Perceptron Algorithm for AND, OR, NAND and NOR

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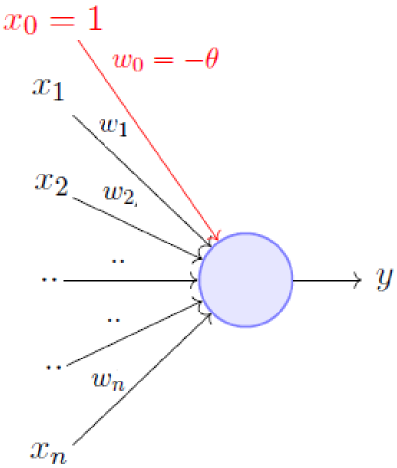
Perceptron Algorithm:

The perceptron model takes an input, aggregates it (weighted sum) and returns 1 only if the aggregated sum is more than some threshold else returns 0. Rewriting the threshold as shown above and making it a constant input with a variable weight, we would end up with something like the following:



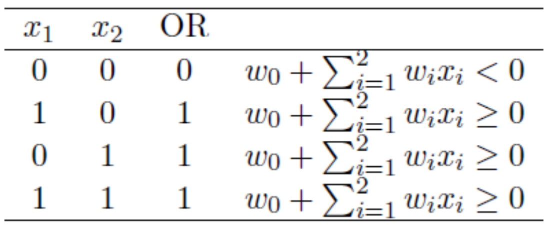
A more accepted convention,

$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

$$where, \quad x_0 = 1 \quad and \quad w_0 = -\theta$$

OR function using a perceptron:

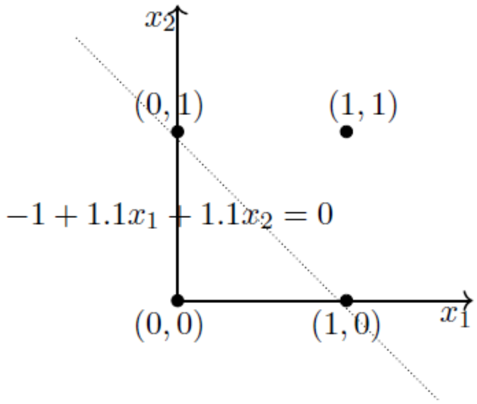


x_1	x_2	OR	
0	0	0	$w_0 + \sum_{i=1}^{2} w_i x_i < 0$
1	0	1	$w_0 + \sum_{i=1}^{2} w_i x_i \geq 0$
0	1	1	$w_0 + \sum_{i=1}^{2} w_i x_i \geq 0$
1	1	1	$w_0 + \sum_{i=1}^{2} w_i x_i \geq 0$

$$w_0 + w_1 \cdot 0 + w_2 \cdot 0 < 0 \implies w_0 < 0$$
$$w_0 + w_1 \cdot 0 + w_2 \cdot 1 \geq 0 \implies w_2 > -w_0$$
$$w_0 + w_1 \cdot 1 + w_2 \cdot 0 \geq 0 \implies w_1 > -w_0$$
$$w_0 + w_1 \cdot 1 + w_2 \cdot 1 \geq 0 \implies w_1 + w_2 > -w_0$$

One possible solution is

$$w_0 = -1, \; w_1 = 1.1, \; w_2 = 1.1$$



Same with AND, NAND and NOR.

Training Data: [[1, 0, 0], [1, 0, 1], [1, 1, 0], [1, 1, 1]] here, x[0] is bias term’s coefficient.

I am taking Initial weights randomly using:

```
w0 = np.random.randn()

w1 = np.random.randn()

w2 = np.random.randn()
```

Observations:

For testing my model, I am specifying some testing data:

test_data = [[0.98, 1],[0.01, 0.97],[0.77, 0.99],[0.912, 1.002],[0.88, 0.11],[0.82, 0.9],[0.8, 1],[0.02, 0.01],[0.21, 0.99],[0.11, 0.2],[0.79, 1],[0.11, 1.02],[0.98, 0.87],[0.2, 1.3],[0.2, 0.003]]

and Actual outputs for the same is:

test_op = [1, 0, 1, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0] # For AND

test_op = [1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 0] # For OR

test_op = [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1] # For NOR

test_op = [0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1] # For NAND

For alpha = 0.1, epochs =1000:

Initial weights -

w0 = -1.3180345632728732 w1 = 1.4075124085250568 w2 = -0.2529847047293955

Final weights -

w0 = 0.4819654367271267 w1 = -0.3924875914749431 w2 = -0.2529847047293955

And Max Accuracy is 100%.