Assignment 1

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Question

What is Lagrange dual problem? Explain weak and strong Duality? Give relevant equations.
Ans. -
The Lagrange dual problem is obtained by forming the Lagrangian of a minimization problem by using non-negative Lagrange multipliers to add the constraints to the objective function, and then minimize the original objective function by solving for the primal variable values.

Let Optimization problem is:

$$\begin{aligned} minimise. \quad & f_0(x) \\ s.t. \quad & f_i(x) \leq 0 \quad i = 1, \dots, m \end{aligned}$$

$$Lagrangian:$$

$$L(x, \lambda) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$$

λ are Langrange multipliers.
So, the supremum over langrange.

$$\max_{\lambda \geq 0} \quad L(x, \lambda) = \max_{\lambda \geq 0}(f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x))$$

$$\max_{\lambda \geq 0} \quad L(x, \lambda) = \begin{cases} f_0(x) & f_i(x) \leq 0 \\ \infty & otherwise \end{cases}$$

So, the Primal form of optimisation problem is:

$$p^* = \min_x \max_{\lambda \geq 0} \quad L(x, \lambda)$$

Langrange dual function is:

$$g(\lambda) = \min_x \quad L(x, \lambda)$$

Now, the Langrangian dual problem is:

$$d^* = \max_{\lambda \geq 0} g(\lambda) = \max_{\lambda \geq 0} \min_x \quad L(x, \lambda)$$

Weak duality states that any feasible solution to the dual problem corresponds to an upper bound on any solution to the primal problem. i.e.,

$$p^* \geq d^*$$

While, Strong duality occurs when the values of the optimal solutions to the primal problem and dual problem are always equal. i.e.,

$$p^* = d^*$$