Assignment 2 – Due March 1, 2023

1) Consider a game where you role three fair dice and count the total. The dice are eight, ten and twelve sided respectively.

   (a) Write an R function that will compute one round of this game. Have it return the values of the three dice and their total. Store the dice roles by the number of sides of the dice (smallest to largest).

   (b) Using the code in (a) write a function that will simulate n rounds of the game.

   (c) Suppose you win another role if the total of the three dice roles is less than or equal to 14. Write an R function that will compute the proportion of roles that meet this criteria after n rounds.

   (d) Run (c) for $n = 1000000$. Repeat this five times and look at the results.

   (e) What does this tell you about the odds of rolling a total of 14 or less?

   (f) Suppose you win \$100 if you role triple sevens. Write an R function that will compute the proportion of roles that meet this criteria after n rounds.

   (g) Run (f) for $n = 1000000$. Repeat this five times and look at the results.

   (h) What does this tell you about the odds of rolling triple sevens?

   (i) Suppose the game ends if you either role triple sevens or run out of free roles (sum of dice > 14). Write an R function that returns a 1 if you win and zero otherwise.

   (j) Using (i) what do you think are the odds of winning this game?

   Submit your R code as *q1.R*. Also, please label each part (a)-(j) with a comment header (i.e. # (a)). Use commented text (#) to answer any textual question.

2) Consider the following joint probability density function

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \mathrm{e}^{-\frac{1}{2(1-\rho^2)}(x^2-2\rho xy+y^2)}, \quad x, y \in \mathbb{R}, \rho \in [-1,1].$$

   A special case of the bivariate normal distribution centered at $(\mu_X, \mu_Y) = (0,0)$, with component variances $(\sigma_X^2, \sigma_Y^2) = (1,1)$ and correlation parameter ρ.

   (a) Write an R function that computes $f(x,y)$ for any value of x, y and ρ.

   (b) Plot (a) with $\rho = 0$ for $x \in [-3,3]$ and $y \in [-3,3]$. Use light blue, values of *phi* and *theta* of 30 and add "f(x,y)" as a label for the z axis.

   (c) Using (b) plot with $phi = theta = i$, $i = 0, 10, 20, 30, \ldots, 100$. Do this in a for loop with a pause of one second between each plot. Hint: Use R's help on *Sys.sleep()*.

   (d) Repeat (b) with $\rho = -0.8, -0.5, 0.0, 0.5, 0.8$. What do you observe?

   (e) Plot $f(x,1)$ on $x \in [-3,3]$ with $\rho = 0$. What does this curve look like to you?

   (f) Plot $f(1,y)$ on $y \in [-3,3]$ with $\rho = 0$.

   Submit your R code as *q2.R* using the same conventions as question 1).