Assignment 11

Vishal Kumar, MIT2019090

Question

Discuss Primal-dual interior-point method and compare it with barrier method.
Ans.-
Let's consider a convex optimisation problem with inequality constraints:

$$\begin{aligned} \min_{x} \quad & f(x) \\ s.t. \quad & h_i(x) \leq 0 \quad for \quad i = 1, ..., m \\ & Ax = B \end{aligned}$$

Primal-dual interior-point method

Start with $x_{(0)}$ such that $h_i(x^{(0)}) < 0$, i = 1, . . . , m, and $u^{(0)} > 0$, $v^{(0)}$. Define $n^{(0)}$ = -$h(x^{(0)})^T u^{(0)}$.
We fix $\mu > 1$, repeat for k = 1, 2, 3 . . .

Now,

- Define t = $\frac{\mu m}{n^(k-1)}$
- Compute primal-dual update direction Δy
- Use backtracking to determine step size s
- Update $y^{(k)} = y^{(k-1)}$ + s Δy
- Compute $\mu^{(k)}$ = -$h(x^{(k)})^T u^{(k)}$
- Stop if $\mu^{(k)} \leq \epsilon$ and $(\parallel r_{prim} \parallel_2^2 + \parallel r_{dual} \parallel_2^2)^1/2 \leq \epsilon$

This method terminates when x is primal feasible, also λ, ν are dual feasible and surrogate gap is smaller than tolerance(ϵ),

In the primal-dual interior-point method the iterates, x^k, λ^k, ν^k are not always feasible escept in the algorithm convergence limit. So, we are not able to easily evaluate a duality gap ν^k in step k of the algorithm like in barrier method. So we define something called surrogate gap, for any x that satisfies $f(x) \prec 0 \; and \; \lambda \succeq 0$ as :

$$\hat{\eta}(x, \lambda) = -f(x)^T \lambda$$

If x is primal feasible and λ, ν are dual feasible, then surrogate gap $\hat{\eta}$ will be the duality gap
Basic difference between Primal-Dual and Barrier Method:

- Both can be motivated in terms of perturbed KKT conditions

- Primal-dual interior-point methods take one Newton step, and move on (no separate inner and outer loops)
- Primal-dual interior-point iterates are not necessarily feasible
- Primal-dual interior-point methods are often more efficient, as they can exhibit better than linear convergence
- Primal-dual interior-point methods are less intuitive ...