Probability: Probability is a mathematical measure of uncertainity.

Experiment: An operation which can produce some well defined outcomes.

2 types of experiments.

(i) Deterministic experiment

(ii) Probabilistic (or) Random experiment.

Deterministic experiment: When we perform an experiments in science and engineering and repeat the same under identical conditions we get the same result every time. This type of experiments called deterministic experiments.

Random experiment: when the experiment is repeated under the identical conditions, if it donot produce the same outcome every time called random experiment.

(i) Tossing a fair coin

(ii) Throwing a dice (unbiased)

Sample space: The set of all possible outcomes in a random experiment is called sample space. It is generally denoted by 'S'.

* Each element of sample space is called a sample point.

(i) In tossing a fair coin there are two possible outcomes head (H) & tail (T)

$S = \{H, T\}$ $n(S) = 2$

Remark: If we have a random experiment with 'n' outcomes $x_1, x_2, x_3 \cdots x_n$

another random experiment with 'm' outcomes $y_1, y_2, y_3, \cdots y_n$

the samplespace $S = \{(x_i, y_j) : 1 \le i \le n, 1 \le j \le m\}$

Event: subset of a sample space satisfying some condition is called an event.

* Events are generally denoted by the letter 'E'.
* If number of events are more we represent by

$E_1, E_2, E_3, E_4, \cdots E_n$.

(1) In tossing a coin, $S = \{H, T\}$

$E$ = getting head, then $E = \{H\}$

Impossible even: $S$ be the sample space. Since $\phi$ (null set) is subset of $S$. so, $\phi$ is called an impossible event.

Sure events: $S$ is subset of $S$. so the event $S$ is called an sure events.

Elementary event: event containing single sample point is called an elementary event (or) simple event

Equally likely events:- The given events are said to be equally likely if none of them is expected to occur in preference to other

In tossing a coin

$E$ = getting head = $\{H\}$  $F$ = getting tail = $\{T\}$

$E$ and $F$ are equally likely events

Mutually exclusive events: two events $E$ and $F$ are said to be mutually exclusive events if $E \cap F = \phi$

sample space = $\{1,2,3,4,5,6\}$

$E = \{1,2,3\}$  $F = \{4,5,6\}$

$E \cap F = \phi$  $\therefore$ $E$ & $F$ are mutually exclusive events.

Mutually exclusive and exhaustive events: two events $E$ & $F$ are said to be mutually exclusive & exhaustive events  $E \cap F = \phi$  $E \cup F = S$

sample space = $S$ = $\{1,2,3,4\}$

$E = \{1,2\}$  $F = \{3,4\}$

$E \cap F = \phi$

$E \cup F = \{1,2,3,4\} = S$

$\therefore$ $E$ & $F$ are mutually exclusive and exhaustive events

Probability of an event: In a random experiment. let S be a sample space and E is an event, then.

The probability of occurence of the event E is defined as

$$P(E) = \frac{\text{no. of fav outcoms}}{\text{total no. of outcomes}} = \frac{n(E)}{n(S)}$$

Properties:

1. Probability of occurence of an event is always non-negative $P(E) \geq 0$.
2. Probability of occurence of an impossible even is '0'

$P(E) = 0$ if $E = \phi$

3. Probability of occurence of sure event is '1'

$P(E) = 1$ if $E = S$.

4. If E and F be two events such that $E \subseteq F$ then

$P(E) \leq P(F)$

5. For a random experiment

$0 \leq P(E) \leq 1$

6. If E&F are mutually exclusive events

$E \cap F = \phi$

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Addition theorem of probability:-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$A \cup B = A \cup (A^c \cap B)$$

$$P(A \cup B) = P(A \cup (A^c \cap B))$$

$$P(A \cup B) = P(A) + P(A^c \cap B)$$

$$P(A \cup B) = P(A) + [P(A^c \cap B) + P(A \cap B) - P(A \cap B)]$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\boxed{P(A) = 1 - P(A^c)}$$

Conditional probability: Let E and F be two events associated with some sample space. Then the probability of occurence of F under the condition that E has already occured and $P(E) \neq 0$, is called conditional probability

* It is denoted by $P(F/E)$

Multiplication theorem: let E and F be two events associated with the same random experiment

$$P(E \cap F) = P(E) \cdot P(F/E) \quad \text{where } P(E) \neq 0$$

$$P(E \cap F) = \frac{n(E \cap F)}{n(E)}$$

$$P(F/E) = \frac{n(E \cap F)}{n(E)} \qquad P(E/F) = \frac{n(E \cap F)}{n(F)}$$

Properties:

① $P(E/E) = 1$

② $E \subseteq F$, then $P(F/E) = 1$ (since $E \cap F = E$)

③ $P(F/E) = 0$ if $E \cap F = \phi$

④ let $E_1, E_2$ and $F$ are three events of sample space S and $E_1 \subseteq E_2$ then $P(E_1/F) \leq P(E_2/F)$.

A die is rolled, if the outcome is odd number then what is the probability that it is a prime?

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

let $E$ = event of getting odd number $= \{1, 3, 5\}$

$P(E) = \frac{3}{6} = \frac{1}{2}$

$F$ = event of getting prime number $= \{2, 3, 5\}$

$P(F) = \frac{3}{6} = 1/2$

$E \cap F = \{3, 5\}$

$P(E \cap F) = \frac{2}{6} = 1/3$

$$P(F/E) = \frac{P(E \cap F)}{P(E)} = \frac{1/3}{1/2} = 2/3.$$

Independent events: we know that two events are independent if the occurence of one does not depend upon the occurence of the other

Independent experiments: Two random experiments are said to be independent if for every pair of events $E$ and $F$, associated with the first and second experiment respectively.

* The probability of the simultaneously occurence of $E$ and $F$ is the product of $P(E)$ & $P(F)$, calculated seperately on the basis of two experiments

$P(E \cap F) = P(E) \cdot P(F)$ $\quad P(E/F) = P(E)$.

* A and B are playing a game by tossing a coin. The game will be over if one of them gets result as heads. If A starts the game then find the respective probabilities for winning the game.

no.of outcomes = {H,T} = S

Getting head

$P(\text{head}) = 1/2 \quad P(\text{tail}) = 1/2$

$P(\text{A- wins the game}) = P[H \text{ or } TTH \text{ or } TTTTH \ldots]$

$= P(H) + P(TTH) \text{ or } P(TTTTH) \text{ or } \ldots$

$= 1/2 + (1/2)^3 + (1/2)^5 + \ldots$

$= 1/2 [1 + (1/2)^2 + (1/2)^4 + \ldots]$

$= 1/2 \left[ \frac{1}{1 - 1/4} \right]$

$= \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

$P(\text{B-wins the game}) = 1 - 2/3 = 1/3$

A can solve 90% of the problems in a book and B can solve 70%. what is the probability that at least one of them will solve a problem, selected at random from the book.

$P(A) = 90/100 \quad P(B) = 70/100$

$P(A \cap B) = P(A) \cdot P(B) = \frac{90 \times 70}{100 \times 100} = 63/100$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{90}{100} + \frac{70}{100} - \frac{63}{100} = \frac{97}{100}$

**Theorem of total probability:** let $B_1, B_2, B_3 \dots, B_k$ constitute a partion of the sample space $S$ with $P(B_i) \neq 0$ for $i = 1, 2, 3, \dots k$. Then for any event $A$ of $S$ with $P(A \cap B_i) \neq 0$

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(B_i)\, P(A/B_i)$$

**Proof:**

since $B_1, B_2, B_3 \dots B_k$

constitute a partition

$S = \bigcup_{i=1}^{k} B_i$ and $B_i \cap B_j = \phi$ for all $i, j$

Now

$$A = A \cap S$$

$$= A \cap \left[\bigcup_{i=1}^{k} B_i\right]$$

$$= A \cap (B_1 \cup B_2 \cup B_3 \cup \dots \cup B_k)$$

$$= (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \dots \cup (A \cap B_k)$$

the sets $A \cap B_1, A \cap B_2, A \cap B_3 \dots A \cap B_k$ are mutually disjoint sets

Applying the additive rule for mutually exclusive events,

$$P(A) = P[(A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \dots (A \cap B_k)]$$

$$= P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) \dots P(A \cap B_k)$$

$$= \sum_{i=1}^{k} P(A \cap B_i)$$

Now applying the multiplicative rule,

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(B_i)\, P(A/B_i)$$

Hence the theorem is proved.

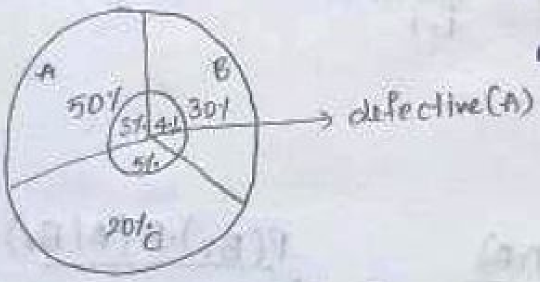
* Three machines A, B and C produce respectively 50%, 30% and 20% of the total number of items of a factory. The percentage of defective output of these machines are 3%, 4% and 5%. If an item is ~~selective~~ selected at random find the probability That the item is defective.

let

A = event of machines produced by A - $50/100$ P(A)

B = $30/100$ P(B)

C $20/100$ P(C)




D - defective item produced by A, B & C

$P(D/A) = 3/100 = 0.03$ $P(D/B) = 4/100 = 0.04$

$P(D/C) = 5/100 = 0.05$

$P(\text{defective}) = P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)$

$= (0.5)(0.03) + (0.3)(0.04) + (0.2)(0.05)$

$= 0.037$.

Baye's theorem: let $B_1, B_2, B_3 \ldots B_K$ constitute a partition of the sample space S with $P(B_i) \neq 0$ for $i = 1, 2, 3, 4, \ldots k$

then for any event A in S with $P(A) \neq 0$,

$$P(B_r/A) = \frac{P(B_r \cap A)}{P(A)} = \frac{P(B_r) \cdot P(A/B_r)}{\sum_{i=1}^{k} P(B_i) \cdot P(A/B_i)}$$

Proof:

from the definition of conditional probability we have

$$P(B_r/A) = \frac{P(B_r \cap A)}{P(A)} \longrightarrow (1)$$

from the theorem of total probability we have

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(B_i) \cdot P(A/B_i) \longrightarrow (2)$$

(2) in (1)

$$P(B_r/A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r) \cdot P(A/B_r)}{\sum_{i=1}^{k} P(B_i) \cdot P(A/B_i)}$$

A business man goes to hotels X, Y and Z, 20%, 50%, & 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbing.

(i) Determine the probability that the bussinessman goes to hotel with faulty plumbing?

(ii) What is the probability that bussinessman's room ~~tiving~~ having faulty plumbing is assigned to hotel Z?

let A = event of faulty plumbing

$B_1$ = bussiness man goes to hotel X, $P(B_1) = \frac{20}{100} = 0.2$

$B_2$ = bussiness man goes to hotel Y, $P(B_2) = \frac{50}{100} = 0.5$

$B_3$ = bussiness man goes to hotel Z, $P(B_3) = \frac{30}{100} = 0.3$

$P(A/B_1) = 5/100 \quad P(A/B_2) = 4/100 \quad P(A/B_3) = 8/100$

$$P(A) = P(B_1) \cdot P(A/B_1) + P(B_2) \cdot P(A/B_2) + P(B_3) \cdot P(A/B_3)$$

$$P(A) = \left(\frac{20}{100}\right)\left(\frac{5}{100}\right) + \left(\frac{50}{100}\right)\left(\frac{4}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{8}{100}\right)$$

$$P(A) = 0.054$$

(ii)

$$P(B_3/A) = \frac{P(B_3 \cap A)}{P(A)} = \frac{P(B_3) \cdot P(A/B_3)}{P(A)}$$

$$= \frac{0.3 \times 0.08}{0.054}$$

$$= \frac{0.024}{0.054} = \frac{24}{54} = \frac{4}{9}$$

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Random variables: Random variable is a function mapping from sample space S to real number set R

i.e:- $X: S \longrightarrow R$, where 'X' is a random variable

Let S be the sample space of some random experiment in which the elements need not be numbers. Now assigning the real values to the outcomes of a random experiment called random variable.

Random variable is a single valued function.

Random variables are two types.

(i) Discrete Random Variables

(ii) Continous Random Variables.

Discrete Random variables: A Random variable 'X' is said to be discrete random variable if its set of possible outcomes of the sample space is countable (finite or an unending sequence with as many elements as there are whole numbers)

Example: Throwing 2 unbaiased dice (R.E)

$n(S) = 36$ = finite  X = sum of the values on faces.

* X is a R.V its value depends on the outcome of the roll of dice.

* The value of X takes are 2,3,4 - . . 12 must be a subset to the sample space

Discrete probability distribution: Each element in a sample space has certain probability (or chance) of occurence (a happening). A formula representing all these probabilities which ~~can~~ a discrete random variables assume is known as the discrete probability distribution.

Example: Tossing two coins simultaneously (R.E)

X = number of heads occuring (random variable)

the sample space, S = {HH, HT, TH, TT}

X takes the values 0, 1, 2

therefore 'X' takes the values 0,1,2

i.e:- Occuring of zero head, one heads two heads.

The probabilities at various values of X are

$P(X=0) = P(\text{zero heads}) = 1/4$

$P(X=1) = P(\text{one head}) = 2/4 = 1/2$

$P(X=2) = P(\text{two heads}) = 1/4$

Continous random variable

| $X = x_i$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X=x_i)$ | 1/4 | 1/2 | 1/4 |

the above table is the probability distribution table

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Probability Mass function: Probability mass function of A discrete random variable 'X' is a function '$f(x)$' $[P(X=x_i) = f(x)]$ satisfying the following conditions.

(i) $f(x) \geq 0$, i.e $P(X=x_i) \geq 0$

(ii) $\sum f(x) = 1$ . i.e $\sum P(X=x_i) = 1$

Cumulative Distribution function:

Cumulative Distribution function of a discrete random variable X is denoted by $F(x)$ and is defined by

$$F(x) = P(X \leq x) = \sum_{x \leq x} f(x)$$

and.

$$P(x_i) = P(X=x_i) = F(x_i) - F(x_{i-1})$$

Example: Consider an experiment of rolling two dice. let X be a random variable which denoted the sum of the numbers on its faces.

So, the probability distribution of X is given by

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = x_i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $P(x = x_i)$ | 0 | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |

Sample space $S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

Computation of cumulative distribution function of X i.e. $F(x)$

$F_x(x=1) = P(x \le 1) = P(x<1) + P(x=0) = 0+0=0$

$F_x(x=2) = P(x \le 2) = P(x=1) + P(x=2) = 0 + 1/36 = 1/36$

$F_x(x=3) = P(x \le 3) = P(x=1) + P(x=2) + P(x=3) = 0 + 1/36 + 2/36 = 3/36$

$F_x(x=4) = P(x \le 4) = P(x=1) + P(x=2) + P(x=3) + P(x=4) = 6/36$

$F_x(x=5) = P(x \le 5) = P(x<5) + P(x=5) = \frac{6}{36} + \frac{4}{36} = \frac{10}{36}$

$F_x(x=6) = P(x \le 6) = P(x<6) + P(x=6) = \frac{10}{36} + \frac{5}{36} = \frac{15}{36}$

$F_x(x=7) = P(x \le 7) = P(x<7) + P(x=7) = \frac{15}{36} + \frac{6}{36} = \frac{21}{36}$

$F_x(x=8) = P(x \leq 8) = P(x<8) + P(x=8) = \frac{21}{36} + \frac{5}{36} = \frac{26}{36}$

$F_x(x=9) = P(x \leq 9) = P(x<9) + P(x=9) = \frac{26}{36} + \frac{4}{36} = \frac{30}{36}$

$F_x(x=10) = P(x \leq 10) = P(x<10) + P(x=10) = \frac{30}{36} + \frac{3}{36} = \frac{33}{36}$

$F_x(x=11) = P(x \leq 11) = P(x<11) + P(x=11) = \frac{33}{36} + \frac{2}{36} = \frac{35}{36}$

$F_x(x=12) = P(x \leq 12) = P(x<12) + P(x=12) = \frac{35}{36} + \frac{1}{36} = \frac{36}{36} = 1$

Continuous Random Variable: The random variable is continous if the range of 'X' is uncountably infinite. Typically an uncountably infinite range results from an 'X' that makes a physical measurement.

Examples: The position, size, time, age, flow, volume, or area of something.

Probability density function: Probability density function of a continous random variable 'X' is a function $f(x)$ satisfying the following conditions.

(1) $f(x) \geq 0$

(2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(3) $P(a<x<b) = \int_a^b f(x)\,dx$ = area under $f(x)$ between ordinates $x=a$ & $x=b$.

Note: (1) $P(a<x<b) = P(a \leq x<b) = P(a<x \leq b) = P(a \leq x<b)$

(2) Probability at a point $x=a$ is

$$P(x=a) = \int_{a-\delta x}^{a+\delta x} f(x)\,dx.$$

Cumulative distribution function: for a continous random variables 'x' with probability density function $f(x)$, the cumulative distribution $F(x)$ is defined as

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt \quad \text{where } -\infty < x < \infty$$

It follows that

$$F(-\infty) = 0,\ F(\infty) = 1, \text{ and } 0 \leq F(x) \leq 1$$

$\Rightarrow$

$$f(x) = \frac{d}{dx} F(x) = F'(x) \geq 0$$

and

$$P(a < x < b) = F(b) - F(a).$$

* Mean (or) expectation (or) expected value of a random variable 'X' is denoted by $E(x)$ or $\mu$, and is defined as

$$\text{mean} = E(X) = \mu \begin{cases} \sum x_i P(X = x_i) & \text{if } X \text{ is discrete R.V} \\ \int_{-\infty}^{\infty} x f(x)\,dx & \text{if } X \text{ is continous R.V} \end{cases}$$

Properties of expectation:

1. $E(kX) = kE(X)$ or $\mu_{kX} = k\mu_k$
2. $E(X+k) = E(X) + k$ or $\mu_{X+k} = \mu_x + k$
3. $E(X+Y) = E(X) + E(Y)$ or $\mu_{X+Y} = \mu_X + \mu_Y$
4. let $X, Y$ are two random variables such that $Y \leq X$ then $E(Y) \leq E(X)$ provided exception are exist.

the above rules are true for both discrete & continous R.V.

Variance characterizes the variability in the distributions, is denoted by $\sigma^2$

$$\sigma^2 = E\left[(x-\mu)^2 f(x)\right] = \begin{cases} \sum (x-\mu)^2 f(x) & \text{for } x \text{ is discrete} \\ \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx & \text{for } x \text{ is continuous} \end{cases}$$

Properties of varience:

① $Var(X+k) = Var(X)$

② $Var(kX) = k^2 Var(X)$

③ $Var(X+Y) = Var(X) + Var(Y)$

④ $Var(X-Y) = Var(X) - Var(Y)$

⑤ $Var(aX+b) = a^2 Var(X)$

⑥ $Var(X) = E(X^2) - \{E(X)\}^2$

<u>Note</u>: $\sigma^2 = E(x^2) - \mu^2$

Proof: By definition of variance we have

$$\sigma^2 = \sum (x-\mu)^2 f(x)$$

$$= \sum (x^2 - 2\mu x + \mu^2) f(x)$$

$$= \sum x^2 f(x) - 2\mu \sum x \cdot f(x) + \mu^2$$

$$= E(x^2) - 2\mu^2 + \mu^2$$

$$= E(x^2) - \mu^2$$

* Similar result follows for continous random variable X.

Problem: Suppose a continous random variable X has the probability density.

$$f(x) = \begin{cases} k(1-x^2) & \text{for } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

find 'k'

(i) find $P(0.1<x<0.2)$ (ii) $P(x>0.5)$

using distribution function, determine the probabilities that

(iv) 'x' less than 0.3 i.e $F(X<0.3)$

(v) Between 0.4 & 0.6 i.e $F(0.4<x<0.6)$

(vi) Calculate mean & variance

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{1} k(1-x^2)\,dx = 1$$

$$k\left[x - \frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[2/3 - 0\right] = 1 \rightarrow \boxed{k = 3/2}$$

$$\begin{array}{r} 0.2 \\ 0.2 \\ \hline 0.04 \end{array} \quad 1-0.04$$

(i) $P(0.1<x<0.2) = F(b) - F(a)$

$\underline{a<x<b}$ $= \frac{3}{2}(1-(0.1)^2) - \frac{3}{2}(1-(0.2)^2) = \frac{3}{2}(0.99 - 0.96)$

$= \frac{3}{2}(0.03) = 0.045$

$$P(X>0.5) = \int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{1} f(x)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$$

$$= \frac{3}{2}\left[\frac{2}{3} - 0.4583\right]$$

$$= 0.31255$$

(iv) Cumulative distribution.

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$F(X<0.3) = \int_{-\infty}^{0.3} f(t)\,dt = \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0}^{0.3}$$

$$= 0.4365$$

(v) $$F(0.4<X<0.6) = \int_{0.4}^{0.6} f(t)\,dt$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.4}^{0.6}$$

$$= \frac{3}{2}\left(0.37866 - 0.528\right)$$

$$= 0.22401$$

(vi) Mean of the function:

$$mean = \mu = \int_{-\infty}^{\infty} x f(x) dx = \int_0^1 x \cdot \frac{3}{2}(1-x^2) dx = \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{2}\left[0 - [0.25]\right] = 3/8 = 0.375$$

(vii) Variance of the function:

$$Varience = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = \int_0^1 \left(x - \frac{3}{8}\right)^2 \frac{3}{2}(1-x^2) dx$$

$$= 0.59375 \qquad = 19/320$$

* If $f(x) = \begin{cases} 0 & x < 2 \\ \frac{2x+3}{18} & 2 \leq x \leq 4 \\ 0 & x > 4 \end{cases}$ then;

(i) Is $f(x)$ a density function

(2) If the above is a density function then find the probability in the closed interval of 2,3.

Since $\int_{-\infty}^{\infty} f(x) dx = 1$

$$\int_{-\infty}^{2} f(x) dx + \int_2^4 f(x) dx + \int_4^{\infty} f(x) dx$$

$$\int_{-\infty}^{2} 0\, dx + \int_2^4 \frac{2x+3}{18} + \int_4^{\infty} 0\, dx.$$

$$\frac{1}{18}\left[\int_2^4 2x + \int_2^4 3\right]$$

$$\frac{1}{18} \cdot \left[2 \cdot \frac{x^2}{2} + 3x\right]_2^4$$

$$\frac{1}{18}\left[x^2 + 3x\right]_2^4$$

$$\frac{1}{18}\left[28 - 12\right]$$

$$18/18 = \underline{1}.$$

∴ it is a probability density function.

$P(2 \leq x \leq 3) \Rightarrow \int_2^3 f(x)\,dx.$

$\Rightarrow \frac{1}{18}\left[x^2+3x\right]_2^3 \Rightarrow \frac{1}{18}\left[18-10\right] = \frac{8}{18} = 4/9 = 0.44$

$$f(x) = \begin{cases} 12x^3 - 21x^2 + 10x & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

(i). $P(x \leq 2)$ (ii) $P(x > 2)$

(iii) find k such that

$P(x \leq k) = 1/2$

i) $P(x \leq 2) = \int_{-\infty}^{2} f(x)\,dx$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$

$= \left[\frac{\cancel{12}^{3} \cdot x^4}{\cancel{4}} - \frac{\cancel{21}^{7} x^3}{\cancel{3}} + \frac{\cancel{10}^{5} x^2}{\cancel{2}}\right]_0^1$

$= \left[3 - 7 + 5\right]$

$= \left[8 - 7\right] = 1$

(iii) $P(x>2)$:

$$= \int_2^\infty f(x)\,dx$$

$$= \int_2^\infty 0\,dx = 0$$

v)

$$3x^4 - 7x^3 + 5x^2 = 1/2$$

$x = 0.45$

$\therefore K = 0.45$

$K = -0.26$

$K = 1.07 + 0.481$

$K = 1.07 - 0.481$

$$\boxed{\begin{aligned} f(K) &= 3x^4 - 7x^3 + 5x^2 - 1/2 \\ f(x) &= 6x^4 - 14x^3 + 10x^2 - 1 \\ K = 0 \quad f(0) &= -1 < 0 \end{aligned}}$$

Bisection methods.

Newton Raphson

ind the mean and varience of the following discrete probab distribution.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x = x_i$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
| $P(x=x_i)$ | $k$ | $0.1$ | $k$ | $0.2$ | $2k$ | $0.4$ | $2k$ |

$k+0.1+k+0.2+2k+0.4+2k = 1$

$6k = 1-0.7$

$k = \frac{0.3}{6} = 0.05$

$$\text{Mean} = E(x) = \sum x_i P(x=x_i)$$

$$= -3(0.05)+(-2)(0.1) + (-1)(0.05) + 1(2\times 0.05) + 2\times 0.4 + 3\times 2\times 0.05$$

$$= 0.8$$

$$V(x) = E(x^2) - \left[E(x)\right]^2 \quad (\text{or}) \quad \sum (x-\mu)^2 f(x)$$

$$E(x^2) = \sum x_i^2 P(x=x_i)$$

$$= (9\times 0.05) + 4(0.1) + 1\times 0.05 + 0 + (1\times 2\times 0.05) + 4(0.4)$$

$$= 3.5$$

$$\text{variance} = 3.5 - (0.8)^2$$

$$= 2.86.$$

$\sigma^2 = \sum (x-\mu)^2 P(x)$

$= (-3-0.8)^2(0.05) + (-2-0.8)^2(0.1) + (-1-0.8)^2(0.05)$

$+ (-0-0.8)^2(0.2) + (1-0.8)^2(0.1)$

$(3-0.8)^2(0.1) \quad + (2-0.8)^2(0.4)$

$-0.722 + -0.784$

$=$

If the probability distribution of a random variable is distribution as below $-2, -1, 0, 1, 2$ $\quad P(x) = 0.2, .1, .3, .3, .1$

find $E(x)$

$x = x_i$ $\quad V(x)$

$E(2x+3)$

$E(2x-3)$

$V(2x+3)$

$V(2x-3)$

Note: (i) In a gambling game, expected value of $E(x)$ of the game is considered to be the value of the game to the player.

The game is favourable to the player if $E>0$.

The game is unfavourable to the player, if $E<0$

The game is fair if $E=0$

(2) Mathematical expectation $E(x) = a_1p_1 + a_2p_2 + \cdots a_kp_k$ where the probabilities of obtaining the amounts $a_1, a_2, a_3 \ldots a_k$ are $p_1, p_2, p_3 \ldots p_k$ respectively.

Example: A player tossess 3 coins, He wins Rs.500 if 3 heads occurs, Rs 300 if 2 heads occurs, Rs.100 if one head occurs. On the other hand, he loses Rs 1500 if 3 tails occurs. Is it favourable. find the value of the game to the player.

let $X$ = no. of head occoring in tosses of a coin. 'X' is a discrete random variable, here

Sample space 'S' = {HHH, HHT, HTH, HTT, THH ... }

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

Probability of 3 head occurs = $P(X=3) = 1/8$

Probability of 2 head occurs = $P(X=2) = 3/8$

Probability of 1 head occurs = $P(X=1) = 3/8$.

∴ discrete probability distribution is

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(x)$ | 1/8 | 3/8 | 3/8 | 1/8 |

$$= 500\left(\frac{1}{8}\right) + 300\left(\frac{3}{8}\right) + 100\left(\frac{3}{8}\right) + 1500\left(\frac{1}{8}\right)$$

$$= 25$$

3/11/2022

## Probability Distributions:

### ① Binomial distribution:

Binomial distribution is a discrete probability distribution develo by James. Bernouli in 1700.

It is also known as Bernoulli distribution.

Properties of binomial or bernoulli distribution:

1. It describes the distribution of probabilities, when there are only two mutually exclusive outcomes for each trial of an experiment.

Eg: tossing a coin, occurrence of head, tail are mutually exclusive.

. Each trial is independent of other trails.

. The probability of success 'p' remains constant from trail to trail similarly the probability of failure 'q' or (1-p) remains constant over all observations.

. The process is performed under the same conditions for a finite number of times or trails, say n.

General model of Binomial distribution:

let 'n' be the finite number of trails, all the trails are independent with the probility of success 'p' for each trial and the probability of failure is 'q' then.

The probability of obtaining exactly 'r' times success out of 'n' trail is given by

$$f(r) = P(X=r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{n-r}$$

for $r = 0, 1, 2, 3 \ldots n$ & $p+q=1$

The probability of 'x' times success in n trails are

$$f(x) = P(X=x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

(or)

$$f(x) = P(X=x) = {}^{n}C_{x} \cdot p^{x} (1-p)^{n-x}$$

* It is denoted by $B(x; n, p)$

$$\sum_{x=0}^{n} B(x; n, p) = 1$$

$${}^{n}C_{0}\, p^{0} q^{n} + {}^{n}C_{1}\, p^{1} q^{n-1} + \cdots \cdots + {}^{n}C_{n}\, p^{n} q^{0} = 1$$

Mean of Binomial distribution:

the mean of the binomial distribution is : $np$

where; $n$ = no of trails

$p$ = probability of success.

Variance of Binomial distribution:

Variance of the binomial distribution is $\sigma^2 = npq$

$n$ = no. of trails

$q$ = probability of failure

① A fair coin is tossed 6 times, find the probability of getting

(i) exactly two heads

(ii) no head

(iii) at least 4 heads

(iv) at least one head.

$n(S) = 2^6 = 64$ $\quad$ $X$ = ~~occurrate~~ occurance of no. of heads

$= 0, 1, 2, 3, 4, 5, 6$

(i) Exactly two heads

$P(X=2) = {}^nC_2 \; p^2 \times q^{n-2}$

$n = 6$

$A \cap B = \phi$ $\begin{cases} A = \text{occurance of head} & P(A) = \frac{1}{2} = p \\ B = \text{occurance of tail} & P(B) = \frac{1}{2} = q \end{cases}$

${}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4$

$= \frac{15}{64}$

(ii) no heads

$P(X=0) = {}^nC_0\, p^0 \times q^{n-0}$

$= q^n$

$A \cap B = \phi$ $\begin{cases} A = \text{occrance of head } P(A) = 0 = P \\ B = \text{occ. of tail} = 1/6 \end{cases}$

$= (1/6)^6\ (1/2)^6 = 1/64$

(iii) $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= 6C_4 (1/2)^4 (1/2)^{6-4} + 6C_5 (1/2)^5 (1/2)$

$+ 6C_6 (1/2)^6 (1/2)^0$

$= \frac{1}{2^6} \left[ 6C_4 + 6C_5 + 6C_6 \right]$

$= 22/64 = 11/32$

(iv) $P(X=1) = P(X=1) + P(X=0)$

$P(X \geq 1)$

$= \cancel{1-1/64}\ 1-$

$1 - P(x=0)$

$P(X=1) = 1 - \frac{1}{64} = \frac{63}{64}$

② A fair die is thrown 7 times. Determine the probability that 5 or 6 appears

(i) exactly 3 times

(ii) never occurs

$n = 7$

probability of sucess $= 2/6 = 1/3$

probability of failure $= 4/6 = 2/3$

(i) $= P(X=3) = {}^7C_3 \cdot (1/3)^3 (2/3)^{7-3} = {}^7C_3 \left(\frac{16}{3^7}\right) = 0.2560$

(ii) $P(X=0) = {}^7C_0 (1/3)^0 (2/3)^7 = \frac{7!}{7!\,0!} \frac{2^7}{3^7} = \frac{2^7}{3^7}$

$= 0.05852$

③ In a sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts?

$20p = 2$

$p = 0.1$

$q = 1 - 0.1 = 0.9$

$1 - [P(X=1) + P(X=2) + P(X=3)]$

$1 - [{}^nC_0 (0.1)^{20} (0.9)^0 + {}^nC_1 (0.1)^{19} (0.9)^1 + {}^nC_2 (0.1)^{20} (0.9)^2]$

$= 0.323$

$\Rightarrow 100(0.323)$

$\rightarrow 323$ samples

itting of Binomial distribution:

Problem: fit a Binomial distribution to the following data

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| f | 7 | 6 | 19 | 35 | 30 | 23 | 7 | 1 |

let x be a random variable, no. of trails $= n = 7$

Mean of the Binomial distribution $= np$.

Mean of the frequency distribution $= \dfrac{\sum f.x}{\sum f}$.

$$\sum fx = 0(7) + 1(6) + 2(19) + 3(35) + 4(30) + 5(23) + 6(7) + 7(1)$$

$$= 433$$

$$\sum f = 7+6+19+35+30+23+7+1 = 128$$

$$\text{mean} = \frac{\sum f.x}{\sum f} = \frac{433}{128} = 3.38$$

$$\mu = np = 3.38$$

$$7p = 3.38$$

$$p = \frac{3.38}{7} = 0.48$$

$$q = 1 - 0.48 = 0.52$$

$\boxed{p = 0.48}$

$\boxed{q = 0.52}$

The expected Binomial frequencies are given by

$P(X=r) = {}^nC_r \cdot p^r q^{n-r} = {}^7C_r\,(0.48)^r\,(0.52)^{n-r}$

for $r = 0,1,2,3,4,5,6,7$

frequencies.

$r=0,\ {}^7C_0(0.48)^0(0.52)^7 = 0.01 \rightarrow f = 128 \times 0.01 = 1$

$r=1,\ {}^7C_1(0.48)^1(0.52)^6 = 0.07 \rightarrow f = 128 \times 0.07 = 9$

$r=2,\ {}^7C_2(0.48)^2(0.52)^5 = 0.19 \rightarrow f = 128 \times 0.19 = 24$

$r=3,\ {}^7C_3(0.48)^3(0.52)^4 = 0.27 \rightarrow f = 128 \times 0.27 = 34$

$r=4,\ {}^7C_4(0.48)^4(0.52)^3 = 0.25 \rightarrow f = 128 \times 0.25 = 32$

$r=5,\ {}^7C_5(0.48)^5(0.52)^2 = 0.17 \rightarrow f = 128 \times 0.17 = 22$

$r=6,\ {}^7C_6(0.48)^6(0.52)^1 = 0.04 \rightarrow f = 128 \times 0.04 = 5$

$r=7,\ {}^7C_7(0.48)^7(0.52)^0 = 0.06 \rightarrow f = 128 \times 0.06 = 1$

Hence fitted binomial distribution is.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| f | 7 | 6 | 19 | 35 | 30 | 23 | 7 | 1 |
| Expected frequencies f | 1 | 9 | 24 | 34 | 32 | 22 | 5 | 1 |

Recurrence relation or Recursive formula for B.D

Shows the relation between 2 consecutive probabilites.

i.e: $P(r+1) = \frac{(n-r)}{r+1} \cdot \frac{p}{q} P(r)$

The ratio of the probabilities of 3 success and 2 success among 5 independent trials is 1/3. Find the probability of getting success 'p'?

$P(x=r) = nC_r \, p^r . q^{n-r}$

$\frac{P(x=3)}{P(x=2)} = \frac{1}{3}$

$\frac{5C_3 . p^3 . q^2}{5C_2 p^2 q^3} = \frac{1}{3}$ $[5C_3 = 5C_2]$

$\frac{p}{q} = \frac{1}{3}$

$\frac{p}{1-p} = \frac{1}{3}$

$3p = 1-p$

$4p = 1$

$\boxed{p = 1/4}$ $\boxed{q = 3/4}$

Poison Distribution:

If the parameters 'n' and 'p' of a binomial distribution are known, where 'n' is very large and 'p' is very small then the application of Binomial distribution becomes difficult.

* However if we assume that as $n \to \infty$ and $p \to 0$ such that 'np' always remains finite (say $\lambda$), we get Poisson Approximation of binomial distribution.

$$\boxed{P(X=r) = \frac{\lambda^r}{r!} e^{-\lambda}, \text{ for } r = 0,1,2,3 \ldots}$$

* This probability distribution is called poison Probability Distribution

* '$\lambda$' is the parameter of the distribution.

* The sum of the poisson probabilites for $r = 0,1,2 \ldots$ is '1'

i.e: $P(0) + P(1) + P(2) + \ldots = e^{-\lambda} + e^{-\lambda}\frac{\lambda}{1!} + e^{-\lambda}\frac{\lambda^2}{2!} + e^{-\lambda}\frac{\lambda^3}{3!} + \ldots$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} \ldots\right]$$

$$= e^{-\lambda} \cdot e^{\lambda} = e^0 = 1$$

Mean and varience of poisson distribution:-

~~mean eleed~~

the mean of the poisson distribotion is "$\lambda$"

Varience of a poisson distribution is also "$\lambda$"

Recurrance relation for Poisson Distribution:

the recurrance relation in the poisson distribution show the relation b/w a consecutive probabilities i.e $P(r+1) = \frac{\lambda}{r+1} P(r)$

Determine the probability that 2 of 100 books bound will be defective if it is known that 5% of books bound at this bindery are defective

(i) Use Binomial distribution

(ii) Use poisson approximation to binomial distribution.

$n = 100$

$p = 5\%$ defective $= 0.05$

$q = 1 - p = 1 - 0.05 = 0.95$

(i) $P(x=2) = nC_2 p^2 q^{n-2} = 100C_2 (0.05)^2 (0.95)^{100-2} = 0.081$

(ii) $\lambda = np = 100 \times (0.05) = 5$

$P(x=2) = \frac{\lambda^2 . e^{-\lambda}}{2!} = \frac{5^2 . e^{-5}}{2!} = \frac{25 . e^{-5}}{2} = 0.084$

All the average 1 person in 1000 makes a ~~xxx~~ numerical error in preparing IT Returns If 10,000 forms are selected at random & examined find the probability that 6 (or) 7 (or) 8 forms will be in error.

$n = 1000$

$P = \frac{1}{1000} = 0.001$

$\lambda = np = 10000 \times 0.001 = 10$

$P(X=6) + P(x=7) + P(x=8) = \frac{\lambda^6 . e^{-\lambda}}{6!} + \frac{\lambda^7 e^{-\lambda}}{7!} + \frac{\lambda^8 e^{-\lambda}}{8!}$

$\frac{1}{6!} ( \lambda^{x} \; = \frac{e^{-\lambda}}{6!} + \frac{e^{-\lambda}}{7!} + \frac{e^{-\lambda}}{8!} =$

$$= \frac{10^6 \cdot e^{-10}}{6!} + \frac{10^7 e^{-10}}{7!} + \frac{10^8 \cdot e^{-10}}{8!}$$

$$\frac{10^6 \cdot e^{-10}}{6!}\left[1 + \frac{10}{7} + \frac{100}{8 \cdot 7}\right]$$

$$= \frac{10^6 e^{-10}}{6!}\left[\frac{1}{1} + \frac{10}{7} + \frac{100}{8 \cdot 7}\right]$$

0.0620 (14.29)

(10.2045)

$$\begin{array}{r} 0.0630 \\ 0.0900 \\ 0.1125 \\ \hline 0.2655 \end{array}$$

$$= \frac{10^6 \cdot e^{-10} \cdot 209}{14 \cdot 6!} = \frac{9488.58532}{10{,}080}$$

$$= 0.94132$$

fitting of a poison distribution:

**Problem:** fit a poison distribution to the following data

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| f | 30 | 62 | 46 | 10 | 2 |

$$\bar{x} = \frac{\sum f \cdot x}{\sum f} = \frac{(0 \times 30) + (1 \times 62) + (2 \times 46) + (10 \times 3) + (4 \times 2)}{150} = \frac{192}{150}$$

$$= 1.28$$

Poisson distribution $P(X=x) = \dfrac{e^{-1.28} \times (1.28)^x}{x!}$

$x = 0$ $P(X=0)$ $\dfrac{e^{-1.28} \times (1.28)^0}{0!} = e^{-1.28}$ $0.2780 \Rightarrow 41.7$

$x = 1$ $P(X=1)$ $e^{-1.28} \times (1.28)^1 = 0.355 \Rightarrow 0.355 \times 150 = 53$

$x = 2$ $P(X=2)$ $e^{-1.28} \times (1.28)^2/2 = 0.2277 \Rightarrow 34.165$

$x = 3$ $P(X=3)$ $e^{-1.28} \times (1.28)^3/6 = 0.0161 \Rightarrow 2.415$

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| f | 30 | 62 | 46 | 10 | 2 |
| | 42 | 54 | 34 | 15 | 5 |

## Normal distribution or Gaussian Distribution:

Normal probability distribution or is the probability distribution of a continuous random variable X, known as normal variable.

It is given by (density function of Normal distribution)

$$N(\mu,\sigma) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

here, $\mu$ = mean

$\sigma$ = standard deviation

The distribution is most important, simple, useful and is the cornerstone of modern statistics because.

(1) Discrete probability distributions such as Binomial, poisson, hyper geometric can be approximated by normal distribution

(2) It is applicable in statistical quality control industry

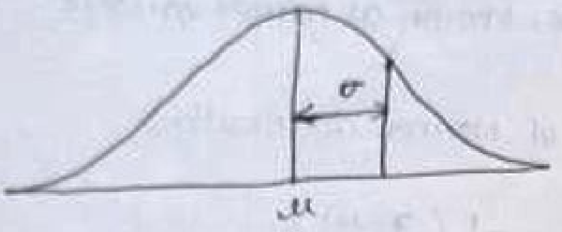
## Characteristic of normal distribution curve:

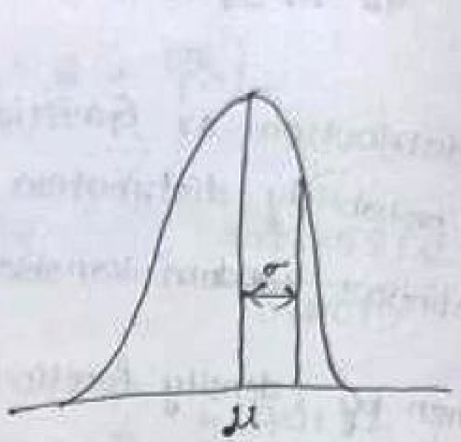
1. The graph of the normal distribution $y = f(x)$ in xy-plane is bell shaped and symmetrical about the line $x = \mu$.

2. The maximum value of gaussian density function is

$$\frac{1}{\sqrt{2\pi\sigma^2}}$$ (it will occur at $x = \mu$)

3. The mean, median and mode are coincide and therefore normal curve has only one maximum point.




* A normal distribution is bell-shaped and symmetric.

* The distribution is determined by the mean($\mu$), and the standard deviation sigma.

* The mean($\mu$) control the center and sigma controls the spreat

4. Normal curve has inflection points at $\mu \pm \sigma$

5. Normal curve is asymptotic to both positive X-axis & negative X-axis.

6. Area under the Normal curve is unity

7. Change of scale from X-axis to Z-axis

$$P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2} dx$$

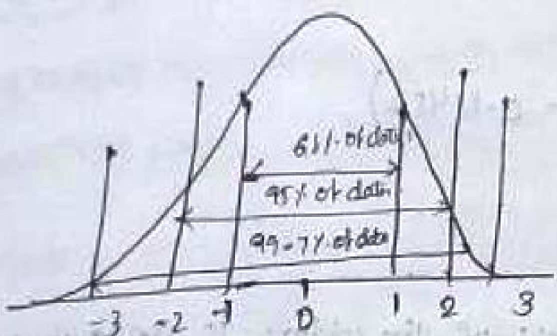
$$P(z_1 \leq Z \leq z_2) = \int_{z_1}^{z_2} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-z^2/2} dz$$

where,

$z_1 = \frac{x_1 - \mu}{\sigma} \quad z_2 = \frac{x_2 - \mu}{\sigma}$

(8) N(0,1) is known as Standard Normal Distribution with mean '0' and standard deviation '1'

(9) Area under the Normal curve is distributed as follows




* the standard normal distribution has mean = 0

Find the areas of a standard normal distribution to the following questions

(i) To the left of $z = -1.27$

(ii) To the left of $z = 0.8$

(iii) To the right of $z = 2.16$

(iv) In between $-1$ to $2$ $(-1 < z < 2)$

(v) To the left of $-1.7$ and to the right of $1.6$

(i) $z = -1.27$

$R.A = 0.5 - \begin{pmatrix} z = 0 \text{ to} \\ z = -1.27 \end{pmatrix}$

$= 0.5 - (0.3980)$

$= 0.102$

(ii)

$R.A = (0.5 + 0.2881) + 0.5$

$= 0.2219 + 0.5$

$= 0.7219 \quad 0.7881$

(iii) R.A = 0.5 − 0.4846

R.A = 0.0154

(iv) R.A = $\begin{pmatrix} z=0 \\ \text{to } z=-1 \end{pmatrix}$ to $\begin{pmatrix} z=0 \\ \text{to } z=2 \end{pmatrix}$

= .3413 + 0.4772

$\boxed{R.A = 0.8185}$

(iv) R.A (0.5 − 0.4654) + (0.5 − 0.4452)

R.A 0.0894

* Assume that the mean height of the solders is 68.22 inches with a variance of 10.8 inches. An regimen has 2000 soldiers then find the no of soldiers whose height is more than 6 feet (we can assume that height follows the standard normal distribution)

Mean = $\mu$ = 68.22 inches

variance − $\sigma^2$ = 10.8 inches

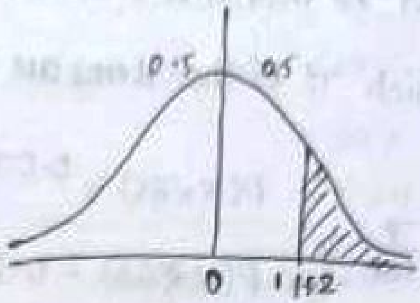
S.d = $\sigma = \sqrt{10.8}$ − 3.286 inches

no of soldiers = 2000

$z = \frac{x-\mu}{\sigma}$   6 feet = 6×12 inches = 72 inches.

$P(x>72) - P\left(z > \frac{72-68.22}{3.286}\right) = P(z > 1.150)$

$R.A = 0.5 - [0.3749]$




$R.A = 0.1251$

No. of soldiers $= 2000 \times 0.1251$

$= 250.2$

Determine the expected no. of boys whose weight is b/w 65 to 70 kgs (ii) >72 kgs. if the weight of 800 boys follows the normal distribution with mean as 66 kg and standard deviation 5 kgs.

Mean $= \mu = 66$ kg

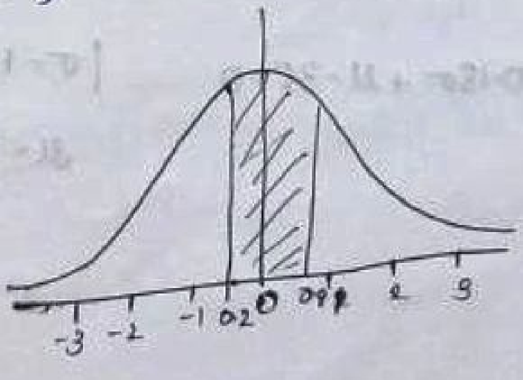
variance $=$ $s.d = 5$ kgs

no. of boys $= 800$

$$z = \frac{x-\mu}{\sigma} = \frac{65 - 66}{5} = -0.2$$

$$\frac{70-66}{5} = 0.8$$

$R.A = 0.0793 + 0.2881$

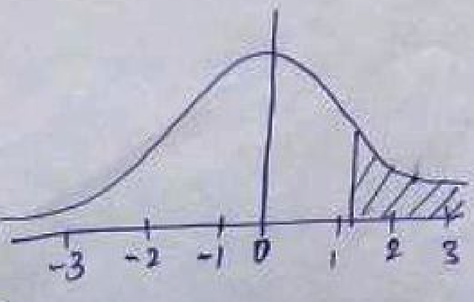



$R.A = 0.3674$

$= 800 \times 0.3674$

$= \underline{294}$

$$R.A = \frac{72-66}{5} = 1.2$$




$R.A = 0.5 - 0.3849$

$R.A = 0.1151$ $\quad = 800 \times \underline{0.1151}$

$= \underline{92}$

* Find the mean and standard deviation of a normal distribution in which 7% of the items are under 35 and 89% under 63.

$P(X<35) = 0.07$

$\frac{7}{100}$ $P(X<63) = 0.89$

$z = \frac{X-\mu}{\sigma} \Rightarrow$ left side area $= \frac{7}{100} = 0.07$

~~0.0714~~

$z = 0.18$

Right side area $= 89\% - 50\%$

$= 39\%$

$= 0.39$

$z = 1.23$

$-0.18 = \frac{35-\mu}{\sigma}$ $\qquad$ $1.23 = \frac{63-\mu}{\sigma}$

$-0.18\sigma = 35-\mu$ $\qquad$ $1.23\sigma + \mu - 63 = 0$

~~850 a1~~

$-0.18\sigma + \mu - 35 = 0$ $\qquad$ $\sigma = 19.86$

$\mu = 38.5744$

$$-1.48 = \frac{35-\mu}{\sigma}$$

$$\boxed{-1.48\sigma + \mu = 35}$$

Right side Area $= 89\% - 50\%$

$= 39\%$

$= 0.39$

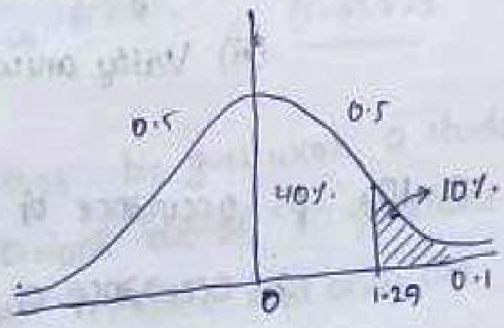
$$\boxed{z = 1.23}$$

$$1.23 = \frac{63-\mu}{\sigma}$$

$$\boxed{1.23\sigma = 63-\mu}$$

*Determine the minimum mark a student must get to recieve A grade if the top 10% of the students are awarded with A grade in an examiniation where mean mark is 72 s.d is 9.

$\mu = 72 \quad s.d = \sigma = 9$




$$z = \frac{x-\mu}{\sigma}$$

$$1.29 = \frac{x-72}{9}$$

$$x = 72 + (1.29)9$$

$$= 72 + 11.61$$

$$= \underline{\underline{83.61}}$$

## Normal approximation to binomial distribution:-

let $x$ be a random variable then from binomial distribution mean $= np$ and standard deviation is $\sqrt{npq}$ to convert the problem in to normal variable the value of

$$\boxed{z = \frac{x - np}{\sqrt{npq}}}$$

**Note:** Normal approximation to binomial distribution is useful only when $np$ or $nq >= 5$

Find the probability for getting 4 to 6 times head out of 10 tosses of a coin. by (i) using normal approximation to binomial dis

(ii) Verify answer with binomial distribution.

$n = 10$, $p =$ occurance of head $= 1/2$

$q =$ Occurance of tail $= 1/2$

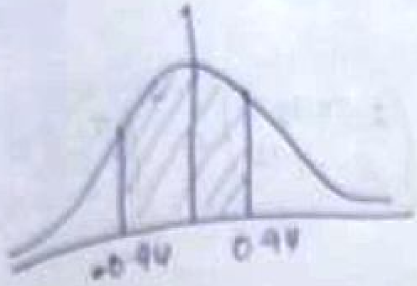
$np = 10 \times 1/2$

$\sigma = \sqrt{npq} = \sqrt{10/4} = 1.58$

(i) $P(4 \le x \le 6) = P(3.5 < z < 6.5)$

$$z_1 = \frac{x - \mu}{\sigma} = \frac{3.5 - 5}{1.58} = -0.94$$

$$z_2 = \frac{6.5 - 5}{1.58} = 0.94$$




$2\begin{pmatrix} z = 0.10 \\ x = 0.94 \end{pmatrix}$

$2(0.3264) = 0.6528$

$P(4 \le x \le 6) = P(x=4) + P(x=5) + P(x=6)$

$= {}^{10}C_4 (1/2)^4 (1/2)^6 + {}^{10}C_5 (1/2)^5 (1/2)^5 + {}^{10}C_6 (1/2)^4 (1/2)^6$

$= \frac{10!}{(6)!(4)!} \times \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^6 + \frac{252}{2^{10}} + \frac{210}{2^{10}}$

$= \frac{210}{2^{10}} \Rightarrow \frac{672}{2^{10}} = \underline{0.65625}$

* Determine the probability that by guess work a student can answer 25 to 30 mc-quiz questions out of 80 questions assume that each question with 4 choice and only one of them is correct and student have no knowledge.

$P(24.5 < x < 30.5)$

$\mu = \underline{np}$ $\quad p = (1/4) \quad q = (3/4)$

$\mu = 80 \times 1/4 = 20$

$\sigma = \sqrt{npq} = \sqrt{80 \times (1/4)(3/4)} = 3.872.$

$z_1 = \frac{24.5 - 20}{3.872}$ $\qquad z_2 = \frac{30.5 - 20}{3.872}$

$z_1 = -1.1621$ $\qquad z_2 = 2.711.$


$R.A = \begin{pmatrix} z = 0 \text{ to} \\ z = -1.16 \end{pmatrix} + \begin{pmatrix} z = 0 \text{ to} \\ z = 2.71 \end{pmatrix}$

$R.A = 0.3770 + 0.4966$

$R.A = 0.8736$

$P(x \leq 24) + P(x \leq 25) + P(x \leq 26)$