Physics-Informed Neural Networks for Heat Transfer Problems - Active sensor Placement

Vishal Bondili

MATH-8070/7070 - INVERSE PROBLEMS

INTRODUCTION

- PINNs are a type of machine learning model that combine neural networks with physical laws and constraints to solve complex problems in fields fluid dynamics, heat transfer, and structural mechanics.
- The sensor placement problem involves identifying the optimal location for sensors to capture relevant information about a system or environment.
- In heat transfer problems, PINNs can be used to learn the temperature distribution and heat flux in a given system, while enforcing the laws of thermodynamics.

MOTIVATION

Cost reduction: Optimal sensor placement can reduce the cost of installation, maintenance, and operation of sensors.

Maximizing sensor efficiency: Sensor placement can impact the efficiency and effectiveness of a system.

Improved decision-making: The accurate and timely data provided by sensors can improve decision-making processes

Real-time monitoring: Optimal sensor placement is necessary to ensure timely and accurate data acquisition

Advantages of PINNs

- PINNs use automatic differentiation to evaluate differential equations without the need for numerical discretization.
- PINNs can handle noisy and incomplete data.
- PINNs can solve ill-posed problems that are difficult or impossible to solve with traditional computational methods.
- The key advantage of using PINNs for heat transfer problems is their ability to learn from data while still incorporating physical constraints.

PROBLEM STATEMENT

Goal: The problem at hand is the determination of optimal sensor locations, which is a costly and often trial-and-error process. To address this issue, This project proposes a method that can dynamically select sensor locations in a way that minimizes the number of temperature measurements required.

Forward Problem and Inverse Problem

Forward Problem:

Predicting the temperature distribution on a surface, given a set of sensor locations and measurements.

Inverse Problem:

Determining the optimal sensor locations that can provide the most accurate prediction of the temperature distribution.

This involves selecting sensor locations that can capture the most significant features of the temperature distribution while minimizing the number of temperature measurements required.

Governing Equations

The governing equations of this problem are the incompressible **Navier–Stokes equation** and the corresponding **Temperature equation**

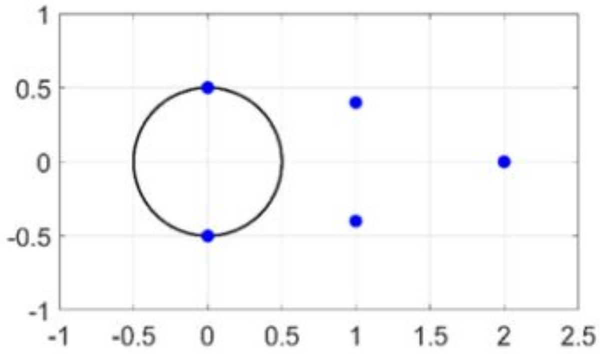
$$\frac{\partial\theta}{\partial t} + (\mathbf{u}\cdot\nabla)\theta = \frac{1}{\mathrm{Pe}}\nabla^2\theta$$
$$\frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} + \mathrm{Ri}\theta$$
$$\nabla\cdot\mathbf{u} = 0$$

where θ, $\mathbf{u} = (u, v)^T$, and p are the dimensionless temperature, velocity, and pressure fields, respectively. Pe, Re, and Ri denote the Peclet, Reynolds, and Richardson numbers, respectively. Note

Simulation Domain

The experiment is performed in a closed enclosure and is classical two-dimensional heat transfer problem of forced heat convection around a circular cylinder in steady-state.

The simulation domain size is [-7.5 D , 22.5 D] x [-10 D , 10 D]consisting of 2094 quadrilateral elements, where D is the diameter of the cylinder. The cylinder center is located at (0,0).



Boundary Conditions

On the **fluid flow part**, the cylinder surface is assumed to be no-slip, no-penetration wall

Inflow Boundary: Uniform velocity ($u = U_{inf}$, $v = 0$) is imposed on the inflow boundary where $x = 7.5 * D$

Outflow Boundary: zero-pressure boundary is prescribed on the outflow boundary where $x = 22.5 * D$

Lateral Boundaries: periodic boundary condition is used on the lateral boundaries where $y = - 10 * D$

For **Heat Transfer part:**

Inflow Boundary: Constant temperature is imposed on the inflow boundary

Outflow Boundary: zero-gradient is used on the outflow boundary $\rightarrow \partial\theta/n = 0$

Lateral Boundaries: periodic boundary is assumed on the lateral boundaries.

Proposed Solution for Sensor Placement

Step 1 Provide an initial sensor configuration.

Step 2 Train the neural network (PINN).

Step 3 Compute the residual on the cylinder boundary inferred by PINN

Residual:

$$e = u\theta_x + v\theta_y - \mathrm{Pe}^{-1}(\theta_{xx} + \theta_{yy})$$

Step 4: Find the position with maximum value of residual.

Step 5: Update the sensor placement and training data, and go back to step 2.

Objective Function

$$\mathcal{L} = \mathcal{L}_r + \mathcal{L}_{ub} + \mathcal{L}_{\theta b} + \mathcal{L}_{\theta}$$

where

$$\mathcal{L}_r = \frac{1}{N_r}\sum_{k=1}^{4}\sum_{i=1}^{N_r} |e_k(x^i, y^i)|^2$$

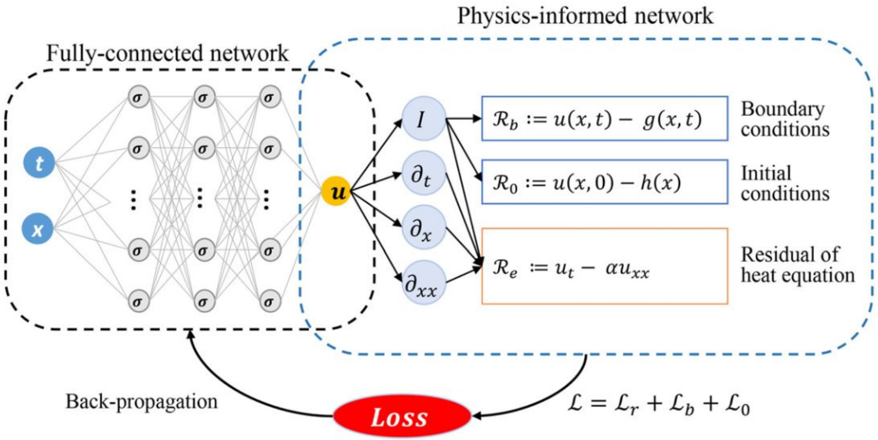
$$\mathcal{L}_{ub} = \frac{1}{N_{ub}}\sum_{i=1}^{N_{ub}} \left[\mathbf{u}(x^i, y^i) - \mathbf{u}_b^i\right]^2$$

$$\mathcal{L}_{\theta b} = \frac{1}{N_{\theta b}}\sum_{i=1}^{N_{\theta b}} |\theta(x^i, y^i) - \theta_b|^2$$

$$\mathcal{L}_{\theta} = \frac{1}{N_{\theta}}\sum_{i=1}^{N_{\theta}} |\theta(x^i, y^i) - \theta_{\text{data}}^i|^2$$

- **L_r** penalizes the governing equations.
- The **L_ub** and **L_θb** are the boundary conditions for velocity and temperature fields, respectively.
- **L_θ** is the mismatch between the inferred temperatures and the in situ measured values. **(Data Fitting Term)**

PINN Design



(a)

Input: Space time coordinates (x,t)

Output: (θ, u, v, p)

Model Evaluation

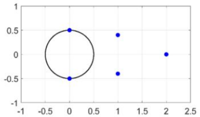
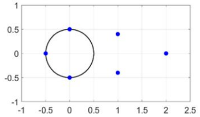
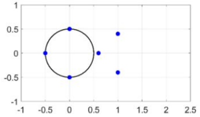
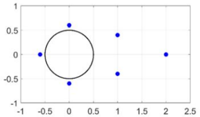
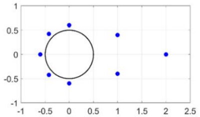
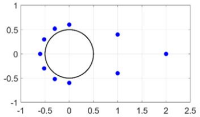
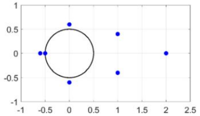
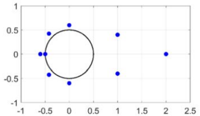
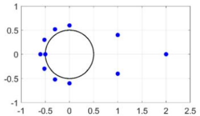
To investigate the performance of the proposed neural network on inferring the temperature, velocity, and pressure fields in the domain, the results obtained by PINNs are quantitatively evaluated against the reference solutions(Numerical Methods)

The model is evaluated on the basis of relative L2 error

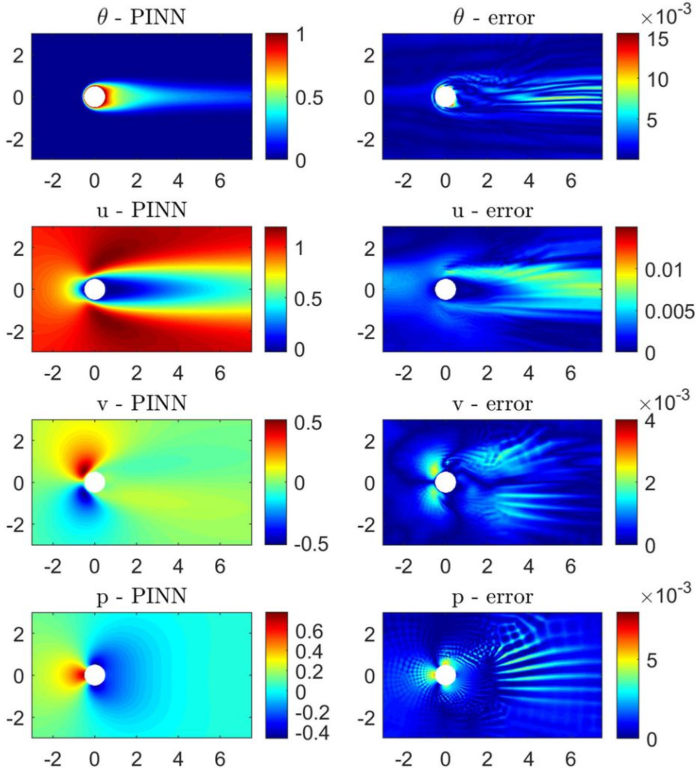
$$\varepsilon_V = \| V - V^* \|_2 / \| V^* \|_2$$

Where **V** is one of the predicted quantities **(θ, u, v, p)**, V* is the corresponding reference solution

Results

Case 1		Case 2		Case 3	
Sensor placement	Errors	Sensor placement	Errors	Sensor placement	Errors
	ϵ_u: 0.30 % ϵ_v: 0.39 % ϵ_p: 0.72 % ϵ_θ: 11.61 %		ϵ_u: 0.50 % ϵ_v: 0.70 % ϵ_p: 1.17 % ϵ_θ: 2.43 %		ϵ_u: 0.30 % ϵ_v: 0.47 % ϵ_p: 0.82 % ϵ_θ: 0.73 %
Case 4		**Case 5**		**Case 6**	
Sensor placement	Errors	Sensor placement	Errors	Sensor placement	Errors
	ϵ_u: 0.51 % ϵ_v: 0.95 % ϵ_p: 1.40 % ϵ_θ: 15.09 %		ϵ_u: 0.30 % ϵ_v: 0.49 % ϵ_p: 0.84 % ϵ_θ: 5.64 %		ϵ_u: 0.29 % ϵ_v: 0.47 % ϵ_p: 0.84 % ϵ_θ: 4.85 %
Case 7		**Case 8**		**Case 9**	
Sensor placement	Errors	Sensor placement	Errors	Sensor placement	Errors
	ϵ_u: 0.22 % ϵ_v: 0.32 % ϵ_p: 0.59 % ϵ_θ: 1.25 %		ϵ_u: 0.33 % ϵ_v: 0.52 % ϵ_p: 0.88 % ϵ_θ: 1.39 %		ϵ_u: 0.27 % ϵ_v: 0.40 % ϵ_p: 0.74 % ϵ_θ: 1.45 %

Results



θ - PINN
θ - error
$\times 10^{-3}$
u - PINN
u - error
v - PINN
v - error
$\times 10^{-3}$
p - PINN
p - error
$\times 10^{-3}$

THANK YOU