Convex Optimization (C3)

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Question 01 [6 marks]

Determine if the functions are convex or not.

- $f(x_1, x_2) = x_1 e^{-(x_1+x_2)}$
- $f(x_1, x_2) = -x_1^2 - 5x_2^2 + 2x_1x_2 + 10x_1 - 10x_2$

Question 02 [4 marks]

Consider the optimization problem

$$\begin{aligned} &\text{minimize} && x^2 + 1 \\ &\text{subject to} && (x-2)(x-4) <= 0 \end{aligned}$$

State the dual problem. is kkt condition satisfied?

Question 03 [6 marks]

1. Let S be a non-empty set in E_n. Show that S is convex iff for each integer $k \geq 2$, the following holds true $x_1 ... x_k \in S$, implies that $\sum_{j=1}^{k} \lambda_j x_j \in S$ where $\sum_{j=1}^{k} \lambda_j = 1$ and $\lambda_j \geq 0$ for $j = 1, ..k$
2. Let S be a convex set in E_n, A be an $m \times n$ matrix and α be a scalar. Show that the following two sets are convex: -
   - $AS = \{y : y = Ax, x \in S\}$
   - $\alpha S = \{\alpha x : x \in S\}$

Question 04 [4 marks]

Apply gradient algorithm for the function $y^2 = 4x$ in the interval $[1, 4]$. Show at least two steps. Explain ADAM Algorithm.