Integrate and Fire Neuron

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I. INTRODUCTION

This is the report for the computational neuroscience's first coursework. This coursework requires to solve seven questions related to integrate and fire neuron. Report is fragmented into different sections, one for each question. Every section will discuss the problem and the solution related to it. But before diving straight into questions, we need to understand what is integrate and fire neuron model. The leaky integrate and fire neuron model is a very popular technique to model a spiky neuron. This model can be summarized into *Equation 1*.

$$\tau_m \frac{dV}{dt} = E_L - V + R_m I_e$$

Equation 1

We achieve the equation from a basic circuit which consist of a capacitor in parallel with a resistance. The V represents the membrane potential and τ_m represents the membrane time constant of the neuron in the equation. If voltage jumps the threshold value then a spike is recorded in neuron and voltage is set to a reset value.

Although this model lacks a lot of dynamics of neuron but still it is very useful in modelling the behavior of a large neural network.

II. QUESTION ONE

Question one asks to simulate an integrate and fire model for one second and with a time step of δt = **1ms**.
Parameters that are given with the question are following.

τ_m = 10ms, E_L = V_r = -70mV,
V_t = -40mV, R_m = 10MOhm, I_e = 3.1nA

As we are given with a constant value of I_e= **3.1nA.** So, *Equation 1* can be solved to give *Equation 2*. This equation is equated using Euler's method. Coursework require us to solve the problem using python. It contains library like numpy and matplotlib which help to solve the complex mathematical problems and plot figures on screen respectively.

$$V(t) = E_L + R_m I_e + [V(0) - E_L - R_m I_e]e^{-t/\tau_m}$$

Equation 2

This question does not talk about both rest voltage and reset voltage. Post each spike the model will set the voltage to$V_r = -70mV$. Question ask to plot the result in voltage as a function of time. The plot is shown in *Figure 1*.

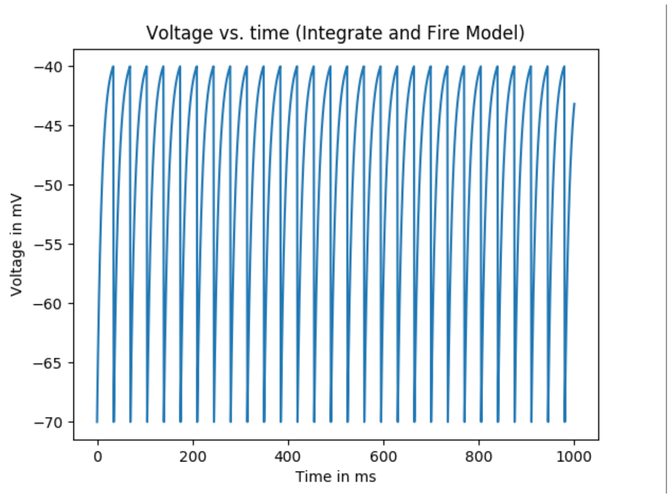

Figure 1

III. QUESTION TWO

Second question ask to analytically calculate the minimum current I_e required for the neuron with parameters as in question one to produce an action potential.

$$\bar{V} = E_L + R_m I_e$$

Equation 3

Here I_e is the equilibrium value where V stops changing. We can calculate this value using the *Equation 3.* We take the value of $\bar{V}$ as -40mV.

$$=> -40mV = -70mV + 10MOhm * I_e$$

$$=> \frac{(-40mV + 70mV)}{10MOhm} = I_e$$

$$=> I_e = 3nA$$

So, $3nA$ is the minimum current required for neuron in question one to produce action potential.

IV. QUESTION THREE

Question three want us to simulate a neuron for input current I_e= **2.9nA**. Which is 0.1 less than minimum current computed in question two.

The approach to this question is completely same as question one but just value of Ie changed from $3.1nA$ to$2.9nA$.

The output is plotted in *Figure 2*. With input current as $2.9nA$, the neuron will never reach the threshold voltage and hence we never observe a spike. Same can be seen in *Figure 2*, we see a straight line instead of spikes as in *Figure 1*.

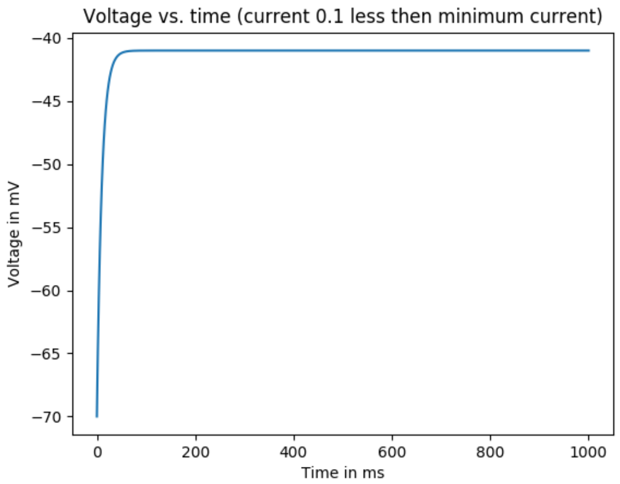

Figure 2

V. QUESTION FOUR

This question requires to simulate a neuron for a current ranging from $2nA$ to $5nA$. For each current value, firing rate is also to be calculated. Firing rate is defined as the number of spikes a neuron experience in 1 second.

The graph is plotted in *Figure 3* showing the firing rate in Hz as a function of current in nA. We can see from the graph that there is a straight line of firing rate with value zero till current is equal to $3nA$. This is because $3nA$ is the minimum current required for the neuron with parameters as in first question to produce an action potential.

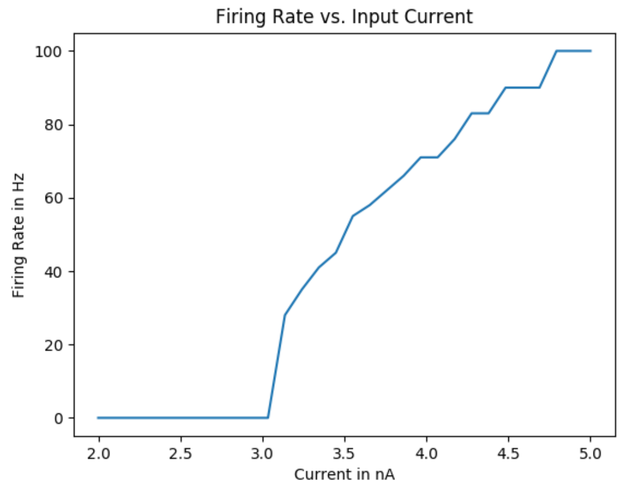

Figure 3

VI. QUESTION FIVE

This question wants us to simulate two neurons having synaptic connection between them. First neuron projects to the second, and the second neuron projects to the first. Parameters given for both neuron are as following.

τ_m **= 20ms,** E_L **= -70mV,** V_r **= -80mV**
V_t **= -54mV,** $R_m I_e$ **= 18mV**
Both neuron have similar synaptic connection and hence same synapse parameter.

$$R_m\overline{G}_s = 0.15, P = 0.5, \tau_s = 10ms$$

We can model a simple synapse using *Equation 4*. This equation is achieved by a time dependent conductor in series with a battery.

$$I_s(t) = \overline{g}_s s(t)(E_s - V)$$

Equation 4

The value of Es determines whether the synapse is excitatory or inhibitory. First part of question five asks to simulate the synapse with $Es = 0.0\ mV$. Hence we observe an excitatory synapse as in *Figure 4*.

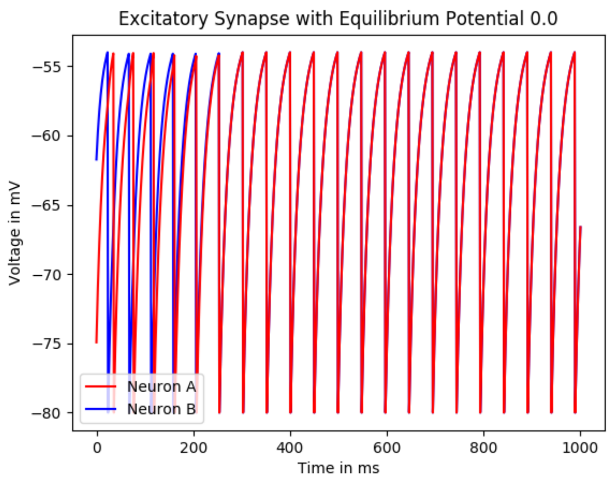

Figure 4

And second part of question five asks to simulate the synapse with $\boldsymbol{Es = -80.0\ mV}$. Hence we observe an inhibitory synapse as in *Figure 5*.

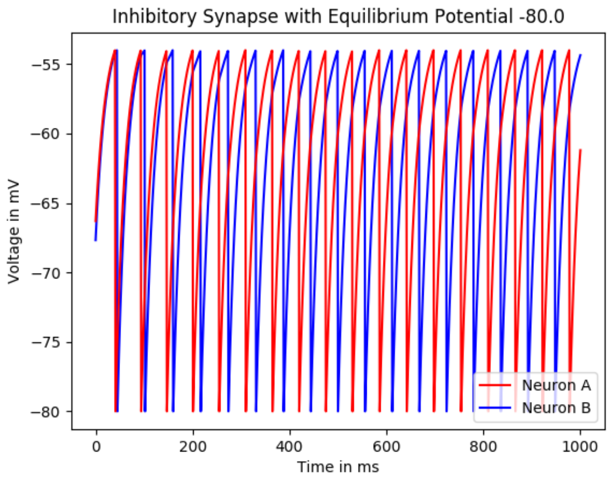

Figure 5

VII. QUESTION SIX

In many real neuron the firing rate falls off after the first few spike. This happens due to the slow potassium current. Parameters for this question are same as question one for neuron. But the parameters for slow potassium current are as follows.

$$\boldsymbol{E_k = -80mV, G_k = 0.005\ (MOhm)^{-1}, \tau = 200ms}$$

The graph is plotted in *Figure 6*. This shows the small decrease in firing rate with time.

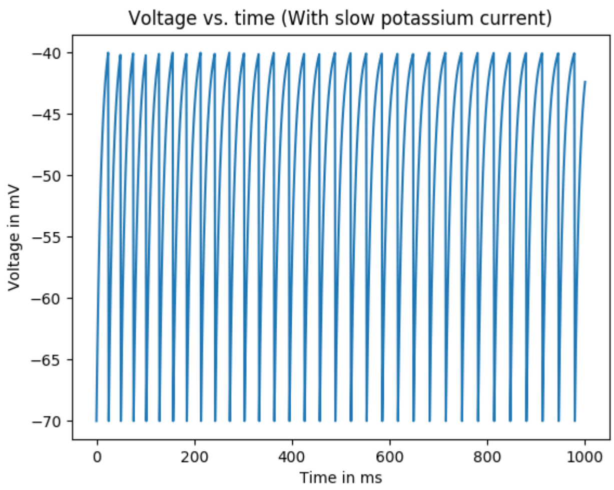

Figure 6

VIII. QUESTION SEVEN

Equation 5 depict the alpha function. The only difference between the single exponential and alpha function is of time.

$$\boldsymbol{s(t) = t\ e^{-t/\tau_s}}$$

Equation 5

So post changing the question 5 values to alpha function we get graphs as in *Figure 7* and *Figure 8*.

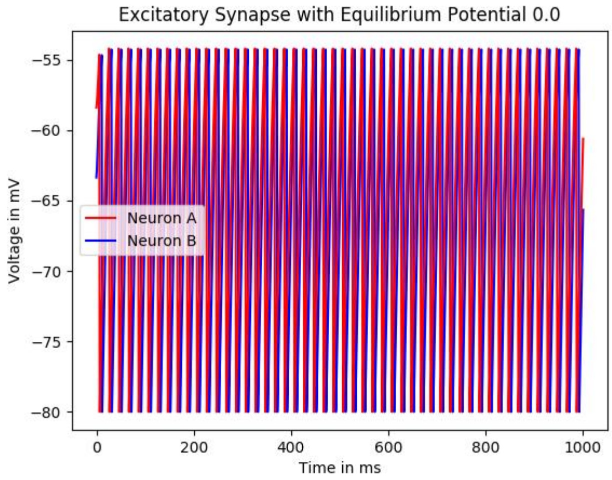

Figure 7

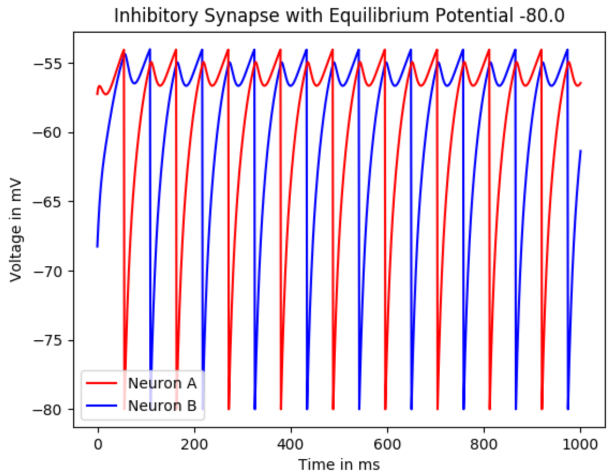

Figure 8