Assignment 2

Vishal Kumar, MIT2019090

Question

What is Slater's constraint qualification? Give relevant equations.
Ans. -
As we know, Strong duality usually holds for convex problems and Constraint qualifications are the conditions for strong duality in convex problems.

Convex problem is:

$$\begin{aligned} &minimise. \quad f_0(x) \\ &subject.to. \quad f_i(x) \leq 0 \quad i = 1, \ldots, m \\ &Lagrangian: \\ &\quad L(x, \lambda) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) \end{aligned}$$

Slater's constraint ensures that strong duality holds for convex problems.
There are two assumptions for slater's theorem:
i) Our objective function f_0 and the constraints should be convex.
ii) Optimal problem is finite.
Slater's theorem states that there should be a slater vector x' for which,

$$f_i(x') < 0 for \quad i = 1, \ldots, m$$

If these conditions fulfilled then we can say that:
i) There is Strong Duality i.e. $p^* = d^*$
ii) Optimal solution of convex problem is bounded and non-empty.