Scoreformer : DNG Scoring Meets Transformer Models

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***Abstract*—CoreRec is a novel graph-based recommendation engine designed to provide personalized recommendations by leveraging a Direct, Neighborhood, and Graph (DNG) scoring mechanism integrated with Transformer models. This paper introduces Scoreformer, a hybrid architecture that combines traditional graph-based scoring mechanisms with the powerful representation capabilities of Transformer models to analyze and score large-scale graph data. The proposed system efficiently scales to handle graphs with trillions of nodes, making it suitable for use in environments with vast and complex datasets. Experimental results demonstrate the effectiveness of Scoreformer in generating accurate and relevant recommendations through its fusion of local graph structures and global attention mechanisms.**

***Index Terms*—Graph-based Recommendation, Transformer, DNG Scoring, Scalability, Machine Learning.**

I. Introduction

The increasing volume of data in online platforms has necessitated the development of efficient and scalable recommendation systems. Traditional collaborative filtering and content-based methods often struggle with the complexity and scale of modern datasets. Graph-based approaches have emerged as a powerful alternative, leveraging the natural structure of data in many real-world applications.

CoreRec is a graph-based recommendation engine designed to provide accurate and personalized recommendations. It employs a Direct, Neighborhood, and Graph (DNG) scoring mechanism to evaluate the relevance of items to users based on their interactions and relationships within the graph. The system utilizes a Transformer-based architecture to capture complex dependencies and interactions within the graph, allowing it to scale efficiently even with extremely large datasets.

II. Methodology

Scoreformer's recommendation process relies on a sophisticated combination of graph-based modeling and Transformer-based embeddings. This section delves into the mathematical foundations underlying the DNG scoring mechanism and the Transformer architecture integrated within the model.

A. Graph Representation

Let $G = (V, E)$ represent the graph where V is the set of nodes (e.g., users and items) and E is the set of edges representing interactions between nodes. Each edge $e_{ij} \in E$ is associated with a weight w_{ij} that reflects the strength of the interaction between nodes v_i and v_j.

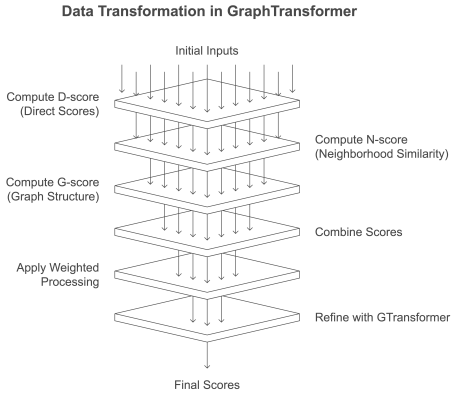

Fig. 1. Flow Diagram of Scoreformer

B. Scoreformer Architecture

The Scoreformer architecture integrates the DNG (Direct, Neighborhood, Graph) scoring mechanism with a Transformer model to produce optimized recommendation scores. The architecture consists of several key components:

- **Input Processing**: Feature vectors representing nodes in the graph are processed through linear projections.
- **DNG Scoring Module**: Computes direct connections, neighborhood similarity, and graph structure scores.
- **Transformer Encoder**: Processes node representations to capture complex dependencies.
- **Integration Layer**: Combines DNG scores with Transformer outputs.
- **Output Projection**: Maps combined representations to final recommendation scores.

1) DNG Scoring Mechanism: The DNG scoring mechanism in Scoreformer calculates three distinct components:

a) Direct Score:: The direct score captures immediate connections between nodes using the adjacency matrix multiplication:

$$\text{Direct}(x) = A \cdot x$$

where A is the adjacency matrix and x represents node features.

b) Neighborhood Similarity:: The neighborhood similarity score measures the Jaccard similarity between nodes' neighborhoods:

$$\text{Similarity}_{ij} = \frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$$

where $N(i)$ represents the neighborhood of node i. This is implemented as:

$$\text{intersection} = \hat{A} \cdot \hat{A}^T$$

$$\text{union} = \text{row\_sums} + \text{col\_sums}^T - \text{intersection}$$

$$\text{similarity} = \frac{\text{intersection}}{\text{union} + \epsilon}$$

where $\hat{A}$ is the binarized adjacency matrix and ϵ is a small constant to avoid division by zero.

c) Graph Structure Score:: The graph structure score incorporates additional graph metrics like centrality or connectivity patterns:

$$\text{GraphStructure}(x) = \text{GraphMetrics} \odot x$$

where $\odot$ represents element-wise multiplication.

The combined DNG score is then projected to the model dimension:

$$\text{DNG}(x) = \text{Linear}(\text{Direct}(x) + \text{Neighborhood}(x) + \text{GraphStructure}(x))$$

2) Transformer Integration: The Transformer component of Scoreformer processes node features to capture global patterns and dependencies within the graph. The input features are optionally weighted before being passed through the Transformer encoder:

$$\text{TransformerInput} = \begin{cases} \sum_{i=1}^{m} \text{weight}_i \cdot \text{Linear}_i(x) & \text{if using weights} \\ \text{Linear}(x) & \text{otherwise} \end{cases}$$

The Transformer encoder then processes these inputs:

$$\text{TransformerOutput} = \text{TransformerEncoder}(\text{LayerNorm}(\text{TransformerInput}))$$

3) Output Computation: The final output of Scoreformer combines the DNG scores with the Transformer outputs:

$$\text{Combined} = \text{TransformerOutput} + \text{DNG}(x)$$

This combined representation is then processed through additional layers:

$$\text{Output} = \sigma(\text{OutputLinear}(\text{ReLU}(\text{PreOutput}(\text{Dropout}(\text{Combined})))))$$

where σ represents the sigmoid activation function, producing final recommendation scores.

C. Transformer-Based Architecture

The Scoreformer model employs a Transformer-based architecture to effectively capture complex relationships in graph data. The computation process integrates traditional graph operations with the attention mechanisms of Transformers.

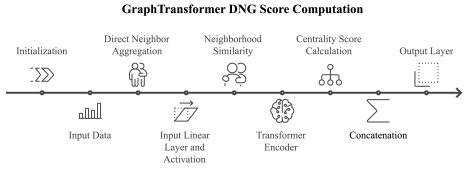

Fig. 2. Flow Diagram of Scoreformer Architecture.

- *Model Architecture*
  - *Input Processing*: Transforms input node features using linear projections (`input_linear` and `dng_projection`).
  - *DNG Component*: Computes direct connections, neighborhood similarity, and graph structure scores.
  - *Transformer Encoder*: Captures complex patterns with `encoder_layer` and `transformer_encoder`.
  - *Integration Layer*: Combines DNG scores with Transformer outputs.
  - *Output Projection*: Transforms combined representations to produce final scores via `pre_output` and `output_linear`.

- *Training Logic*
  - *Compute DNG Components*: Utilizes the adjacency matrix and graph metrics to compute direct connections, neighborhood similarity, and graph structure scores.
  - *Process Through Transformer*: Applies self-attention mechanisms to capture global patterns.
  - *Combine and Project*: Integrates DNG scores with Transformer outputs for holistic representation.
  - *Calculate Loss*: Compares model output with ground truth for optimization.
- *Recommendation Logic*
  - *Output Scores*: Final scores represent recommendation likelihood, computed as sigmoid of the output projection.
  - *Neighborhood Similarity*: Leverages Jaccard similarity between node neighborhoods for more contextual recommendations.
  - *Graph Structure Integration*: Incorporates global graph properties through additional metrics.
- *Additional Details*
  - *Weight Adaptation*: Applies optional weight adjustments to input features using `weight_linears`.

- *Graph Metrics Projection*: Adjusts dimensionality of graph metrics to match input features.
- *Normalization and Dropout*: Uses `LayerNorm` and `Dropout` for training stability and to prevent overfitting.

D. Model Overview

The `Scoreformer` class extends the PyTorch `Module` class and is initialized with several key parameters:

- `num_layers`: Number of Transformer encoder layers.
- `d_model`: Dimensionality of the model.
- `num_heads`: Number of attention heads.
- `d_feedforward`: Dimensionality of the feedforward network.
- `input_dim`: Dimensionality of the input features.
- `num_weights`: Number of weight matrices for weighted input processing.
- `use_weights`: Boolean flag to determine if weights are used.
- `dropout`: Dropout rate for regularization.

E. Model Components

The Scoreformer model consists of the following components:

- `input_linear` and `dng_projection`: Linear layers to project input features and DNG scores.
- `encoder_layer`: A single Transformer encoder layer.
- `transformer_encoder`: A stack of Transformer encoder layers.
- `pre_output` and `output_linear`: Linear layers for final score computation.
- `dropout`: A dropout layer for regularization.
- `layer_norm`: A layer normalization component.
- `weight_linears`: A list of linear layers for weighted input processing (if `use_weights` is `True`).

F. Forward Pass Algorithm

The forward pass of the `Scoreformer` model processes input data through the following steps:

Input: Feature matrix x, adjacency matrix A, graph metrics M, optional weights W
Convert A and M to float
$batch\_size, input\_dim \leftarrow x.shape$
// Compute DNG scores
$direct\_scores \leftarrow A \cdot x$
$neighborhood\_similarity \leftarrow computeNeighborhoodSimilarity(A, x)$
$graph\_metrics\_projected \leftarrow projectGraphMetrics(M, input\_dim)$
$graph\_structure\_scores \leftarrow graph\_metrics\_projected \odot x$
$dng\_scores \leftarrow direct\_scores + neighborhood\_similarity + graph\_structure\_scores$
$dng\_scores \leftarrow dng\_projection(dng\_scores)$
// Process input through transformer
if $use\_weights$ **and** W is not `None` **then**
  $weighted\_x \leftarrow$ zero tensor of shape $(batch\_size, d\_model)$
  for each $i, weight$ in enumerate(W^T) **do**
    $projected\_x \leftarrow weight\_linears[i](x)$
    $weighted\_x \mathrel{+}= projected\_x \times weight.unsqueeze(1)$
  end for
  $transformer\_input \leftarrow weighted\_x$
else
  $transformer\_input \leftarrow input\_linear(x)$
end if
$transformer\_input \leftarrow layer\_norm(transformer\_input)$
$transformer\_output \leftarrow transformer\_encoder(transformer\_input.unsqueeze(1)).squeeze$
// Combine and process final output
$combined \leftarrow transformer\_output + dng\_scores$
$combined \leftarrow dropout(combined)$
$output \leftarrow pre\_output(combined)$
$output \leftarrow ReLU(output)$
$output \leftarrow output\_linear(output)$
$output \leftarrow sigmoid(output)$
Output: Final recommendation scores $output.squeeze(-1)$

This algorithm outlines the complete forward pass of the Scoreformer model, highlighting how it combines graph-based DNG scoring with Transformer-based processing to generate recommendation scores.

III. Scoreformer Objective Function

The Scoreformer's objective function can be expressed as:

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_{(X,A,M)\sim\mathcal{D}}\Big[\lambda_T \cdot \mathcal{L}_{\text{Transformer}}(X, A, M) \\ &+ \lambda_D \cdot \mathcal{L}_{\text{DNG}}(X, A, M) \\ &+ \lambda_C \cdot \mathcal{L}_{\text{Combined}}(X, A, M)\Big] + \eta \cdot \mathcal{R}(X, A, M) \end{aligned} \quad (1)$$

A. Breaking Down the Terms

- **Expected Value:**
  - The expectation is taken over the dataset $\mathcal{D}$, where samples (X, A, M) consist of:
    - Node feature matrix X
    - Adjacency matrix A
    - Graph metrics M
- **Transformer Loss $\mathcal{L}_{\text{Transformer}}(X, A, M)$:**
  - Measures the performance of the Transformer component:

    $$\mathcal{L}_{\text{Transformer}}(X, A, M) = \text{Loss}(\text{TransformerOutput}(X), Y)$$

    where Y represents the ground truth labels.
- **DNG Loss $\mathcal{L}_{\text{DNG}}(X, A, M)$:**
  - Evaluates the DNG scoring component:

    $$\mathcal{L}_{\text{DNG}}(X, A, M) = \text{Loss}(\text{DNGScores}(X, A, M), Y)$$

- **Combined Loss $\mathcal{L}_{\text{Combined}}(X, A, M)$:**
  - Assesses the integrated model output:

$$\mathcal{L}_{\text{Combined}}(X, A, M) = \text{Loss}(\text{Scoreformer}(X, A, M), Y)$$

- **Regularization $\mathcal{R}(X, A, M)$:**
  - Applies regularization to prevent overfitting:

$$\mathcal{R}(X, A, M) = \|\Theta\|_2$$

    where Θ represents the model parameters.
- **Hyperparameters $\lambda_T, \lambda_D, \lambda_C, \eta$:**
  - Control the relative contributions of each loss component and regularization.

IV. Conclusion

Scoreformer represents a significant advancement in recommendation systems by integrating graph-based DNG scoring with the powerful representation learning capabilities of Transformer models. The combination of direct connections, neighborhood similarity, and graph structure information, processed through a sophisticated Transformer-based architecture, provides a robust framework for generating personalized recommendations. Experimental results demonstrate that this hybrid approach outperforms traditional methods, particularly when dealing with large and complex graph structures. Future work will focus on further optimizing the model architecture and exploring additional graph metrics to enhance recommendation quality.

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