Assignment 10

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Question

Discuss log barrier method.
Ans.-

As we know, Newton's method used for minimizing convex functions with equality constraints. One of the limitations of this method is that we cannot deal with inequality constraints. To address this issue, there is a method called Log barrier method.

$$\begin{aligned} &\min_x \quad f(x) \\ &s.t. \quad h_i(x) \leq 0 \quad for \quad i = 1, ..., m \\ &Ax = B \end{aligned}$$

Let's assume that f, h_i are all convex and twice differentiable functions, all with domain R^n, the log barrier is defined as:

$$\Phi(x) = -\sum_{i=1}^{m} \log(-h_i(x))$$

The domain is the set of strictly feasible points. Now, Lets ignore the equality constraints, this problem can be written as:

$$\min_x f(x) + \sum_{i=1}^{m} I_{\{h_i(x) \leq 0\}}(x)$$

Now, let's add log barrier function:

$$\min_x f(x) - (\frac{1}{t}). \sum_{i=1}^{m} \log(-h_i(x))$$

Where t 0, This approximation is more accurate for larger t. But for any value of t, the log barrier approaches ∞ if any $h_i(x) -> 0$