Assignment 5

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Question

Discuss Quasi-Newton Condition with DFP and BFGS updates.
Ans. -
Newton's method for finding an extreme point is:

$$x_+ = x - H^{-1}(x)g(x)$$

Quasi-Newton methods repeat updates of the form:

$$x_+ = x + ts$$

where, s is defined by linear system:

$$H(x)s = -g(x)$$

Where, H(x) is hessian and g(x) is gradient of f.

Algorithm:
Let x ∈ R, H > 0 for k=1,2,3.,.,.
Repeat
Solve $H^{k-1}s^{k-1}$ = -g^{k-1}
Update x^k = x^{k-1} + $t_k s^{k-1}$
Compute H^k from H^{k-1}

Different Quasi-Newton methods implement Step 3 differently. As we will see, commonly we can compute: $(H^k)^{-1}$ from $(H^{k-1})^{-1}$

Davidon-Fletcher-Powell (DFP) update:
In this method, they used inverse C:

$$C_+ = C + auu^T + bvv^T$$

Multiplying by y, using the secant equation s = $C_+ y$, and solving for a, b, yields:

$$C_+ = C - \frac{Cyy^T C}{y^T Cy} + \frac{ss^T}{y^T s}$$

then It shows:

$$H^k = (I - \frac{ys^T}{y^T s})H^{k-1}(I - \frac{sy^T}{y^T s}) + \frac{yy^T}{y^T s}$$

This is the Davidon-Fletcher-Powell (DFP) update.
O(n^2), preserves positive definiteness.

Broyden-Fletcher-Goldfarb-Shanno(BFGS) update:
Let's now try a rank-two update:

$$H^k = H^{k-1} + auu^T + bvv^T$$

The secant equation y = $H^k s$ yields

$$y - H^k s = (au^T s)u + (bv^T s)v$$

Putting u = y, v = $H^{k-1}s$, and solving for a, b we get:

$$H^k = H^{k-1} - \frac{H^{k-1}ss^T H^{k-1}}{s^T H^{k-1} s} + \frac{yy^T}{y^T s}$$

called the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update.
Importantly, BFGS update preserves positive definiteness