} = cost to ship one product unit from i^{th} to j^{th} location
x_{ij} = quantity of product to ship from i^{th} to j^{th} location

Formulation:

$$\begin{aligned}
&\min && \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \\
&s.t. && \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\
&\forall j = 1, 2, \dots \dots n && \sum_{i=1}^{m} x_{ij} = b_j \\
&\forall i = 1, 2, \dots \dots m && \sum_{j=1}^{n} x_{ij} \le a_i \\
&\forall i = 1, 2, \dots \dots m, \forall j = 1, 2, \dots \dots n && x_{ij} \ge 0
\end{aligned}$$

(c) Suppose that there are 2 sources that generate waste and 3 disposal sites. The amount of waste generated at source A is 200 and at source B is 400. The capacity of disposal sites 1, 2 and 3 are 250, 150 and 100, respectively. The unit shipping costs from waste source A to disposal sites 1, 2 and 3 are 35, 15, and 20, respectively. The unit shipping costs from waste source B to disposal sites 1, 2 and 3 are 20, 5, and 15, respectively. Write the formulation for determining the optimal assignment that minimizes transportation costs.

Answer 1(c):

Source	Amount
A	200
B	400

Destination	Capacity
1	250
2	150
3	100

Cost from source	To 1	To 2	To 3
A	35	15	20
B	20	5	15

Let :
X_i be the amount shipped from A to i^{th} destination
Y_i be the amount shipped from B to i^{th} destination
Formulation :

$$\begin{aligned}
&min && 35\,X_1 + 15\,X_2 + 20\,X_3 + 20\,Y_1 + 5\,Y_2 + 15\,Y_3 \\
&s.t. && X_1 + X_2 + X_3 = 200 \\
& && Y_1 + Y_2 + Y_3 = 400 \\
& && X_1 + Y_1 \le 250 \\
& && X_2 + Y_2 \le 150 \\
& && X_3 + Y_3 \le 100 \\
& && X_1, X_2, X_3, Y_1, Y_2, Y_3 \ge 0
\end{aligned}$$

Problem 2:

Stanford Dining is thinking of catering Magical 24-hour Energy Donuts that tastes like regular donuts but when consumed, keeps students fully awake the whole day. Based on surveys, these donuts have an anticipated steady demand of about 24,000 donuts per year. The fixed cost per unit is $3. It costs $250 to place an order. Inventory holding costs are based on an annual interest rate of 15%.

Given parameters:	**Known equations:**
λ = 24000 /year i = 0.15 c = $3 / unit K = $250	$h = I\,c$ $Q^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2K\lambda}{i\,c}}$ $TC\,(Q^*) = \left(\frac{\lambda}{Q^*}\right)K + \left(\frac{Q^*}{2}\right)ic + \lambda\,c$

(a) What is the optimal order quantity?

Solution 2(a):

$$Q^* = \sqrt{\frac{2 * 24000 * 250}{0.15 * 3}} = 5163.97$$

Optimal Order quantity : 5163

(b) Suppose that Lala Land, the producer of Magical Donuts, provides an all-unit discount with the following schedule:

i. $3Q for Q ≤ 5,000

ii. $2Q for 5,000 < Q < 6,250

iii. $1Q for Q ≥ 6,250

What is the optimal order quantity in this case? Using your optimal order quantity, what is the total cost?

Solution 2(b):

Calculations for (i):

$$Q^* = \sqrt{\frac{2 * 24000 * 250}{0.15 * 3}} = 5163.97$$

5163.97 is out of range so we will consider the boundary value 5000.

$$TC\,(Q^*) = \left(\frac{24000}{5000}\right)250 + \left(\frac{5000}{2}\right)0.15 * 3 + 24000 * 3$$

$$TC\,(Q^*) = 1200 + 1125 + 72000 = \$\,74325$$

Calculations for (ii):

$$Q^* = \sqrt{\frac{2 * 24000 * 250}{0.15 * 2}} = 6324.55$$

Since 6324 is out of range we can consider the boundary value 6249

$$TC\ (Q^*)\ = \left(\frac{24000}{6249}\right) 250\ +\ \left(\frac{6249}{2}\right) 0.15\ *\ 2\ + 24000\ *\ 2$$

$$TC\ (Q^*)\ = 960.15 +\ 937.35\ + 48000\ \ =\ \$\ 49897.5$$

Calculations for (iii):

$$Q^*\ =\ \sqrt{\frac{2\ *\ 24000\ *\ 250}{0.15\ *\ 1}}\ =\ 8944.27$$

$$TC\ (Q^*)\ = \left(\frac{24000}{8944}\right) 250\ +\ \left(\frac{8944}{2}\right) 0.15\ *\ 1\ + 24000\ *\ 1$$

$$TC\ (Q^*)\ = 670.84 +\ 670.8\ + 24000\ \ =\ \$\ 25341.64$$

As the lowest total cost is for case (iii),
Optimal order quantity = 8944
Optimal total cost = $25341.64

(c) Now suppose that Lala Land provides an incremental discount as follows:

i. $3Q for Q ≤ 6,000
ii. $18,000 + $2.90(Q – 6,000) for 6,000 < Q < 10,000
iii. $30,000 + $2.80(Q – 10,000) for Q ≥ 10,000

What is the optimal order quantity in this case? Using your optimal order quantity, what is the total cost?

Calculations for (i):

$$Q^*\ =\ \sqrt{\frac{2\ *\ 24000\ *\ 250}{0.15\ *\ 3}}\ =\ 5163.97$$

$$TC\ (Q^*)\ = \left(\frac{24000}{5163}\right) 250\ +\ \left(\frac{5163}{2}\right) 0.15\ *\ 3\ + 24000\ *\ 3$$

$$TC\ (Q^*)\ = 1162.11\ +\ 1161.67\ + 72000\ \ =\ \$\ 74323.78$$

Calculations for (ii):

$$c_{avg}\ = \frac{c\ (Q)}{Q}\ = \frac{18000\ +\ 2.9\ (\ Q\ -\ 6000\ )}{Q}\ = \frac{18000\ + 2.9\ Q\ -\ 17400}{Q}\ = 2.9\ +\ \frac{600}{Q}$$

$$TC\ (Q)\ = \left(\frac{24000}{Q}\right) 250\ +\ \left(\frac{Q}{2}\right) 0.15\ \left(2.9 + \frac{600}{Q}\right)\ + 24000\ *\ \left(2.9 + \frac{600}{Q}\right)$$

$$TC\ (Q) = \frac{24000 * 250}{Q} + 0.2175\,Q + 45 + 69600 + \frac{24000 * 600}{Q}$$

$$TC\ (Q) = \frac{24000 * 850}{Q} + 0.2175\,Q + 69645$$

$$\frac{d\,[TC\ (Q)]}{dQ} = -\frac{24000 * 850}{Q^2} + 0.2175 = \emptyset$$

$$\frac{24000 * 850}{Q^2} = 0.2175$$

$$Q^* = \sqrt{\frac{24000 * 850}{0.2175}} = 9684.68$$

Putting the Q^* value in TC calculated above:

$$TC\ (Q^*) = \left(\frac{24000}{9684}\right)250 + \left(\frac{9684}{2}\right)0.15\left(2.9 + \frac{600}{9684}\right) + 24000 * \left(2.9 + \frac{600}{9684}\right)$$

$$TC\ (Q^*) = 619.57 + 210.27 + 45 + 69600 + 1486.98 = \$73857.82$$

Calculations for (Iii):

$$c_{avg} = \frac{c\ (Q)}{Q} = \frac{30000 + 2.8\,(\,Q - 10000\,)}{Q} = 2.8 + \frac{2000}{Q}$$

$$TC\ (Q) = \left(\frac{24000}{Q}\right)250 + \left(\frac{Q}{2}\right)0.15\left(2.8 + \frac{2000}{Q}\right) + 24000 * \left(2.8 + \frac{2000}{Q}\right)$$

$$TC\ (Q) = \frac{24000 * 250}{Q} + 0.21\,Q + 150 + 67200 + \frac{24000 * 2000}{Q}$$

$$TC\ (Q) = \frac{24000 * 2250}{Q} + 0.21\,Q + 67350$$

$$\frac{d\,[TC\ (Q)]}{dQ} = -\frac{24000 * 2250}{Q^2} + 0.21 = \emptyset$$

$$\frac{24000 * 2250}{Q^2} = 0.21$$

$$Q^* = \sqrt{\frac{24000 * 2250}{0.21}} = 16035.97$$

Putting the Q^* value in TC calculated above:

$$TC\ (Q^*) = \left(\frac{24000}{16035}\right) 250 + \left(\frac{16035}{2}\right) 0.15 \left(2.9 + \frac{600}{16035}\right) + 24000 * \left(2.9 + \frac{600}{16035}\right)$$

$$TC\ (Q^*) = 3367.633 + 3367.35 + 67350 = \$74085.983$$

Since using the discount for case (ii) gives us lowest total cost we can say
Optimal order quantity = 9684
Optimal total cost = $73857.82

Problem 3:

The Batu Co. manufactures car engine spare parts. One particular spare part has a known and constant demand rate of 1,600 units per year. The fixed cost of the setup for each production run is $200 and the inventory holding cost is $4 per unit per year. There is a lead time of 1 week. Assuming that there is infinite production capacity, compute:

Given parameters:	**Known equations:**
λ = 1600 units/year h = $4 K = $200	$\mathrm{H} = maximum\ inventory\ level = Q\left(1 - \frac{\lambda}{\Psi}\right)$ $Q^* = \sqrt{\frac{2\,K\,\lambda}{h\left(1 - \frac{\lambda}{\Psi}\right)}}$ $TC\ (Q^*) = \left(\frac{\lambda}{Q^*}\right) K + \left(\frac{Q^*}{2}\right)\left(1 - \frac{\lambda}{\Psi}\right) h + \lambda\, \mathrm{c}$

(a) The economic order quantity

Since the production rate here is infinite

$$Q^* = \sqrt{\frac{2\,K\,\lambda}{h}} = \sqrt{\frac{2 * 200 * 1600}{4}} = 400$$

(b) The optimal reorder point

(Be careful about the time unit when multiplying, they should be the same)

We have constant demand of 1600 units per year.
Or we have constant demand of 1600/52 per week = 30.76 units / week.

Considering 1 week of lead time, we need to place the order when there are 31 units left in the inventory to prevent any stock outs.

(c) The resulting annual setup cost

$$\text{Annual Setup cost} = \left(\frac{\lambda}{Q^*}\right) K = \left(\frac{1600}{400}\right) 200 = 800$$

Now assume that there was a finite production rate of 8,000 units per year. Compute:

(d) The EOQ

$$Q^* = \sqrt{\frac{2\,K\,\lambda}{h\left(1-\frac{\lambda}{\Psi}\right)}} = \sqrt{\frac{2 * 200 * 1600}{4\left(1-\frac{1600}{8000}\right)}} = \sqrt{\frac{160000}{0.8}} = 447.21$$

(e) The maximum inventory level

$$\text{Maximum inventory level} = \mathrm{H} = Q^*\left(1-\frac{\lambda}{\Psi}\right) = 447.21\left(1-\frac{1600}{8000}\right) = 357.768$$

(f) The total annual holding cost

$$\text{Total annual holding cost} = \left(\frac{Q^*}{2}\right)\left(1-\frac{\lambda}{\Psi}\right)h = \left(\frac{447.21}{2}\right)\left(1-\frac{1600}{8000}\right)4 = 715.536$$

Problem 4:

Read the following articles:

- https://www.defensenews.com/air/2017/04/24/gao-urges-caution-on-economic-order-quantity-buy-for-f-35-jet/
- http://scdigest.com/experts/DrWatson_19-02-19.php?cid=15200

Select one of these two articles, and do the following:

1. Briefly summarize the article.
2. Provide a few interpretive thoughts on the article, using what you have learned from class.
3. Provide one recommendation on how the dilemma posed in the article could be resolved.

Some notes:

- Please limit your responses to one page, double spaced, 12 point font.
- There is no right answer to this question. We are evaluating your ability to apply what you learn in class to practical applications.
- This question is not intended to be free points. If you do not demonstrate a sufficient level of critical thinking, full credit will not be awarded.

The article by Dr. Michael Watson, about "EOQ Model and the Hidden Costs of Fixed Costs" majorly deals with how to save on fixed costs by filtering the type of tasks in separate buckets and batching the execution accordingly. Let's say there is a process inclusive of 50 tasks of 10 types distributed evenly. Also there is a setup cost of 1 and execution cost of 2 for each task. Worst case cost for non-batched execution is 1*50 + 2*50 = 150. While if we batch the type of tasks the worst case would be 1*(50/10) + 2*50 = 105. Thus batching tasks/production can save us lot a lot of setup cost and finally in the overall cost.

While gardening in my backyard I perform bush trimming, leaves racking and lawn mowing etc. Changing my clothes, setting up all the equipment's and later re-organizing tools and cleanup are a part of setup cost for that job. If I perform all my gardening tasks together, I will have to pay my setup cost once. But not all plants/trees/grass need care at same intervals, also I will get tired after a some tasks. Similar example is mentioned in the article about batching software engineering tasks such as checking email, coding, testing, etc. Being a software engineer I practice that batching myself. But again here, there are times when customer or your team is dependent on your response via email/slack or ticketing systems.

Considering the caveats, it will be a good idea to batch a subset of gardening tasks together (based on amount of labor needed and plant's growth rate) or batch certain parts of the software engineering code together. While batching is important, it is also important to find an optimal switching time so to not lock, exhaust or stock outs on others. In real world almost every job/process is a combination of different types of tasks and all needs to be performed. Thus, while we might not be able to change our cost from 150 to 105, finding an optimal ~125 solution will help in cost savings and preventing blocks.