Assignment 9

Vishal Kumar, MIT2019090

Question

What is Self-concordant Function? Discuss its relation with Newton's method.
Ans.-

For a optimization problem, a self-concordant function is a function f : R − > R for which,

$$|f^{'''}(x)| \leq 2f^{''}(x)^{3/2}$$

0r, a function f : R − > R that, wherever $f^{''}(x) > 0$, satisfies

$$| \frac{d}{dx} \frac{1}{\sqrt{f''(x)}} | \leq 1$$

and which satisfies $f^{'''}(x) = 0$ elsewhere.
Examples:
i.) Linear and Quadratic functions.
ii.) Negative Logarithm i.e. f(x) = -log(x)

Advantages:
i.) Possesses affine invariant property
ii.) Provides a new tool for analyzing Newton's method that exploits the affine invariance of the method.
iii.) Results in a practical upper bound on the Newton's iterations
iv.) Plays a crucial role in performance analysis of interior point method

Newton's method using Self-concordant function:

Let f : R^n − > R be self-concordant and $\Delta^2 f(x) \geq 0$ for all x L_0

Self-concordance replaces strict convexity and Lipschitz Hessian assumptions and Newton decrement replaces the role of the gradient norm.
Using Self-concordant, Newton direction is:

For Newton's decrement $\lambda(x)$ and any $v \in R^n$

$$\lambda(x) = \sup_{v \neq 0} \frac{-v^T \Delta f(x)}{(v^T \Delta^2 f(x) v)^{1/2}}$$

with the supremum attainment at v = $-[\Delta^2 f(x)]^{-1} \Delta f(x)$