SYLLABUS

UNIT – I

PRECESSION : Gyroscopes, effect of precession motion on the stability of moving vehicles such as motor car, motor cycle, aero planes and ships. Static and dynamic force analysis of planar mechanisms.

UNIT – II

FRICTION : Inclined plane, friction of screw and nuts, pivot and collar, uniform pressure, uniform wear, friction circle and friction axis : lubricated surfaces, boundary friction, film lubrication.

UNIT –III

Clutches: Friction clutches- Single Disc or plate clutch, Multiple Disc Clutch, Cone Clutch, Centrifugal Clutch.

BRAKES AND DYNAMOMETERS : Simple block brakes, internal expanding brake, band brake ofvehicle. Dynamometers – absorption and transmission types. General description and methods of operations.

UNIT – IV

TURNING MOMENT DIAGRAM AND FLY WHEELS : Turning moment – Inertia Torque connecting rod angular velocity and acceleration, crank effort and torque diagrams – Fluctuation of energy – Fly wheels and their design.

UNIT-V

GOVERNERS : Watt, Porter and Proell governors. Spring loaded governors – Hartnell and hartung with auxiliary springs. Sensitiveness, isochronism and hunting.

UNIT – VI

BALANCING : Balancing of rotating masses Single and multiple – single and different planes.

UNIT –VII

Balancing of Reciprocating Masses: Primary, Secondary, and higher balancing of reciprocating masses. Analytical and graphical methods. Unbalanced forces and couples – examination of "V" multi cylinder in line and radial engines for primary and secondary balancing, locomotive balancing – Hammer blow, Swaying couple, variation of tractive efforts.

UNIT – VIII

VIBRATION : Free Vibration of mass attached to vertical spring – oscillation of pendulums, centers of oscillation and suspension. Transverse loads, vibrations of beams with concentrated and distributed loads. Dunkerly's methods, Raleigh's method. Whirling of shafts, critical speeds, torsional vibrations, two and three rotor systems. Simple problems on forced damped vibration Vibration Isolation & Transmissibility

TEXT BOOKS :

1. Theory of Machines / S.S Ratan/ Mc. Graw Hill Publ.
2. Theory of Machines / Jagadish Lal & J.M.Shah / Metropolitan.

REFERENCES:

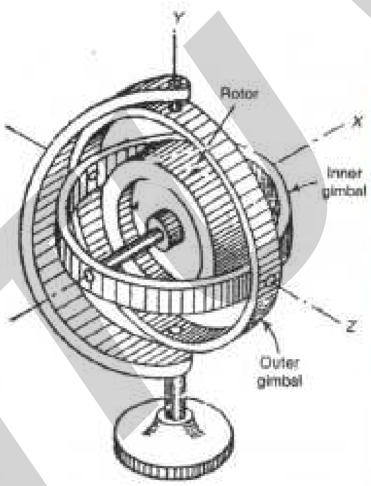
1. Mechanism and Machine Theory / JS Rao and RV Dukkipati / New Age
2. Theory of Machines / Shiegly / MGH
3. Theory of Machines / Thomas Bevan / CBS Publishers
4. Theory of machines / Khurmi/S.Chand.

UNIT – I
PRECESSION

Introduction

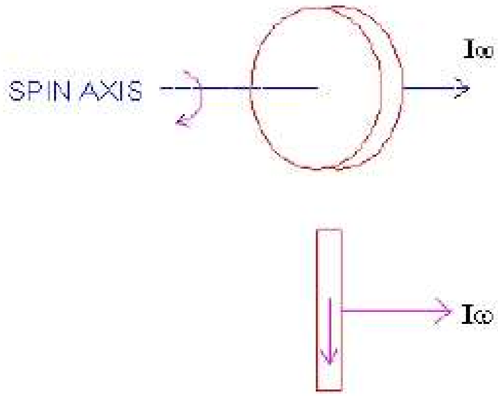
'Gyre' is a Greek word, meaning 'circular motion' and Gyration means the whirling motion. A gyroscope is a spatial mechanism which is generally employed for the study of precessional motion of a rotary body. Gyroscope finds applications in gyrocompass, used in aircraft, naval ship, control system of missiles and space shuttle. The gyroscopic effect is also felt on the automotive vehicles while negotiating a turn.

A gyroscope consists of a rotor mounted in the inner gimbal. The inner gimbal is mounted in the outer gimbal which itself is mounted on a fixed frame as shown in Fig.. When the rotor spins about X-axis with angular velocity ω rad/s and the inner gimbal precesses (rotates) about Y-axis, the spatial mechanism is forced to turn about Z-axis other than its own axis of rotation, and the gyroscopic effect is thus setup. The resistance to this motion is called gyroscopic effect.



ANGULAR MOTION

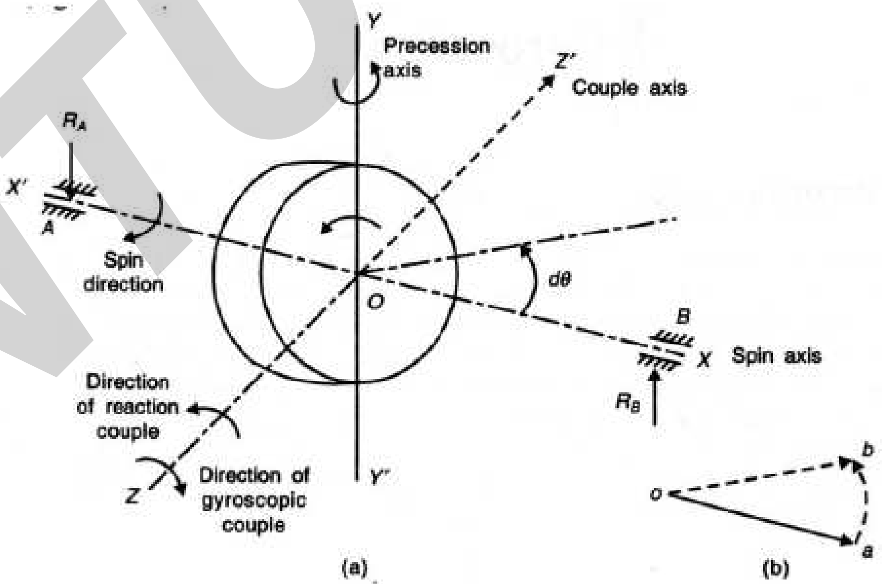
A rigid body, (Fig.) spinning at a constant angular velocity ω rad/s about a spin axis through the mass centre. The angular momentum 'H' of the spinning body is represented by a **vector** whose magnitude is 'Iω'. I represents the mass amount of inertia of the rotor about the axis of spin.



The direction of the angular momentum can be found from the right hand screw rule or the right hand thumb rule. Accordingly, if the fingers of the right hand are bent in the direction of rotation of rotor, then the thumb indicates the direction of momentum.

GYROSCOPIC COUPLE

Consider a rotary body of mass *m* having radius of gyration *k* mounted on the shaft supported at two bearings. Let the rotor spins (rotates) about X-axis with constant angular velocity ω rad/s. The X-axis is, therefore, called spin axis, Y-axis, precession axis and Z-axis, the couple or torque axis (Fig.).



The angular momentum of the rotating mass is given by,

$$H = mk_2\ \omega = I\omega$$

Now, suppose the shaft axis (X-axis) precesses through a small angle δθ about Y-axis in the plane *XOZ*, then the angular momentum varies from *H* to *H* + δ*H*, where δ*H* is the change in the angular momentum, represented by vector ab [Figure 15.2(b)]. For the small value of angle of rotation 5$_0$, we can write

$$\begin{aligned} ab &= oa \times \delta\theta \\ \delta H &= H \times \delta\theta \\ &= I\omega\delta\theta \end{aligned}$$

However, the rate of change of angular momentum is:

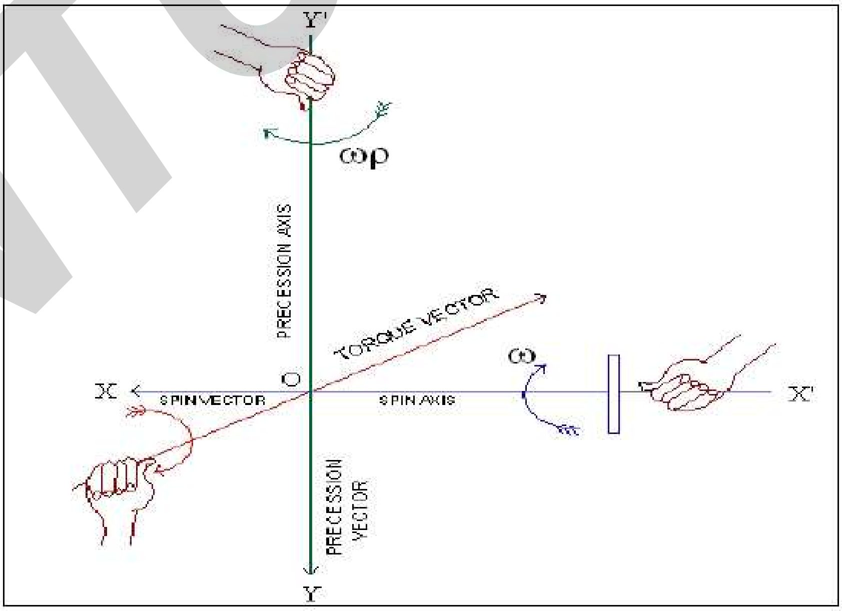
$$C = \frac{dH}{dt} = \lim_{\delta t \to 0}\left(\frac{I\omega\,\delta\theta}{\delta t}\right)$$

$$= I\omega\frac{d\theta}{dt}$$

$$\mathbf{C = I\omega\omega_p}$$

Direction of Spin vector, Precession vector and Couple/Torque vector with forced precession

To determine the direction of spin, precession and torque/couple vector, right hand screw rule or right hand rule is used. The fingers represent the rotation of the disc and the thumb shows the direction of the spin, precession and torque vector (Fig.).

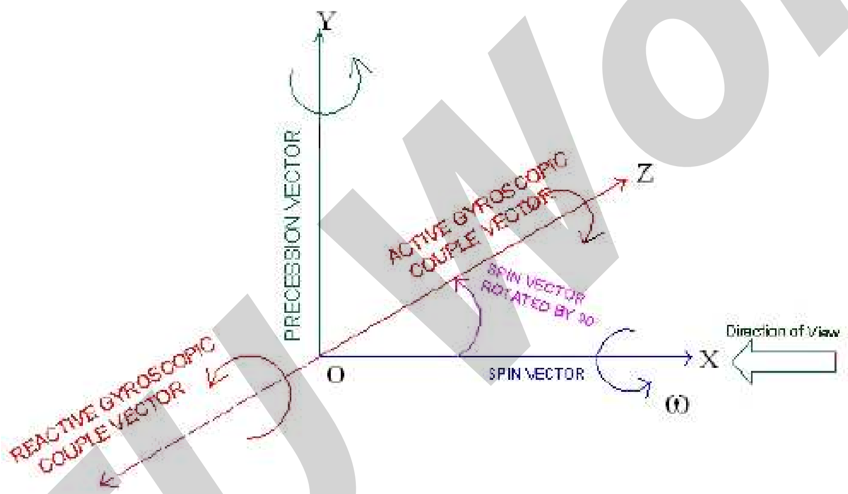


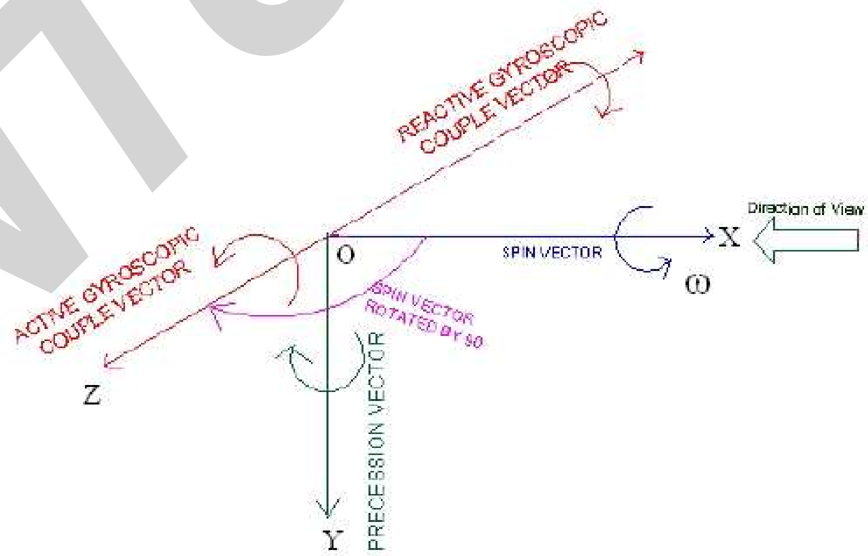
The method of determining the direction of couple/torque vector is as follows

Case (i):

Consider a rotor rotating in anticlockwise direction when seen from the right (Fig.5 and Fig. 6), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector, the following procedure is used.

- Turn the spin vector through 90$_0$ in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction

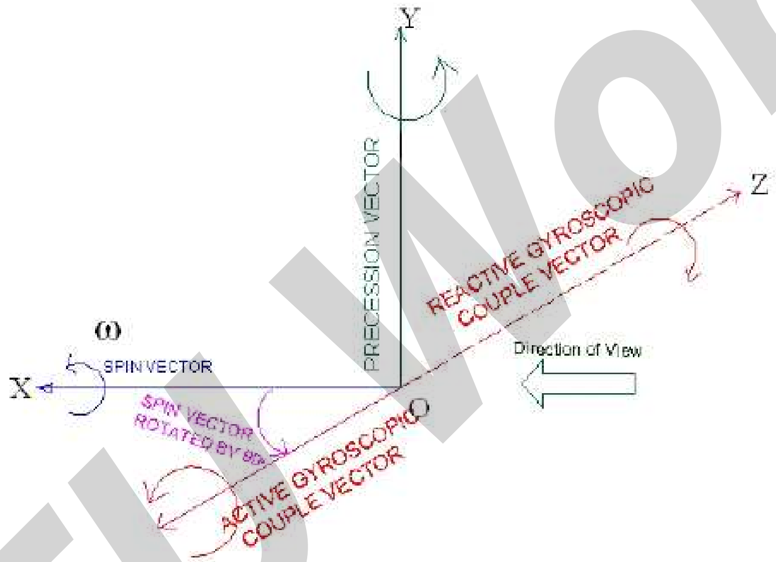


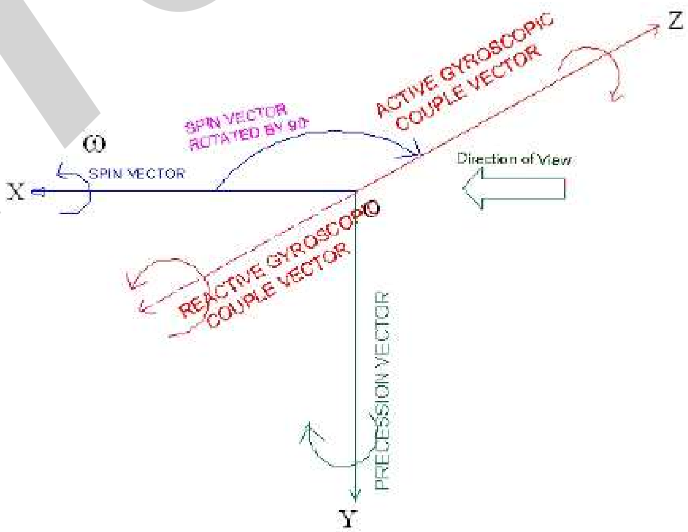


Case (ii):

Consider a rotor rotating in clockwise direction when seen from the right (Fig.7 and Fig. 8), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector,

- Turn the spin vector through 90^0 in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction





The resisting couple/ reactive couple will act in the direction opposite to that of the gyroscopic couple. This means that, whenever the axis of spin changes its direction, a **gyroscopic couple** is applied to it through the bearing which supports the spinning axis.

GYROSCOPIC EFFECT ON SHIP

Gyroscope is used for stabilization and directional control of a ship sailing in the rough sea. A ship, while navigating in the rough sea, may experience the following three different types of motion:

(i) Steering—The turning of ship in a curve while moving forward

(ii) Pitching—The movement of the ship up and down from horizontal position in a vertical plane about transverse axis

(iii)Rolling—Sideway motion of the ship about longitudinal axis

For stabilization of a ship against any of the above motion, the major requirement is that the gyroscope shall be made to precess in such a way that reaction couple exerted by the rotor opposes the disturbing couple which may act on the frame.

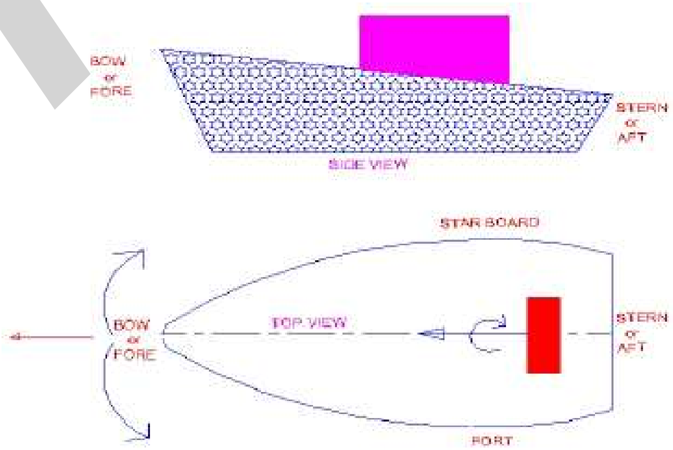
Ship Terminology

(i) Bow – It is the fore end of ship

(ii) Stern – It is the rear end of ship

(iii) Starboard – It is the right hand side of the ship looking in the direction of motion

(iv) Port – It is the left hand side of the ship looking in the direction of motion

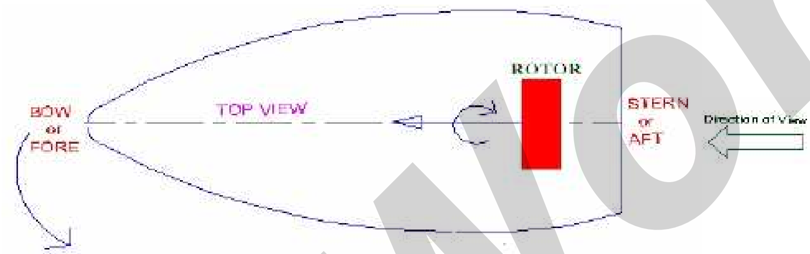


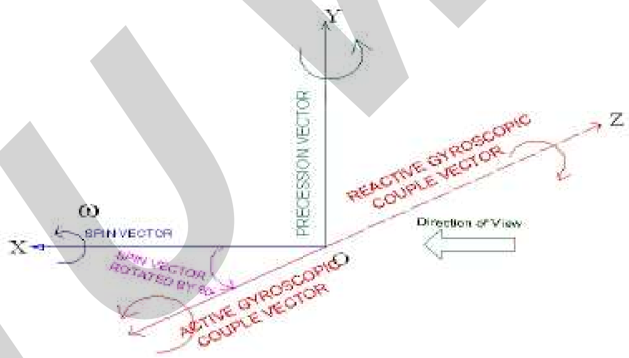
Consider a gyro-rotor mounted on the ship along longitudinal axis (X-axis) as shown in Fig.10 and rotate in clockwise direction when viewed from rear end of the ship. The angular speed of the rotor is ω rad/s. The direction of angular momentum vector *oa*, based on direction of rotation of rotor, is decided using right hand thumb rule as discussed earlier. The gyroscopic effect during the three types of motion of ship is discussed.

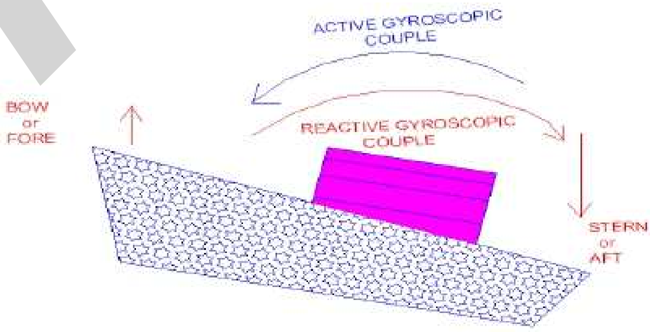
Gyroscopic effect on Steering of ship

(i) Left turn with clockwise rotor

When ship takes a left turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way.



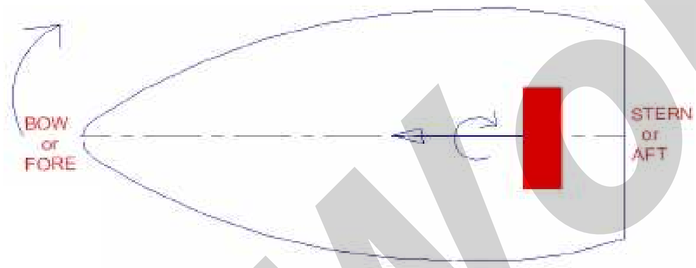


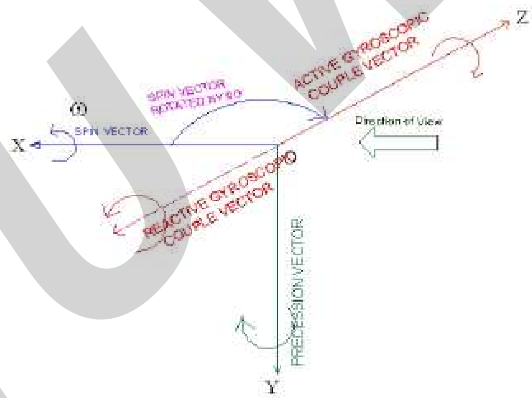


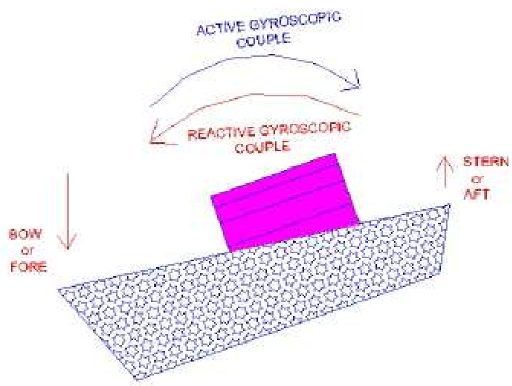
Note that, always reactive gyroscopic couple is considered for analysis. From the above analysis (Fig.), the couple acts over the ship between stern and bow. This reaction couple tends to raise the front end (bow) and lower the rear end (stern) of the ship.

(ii) Right turn with clockwise rotor

When ship takes a right turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple acts on the ship is analyzed (Fig 14). Again, the couple acts in vertical plane, means between stern and bow. Now the reaction couple tends to lower the bow of the ship and raise the stern.

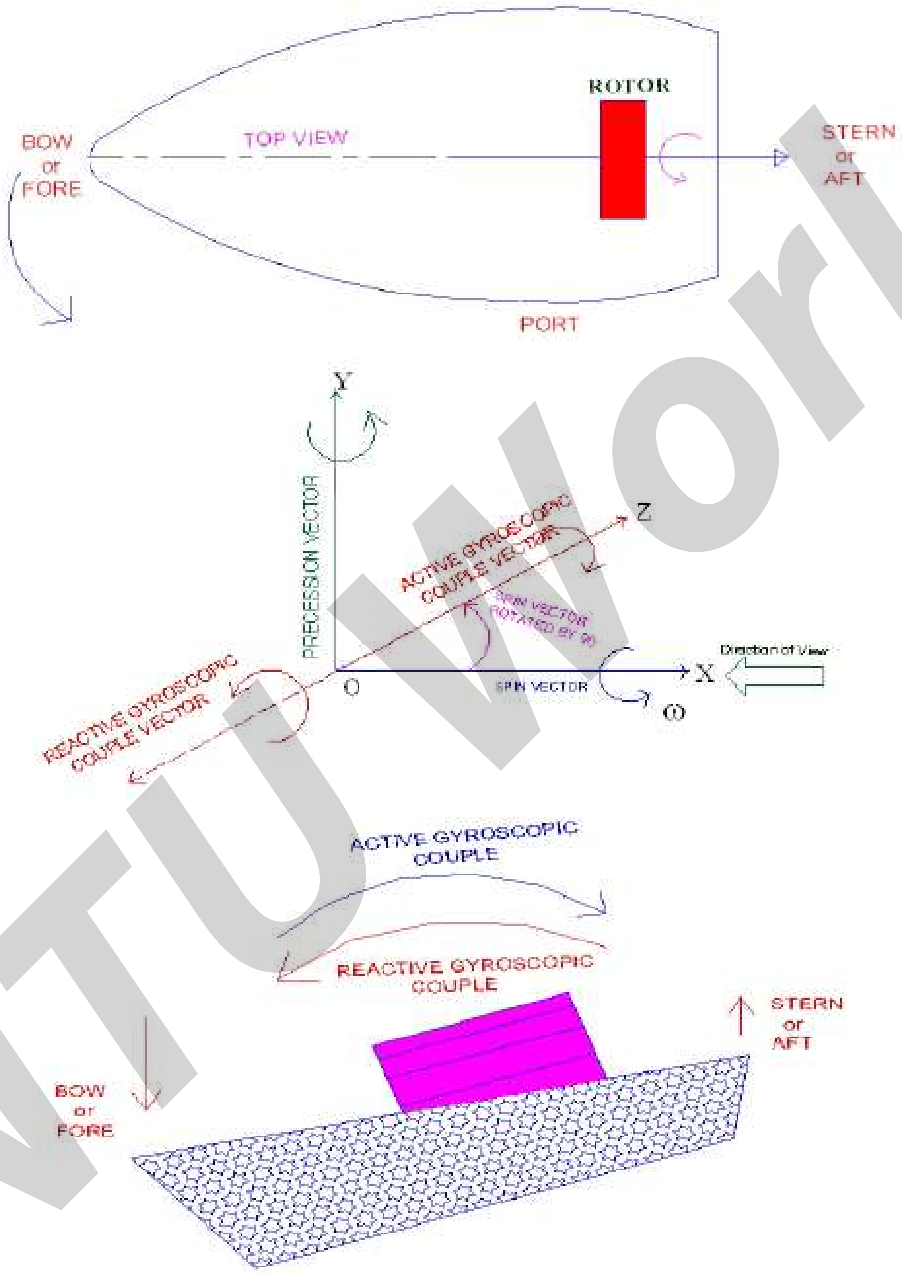






(iii)***Left turn with anticlockwise rotor***

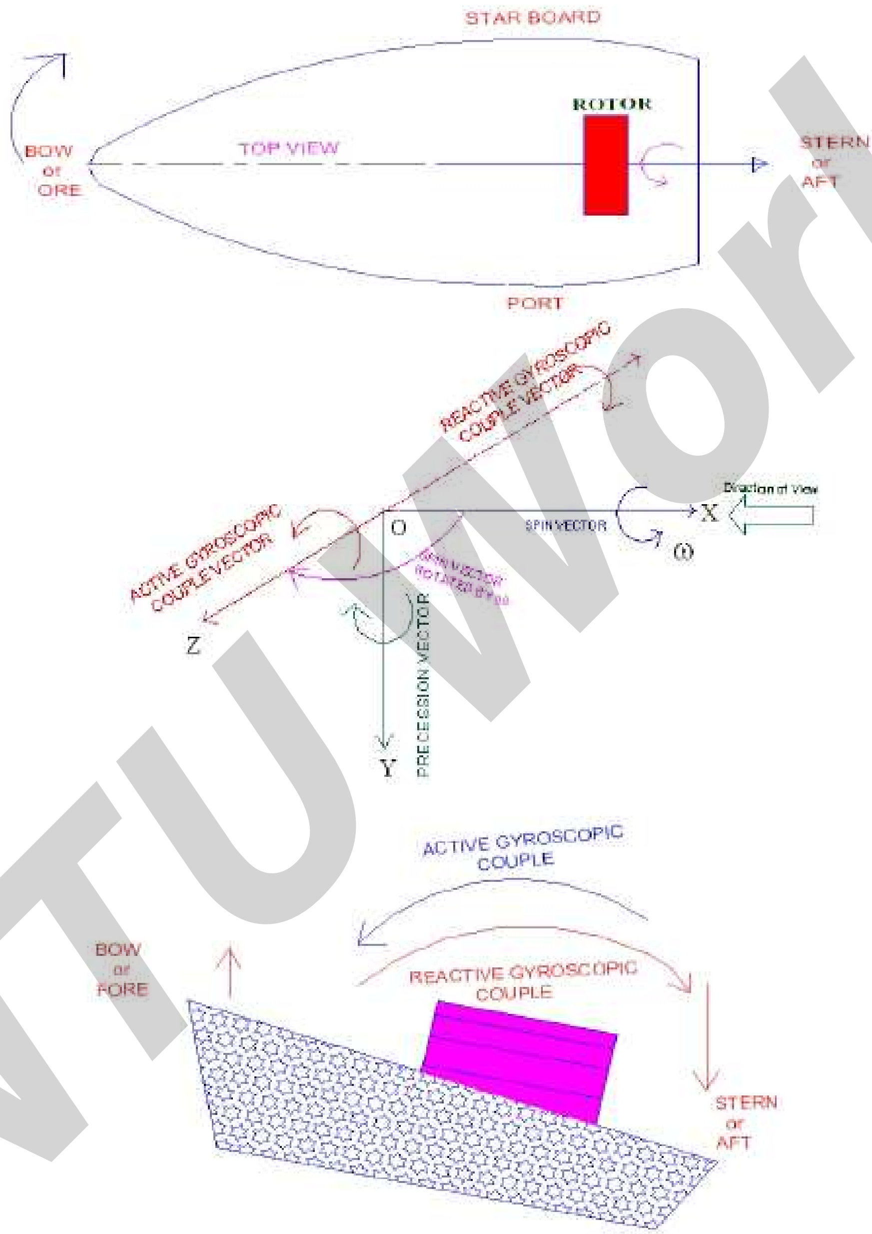
When ship takes a left turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way (Fig.).



The couple acts over the ship is between stern and bow. This reaction couple tends to press or dip the front end (bow) and raise the rear end (stern) of the ship.

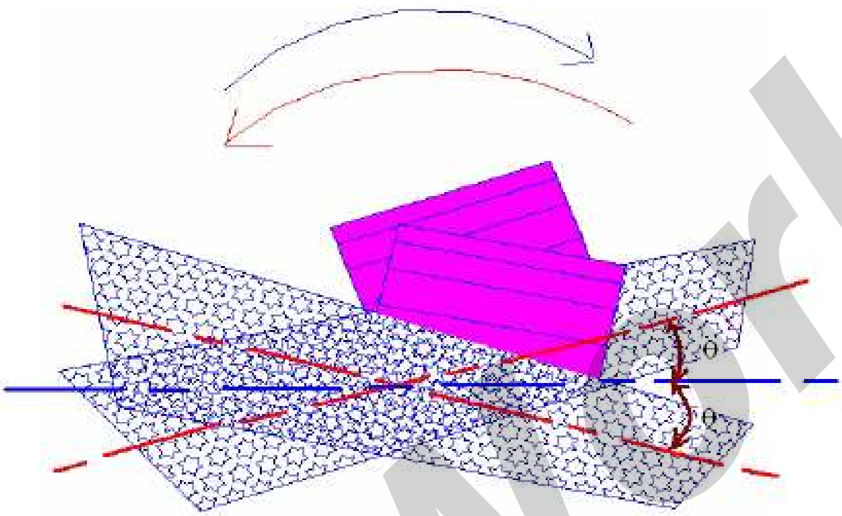
(iv) Right turn with anticlockwise rotor

When ship takes a right turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is according to Fig 20. Now, the reaction couple tends to raise the bow of the ship and dip the stern



Gyroscopic effect on Pitching of ship

The pitching motion of a ship generally occurs due to waves which can be approximated as sine wave. During pitching, the ship moves up and down from the horizontal position in vertical plane (Fig.)



Let θ = angular displacement of spin axis from its mean equilibrium position
A = amplitude of swing
(= angle in degree $\times \frac{2\pi}{360°}$)

and ω_0 = angular velocity of simple hormonic motion $\left(= \frac{2\pi}{\text{time period}}\right)$

The angular motion of the rotor is given as

$$\theta = A \sin \omega_0 t$$

Angular velocity of precess: $\omega_p = \frac{d\theta}{dt}$

$$= \frac{d}{dt}(A \sin \omega_0 t)$$

or $\omega_p = A\omega_0 \cos \omega_0 t$

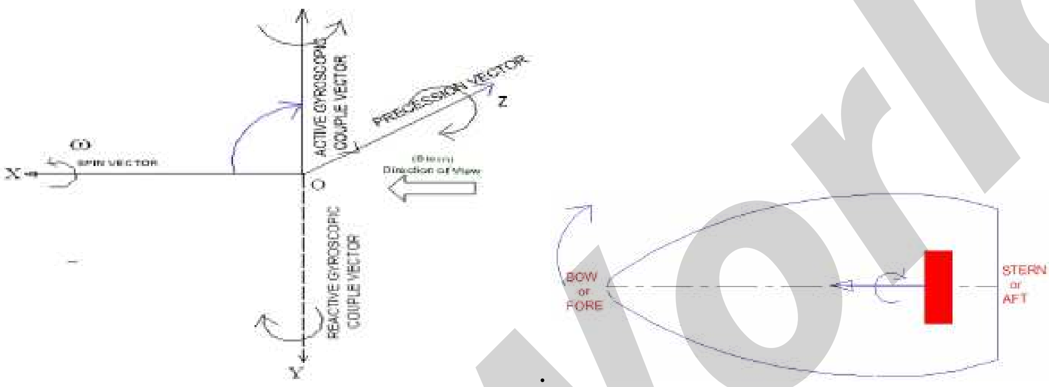
The angular velocity of precess will be maximum when $\cos \omega_0 t = 1$

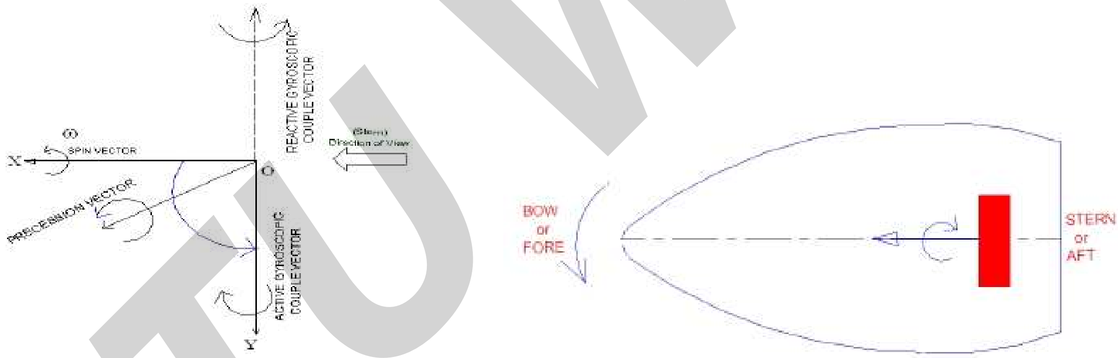
or $\omega_{p\max} = A\omega_0$

$$= A \times \frac{2\pi}{t}$$

Thus the gyroscopic couple: $C = I \, \omega \, \omega_p$

Consider a rotor mounted along the longitudinal axis and rotates in clockwise direction when seen from the rear end of the ship. The direction of momentum for this condition is shown by vector *ox* (Fig.24). When the ship moves up the horizontal position in vertical plane by an angle $\delta\theta$ from the axis of spin, the rotor axis (X-axis) processes about Z- axis in XY-plane and for this case Z-axis becomes precession axis. The gyroscopic couple acts in anticlockwise direction about Y-axis and the reaction couple acts in opposite direction, i.e. in clockwise direction, which tends to move towards **right side** (Fig.25). However, when the ship pitches down the axis of spin, the direction of reaction couple is reversed and the ship turns towards **left side** (Fig.)





Similarly, for the anticlockwise direction of the rotor viewed from the rear end (Stern) of the ship, the analysis may be done.

Gyroscopic effect on Rolling of ship.

The axis of the rotor of a ship is mounted along the longitudinal axis of ship and therefore, there is **no** precession of this axis. Thus, no effect of gyroscopic couple on the ship frame is formed when the ship rolls

Gyroscopic Effect on Aeroplane

Aeroplanes are subjected to gyroscopic effect when it taking off, landing and negotiating left or right turn in the air.

Let

ω = Angular velocity of the engine rotating parts in rad/s,

m = Mass of the engine and propeller in kg,

r_W = Radius of gyration in m,

I = Mass moment of inertia of engine and propeller in kg m^2,

V = Linear velocity of the aeroplane in m/s,

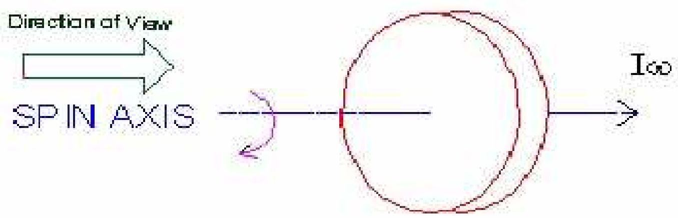
R = Radius of curvature in m,

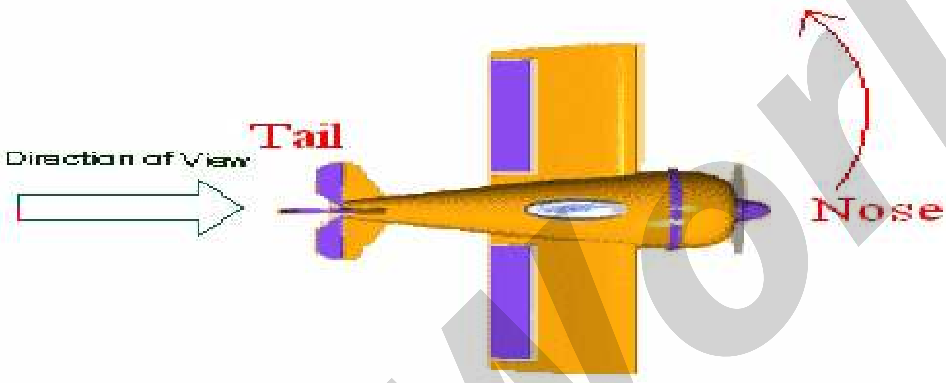
ω_P =Angular velocity of precession =v/R rad/s

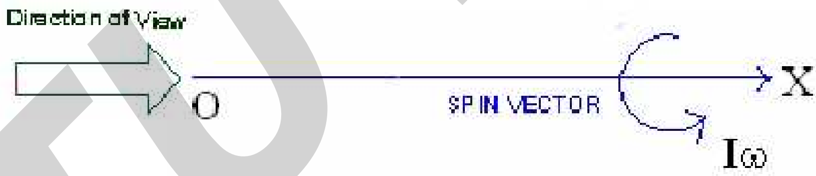
Gyroscopic couple acting on the aero plane **= C = I** $\omega\ \omega_p$

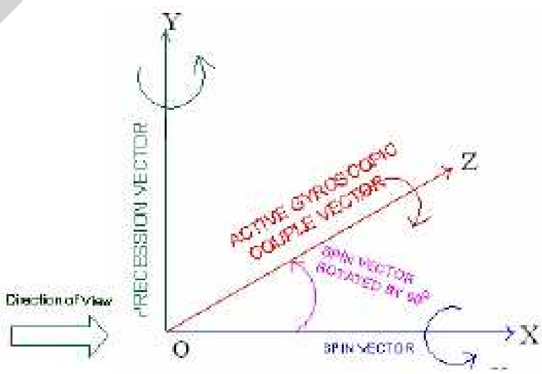
Let us analyze the effect of gyroscopic couple acting on the body of the aero plane for various conditions.

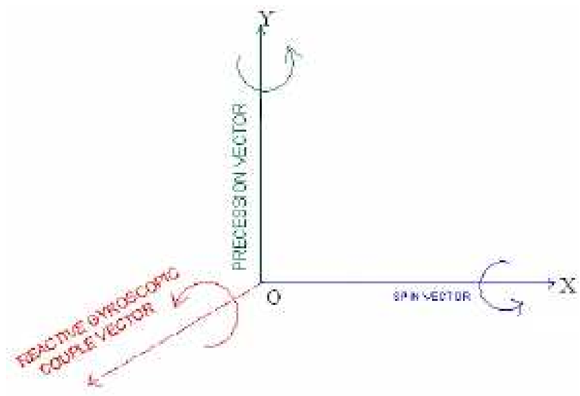
Case (i): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT


Direction of View
SPIN AXIS
Iω


Direction of View
Tail
Nose

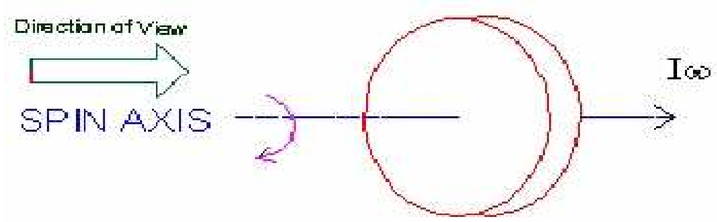

Direction of View
O
SPIN VECTOR
X
Iω

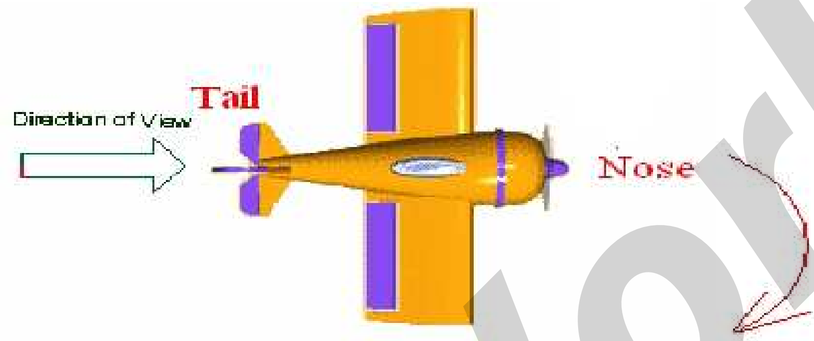

Y
PRECESSION VECTOR
Z
ACTIVE GYROSCOPIC
COUPLE VECTOR
SPIN VECTOR
ROTATED BY 90°
Direction of View
O
SPIN VECTOR
X

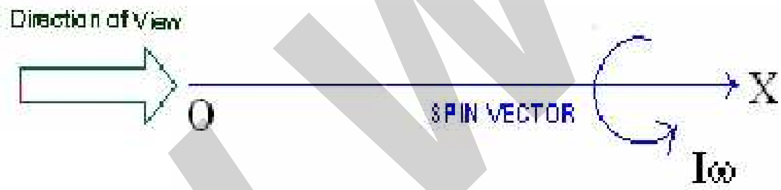


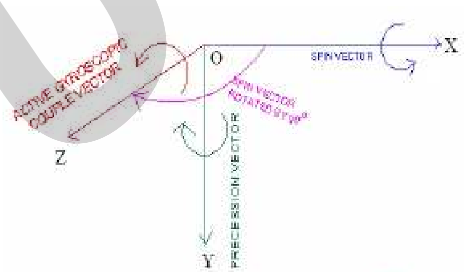
According to the analysis, the reactive gyroscopic couple tends to dip the tail and raise the nose of aeroplane.

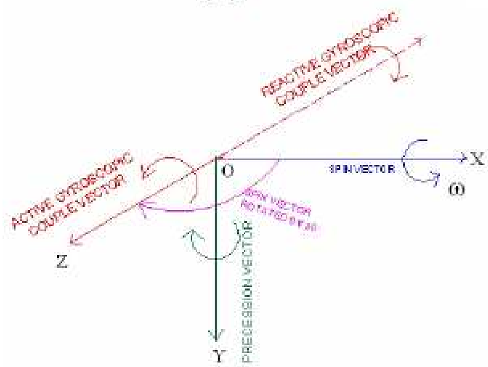
Case (ii): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards RIGHT


Direction of View
SPIN AXIS
Iω

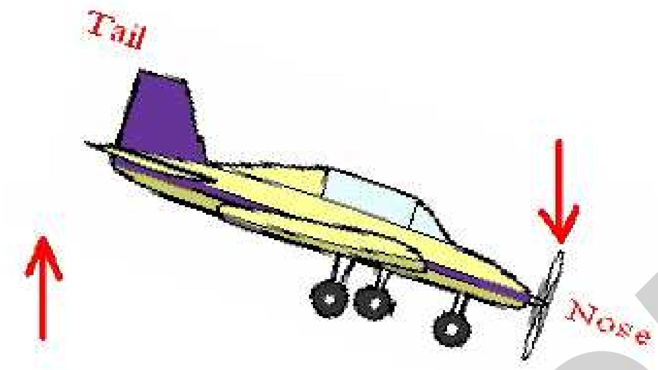

Tail
Direction of View
Nose


Direction of View
O
SPIN VECTOR
X
Iω

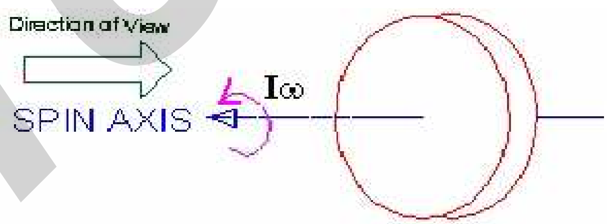

O
SPIN VECTOR
X
ACTIVE GYROSCOPIC COUPLE VECTOR
SPIN VECTOR ROTATED BY 90°
Z
PRECESSION VECTOR
Y

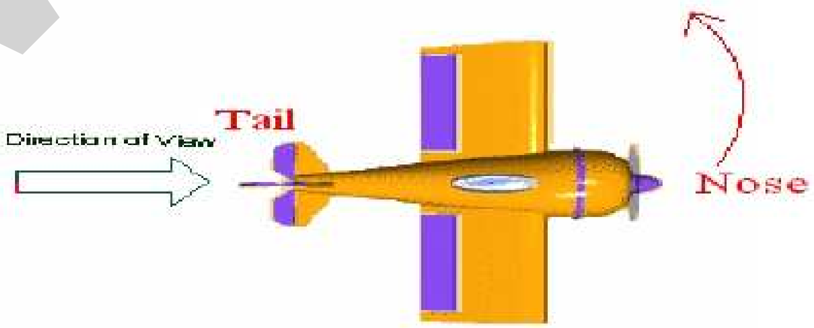

REACTIVE GYROSCOPIC COUPLE VECTOR
ACTIVE GYROSCOPIC COUPLE VECTOR
O
SPIN VECTOR
X
ω
Z
PRECESSION VECTOR
Y

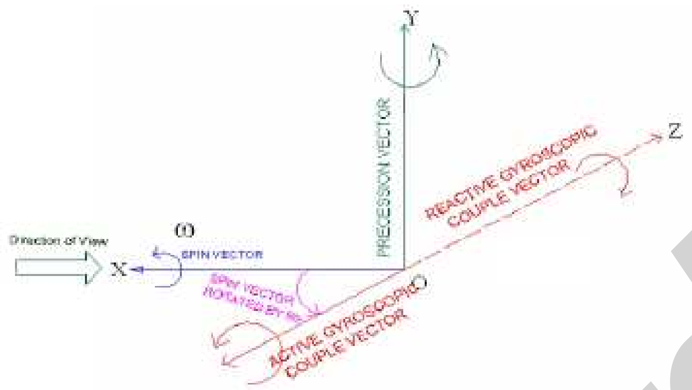
According to the analysis, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.



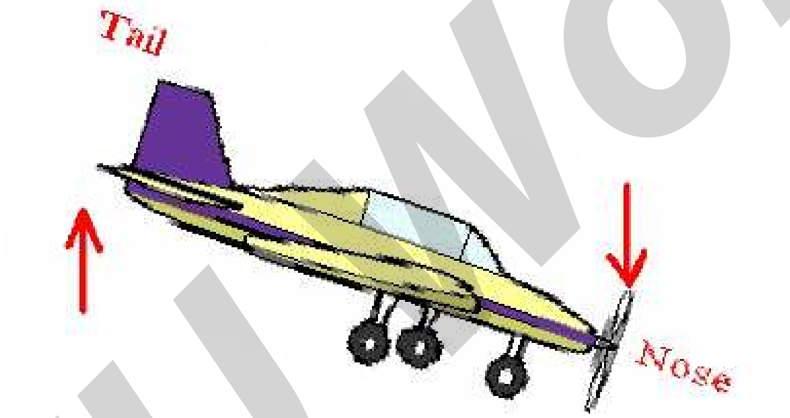
Case (iii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT



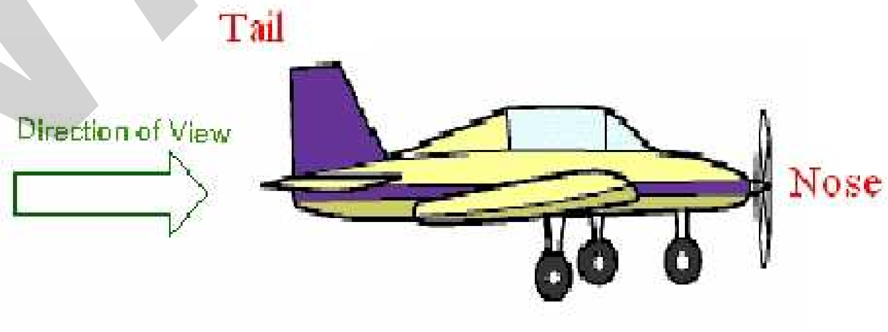


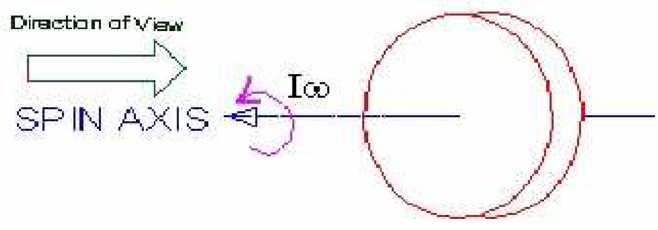


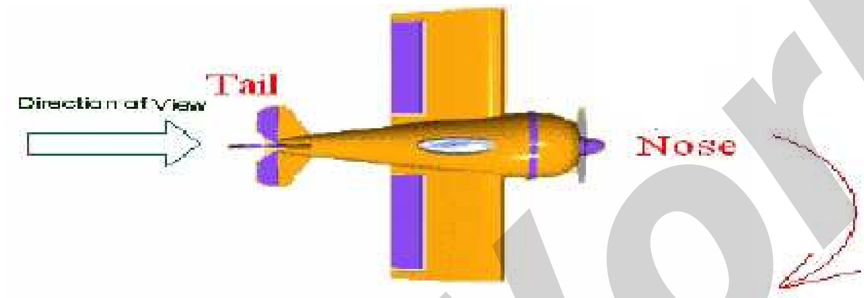
The analysis indicates, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.

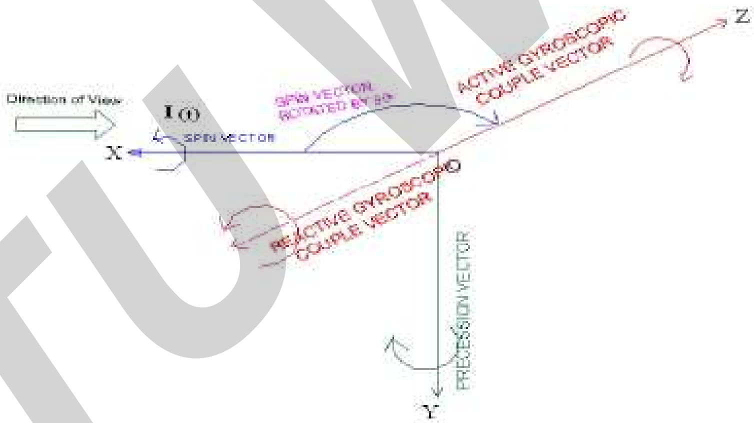


Case (iv): **PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane turns towards RIGHT**

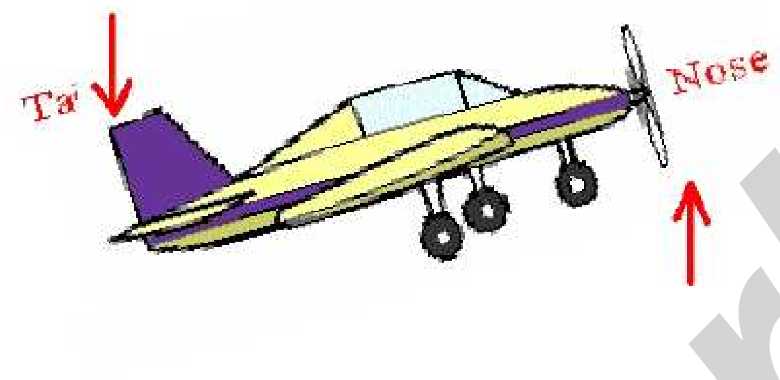




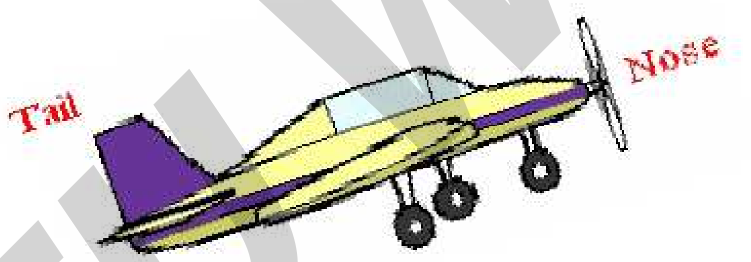


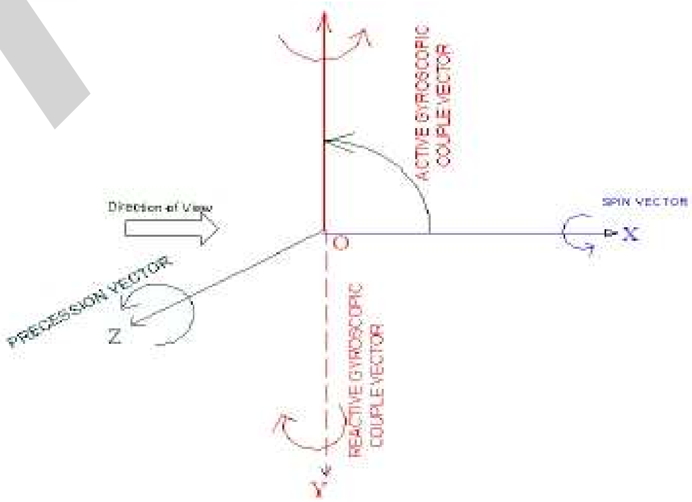


The analysis shows, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.

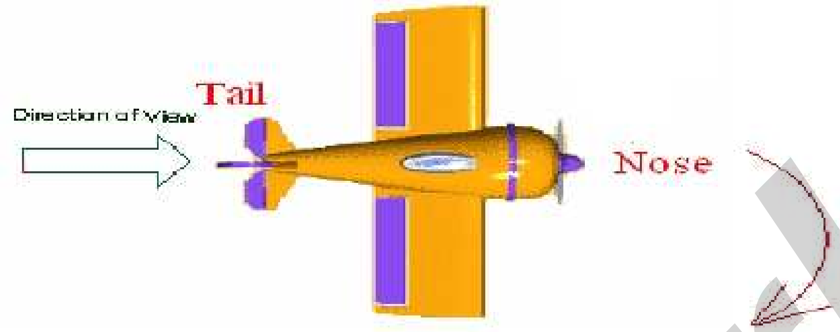


Case (v): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane takes off or nose move upwards

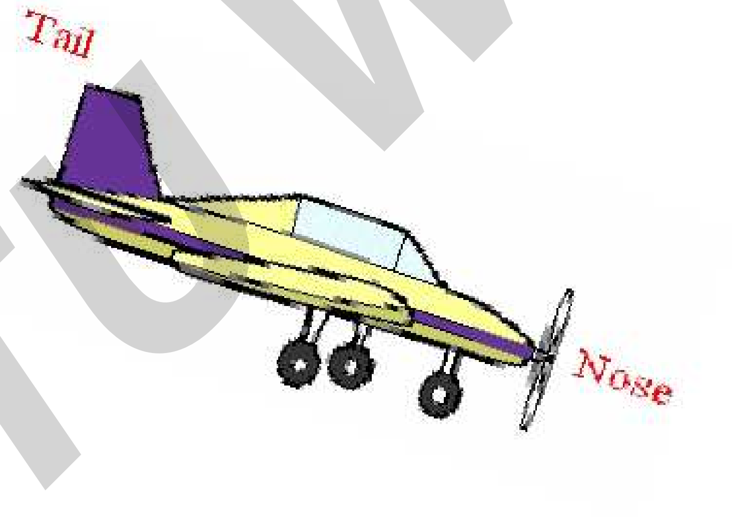


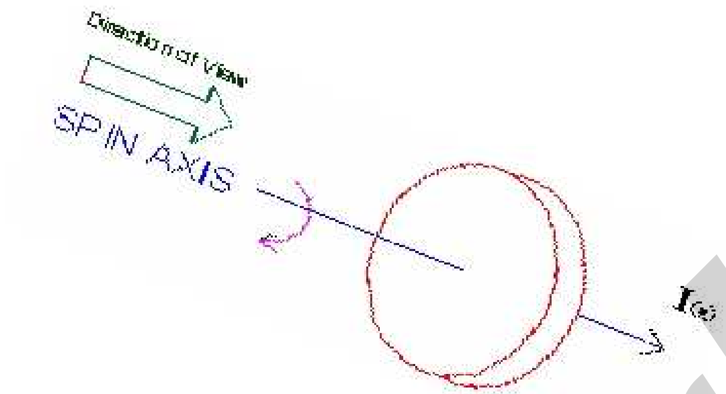


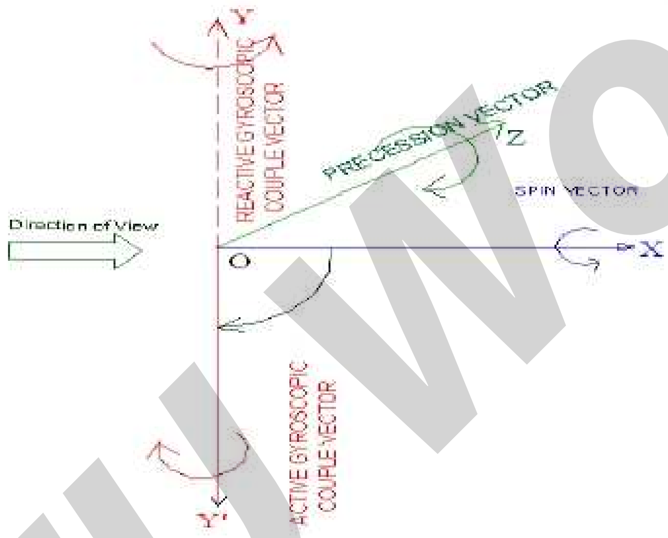
The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right



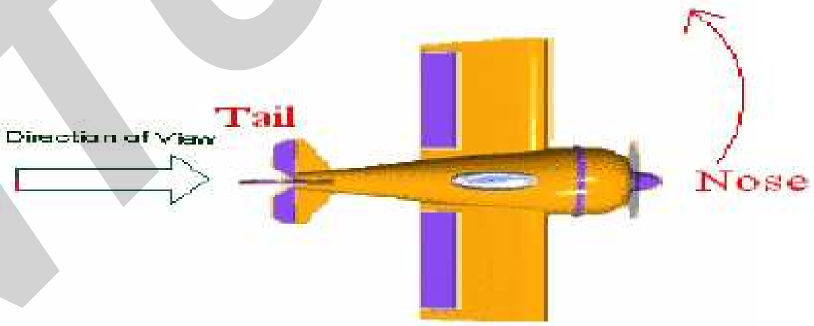
Case (vi): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards



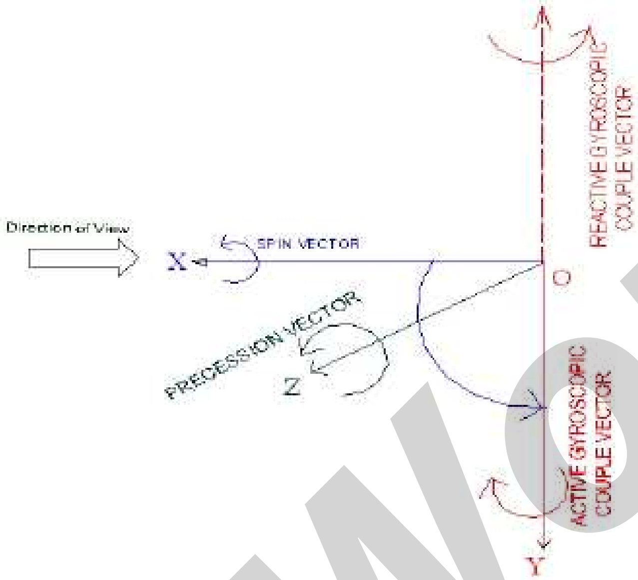




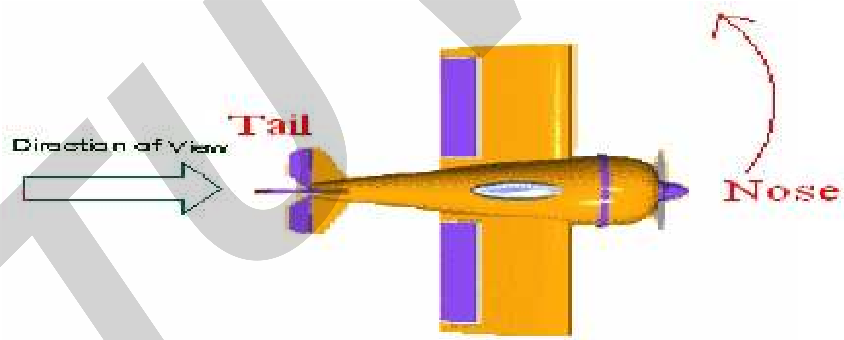
The reactive gyroscopic couple tends to turn the nose of aeroplane toward left



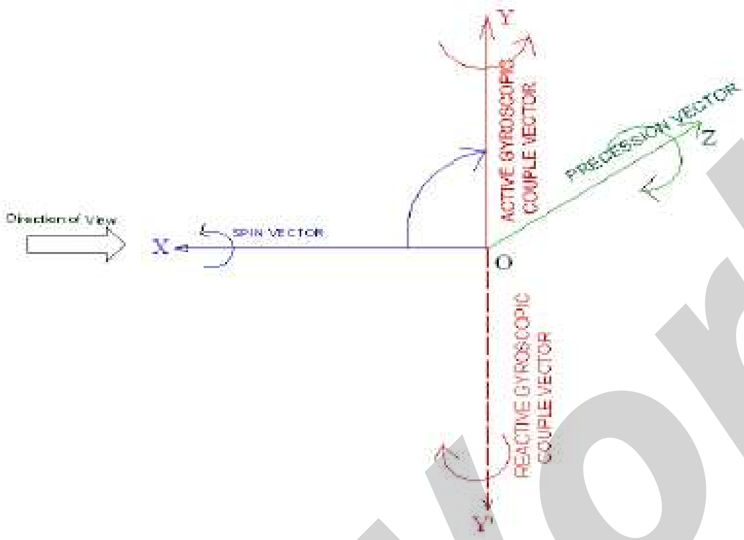
Case (vii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane takes off or nose move upwards



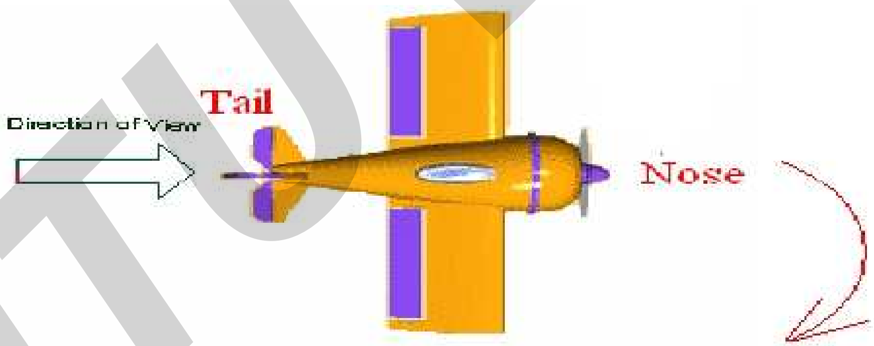
The reactive gyroscopic couple tends to turn the nose of aeroplane toward left



Case (viii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards



The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right



Stability of Automotive Vehicle

A vehicle running on the road is said to be stable when no wheel is supposed to leave the road surface. In other words, the resultant reactions by the road surface on wheels should act in upward direction. For a moving vehicle, one of the reaction is due to gyroscopic couple produced by the rotating wheels and rotating parts of the engine. Let us discuss stability of two and four wheeled vehicles when negotiating a curve/turn.

Stability of Two Wheeler negotiating a turn

Fig shows a two wheeler vehicle taking **left turn** over a curved path. The vehicle is inclined to the vertical for equilibrium by an angle θ known as angle of heel.

Let

m = *Mass of the vehicle and its* rider in kg,

W = *Weight of the vehicle and* its rider in newtons = $m.g$,

h = *Height of the centre of* gravity of the vehicle and rider,

r_W = *Radius of the wheels,*

R = *Radius of track or* curvature,

I_W = *Mass moment of inertia of each wheel,*

I_E = *Mass moment of inertia of the rotating parts of the engine,*

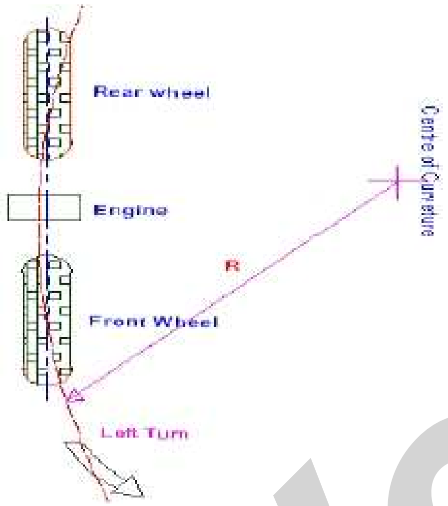
ω_W = Angular velocity of the wheels,

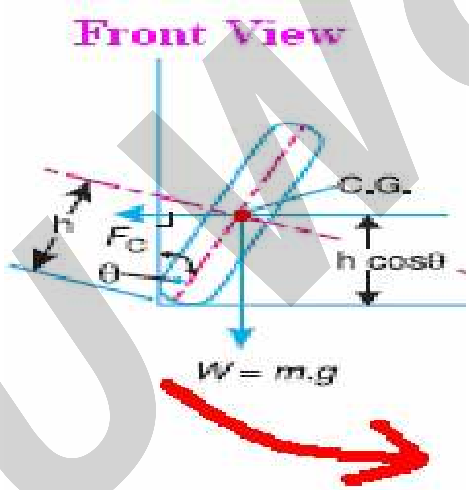
ω_E = Angular velocity of the engine rotating parts,

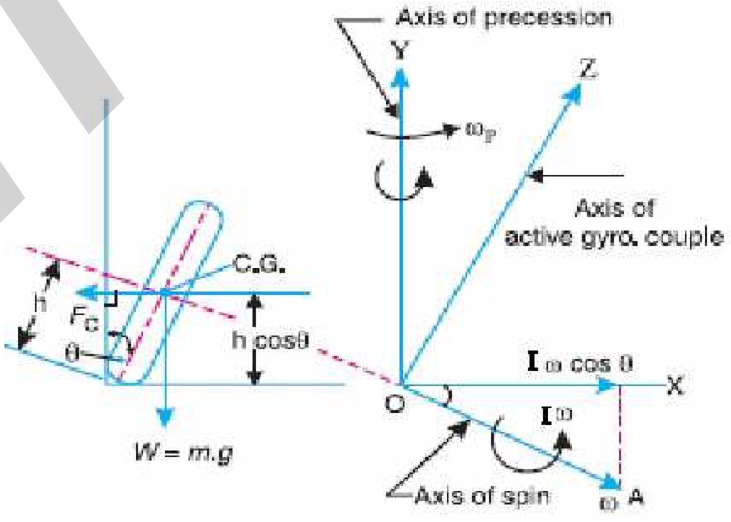
G = *Gear ratio* = ω_E / ω_W,

v = Linear velocity of the vehicle = $\omega w \times rw$,

θ = Angle of heel. It is inclination of the vehicle to the vertical for equilibrium







Let us consider the effect of the gyroscopic couple and centrifugal couple on the wheels.

1. Effect of Gyroscopic Couple

We know that, $V = \omega_W \times r_W$

$\omega_E = G \,.\omega_W$ or

Angular momentum due to wheels = $2 I_W \omega_W$

Angular momentum due to engine and transmission = $I_E \omega_E$

Total angular momentum ($I x\omega$) = $2 I_W \omega_W \pm I_E \omega_E$

$$= 2I_W \frac{v}{r_w} \pm I_E G \frac{v}{r_w}$$

$$= \frac{v}{r_w}(2I_W \pm GI_E)$$

Velocity of precession = ω_p

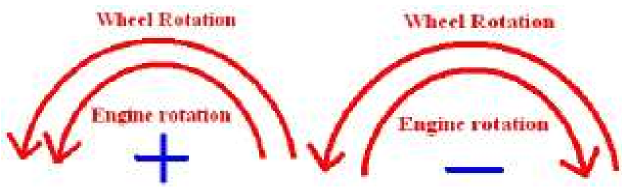
It is observed that, when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in Fig… This angle is known as 'angle of heel'. In other words, the axis of spin is inclined to the horizontal at an angle θ , as shown in Fig.73 Thus, the angular momentum vector I ω due to spin is represented by OA inclined to OX at an angle θ. But, the precession axis is in vertical. Therefore, the spin vector is resolved along OX.

Gyroscopic Couple,

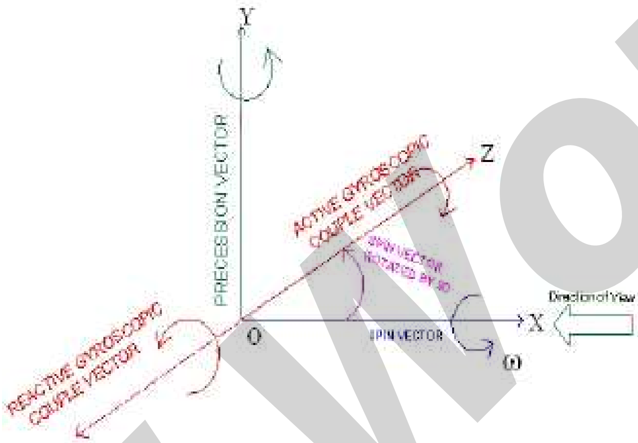
$$C_g = (I\omega)\cos\theta \times \omega_p$$

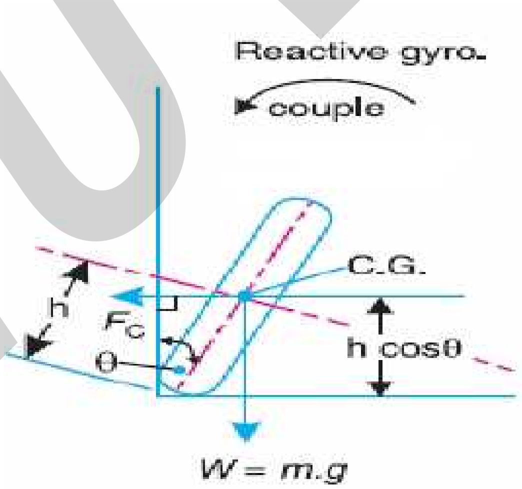
$$C_g = \frac{v^2}{Rr_w}(2I_W \pm GI_E)\cos\theta$$

Note: When the engine is rotating in the same direction as that of wheels, then the positive sign is used in the above equation. However, if the engine rotates in opposite direction to wheels, then negative sign is used.

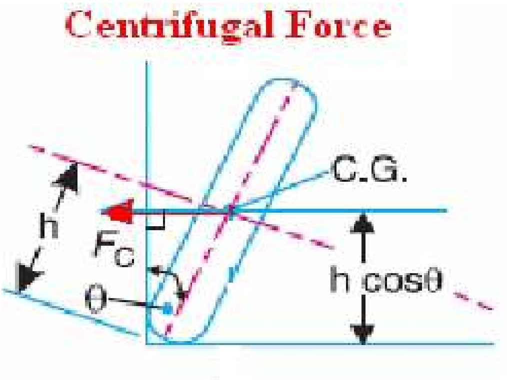


The gyroscopic couple will act over the vehicle outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown in Fig…



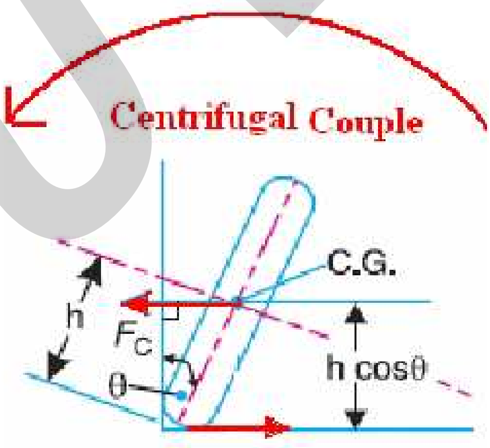


2. Effect of Centrifugal Couple



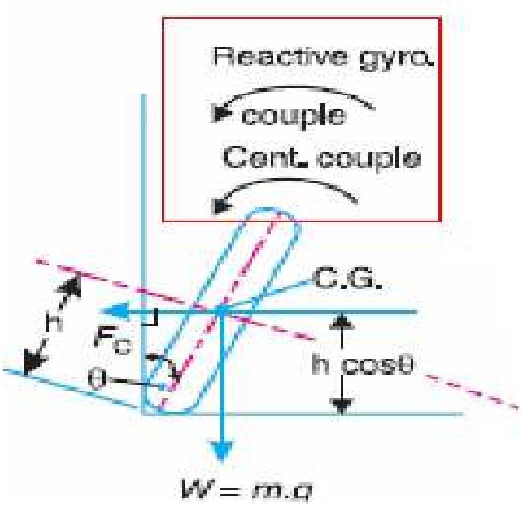
Centrifugal force, $$F_c = \frac{mv^2}{R}$$

Centrifugal Couple $$C_c = F_c \times h\cos\theta$$
$$= \frac{mv^2}{R} h\cos\theta$$



The Centrifugal couple will act over the two wheeler outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown in Fig.

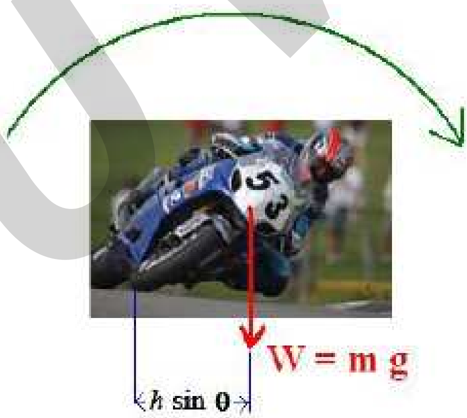
Therefore, the total Over turning couple: $C = C_g + C_c$



$$C = \frac{v^2}{Rr}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R}h\cos\theta$$

For the vehicle to be in equilibrium, overturning couple should be equal to balancing couple acting in clockwise direction due to the weight of the vehicle and rider.

$$C = mgh \sin\theta$$



For the stability, overturning couple must be equal to balancing couple,

$$\boxed{\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R}h\cos\theta = mgh\sin\theta}$$

Therefore, from the above equation, the value of angle of heel (θ) may be determined, so that the vehicle does not skid. Also, for the given value of θ, the maximum vehicle speed in the turn with out skid may be determined.

Stability of Four Wheeled Vehicle negotiating a turn.

Consider a four wheels automotive vehicle as shown in Figure 82. The engine is mounted at the rear with its crank shaft parallel to the rear axle. The centre of gravity of the vehicle lies vertically above the ground where total weight of the vehicle is assumed to be acted upon.

Let

m = *Mass of the vehicle* (kg)

W = *Weight of the vehicle* (N) = $m.g$,

h = *Height of the centre of* gravity of the vehicle (m)

r_W = *Radius of the wheels (m)*

R = *Radius of track or* curvature (m)

I_W = *Mass moment of inertia of each wheel* ($kg\text{-}m^2$)

I_E = *Mass moment of inertia of the rotating parts of the engine* ($kg\text{-}m^2$)

ω_W = Angular velocity of the wheels (rad/s)

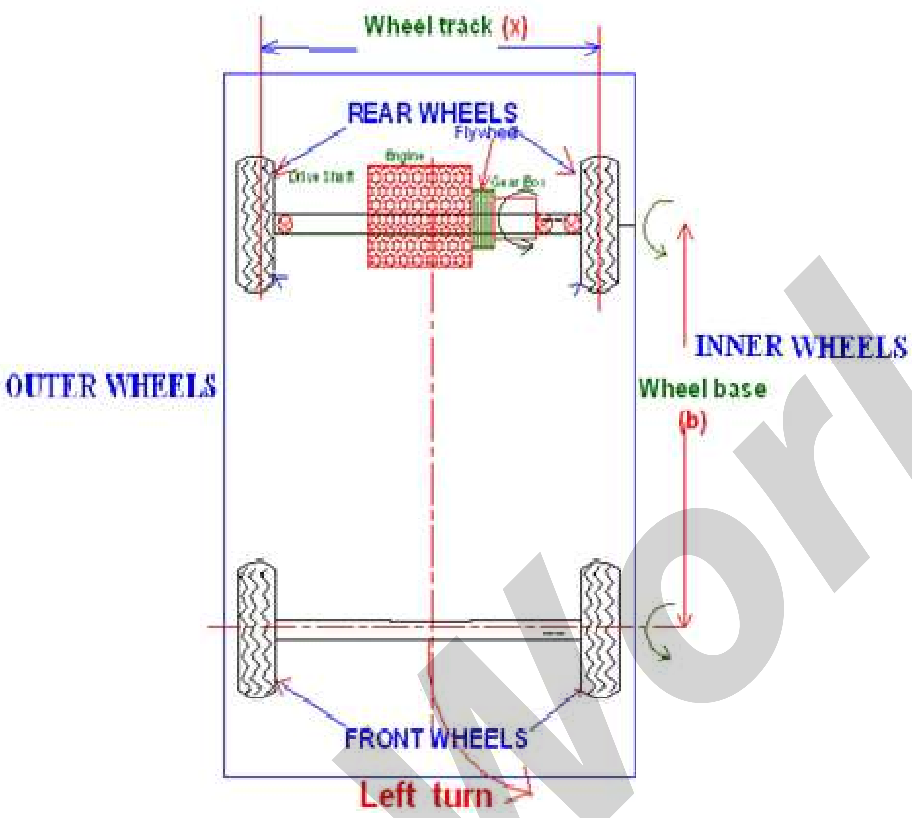
ω_E = Angular velocity of the engine (rad/s)

G = *Gear ratio* = ω_E / ω_W,

v = *Linear velocity of the vehicle (m/s)*= $\omega_W \times r_W$,

x = Wheel track (m)

b = Wheel base (m)



(i) **Reaction due to weight of Vehicle**

Weight of the vehicle. Assuming that weight of the vehicle *(mg)* is equally distributed over four wheels. Therefore, the force on each wheel acting downward is m*g/4* and the reaction by the road surface on the wheel acts in upward direction.

$$R_w = \frac{mg}{4}$$

(ii) **Effect of Gyroscopic couple due to Wheel**

Gyroscopic couple due to four wheels is,

$$C_W = 4\, I_W \omega \omega_p$$

(iii) **Effect of Gyroscopic Couple due to Engine**

Gyroscopic couple due to rotating parts of the engine

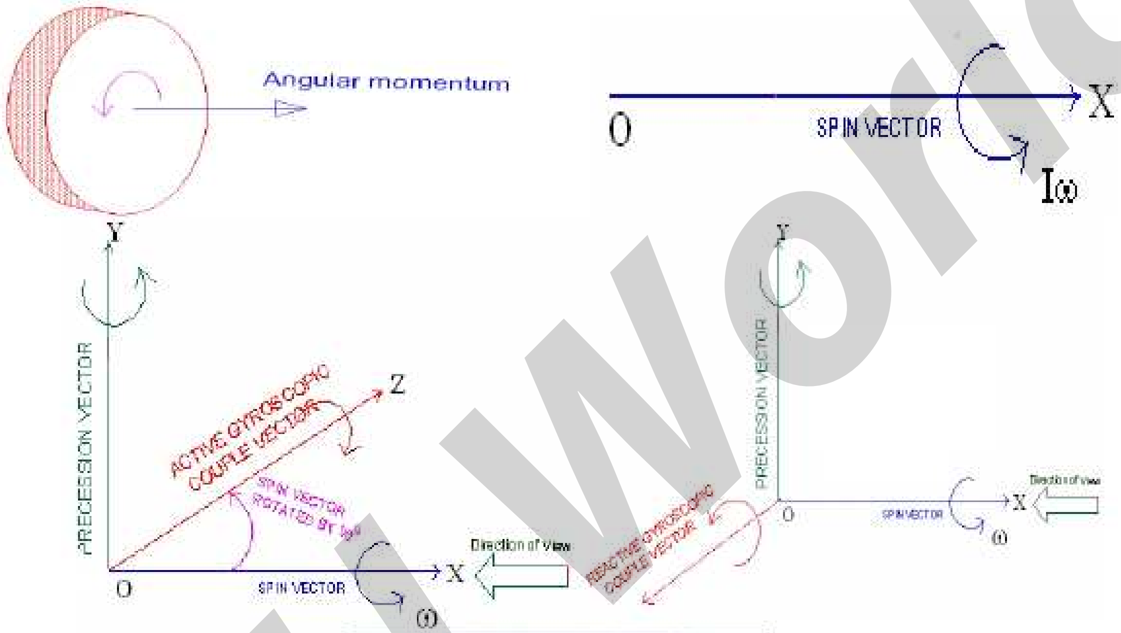
$$C_E = I_E\, \omega\, \omega_p = I_E\, G\, \omega\, \omega_p$$

Therefore, Total gyroscopic couple:

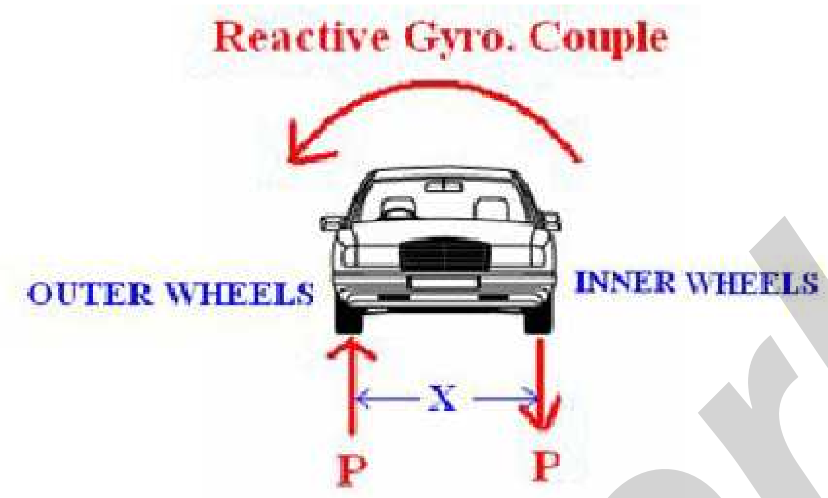
$$C_g = C_W + C_E = \omega\,\omega_p\,(4I_W \pm I_E G)$$

When the wheels and rotating parts of the engine rotate in the same direction, then positive sign is used in the above equation. Otherwise negative sign should be considered.

Assuming that the vehicle takes a left turn, the reaction gyroscopic couple on the vehicle acts between outer and inner wheels.



This gyroscopic couple tends to press the outer wheels and lift the inner wheels



Due to the reactive gyroscopic couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwords on the outer wheels and vertically downwords on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be P Newtons, then,

$$\mathbf{P} \, x \, X = C_g$$

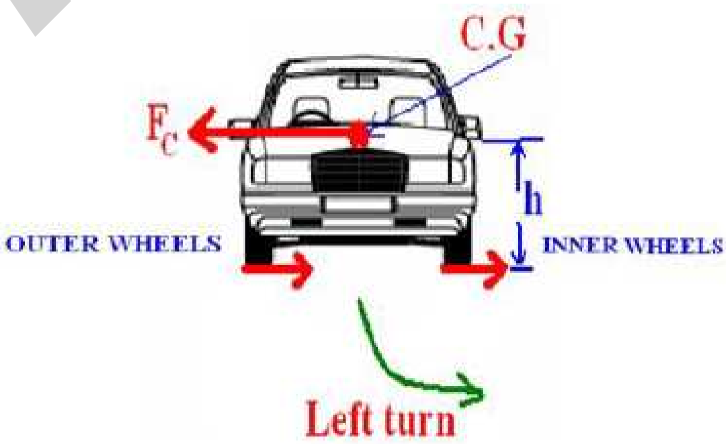
$$\mathbf{P} = \frac{C_g}{x}$$

Road reaction on each outer/Inner wheel,

$$\frac{P}{2} = \frac{C_g}{2X}$$

(iii)**Effect of Centrifugal Couple**

When a vehicle moves on a curved path, a centrifugal force acts on the vehicle in outward direction through the centre of gravity of the vehicle(Fig…)



Centrifugal force,

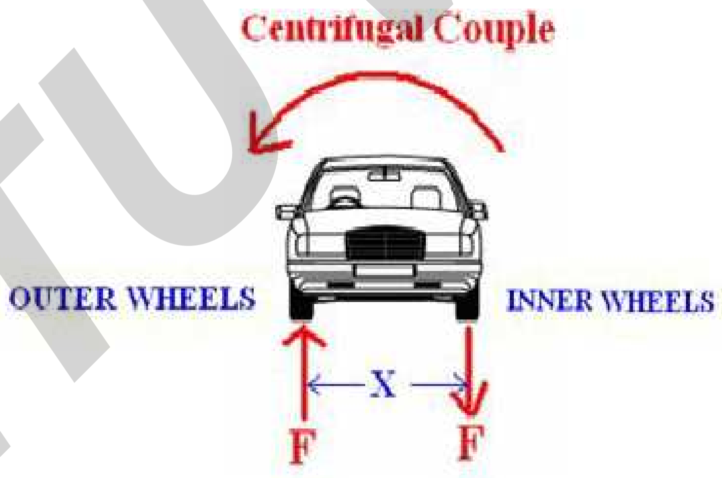
$$F_c = m\omega_p^2 R = \frac{mv^2}{R}$$

This force forms a Centrifugal couple.

$$C_c = \frac{mv^2 h}{R}$$

This centrifugal couple tends to press the outer and lift the inner

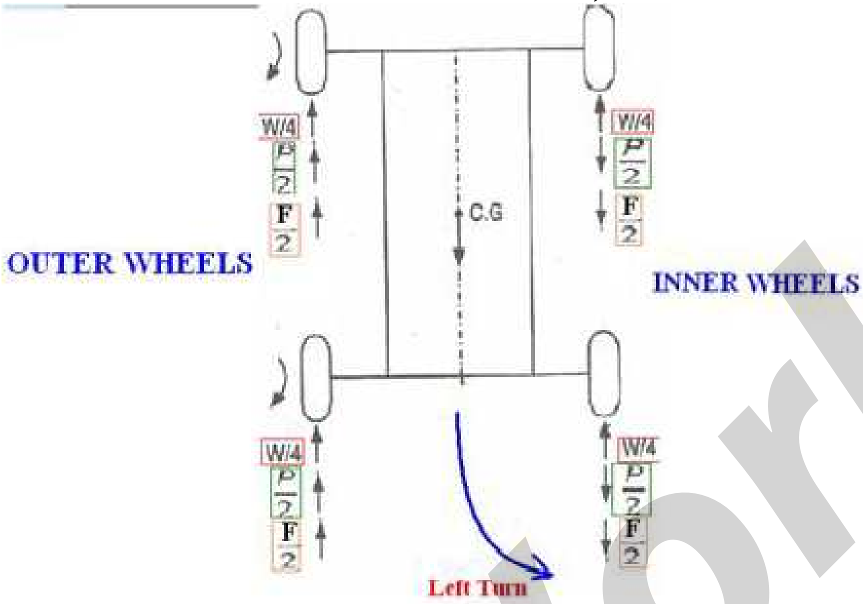
Due to the centrifugal couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwords on the outer wheels and vertically downwords on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be F Newtons, then,



Road reaction on each outer/Inner wheel,

$$\frac{F}{2} = \frac{C_c}{2X}$$

The reactions on the outer/inner wheels are as follows,



Total vertical reaction at each outer wheels

$$P_o = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

Total vertical reaction at each inner wheels

$$P_i = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

UNIT – II
FRICTION

Introduction

It has been established since long, that the surfaces of the bodies are never perfectly smooth. When, even a very smooth surface is viewed under a microscope, it is found to have roughness and irregularities, which may not be detected by an ordinary touch. If a block of one substance is placed over the level surface of the same or of different material, a certain degree of interlocking of the minutely projecting particles takes place. This does not involve any force, so long as the block does not move or tends to move. But whenever one block moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the upper block, is called the ***force of friction*** or simply ***friction***. It thus follows, that at every joint in a machine, force of friction arises due to the relative motion between two parts and hence some energy is wasted in overcoming the friction. Though the friction is considered undesirable, yet it plays an important role both in nature and in engineering *e.g.* waling on a road, motion of locomotive on rails, transmission of power by belts, gears etc. The friction between the wheels and the road is essential for the car to move forward.

Types of Friction

In general, the friction is of the following two types :

1. *Static friction.* It is the friction, experienced by a body, when at rest.

2. *Dynamic friction.* It is the friction, experienced by a body, when in motion. The dynamic friction is also called ***kinetic friction*** and is less than the static friction. It is of the following three types :

(*a*) ***Sliding friction.*** It is the friction, experienced by a body, when it ***slides*** over another body.

(*b*) ***Rolling friction.*** It is the friction, experienced between the surfaces which has ***balls*** or ***rollers*** interposed between them.

(*c*) ***Pivot friction.*** It is the friction, experienced by a body, due to the ***motion of rotation*** as in case of foot step bearings.

The friction may further be classified as :

1. Friction between unlubricated surfaces, and

2. Friction between lubricated surfaces.

Friction Between Unlubricated Surfaces

The friction experienced between two dry and unlubricated surfaces in contact is known as ***dry*** or ***solid friction***. It is due to the surface roughness. The dry or solid friction includes the sliding friction and rolling friction as discussed above.

Friction Between Lubricated Surfaces

When lubricant (*i.e.* oil or grease) is applied between two surfaces in contact, then the friction may be classified into the following two types depending upon the thickness of layer of a lubricant.

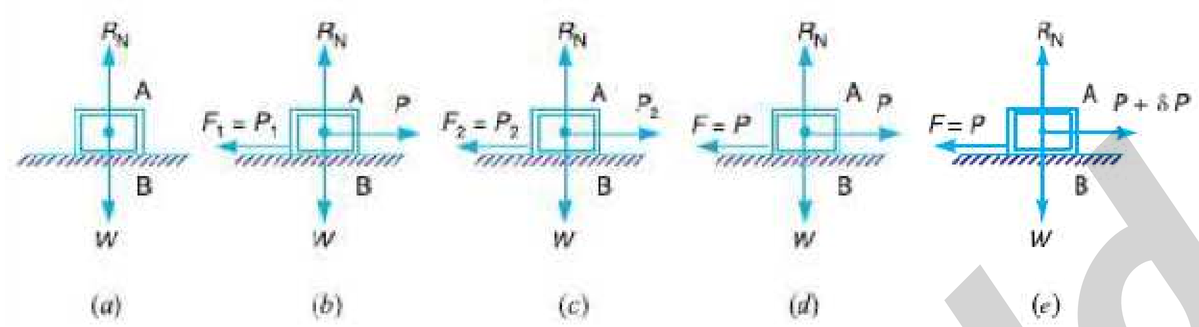
1. *Boundary friction* (*or greasy friction or non-viscous friction*). It is the friction, experienced between the rubbing surfaces, when the surfaces have a very thin layer of lubricant. The thickness of this very thin layer is of the molecular dimension. In this type of friction, a thin layer of lubricant forms a bond between the two rubbing surfaces. The lubricant is absorbed on the surfaces and forms a thin film. This thin film of the lubricant results in less friction between them. The boundary friction follows the laws of solid friction.

2. *Fluid friction* (*or film friction or viscous friction*). It is the friction, experienced between the rubbing surfaces, when the surfaces have a thick layer of the lubrhicant. In this case, the actual surfaces do not come in contact and thus do not rub against each other. It is thus obvious that fluid friction is not due to the surfaces in contact but it is due to the ***viscosity*** and ***oiliness*** of the lubricant. **Note :** The ***viscosity*** is a measure of the resistance offered to the sliding one layer of the lubricant over an adjacent layer. The absolute viscosity of a lubricant may be defined as the force required to cause a plate of unit area to slide with unit velocity relative to a parallel plate, when the two plates are separated by a layer of lubricant of unit thickness.

The ***oiliness*** property of a lubricant may be clearly understood by considering two lubricants of equal viscosities and at equal temperatures. When these lubricants are smeared on two different surfaces, it is found that the force of friction with one lubricant is different than that of the other. This difference is due to the property of the lubricant known as oiliness. The lubricant which gives lower force of friction is said to have greater oiliness.

Limiting Friction

Consider that a body A of weight W is lying on a rough horizontal body B as shown in Fig. In this position, the body A is in equilibrium under the action of its own weight W, and the normal reaction R_N (equal to W) of B on A. Now if a small horizontal force P_1 is applied to the body A acting through its centre of gravity as shown in Fig., it does not move because of the frictional force which prevents the motion. This shows that the applied force P_1 is exactly balanced by the force of friction F_1 acting in the opposite direction If we now increase the applied force to P_2 as shown in Fig. 10.1 (*c*), it is still found to be in equilibrium. This means that the force of friction has also increased to a value $F_2 = P_2$. Thus every time the effort is increased the force of friction also increases, so as to become exactly equal to the applied force. There is, however, a limit beyond which the force of friction cannot increase as shown in Fig. 10.1 (*d*). After this, any increase in the applied effort will not lead to any further increase in the force of friction, as shown in Fig. 10.1 (*e*), thus the body A begins to move in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as ***limiting force of friction*** or simply ***limiting friction***. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction into play is called **static friction** which may have any value between zero and limiting friction.



Laws of Static Friction

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.

2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.

3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = \text{constant}$$

4. The force of friction is independent of the area of contact, between the two surfaces.

5. The force of friction depends upon the roughness of the surfaces.

Laws of Kinetic or Dynamic Friction

Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.

2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.

3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Laws of Solid Friction

Following are the laws of solid friction :

1. The force of friction is directly proportional to the normal load between the surfaces.

2. The force of friction is independent of the area of the contact surface for a given normal load.

3. The force of friction depends upon the material of which the contact surfaces are made.

4. The force of friction is independent of the velocity of sliding of one body relative to the other body.

Laws of Fluid Friction

Following are the laws of fluid friction :

1. The force of friction is almost independent of the load.

2. The force of friction reduces with the increase of the temperature of the lubricant.

3. The force of friction is independent of the substances of the bearing surfaces.

4. The force of friction is different for different lubricants.

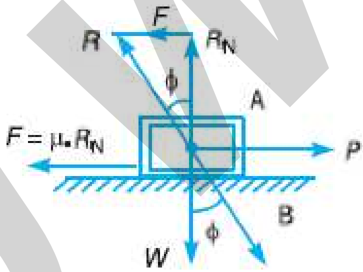
Coefficient of Friction

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is generally denoted by $\propto$. Mathematically, coefficient of friction,

$$\propto = F/R_N$$

Limiting Angle of Friction

Consider that a body A of weight (W) is resting on a horizontal plane B, as shown in Fig. 10.2. If a horizontal force P is applied to the body, no relative motion will take place until the applied force P is equal to the force of friction F, acting opposite to the direction of motion. The magnitude of this force of friction is $F = \propto .W = \propto .R_N$, where R_N is the normal reaction. In the limiting case, when the motion just begins, the body will be in equilibrium under the action of the following three forces:



1. Weight of the body (W),

2. Applied horizontal force (P), and

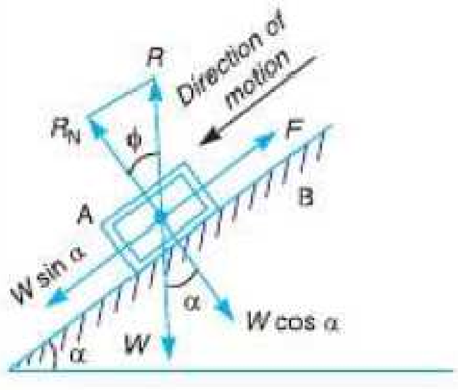
3. Reaction (R) between the body A and the plane B.

The reaction R must, therefore, be equal and opposite to the resultant of W and P and will be inclined at an angle ⟩ to the normal reaction R_N. This angle ⟩ is known as the ***limiting angle of friction***. It may be defined as the angle which the resultant reaction R makes with the normal reaction R_N.

From Fig. 10.2, tan ⟩ = $F/R_N = \propto R_N / R_N = \propto$

Angle of Repose

Consider that a body A of weight (W) is resting on an inclined plane B, as shown in Fig. 10.3. If the angle of inclination ⟨ of the plane to the horizontal is such that the body begins to move down the plane, then the angle ⟨ is called the ***angle of repose***. A little consideration will show that the body will begin to move down the plane when the angle of inclination of the plane is equal to the angle of friction (*i.e.* ⟨ = ⟩). This may be proved as follows :



The weight of the body (W) can be resolved into the following two components :

1. $W \sin \alpha$, parallel to the plane B. This component tends to slide the body down the plane.

2. $W \cos \alpha$, perpendicular to the plane B. This component is balanced by the normal reaction (R_N) of the body A and the plane B.

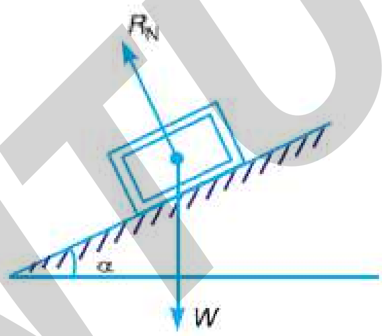
The body will only begin to move down the plane, when

$$W \sin \alpha = F = \mu.R_N = \mu.W \cos \alpha$$

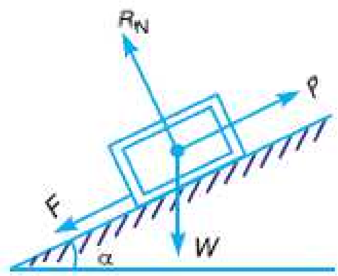
$$\tan \alpha = \mu = \tan \phi \text{ or } \alpha = \phi$$

Friction of a Body Lying on a Rough Inclined Plane

Consider that a body of weight (W) is lying on a plane inclined at an angle α with the horizontal, as shown in Fig. (*a*) and (*b*).



(*a*) Angle of inclination less than angle of friction.



(*b*) Angle of inclination more than angle of friction.

little consideration will show that if the inclination of the plane, with the horizontal, is less than the angle of friction, the body will be in equilibrium as shown in Fig. 10.6 (*a*). If,in this condi- tion, the body is required to be moved upwards and downwards, a corresponding force is required for the same. But, if the inclination of the plane is more than the angle of friction, the body will move down and an upward force (P) will be required to resist the body from moving down the plane as shown in Fig. (*b*).

Let us now analyse the various forces which act on a body when it slides either up or down an inclined plane.

1. ***Considering the motion of the body up the plane***

Let W = Weight of the body,

α = Angle of inclination of the plane to the horizontal,

ϕ = Limiting angle of friction for the contact surfaces,

P = Effort applied in a given direction in order to cause the body to slide with uniform velocity parallel to the plane, considering friction,

P_0 = Effort required to move the body up the plane neglecting friction,

θ = Angle which the line of action of P makes with the weight of the body W,

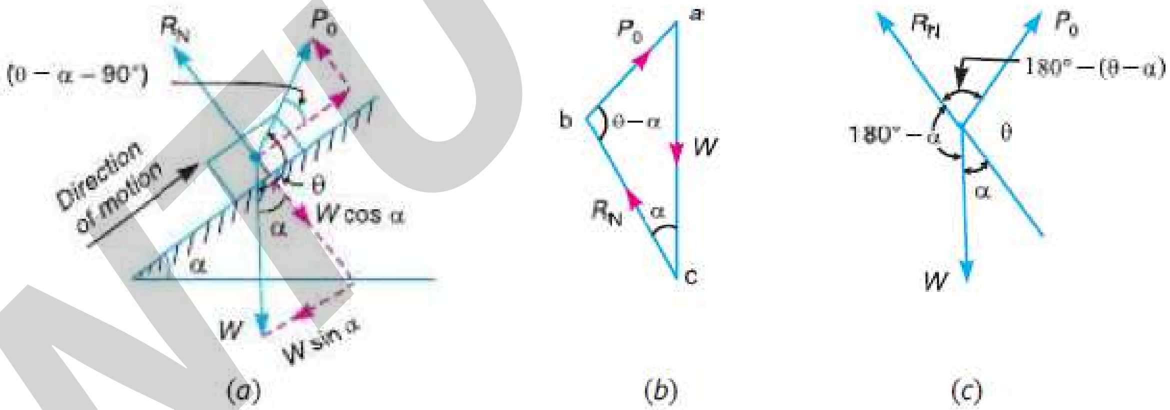
μ = Coefficient of friction between the surfaces of the plane and the body,

R_N = Normal reaction, and

R = Resultant reaction

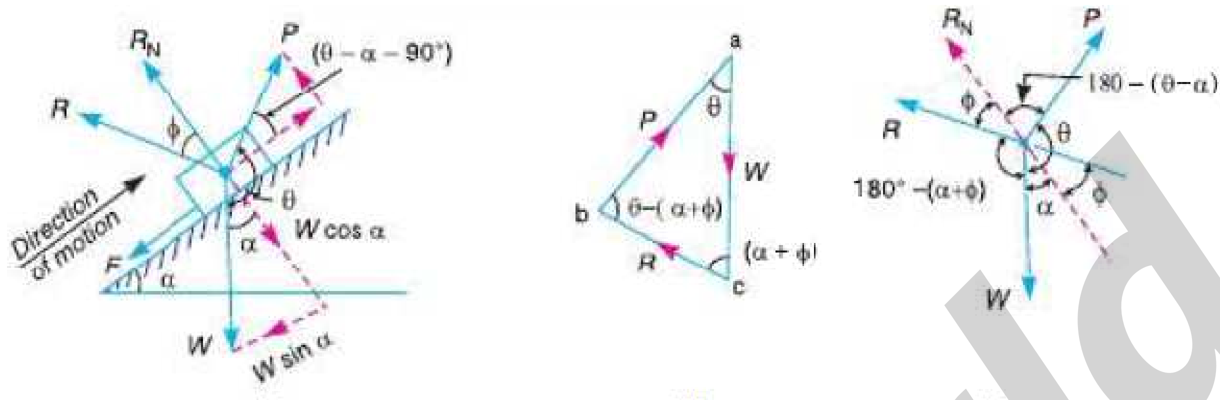
When the friction is neglected, the body is in equilibrium under the action of the three forces, *i.e.* P_0, W and R_N, as shown in Fig. 10.7 (*a*). The triangle of forces is shown in Fig. 10.7 (*b*). Now applying sine rule for these three concurrent forces,

$$\frac{P_0}{\sin\alpha} = \frac{W}{\sin(\theta - \alpha)} \quad \text{or} \quad P_0 = \frac{W\sin\alpha}{\sin(\theta - \alpha)}$$



When friction is taken into account, a frictional force $F = \mu.R_N$ acts in the direction opposite to the motion of the body, as shown in Fig. 10.8 (*a*). The resultant reaction R between the plane and the body is inclined at an angle ϕ with the normal reaction R_N. The triangle of forces is shown in Fig. (*b*). Now applying sine rule,

$$\frac{P}{\sin(\alpha + \phi)} = \frac{W}{\sin[\theta - (\alpha + \phi)]}$$

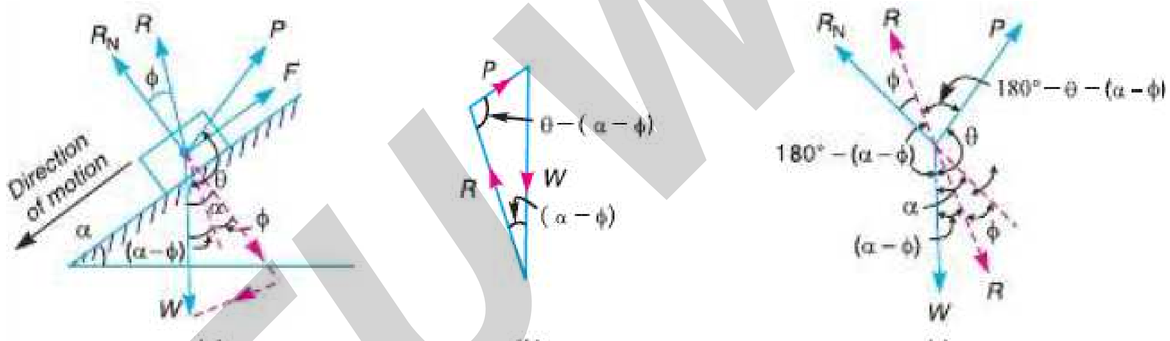


(a) (b) (c)

2. *Considering the motion of the body down the plane*

Neglecting friction, the effort required for the motion down the plane will be same as for the motion up the plane, *i.e.*

$$P_0 = \frac{W \sin \alpha}{\sin (\theta - \alpha)}$$



(a) (b) (c)

When the friction is taken into account, the force of friction $F = \propto . R_N$ will act up the plane and the resultant reaction R will make an angle ϕ with R_N towards its right as shown in Fig. (*a*). The triangle of forces is shown in Fig. (*b*). Now from sine rule,

$$\frac{P}{\sin (\alpha - \phi)} = \frac{W}{\sin[\theta - (\alpha - \phi)]}$$

$$P = \frac{W \sin (\alpha - \phi)}{\sin[\theta - (\alpha - \phi)]}$$

Efficiency of Inclined Plane

The ratio of the effort required neglecting friction (*i.e.* P_0) to the effort required considering friction (*i.e.* P) is known as efficiency of the inclined plane. Mathematically, efficiency of the inclined plane,

$$\eta = P_0 / P$$

Let us consider the following two cases :

1. ***For the motion of the body up the plane***

$$\text{Efficiency,} \quad \eta = \frac{P_0}{P} = \frac{W \sin \alpha}{\sin(\theta - \alpha)} \times \frac{\sin[\theta - (\alpha + \phi)]}{W \sin(\alpha + \phi)}$$

$$= \frac{\sin \alpha}{\sin \theta \cos \alpha - \cos \theta \sin \alpha} \times \frac{\sin \theta \cos(\alpha + \phi) - \cos \theta \sin(\alpha + \phi)}{\sin(\alpha + \phi)}$$

Multiplying the numerator and denominator by $\sin(\alpha + \phi) \sin \theta$, we get

$$\eta = \frac{\cot(\alpha + \phi) - \cot \theta}{\cot \alpha - \cot \theta}$$

2. ***For the motion of the body down the plane***

Since the value of P will be less than P_0, for the motion of the body down the plane, therefore in this case,

$$\eta = \frac{P}{P_0} = \frac{W \sin(\alpha - \phi)}{\sin[\theta - (\alpha - \phi)]} \times \frac{\sin(\theta - \alpha)}{W \sin \alpha}$$

$$= \frac{\sin(\alpha - \phi)}{\sin \theta \cos(\alpha - \phi) - \cos \theta \sin(\alpha - \phi)} \times \frac{\sin \theta \cos \alpha - \cos \theta \sin \alpha}{\sin \alpha}$$

Multiplying the numerator and denominator by $\sin(\alpha - \phi) \sin \theta$, we get

$$\eta = \frac{\cot \alpha - \cot \theta}{\cot(\alpha - \phi) - \cot \theta}$$

Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as ***external threads***. But if the threads are cut on the internal surface of a hollow rod, these are known as ***internal threads***. The screw threads are mainly of two types *i.e.* V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V- threads are used for the purpose of tightening pieces together *e.g.* bolts and nuts etc. But the square threads are used in screw jacks, vice screws etc. The following terms are important for the study of screw :

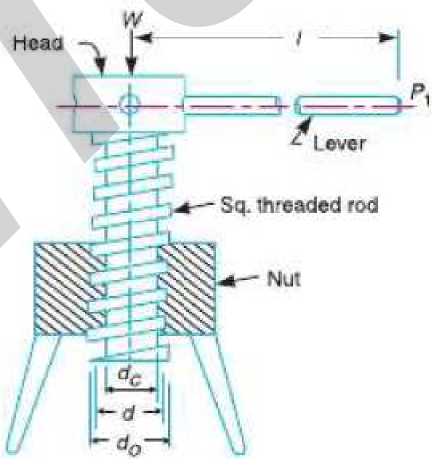
1. ***Helix.*** It is the curve traced by a particle, while describing a circular path at a uniformspeed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced bya particle while moving along a screw thread.
2. ***Pitch***. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw.
3. ***Lead***. It is the distance, a screw thread advances axially in one turn.
4. ***Depth of thread***. It is the distance between the top and bottom surfaces of a thread (also known as **crest** and **root** of a thread).
5. ***Single-threaded screw***. If the lead of a screw is equal to its pitch, it is known as single threaded screw.
6. ***Multi-threaded screw.*** If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw ***e.g.*** in a double threaded screw, two threads are cut in one lead length. In such cases, all the threads run independently along the length of the rod. Mathematically,

   Lead = Pitch × Number of threads
7. ***Helix angle.*** It is the slope or inclination of the thread with the horizontal. Mathematically,

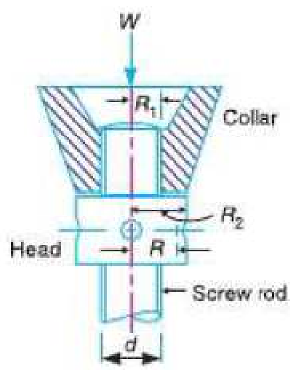
$$\tan\alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$

Screw Jack

The screw jack is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works is similar to that of an inclined plane.



(a) Screw jack.

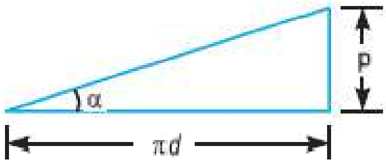


(b) Thrust collar.

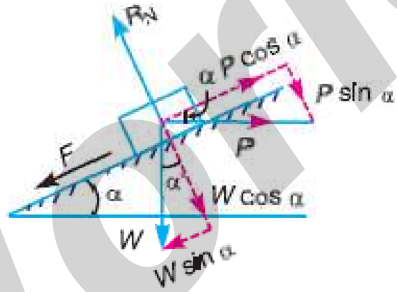
Fig. (*a*) shows a common form of a screw jack, which consists of a square threaded rod (also called screw rod or simply screw) which fits into the inner threads of the nut. The load, to be raised or lowered, is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

Torque Required to Lift the Load by a Screw Jack

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. (*a*).



(*a*) Development of a screw.



(*b*) Forces acting on the screw.

Let p = Pitch of the screw,

d = Mean diameter of the screw,

〈 = Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

∝ = Coefficient of friction, between the screw and nut = tan 〉,where 〉 is the friction angle.

From the geometry of the Fig. 10.12 (*a*), we find that

tan 〈 = p/□ d

Since the principle on which a screw jack works is similar to that of an inclined plane, there- fore the force applied on the lever of a screw jack may be considered to be horizontal as shown in Fig. (*b*).

Since the load is being lifted, therefore the force of friction (F = ∝.R_N) will act downwards. All the forces acting on the screw are shown in Fig.(*b*).

Resolving the forces along the plane,

P cos 〈 = W sin 〈 + F = W sin 〈 + ∝.R_N

and resolving the forces perpendicular to the plane,

R_N = P sin 〈 + W cos 〈

Substituting this value of R_N in equation (*i*),

P cos 〈 = W sin 〈 + ∝ (P sin 〈 + W cos 〈)

= W sin 〈 + ∝ P sin 〈 + ∝ W cos 〈

4 $$P = W \cdot \frac{\sin \langle + \propto \cos \langle}{\cos \langle \updownarrow \propto \sin \langle}$$

Substituting the value of $\propto$ = tan $\rangle$ in the above equation, we get

$$P = W \cdot \frac{\sin \langle + \tan \rangle \cos \langle}{\cos \langle \updownarrow \tan \rangle \sin \langle}$$

Multiplying the numerator and denominator by cos $\rangle$,

$$P = W \cdot \frac{\sin \langle \cos \rangle + \sin \rangle \cos \langle}{\cos \langle \cos \rangle \updownarrow \sin \langle \sin \rangle} \quad W \quad \frac{\sin (\langle + \rangle)}{\cos (\langle + \rangle)}$$

$$= W \tan (\langle + \rangle)$$

4 Torque required to overcome friction between the screw and nut,

$$T_1 = P \cdot \frac{d}{2} = W \tan (\langle + \rangle) \frac{d}{2}$$

When the axial load is taken up by a thrust collar or a flat surface, as shown in Fig. 10.11 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \propto_1 W \left(\frac{R_1 + R_2}{2} \right) = \propto_1 . W . R$$

where R_1 and R_2 = Outside and inside radii of the collar,

R = Mean radius of the collar, and

$\propto_1$ = Coefficient of friction for the collar.

4 Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2 = P \times \frac{d}{2} + \propto_1 . W . R$$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever, *i.e.*

$$T = P \cdot \frac{d}{2} = P_1 . l$$

Torque Required to Lower the Load by a Screw Jack

We have discussed in Art. 10.18, that the principle on which the screw jack works is similar to that of an inclined plane. If one complete turn of a screw thread be imagined to be unwound from the body of the screw and developed, it will form an inclined plane as shown in Fig. (*a*).

Let p = Pitch of the screw,

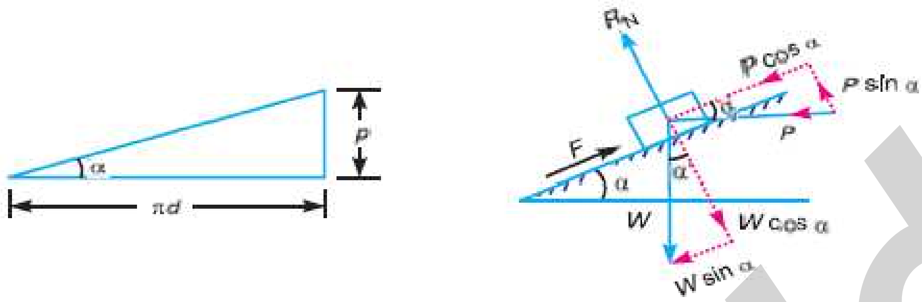
d = Mean diameter of the screw,

$\langle$ = Helix angle,

P = Effort applied at the circumference of the screw to lower the load,

W = Weight to be lowered, and

$\propto$ = Coefficient of friction between the screw and nut = tan $\rangle$, where $\rangle$ is the friction angle.



(*a*) (*b*)

From the geometry of the figure, we find that

$$\tan \alpha = p/\pi d$$

Since the load is being lowered, therefore the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the screw are shown in Fig. (*b*).

Resolving the forces along the plane,

$$P \cos \alpha = F - W \sin \alpha = \mu.R_N - W \sin \alpha$$

and resolving the forces perpendicular to the plane,

$$R_N = W \cos \alpha - P \sin \alpha$$

Substituting this value of R_N in equation ***(i)***,

$$P \cos \alpha = \mu\,(W \cos \alpha - P \sin \alpha) - W \sin \alpha$$

$$= \mu.W \cos \alpha - \mu.P \sin \alpha - W \sin \alpha$$

$$P = W \cdot \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \cdot \frac{(\tan \phi \cos \alpha - \sin \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P = W \cdot \frac{(\sin \phi \cos \alpha - \sin \alpha \cos \phi)}{(\cos \alpha \cos \phi + \sin \phi \sin \alpha)} = W \cdot \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)}$$

$$= W \tan(\phi - \alpha)$$

Torque required to overcome friction between the screw and nut,

$$T = P \times \frac{d}{2} = W \tan(\phi - \alpha)\frac{d}{2}$$

Over Hauling and Self Locking Screws

We have seen in Art. 10.20 that the effort required at the circumference of the screw to lower the load is

$$P = W \tan (\rangle - \langle)$$

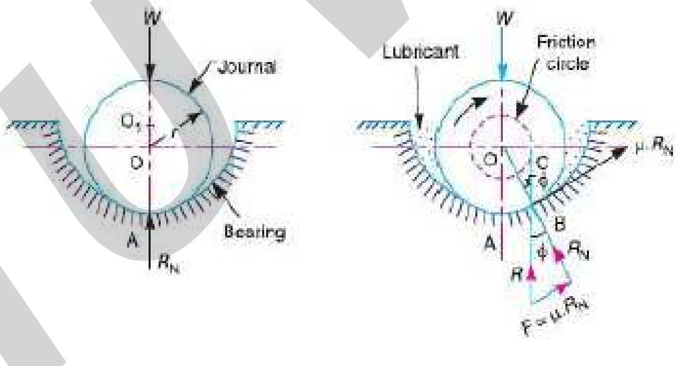
and the torque required to lower the load

$$T = P \cdot \frac{d}{2} = W \tan (\rangle - \langle) \frac{d}{2}$$

In the above expression, if $\rangle < \langle$, then torque required to lower the load will be ***negative.*** In other words, the load will start moving downward without the application of any torque. Such a condition is known as ***over haulding of screws.*** If however, $\rangle > \langle$, the torque required to lower the load will ***positive***, indicating that an effort is applied to lower the load. Such a screw is known as ***self locking screw.*** In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle *i.e.* $\propto$ or $\tan \rangle > \tan \langle$.

Friction in Journal Bearing-Friction Circle

A journal bearing forms a turning pair as shown in Fig. 10.15 (*a*). The fixed outer element of a turning pair is called a ***bearing*** and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a ***journal.*** The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing.



When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig. 10.15 (*a*). The load W on the journal and normal reaction R_N (equal to W) of the bearing acts through the centre. The reaction R_N acts vertically upwards at point A. This point A is known as ***seat*** or ***point of pressure.***

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig. 10.15 (*b*). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction.Therefore, the reaction R does not act vertically upward, but acts at another point of pressure B. This is due to the fact that when shaft rotates, a frictional force $F = \mu R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point A to point B.

In order that the rotation may be maintained, there must be a couple rotating the shaft

.Let θ = Angle between R (resultant of F and R_N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in metres.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$$R = W, \text{ and } T = W \times OC = W \times OB \sin\theta = W.r \sin\theta$$

Since θ is very small, therefore substituting $\sin\theta = \tan\theta$

$$T = W.r \tan\theta = \mu.W.r$$

If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$$P = T.\omega = T \times 2\pi N/60 \text{ watts}$$

Flat Pivot Bearing

When a vertical shaft rotates in a flat pivot bearing (known as **foot step bearing**), as shown in Fig. 10.17, the sliding friction will be along the surface of contact between the shaft and the bearing.

Let W = Load transmitted over the bearing surface,

R = Radius of bearing surface,

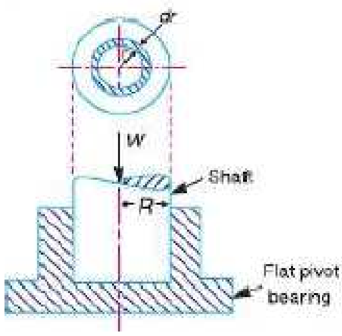
p = Intensity of pressure per unit area of bearing surface between rubbing surfaces, and

μ = Coefficient of friction.

We will consider the following two cases :

1. When there is a uniform pressure ; and

2. When there is a uniform wear.



1. *Considering unifrom pressure*

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

Consider a ring of radius r and thickness dr of the bearing area.

Area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring,

$$\delta W = p \times A = p \times 2\pi r.dr$$

Frictional resistance to sliding on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu \, p \times 2\pi r.dr = 2\pi \mu.p.r.dr$$

Frictional torque on the ring,

$$T_r = F_r \times r = 2\pi \mu \, p \, r.dr \times r = 2\pi \mu \, p \, r^2 \, dr$$

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

2. *Considering uniform wear*

We have already discussed that the rate of wear depends upon the intensity of pressure (p) and the velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (*i.e.* $p.v.$.). Since the velocity of rubbing surfaces increases with the distance (*i.e.* radius r) from the axis of the bearing, therefore for uniform wear

$$p.r = C \text{ (a constant)}$$

and the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr$$

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi \, C.dr = 2\pi \, C \, [r]_0^R = 2\pi C.R \text{ or } C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi \mu \, p \, r^2 dr = 2\pi \mu \cdot \frac{C}{r} \cdot r^2 dr$$

$$= 2\pi \mu.C.r \, dr$$

Conical Pivot Bearing

The conical pivot bearing supporting a shaft carrying a load *W* is shown in Fig

P_0= Intensity of pressure normal to the cone,

⟨= Semi angle of the cone,

∝= Coefficient of friction between the shaft and the bearing, and

R= Radius of the shaft.

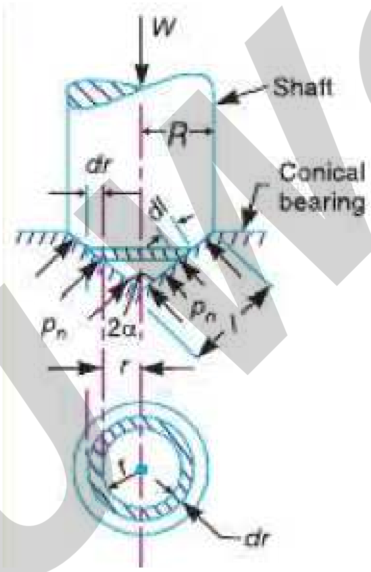
Consider a small ring of radius *r* and thickness *dr*. Let *dl* is the length of ring along the cone, such that

dl = *dr* cosec ⟨

Area of the ring,

A = 2□*r*.*dl* = 2□*r*.*dr* cosec ⟨

...(∵ *dl* = *dr* cosec ⟨)



1. *Considering uniform pressure*

We know that normal load acting on the ring,

W_n = Normal pressure × Area

= p_n × 2□*r*.*dr* cosec ⟨

and vertical load acting on the ring,

*™*W* = Vertical component of ™W_n = ™W_n.sin ⟨

=p_n × 2□*r*.*dr* cosec ⟨. sin ⟨ = p_n × 2□ *r*.*dr*

p_n = *W* / □R_2

We know that frictional force on the ring acting tangentially at radius *r*,

$$F_r = \propto .\text{™}W_n = \propto .\ p_n .2\square r.dr \text{ cosec} \langle = 2\square \propto .\ p_n .\text{cosec} \langle .r.dr$$

and frictional torque acting on the ring,

$$T_r = F_r \cdot r = 2\ \square \propto .\ p_n .\text{cosec} \langle .r.dr \cdot r = 2\square \propto .\ p_n \text{ cosec} \langle .r_2 .dr$$

Integrating the expression within the limits from 0 to R for the total frictional torque on the conical pivot bearing

2. *Considering uniform wear*

In Fig., let p_r be the normal intensity of pressure at a distance r from the central axis. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance

$$p_r.r = C \text{ (a constant) or } p_r = C/r$$

and the load transmitted to the ring,

$$\text{™}W = p_r \cdot 2\square r.dr = \frac{¦}{r} \cdot 2\square r.dr = 2\square C.dr$$

4 Total load transmitted to the bearing,

$$W = \int_0^R 2\square C.dr = 2\square C\,[r]_0^R = 2\square C.R \text{ or } C = \frac{W}{2\square R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\square \propto .\ p_r .\text{cosec} \langle .r_2 .dr = 2\square \propto \cdot \frac{¦}{r} \cdot \text{cosec} \langle .r_2 .dr$$

$$= 2\square \propto .C.\text{cosec} \langle .r.dr$$

4 Total frictional torque acting on the bearing,

$$T = \int_0^R 2\square \propto .C\ .\text{cosec} \langle .r.dr = 2\square \propto .C.\text{cosec} \langle \left| \frac{r_2}{2} \right|_0^R$$

$$= 2\square \propto .C.\text{cosec} \langle \cdot \frac{R_2}{2} = \square \propto .C.\text{cosec} \langle .R_2$$

Substituting the value of C, we have

$$T = \square \propto \cdot \frac{W}{2\square R} \cdot \text{cosec} \langle .R_2 = \frac{1}{2} \cdot \propto .W.R \text{ cosec} \langle = \frac{1}{2} \cdot \propto .W\ L$$

UNIT –III

1. Clutches

Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the driven shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually brings the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.

2. The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.

3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

The friction clutches of the following types are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),

2. Cone clutches, and

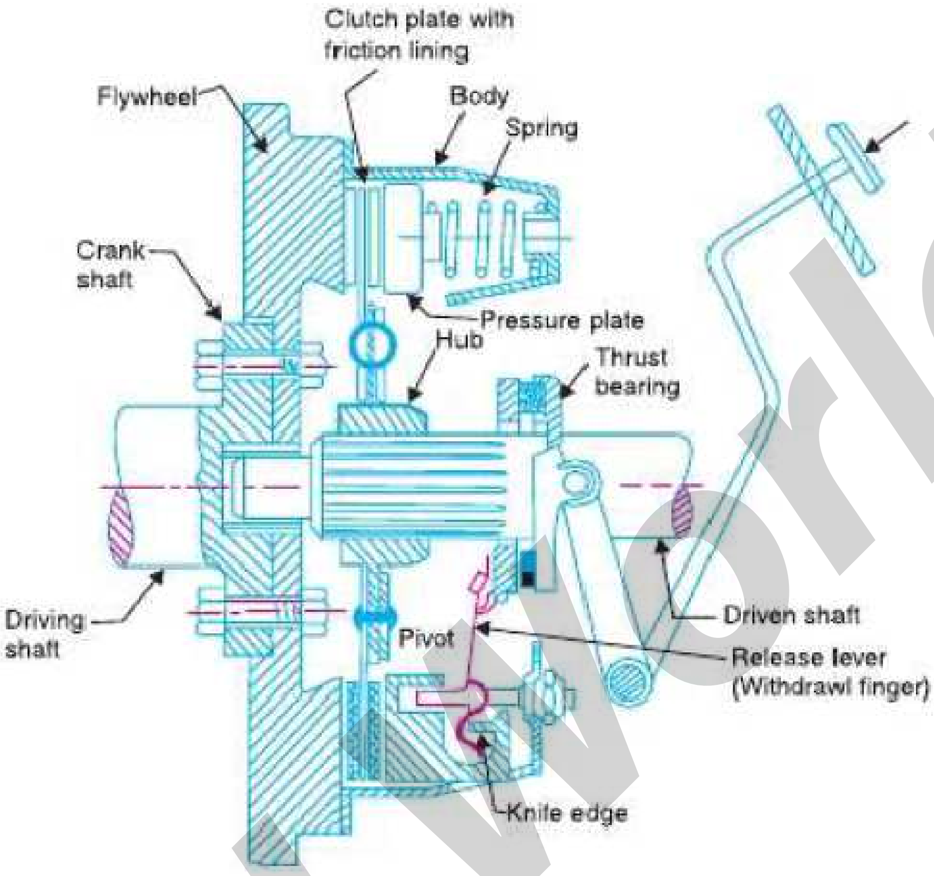
3. Centrifugal clutches.

We shall now discuss, these clutches, in detail, in the following pages. It may be noted that the disc and cone clutches are based on the same theory as the pivot and collar bearings.

Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.



The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

Now consider two friction surfaces, maintained in contact by an axial thrust W, as shown in Fig. (a).

T = Torque transmitted by the clutch

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces, and

$\propto$ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. (b).

We know that area of contact surface or friction surface,

$= 2 \square r.dr$

Normal or axial force on the ring,

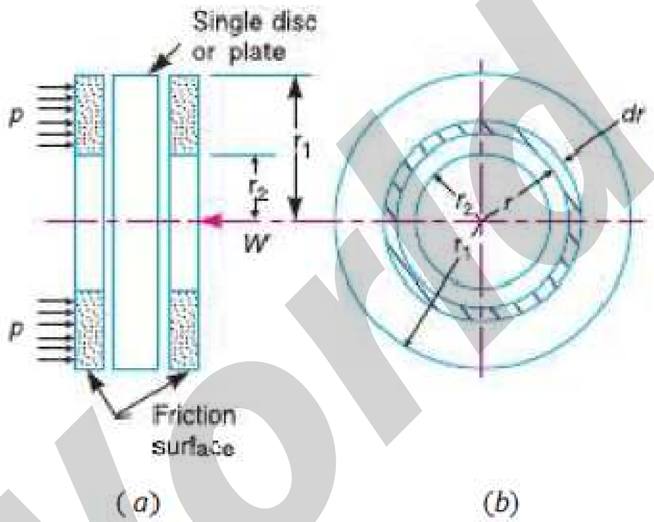
$™W$ = Pressure × Area = $p \times 2 \square r.dr$

and the frictional force on the ring acting tangentially at radius r,

$F_r = \propto.™W = \propto.p \times 2 \square\, r.dr$

Frictional torque acting on the ring,

$T_r = F_r \times r = \propto.p \times 2 \square\, r.dr \times r = 2 \square \times \propto .p.r_2\, dr$



We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\square[(r_1)^2 \text{ⅰ} (r_2)^2]}$$

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

4 Total frictional torque acting on

$$T = \int_{r_1}^{r_2} 2 \square \propto p.r^2 .dr$$

Substituting the value of p from e

$$T = 2 \square \propto \cdot \frac{}{\square[(r_1)}$$

$T_r = 2 \square \propto .p.r_2\, dr$

$$= \frac{2}{3} \cdot \propto .W \left| \frac{(r_1}{\square (} \right.$$

R = Mean radiu

$$= \frac{2}{3} \left| \frac{(r_1)^3}{\square (r_1)^2} \right.$$

2. *Considering uniform wear*

In Fig. 10.22, let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r = C \text{ (a constant) or } p = C/r \qquad \text{...}(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi\, r.dr = \frac{C}{r}\cdot 2\pi\, C.dr = 2\pi C.dr$$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\, C\, dr = 2\pi\, C\,[r]_{r_2}^{r_1} = 2\pi\, C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\,(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.\, p\, r^2 .dr = 2\pi\mu \cdot \frac{C}{r} r^2 .dr = 2\pi\mu.C.r.dr$$

$$\text{...}(\because p = C/r)$$

∴ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C\,[(r_1)^2 - (r_2)^2] = \pi\mu \cdot \frac{W}{2\pi\,(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

$$= \frac{1}{2}\cdot \mu.W\,(r_1 + r_2) = \mu.W.R$$

where R = Mean radius of the friction surface $= \frac{r_1 + r_2}{2}$

Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 10.23, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let

n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

4 Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

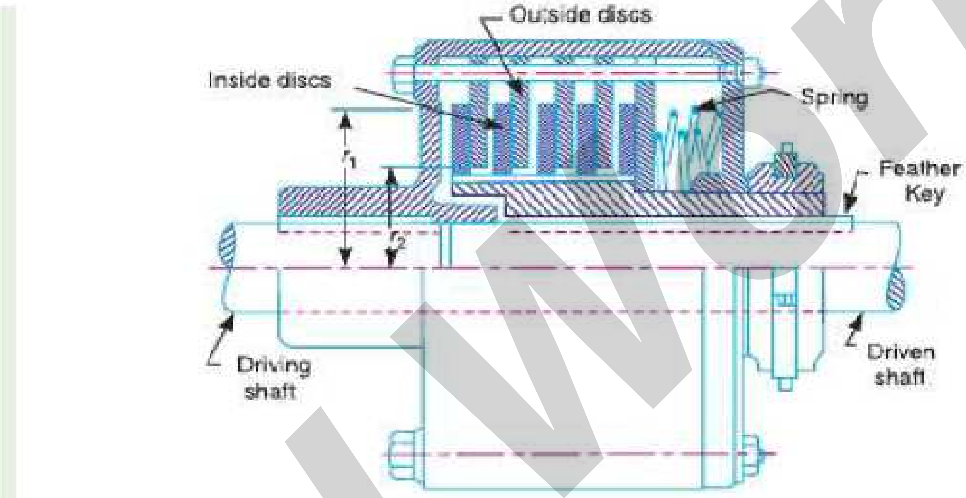
and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of the friction surfaces

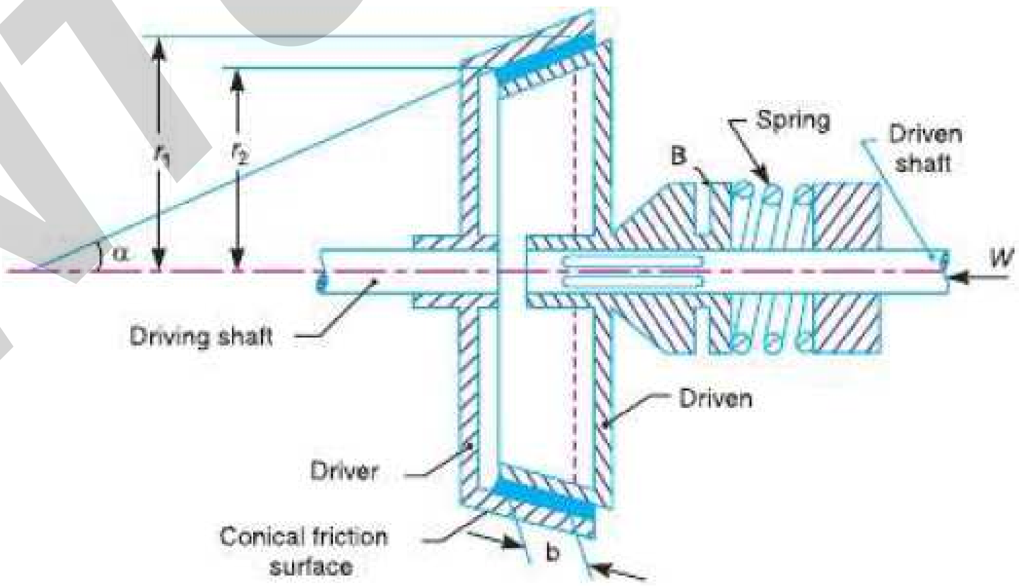
$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{...(For uniform wear)}$$

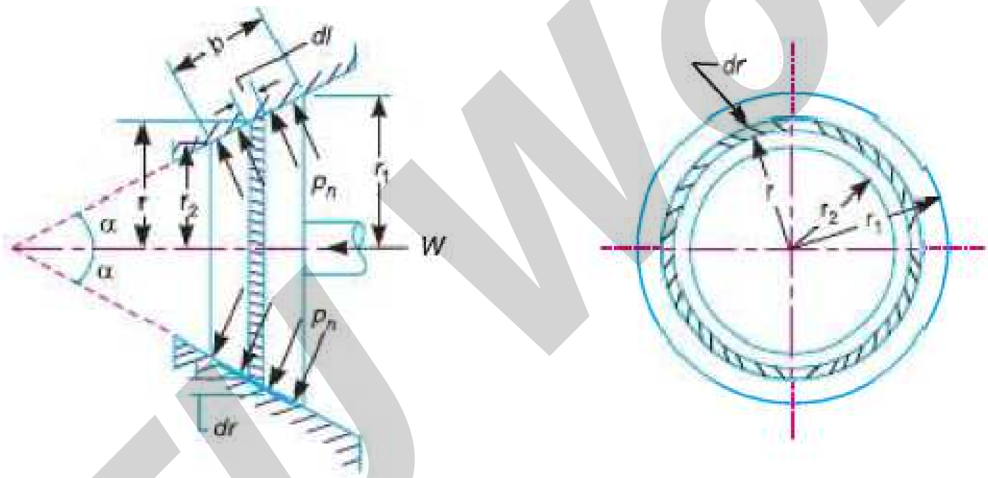


Cone Clutch

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch



It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction. Consider a pair of friction surface as shown in Fig. 10.25 (*a*). Since the area of contact of a pair of friction surface is a frustrum of a cone, therefore the torque transmitted by the cone clutch may be determined in the similar manner as discussed for conical pivot bearings in Art.



(*a*) (*b*)

p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between contact surfaces),

r_1 and r_2 = Outer and inner radius of friction surfaces respectively.

R = Mean radius of the friction surface

〈 = Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

∝ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).

Consider a small ring of radius *r* and thickness *dr*, as shown in Fig. 10.25 (*b*). Let *dl* is length of ring of the friction surface, such that

$$dl = dr.\text{cosec}\,\alpha$$

Area of the ring,

$$A = 2\pi\, r.dl = 2\pi r.dr\ \text{cosec}\,\alpha$$

We shall consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. *Considering uniform pressure*

We know that normal load acting on the ring,

$$W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi\, r.dr.\text{cosec}\,\alpha$$

and the axial load acting on the ring,

$$W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin\alpha = p_n \times 2\pi\, r.dr.\ \text{cosec}\,\alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi\, p_n r.dr = 2\pi p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n \left[(r_1)^2 - (r_2)^2\right]$$

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

$$W = \int_{r_2}^{r_1} 2\pi\, p_n r.dr = 2\pi p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n \left[(r_1)^2 - (r_2)^2\right]$$

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi\, r.dr.\text{cosec}\,\alpha$$

Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi\, r.dr.\ \text{cosec}\,\alpha.r = 2\pi\,\mu.p_n.\text{cosec}\,\alpha.r^2\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu \cdot p_n \cdot \text{cosec}\,\alpha \cdot r^2 \cdot dr = 2\pi\mu\, p_n \cdot \text{cosec}\,\alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\, p_n \cdot \text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation ***(i)***, we get

$$T = 2\pi\mu \cdot \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \cdot \text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \cdot \mu W \cdot \text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering uniform wear*

In Fig. 10.25, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$\therefore$ $p_r \cdot r = C$ (a constant) or $p_r = C / r$

We know that the normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r \cdot dr\ \text{cosec}\,\alpha$$

and the axial load acting on the ring ,

$$\delta W = \delta W_n \times \sin\alpha = p_r \cdot 2\pi r \cdot dr \cdot \text{cosec}\,\alpha \cdot \sin\alpha = p_r \times 2\pi r \cdot dr$$

$$= \frac{C}{r} \cdot 2\pi r \cdot dr = 2\pi C \cdot dr \qquad \ldots(\because p_r = C/r)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C\, [r]_{r_2}^{r_1} = 2\pi C\,(r_1 - r_2)$$

or $$C = \frac{W}{2\pi(r_1 - r_2)} \qquad \ldots\textit{(iii)}$$

We know that frictional force acting on the ring,

$$F_r = \mu \cdot \delta W_n = \mu \cdot p_r \times 2\pi r \times dr\ \text{cosec}\,\alpha$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = \mu\, p_r \times 2\pi r \cdot dr \cdot \text{cosec}\,\alpha \times r$$

$$= \mu \cdot \frac{C}{r} \cdot 2\pi r^2 \cdot dr \cdot \text{cosec}\,\alpha = 2\pi\mu \cdot C\ \text{cosec}\,\alpha \cdot r\, dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\text{cosec}\,\alpha.r\,dr = 2\pi\mu.C.\text{cosec}\,\alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\text{cosec}\,\alpha\left[\frac{(r_1)^2-(r_2)^2}{2}\right]$$

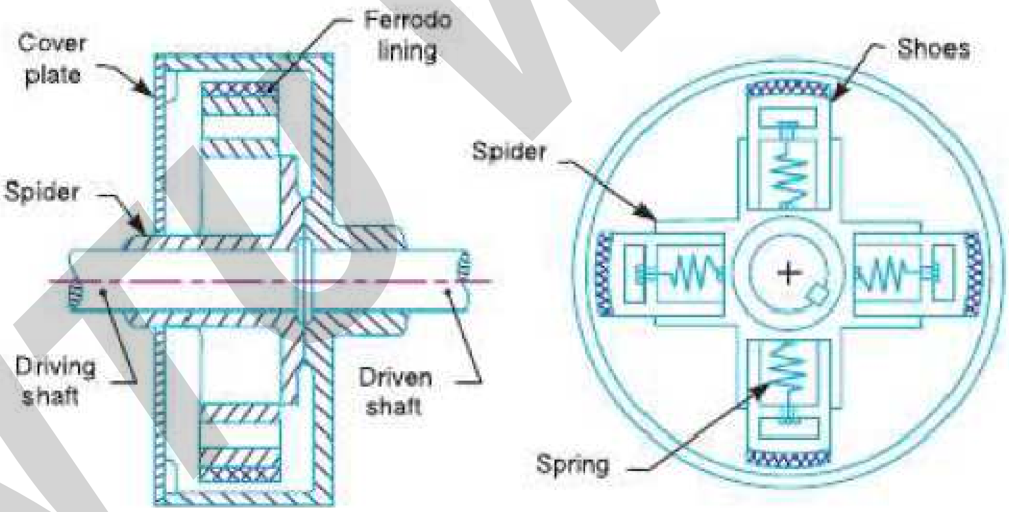
Substituting the value of C from equation (*i*), we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1-r_2)} \times \text{cosec}\,\alpha\left[\frac{(r_1)^2-(r_2)^2}{2}\right]$$

$$= \mu.W\,\text{cosec}\,\alpha\left(\frac{r_1+r_2}{2}\right) = \mu.W.R\,\text{cosec}\,\alpha$$

Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 10.28. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held



against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder

and enables more torque to be transmitted.

In order to determine the mass and size of the shoes, the following procedure is adopted :

1. *Mass of the shoes*

Consider one shoe of a centrifugal clutch as shown in Fig

Let m = Mass of each shoe,

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s = $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

$\propto$ = Coefficient of friction between the shoe and rim.

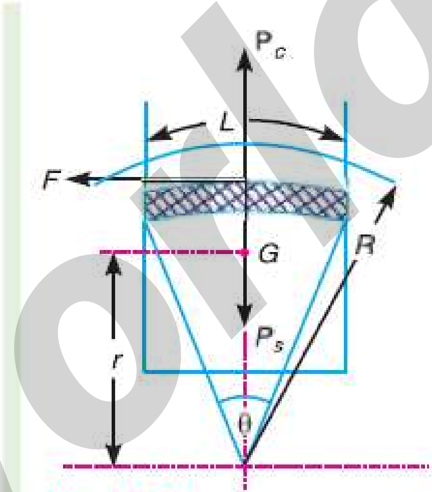


Fig. 10.29. Forces on a shoe of centrifugal clutch.

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$$P_s = m\,(\omega_1)^2\,r$$

The net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s$$

and the frictional force acting tangentially on each shoe,

$$F = \propto (P_c - P_s)$$

Frictional torque acting on each shoe,

$$= F \times R = \propto (P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \propto (P_c - P_s)\, R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. *Size of the shoes*

l = Contact length of the shoes,

b = Width of the shoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, the intensity of pressure may be taken as 0.1 N/mm2

Area of contact of the shoe,

$$A = l.b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is ($P_c - P_s$), therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

2. Brakes and Dynamometers

Introduction

A ***brake*** is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The capacity of a brake depends upon the following factors :

1. The unit pressure between the braking surfaces,

2. The coefficient of friction between the braking surfaces,

3. The peripheral velocity of the brake drum,

4. The projected area of the friction surfaces, and

5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

Materials for Brake Lining

The material used for the brake lining should have the following characteristics

1. It should have high coefficient of friction with minimum fading. In other words, the coeffi- cient of friction should remain constant with change in temperature.

2. It should have low wear rate.

3. It should have high heat resistance.

4. It should have high heat dissipation capacity.

5. It should have adequate mechanical strength.

6. It should not be affected by moisture and oil.

The materials commonly used for facing or lining of brakes and their properties are shown in the following table.

Types of Brakes

The brakes, according to the means used for transforming the energy by the braking elements, are classified as :

1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,

2. Electric brakes *e.g.* generators and eddy current brakes, and

3. Mechanical brakes.

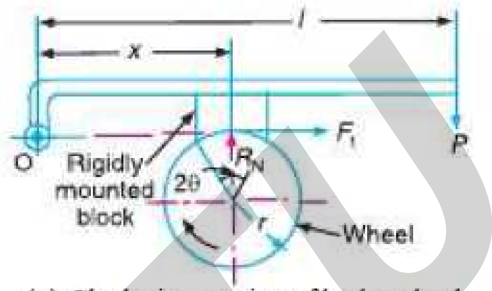
The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel. The mechanical brakes, according to the direction of acting force, may be divided into the following two groups :

(a) ***Radial brakes.*** In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into ***external brakes*** and ***internal brakes.*** According to the shape of the friction elements, these brakes may be ***block*** or ***shoe brakes*** and ***band brakes.***

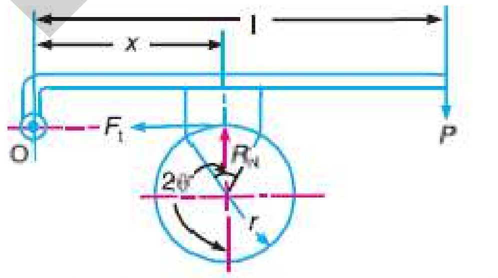
(b) ***Axial brakes.*** In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches. Since we are concerned with only mechanical brakes, therefore, these are discussed, in detail, in the following pages.

Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. 19.1. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 19.1. The other end of the lever is pivoted on a fixed fulcrum O.



(*a*) Clockwise rotation of brake wheel



(*b*) Anticlockwise rotation of brake wheel

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu . R_N$$

and the braking torque, $T_B = F_t . r = \mu . R_N . r$

Let us now consider the following three cases :

Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum O of the lever, and the brake wheel rotates clockwise as shown in Fig. (*a*), then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \cdot x = P \cdot l \text{ or } R_N = \frac{P \cdot l}{x}$$

Braking torque,

$$T_B = \propto.R_N.r = \propto \cdot \frac{P.l}{x} \cdot r = \frac{\propto.P.l.r}{x}$$

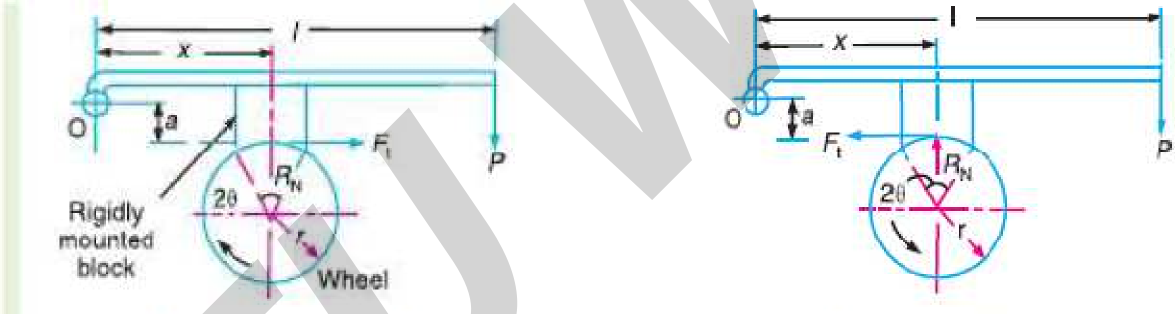
It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. (*b*), then the braking torque is same, *i.e*

$$T_B = \propto.R_N.r = \frac{\propto.P.l.r}{x}$$

Case 2. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. (*a*), then for equilibrium, taking moments about the fulcrum *O*,

$$R_N \times x + F_t \times a = P.l \quad \text{or } R_N \times x + \mu\, R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \propto.a}$$

and braking torque, $T_B = \propto R_N\ r = \dfrac{\propto.\ p.l.r}{x + \propto.a}$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

When the brake wheel rotates anticlockwise, as shown in Fig. 19.2 (*b*), then for equilibrium.

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

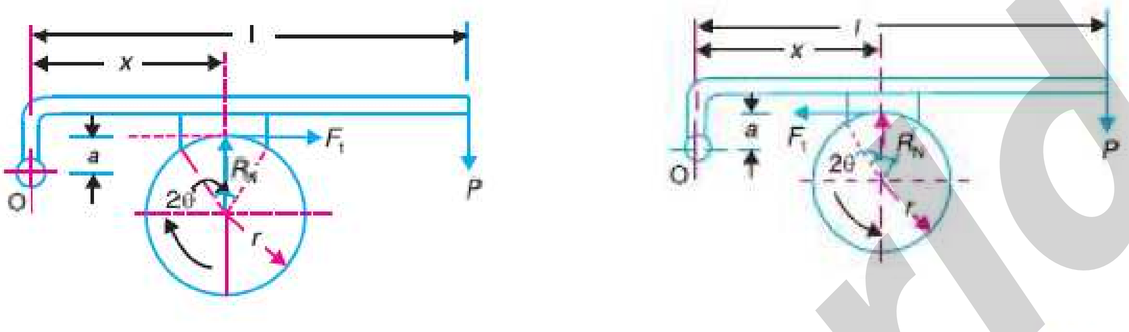
or $\quad R_N (x - \mu.a) = P.l \quad$ or $\quad R_N = \dfrac{P.l}{x - \propto.a}$

and braking torque, $\quad T_B = \propto.R_N.r = \dfrac{\propto.P.l.r}{x - \propto.a}$

Case 3. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' above the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 19.3 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N.x = P.l + F_t\,.\,a = P.l + \mu.R_N.a$$

or $\quad R_N\,(x - \mu.a) = P.l \quad$ or $\quad R_N = \dfrac{P.l}{x \updownarrow \propto.a}$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

and braking torque,

$$T_B = \mu.R_N.r = \frac{\propto.P.l.r}{x \updownarrow \propto.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l \quad \text{or} \quad R_N \times x + \mu.R_N \times a = P.l \text{ or } R_N = \frac{P.l}{x + \propto.a}$$

and braking torque, $\quad T_B = \mu.R_N.r = \dfrac{\propto.P.l.r}{x + \propto.a}$

Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever, as shown in Fig. 19.4, instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when 2θ > 60°) is

given by

$$T_B = F_t \cdot r = \propto_2.R_N.r$$

where $\quad \propto_2$ = Equivalent coefficient of friction = $\dfrac{4\propto \sin\theta}{2\theta + \sin 2\theta}$, and

μ = Actual coefficient of friction.

These brakes have more life and may provide a higher braking torque.

Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes,or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig., is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum. When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :

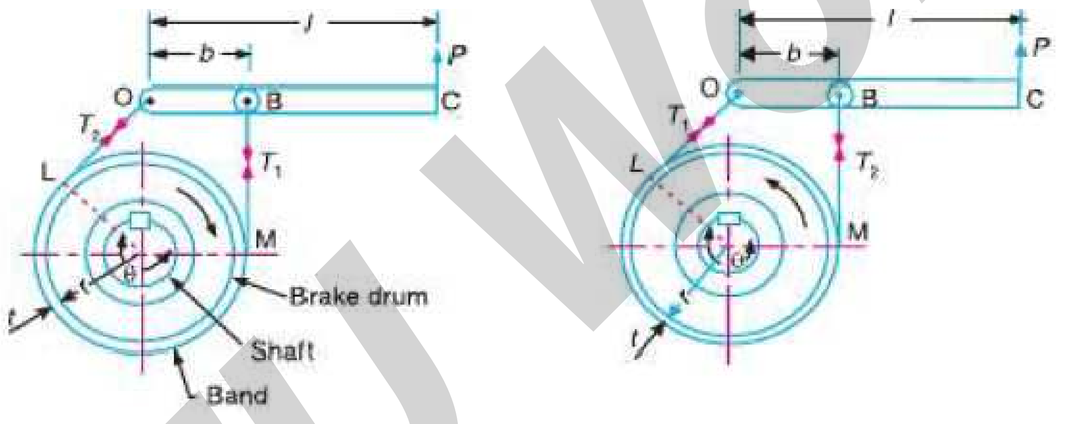
θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{or} \quad 2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

and braking force on the drum = $T_1 - T_2$

Braking torque on the drum,

$T_B = (T_1 - T_2)\, r$ . . . (Neglecting thickness of band)

$= (T_1 - T_2)\, r_e$ . . . (Considering thickness of band)

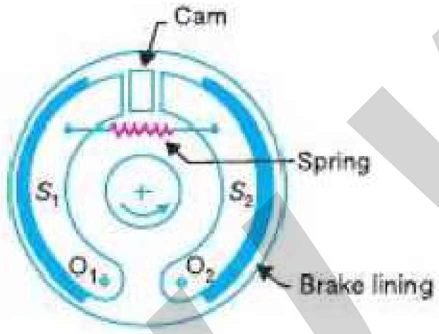
Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig.(*a*), the end of the band attached to the fulcrum O will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig.(*b*), the tensions in the band will reverse, *i.e.* the end of the band attached to the fulcrum *O* will be tight with tension T_1 and the end of the band attached to *B* will be slack with tension T_2. Now taking moments about the fulcrum *O*, we have

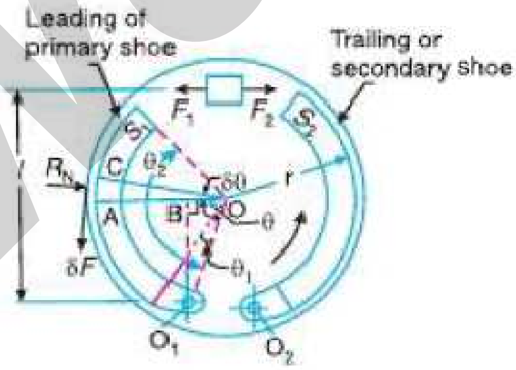
$P.l = T_1.b$ . . . (For clockwise rotation of the drum)

$P.l = T_2.b$ . . . (For anticlockwise rotation of the drum)

Internal Expanding Brake

An internal expanding brake consists of two shoes S_1 and S_2 as shown in Fig.. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are





normally held in off position by a spring as shown in Fig. 19.24. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 19.25. It may be noted that for the anticlockwise direction, the left hand shoe is known as ***leading*** or ***primary shoe*** while the right hand shoe is known as ***trailing*** or ***secondary shoe.***

Let r = Internal radius of the wheel rim,

b = Width of the brake lining,

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Consider a small element of the brake lining AC subtending an angle $\delta\theta$ at the centre. Let OA makes an angle θ with OO_1 as shown in Fig. 19.25. It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about O_1, therefore the rate of wear of the shoe lining at A will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from O_1 to OA, *i.e.* O_1B. From the geometry of the figure,

$$O_1B = OO_1 \sin\theta$$

and normal pressure at A,

$$p_N \propto \sin\theta \text{ or } p_N = p_1 \sin\theta$$

Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N\,(b.r.\delta\theta) = p_1 \sin\theta\,(b.r.\delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu . \delta R_N = \mu . p_1 \sin\theta\,(b.r.\delta\theta)$$

$\therefore$ Braking torque due to the element about O,

$$\delta T_B = \delta F . r = \mu . p_1 \sin\theta\,(b.r.\delta\theta)\,r = \mu . p_1 b\, r^2 (\sin\theta . \delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\, p_1 b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1 b\, r^2 \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b\, r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N . O_1B = \delta R_N\,(OO_1 \sin\theta)$$

$$= p_1 \sin\theta\,(b.r.\delta\theta)\,(OO_1 \sin\theta) = p_1 \sin^2\theta\,(b.r.\delta\theta)\,OO_1$$

$\therefore$ Total moment of normal forces about the fulcrum O_1,

$$= p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1 - \cos 2\theta)\,d\theta \qquad \ldots \left[\because \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)\right]$$

$$= \frac{1}{2}\, p_1.b.r.OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2}\, p_1.b.r.OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2}\, p_1.b.r.OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \cdot AB = \delta F\,(r - OO_1 \cos\theta) \qquad \ldots (\because AB = r - OO_1 \cos\theta)$$

$$= \mu\, p_1 \sin\theta\,(b.r.\delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu.\,p_1.b.r\,(r \sin\theta - OO_1 \sin\theta \cos\theta)\,\delta\theta$$

$$= \mu.\,p_1.b.r\left(r \sin\theta - \frac{OO_1}{2}\sin 2\theta\right)\delta\theta \qquad \ldots (\because 2\sin\theta\cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu\, p_1 b\, r \int_{\theta_1}^{\theta_2}\left(r \sin\theta - \frac{OO_1}{2}\sin 2\theta\right) d\theta$$

$$= \mu\, p_1 b\, r\left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b\, r\left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1\right]$$

$$= \mu\, p_1 b\, r\left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

Types of Dynamometers

Following are the two types of dynamometers, used for measuring the brake power of an engine.

1. Absorption dynamometers, and

2. Transmission dynamometers.

In the *absorption* ***dynamometers,*** the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the ***transmission dynamometers,*** the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

Classification of Absorption Dynamometers

The following two types of absorption dynamometers are important from the subject point of view :

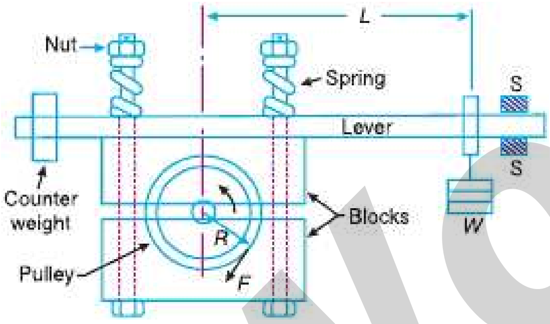
1. Prony brake dynamometer, and

2. Rope brake dynamometer.

These dynamometers are discussed, in detail, in the following pages.

Prony Brake Dynamometer

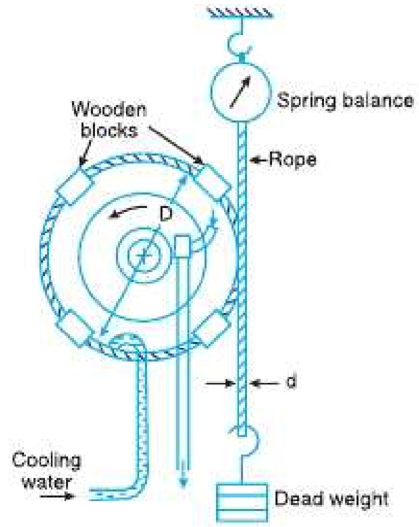
A simplest form of an absorption type dynamometer is a prony brake dynamometer, as shown in Fig. 19.31. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig.. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight W at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever



When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight W must balance the mo- ment of the frictional resistance between the blocks and the pulley.

Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measur- ing the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel. In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.



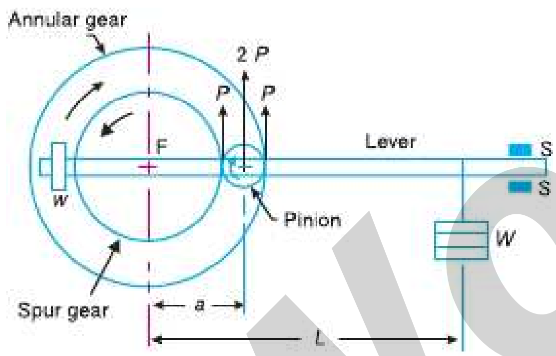
Classification of Transmission Dynamometers

The following types of transmission dynamometers are important from the subject point of view :

1. Epicyclic-train dynamometer,
2. Belt transmission dynamometer, and
3. Torsion dynamometer.

We shall now discuss these dynamometers, in detail, in the following pages.

Epicyclic-train Dynamometer



An epicyclic-train dynamometer, as shown in Fig. 19.33, consists of a simple epicyclic train of gears, *i.e.* a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (*i.e.* driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight w is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the friction of the pin on which the pinion rotates is neglected, then the tangential effort P exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal.

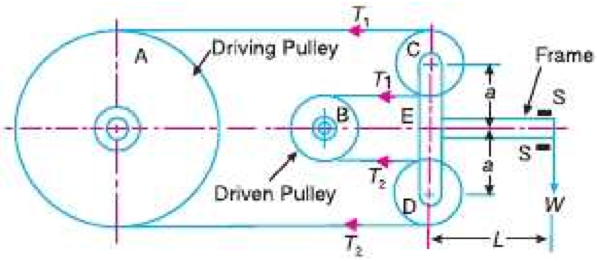
Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is $2P$. This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight W at the end of the lever. The stops S, S are provided to control the movement of the lever.

For equilibrium of the lever, taking moments about the fulcrum F,

$$2P \times a = W.L \qquad \text{or} \qquad P = W.L / 2a$$

Belt Transmission Dynamometer-Froude or Throneycroft Transmission Dynamometer

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt. A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it is running.



A belt transmission dynamometer, as shown in Fig. 19.34, is called a Froude or Throneycroft transmission dynamometer. It consists of a pulley A (called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley B (called driven pulley) mounted on another shaft to which the power from pulley A is transmitted. The pulleys A and B are connected by means of a continuous belt passing round the two loose pulleys C and D which are mounted on a T-shaped frame. The frame is pivoted at E and its movement is controlled by two stops S,S. Since the tension in the tight side of the belt (T_1) is greater than the tension in the slack side of the belt (T_2), therefore the total force acting on the pulley C (*i.e.* $2T_1$) is greater than the total force acting on the pulley D (*i.e.* $2T_2$). It is thus obvious that the frame causes movement about E in the anticlockwise direction. In order to balance it, a weight W is applied at a distance L from E on the frame as shown in Fig.

Now taking moments about the pivot E, neglecting friction,

$$2T_1 \cdot a = 2T_2 \cdot a + W .$$

Torsion Dynamometer

A torsion dynamometer is used for measuring large powers particularly the power transmit- ted along the propeller shaft of a turbine or motor vessel. A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (T), length of the shaft (l), diameter of the shaft (D) and modulus of rigidity (C) of the material of the shaft. We know that the torsion equation is

UNIT – IV

TURNING MOMENT DIAGRAM AND FLY WHEELS

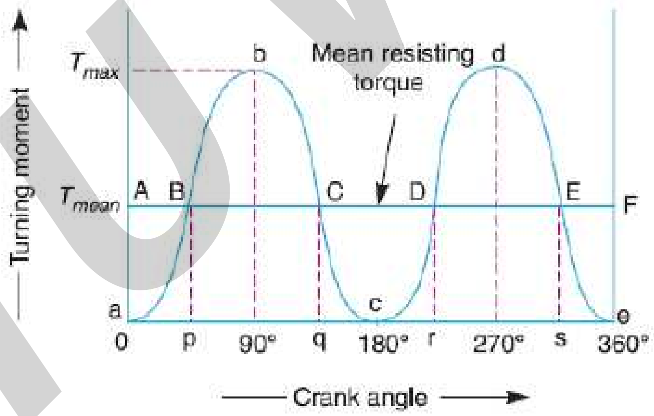
Turning Moment Diagram: The turning moment diagram is graphical representation of the turning moment or crank effort for various positions of crank.

Single cylinder double acting engine:

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.

the turning moment on the crankshaft,

$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$



Turning moment diagram for a single cylinder, double acting steam engine.

where F_P = Piston effort,

r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and

θ = Angle turned by the crank from inner dead centre.

From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.

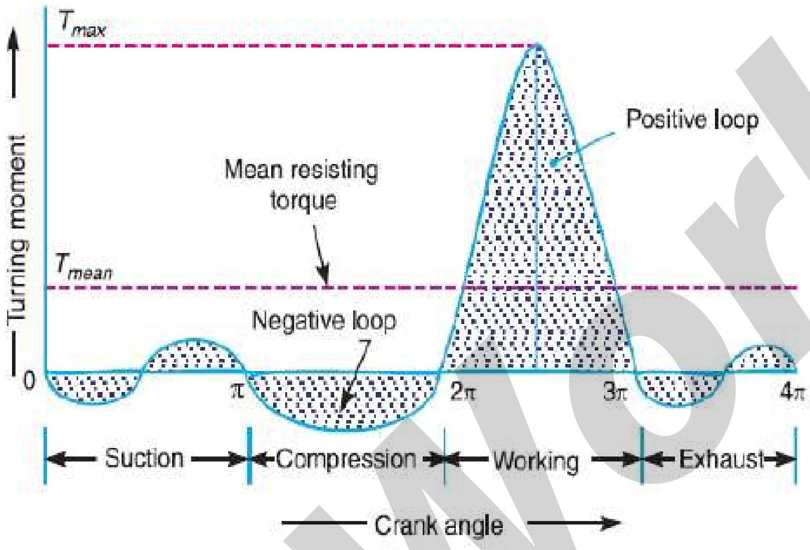
This is shown by the curve *abc* in Fig. and it represents the turning moment diagram for outstroke. The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.

Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. The height of the ordinate *aA* represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aAFe* is proportional to the work done against the mean resisting torque.

For flywheel, have a look at your tailor's manual sewing machine.

Turning moment diagram for 4-stroke I.C engine:

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, *i.e.* 720° (or 4 π radians).

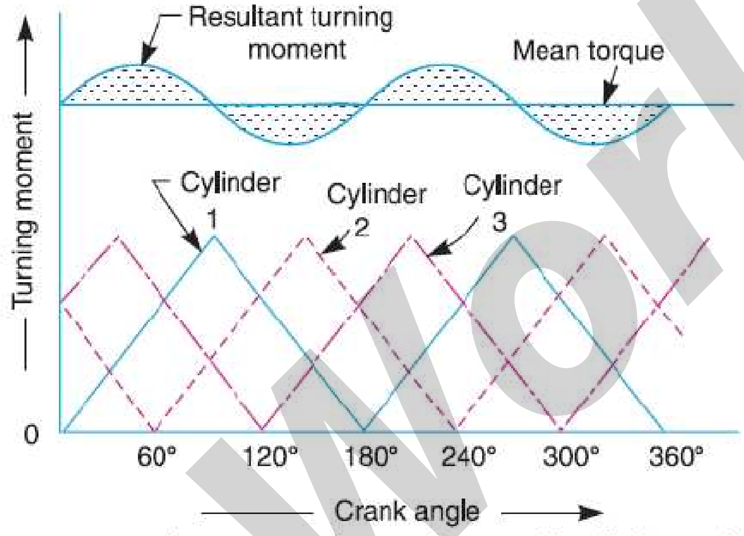


Turning moment diagram for a four stroke cycle internal combustion engine.

Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. 16.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig.

Turning moment diagram for a multi cylinder engine:

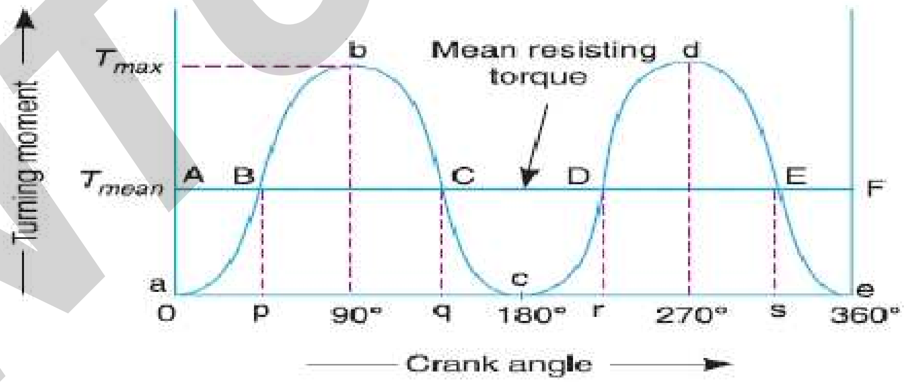
A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders, are usually placed at 120° to each other.



Turning moment diagram for a multi-cylinder engine.

Fluctuation of Energy:

The difference in the kinetic energies at the point is called the maximum fluctuation of energy.



The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig. We see that the mean resisting torque line AF cuts the turning moment diagram at points B, C, D and E. When the crank moves from a to p, the work done by the engine is equal to the area aBp, whereas the energy required is represented by the area $aABp$. In other words, the engine has done less work (equal to the area $a\,AB$) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from p to q, the work done by the engine is equal to the area $pBbCq$, whereas the requirement of energy is represented by the area $pBCq$. Therefore, the engine has done more work than the requirement. This excess work (equal to the area BbC) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from p to q.

Similarly, when the crank moves from q to r, more work is taken from the engine than is developed. This loss of work is represented by the area $C\,c\,D$. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from q to r. As the crank moves from r to s, excess energy is again developed given by the area $D\,d\,E$ and the speed again increases. As the piston moves from s to e, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuations of energy***. The areas BbC, CcD, DdE, etc. represent fluctuations of energy.

A little consideration will show that the engine has a maximum speed either at q or at s. This is due to the fact that the flywheel absorbs energy while the crank moves from p to q and from r to s. On the other hand, the engine has a minimum speed either at p or at r. The reason is that the flywheel gives out some of its energy when the crank moves from a to p and q to r. The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy***.

Fluctuation of Speed:

This is defined as the ratio of the difference between the maximum and minimum angular speeds during a cycle to the mean speed of rotation of the crank shaft.

Maximum fluctuation of energy:

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. The horizontal line AG represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

Let the energy in the flywheel at $A = E$, then from Fig. we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (*i.e.* cycle repeats after G)

A flywheel stores energy when the supply is in excess and releases energy when energy is in deficit.

Let us now suppose that the greatest of these energies is at B and least at E. Therefore,

Maximum energy in flywheel

$$= E + a_1$$

Minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

Coefficient of fluctuation of energy:

It may be defined as the **ratio of the maximum fluctuation of energy to the work done per cycle.** Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The work done per cycle (in N-m or joules) may be obtained by using the following two relations :

1. Work done per cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned (in radians), in one revolution.

= 2π, in case of steam engine and two stroke internal combustion engines

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation :

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N/60$

2. The work done per cycle may also be obtained by using the following relation :

$$\text{Work done per cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute,

= N, in case of steam engines and two stroke internal combustion engines,

= $N/2$, in case of four stroke internal combustion engines.

Coefficient of fluctuation of speed:

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed***. The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed***.

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

N = Mean speed in r.p.m. $= \frac{N_1 + N_2}{2}$

∴ Coefficient of fluctuation of speed,

$$C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

Energy stored in flywheel:

A flywheel is a rotating mass that is used as an energy reservoir in a machine. It absorbs energy in the form of kinetic energy, during those periods of crank rotation when actual turning moment is greater than the resisting moment and release energy, by way of parting with some of its K.E, when the actual turning moment is less than the resisting moment.

UNIT-V
GOVERNERS

Introduction

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g.* when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of work- ing fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

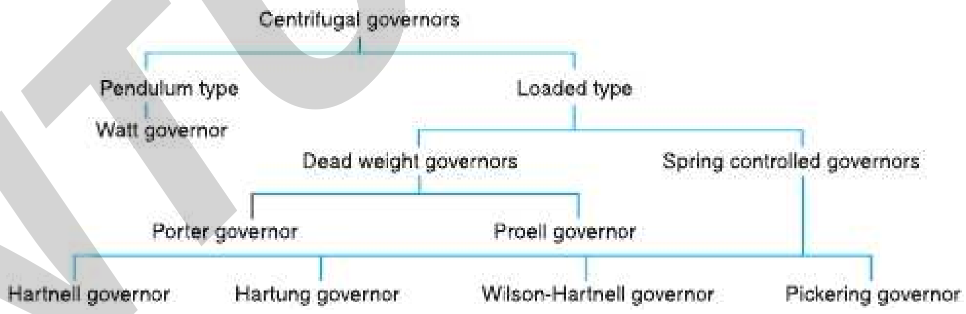
A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ; ***conversely,*** when the load decreases, the engine speed in- creases and the governor decreases the supply of working fluid.

Note : We have discussed in Chapter 16 (Art. 16.8) that the func- tion of a flywheel in an engine is entirely different from that of a governor. It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation. It does not control the speed variations caused by a varying load. The varying demand for power is met by the governor regulating the supply of working fluid.

Types of Governors

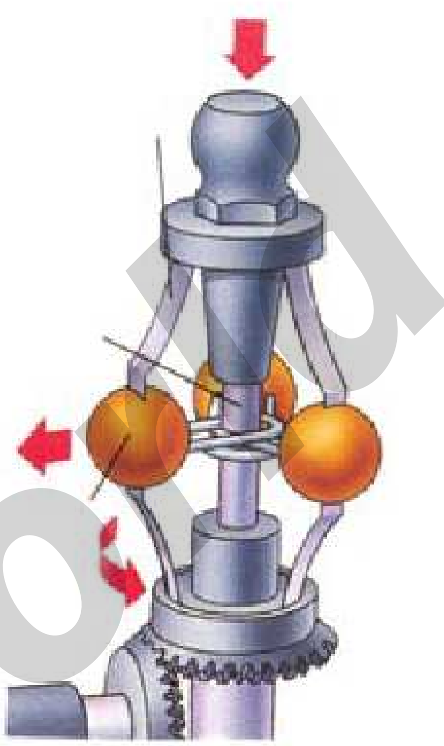
The governors may, broadly, be classified as

1. Centrifugal governors, and
2. Inertia governors.

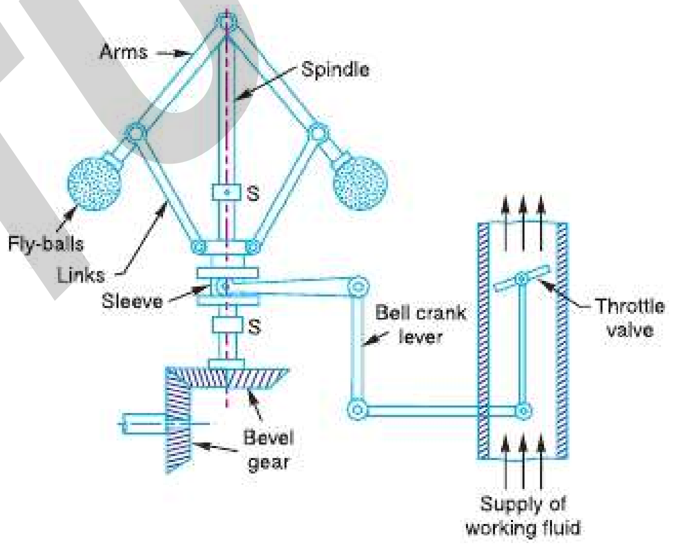


Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force****.It consists of two balls of equal mass, which are attached to the arms as shown in Fig. 18.1. These balls are known as ***governor balls* or *fly balls.*** The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve re- volves with the spindle ; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops S, S are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid de- creases when the sleeve rises and increases when it falls.



When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves down- wards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.



Terms Used in Governors

The following terms used in governors are important from the subject point of view ;

1. *Height of a governor.* It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.

2. *Equilibrium speed.* It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. *Mean equilibrium speed.* It is the speed at the mean position of the balls or the sleeve.

4. *Maximum and minimum equilibrium speeds.* The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and mini- mum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the mini- mum equilibrium speeds

5. *Sleeve lift.* It is the vertical distance which the sleeve travels due to change in equilibrium speed.

3.4 Gravity Loaded Controlled Governors

(a) Watt Governor

This type of governor is shown in fig-3.1 (a). It is the original form of governor as used by Watt on some of his early steam engines. In this type of governor, each ball is attached to an arm, which is pivoted on the axis of rotation. The sleeve is attached to the governor balls by arms, pin-jointed at both ends, and is free to slide along the governor shaft.

The upper arm may be suspended from the vertical spindle in three ways as shown in fig-3.3.

(i) From the axis of the spindle as shown in fig-3.3 (a).

(ii) From a point attached to a collar on the spindle so that the arm produced intersects the spindle as shown in fig-3.3 (b).

(iii) From a point to a collar so that the arm crosses the spindle as shown in fig 3.3(c).

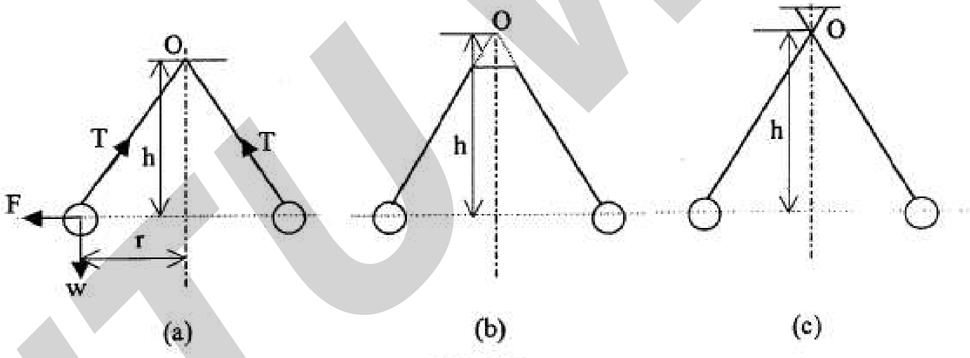


Fig-3.3

The height of the governor, which is donated by 'h' in figure, is the distance from the center of the mass to the point of intersection between the arm and the axis of the spindle.

Let 'w' be the weight of the ball, 'T' the tension in the arm and 'F' the centrifugal force when the radius to the center of the ball is 'r' and the angular velocity of the arm and the ball about the spindle axis is 'ω'.

For the simplified analysis, which follows, the weights of the sleeve, the upper ball arms, the lower links and friction are all neglected. As the weight of the lower arms and sleeve is neglected, the tensions in the lower links are negligible and hence only three forces are acting on each rotating ball.

(i) The weight 'w' acting vertically downwards

(ii) The centrifugal force '$F = \frac{w}{g}\omega^2 r$' acting radially outwards

(iii) The tension 'T' in the upper arm.

Taking moment about O, the point of intersection of the arm and the axis of the spindle, for the forces acting on the governor balls, we get

$$\frac{w}{g}\omega^2 r \times h = w \times r$$

$$h = \frac{g}{\omega^2} \qquad \text{(i)}$$

The equation (i) shows that neither the weight of the balls nor the length of the supporting arms has any influence on the height of the governor. It varies inversely as the square of the speed.

When 'g' is in cm/s^2 and 'ω' is in radian/s, then 'h' is in cm.

Let 'N' be the speed in rpm, then

$$\omega = \frac{2\pi N}{60} = \frac{\pi N}{30}$$

$$\therefore h = \frac{900g}{\pi^2 N^2} = \frac{900 \times 981}{\pi^2 N^2} = \frac{89560}{N^2} \text{ cm.}$$

Since the height of the governor is inversely proportional to the square of the speed it is small at high speeds and at such speeds the change in height corresponding to a small change in speed is insufficient to enable a governor of the Watt type to operate the mechanism to give the necessary change in the fuel supply or steam supply.

From the table given below it can be seen that the height diminishes very rapidly as the speed of rotation increases.

N (rpm)	40	60	80	100	120	150	220
h (cm)	55.98	24.88	13.98	8.96	6.22	3.98	2.24

Thus, this governor is suitable only for low speeds of rotation not exceeding 75 rpm. It might then be suggested that a speed reduction gear between engine shaft and the governor spindle would allow this governor to be used with higher speed engines. However, it should be noted that this is not a satisfactory remedy.

(b) Porter Governor

The type of governor, which is illustrated at fig-3.1 (b), is known as the Porter governor. The only respect in which it differs from the Watt governor is in the use of a heavily weighted sleeve. The additional downward force increases the speed of revolution required to enable the balls to rise to any pre-determined level.

Let 'w' be the weight of each ball and 'W' be the weight of the central load. T_1 be the tension in the upper arm and T_2 the tension in the suspension link. α and β be the inclinations to the vertical of the upper arm and suspension links respectively. The weight of arms and weight of suspension links and the effect of friction to the movement of the sleeve are neglected.

There are several ways of determining the relation between the height 'h' and the speed 'ω'. In this chapter, two methods are used to derive the relation.

(i) Instantaneous Center Method

Consider the equilibrium of the forces acting on the suspension link 'AC', which is shown in fig-3.4. These forces are 'F, w and T_1 at C and $\frac{W}{2}$ and Q at A. The equation connecting 'F, w and W' is derived by taking moment about I, the point of intersection of the lines of action of forces T_1 and Q. This point of intersection I is also the instantaneous center of the link AC. The point I lies at the point of intersection of BC produce and a line drawn through A perpendicular to the axis of the governor spindle.

Taking moment about I,

$$F \times CD = w \times ID + \frac{W}{2}(ID + DA)$$

$$F = w \times \frac{ID}{CD} + \frac{W}{2}\left(\frac{ID}{CD} + \frac{DA}{CD}\right)$$

$$= w \tan\alpha + \frac{W}{2}(\tan\alpha + \tan\beta)$$

$$= \left\{\frac{W}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) + w\right\}\tan\alpha$$

$$= \left\{\frac{W}{2}(1+k) + w\right\}\tan\alpha$$

where $k = \frac{\tan\beta}{\tan\alpha}$.

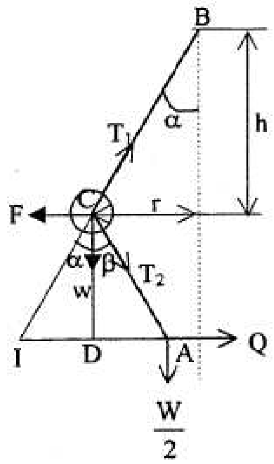


Fig-3.4

If 'h' be the height of the governor, then $\tan\alpha = \frac{r}{h}$. Further, we have $F = \frac{w}{g}\omega^2 r$.

Therefore, we get

$$\frac{w}{g}\omega^2 r = \left[\frac{W}{2}(1+k) + w\right]\frac{r}{h} \qquad \text{(or)}$$

$$\omega^2 = \left[\frac{\frac{W}{2}(1+k) + w}{w}\right]\frac{g}{h} \qquad \text{(i)}$$

When the length of the arms and the suspension links are of equal length and the axis of the joints at B and A either intersect the governor spindle or are at equal distances from the governor spindle the value 'k' is equal to 1 and the equation (i) reduces to the form

$$\omega^2 = \left(\frac{W+w}{w}\right)\frac{g}{h} \qquad \text{(ii)}$$

When the lengths of the arms are unequal and the axes of the joints at B and A are at different distances from the governor spindle the k will have a different value for each radius of rotation of the governor balls, This value of 'k' can be best found by calculating the value of α and β. It should be noted that when 'k' is not equal to 1, its value changes as the height of the governor changes.

For the simple Watt governor, the weight of the sleeve W is negligible and we have either from equation (i) or (ii) the relation $\omega^2 = \frac{g}{h}$ which has derived earlier.

(ii) Equilibrium Method

The governor sleeve, which is loaded by the weight W is in equilibrium under a system of three forces, W the load on the sleeve and the tensions T_2 in the two lowered suspension links. As the system of forces is in equilibrium, the force triangle drawn for these forces must be a closed one as shown in fig-3.5 (a).

The pin joint C between the upper arm and the lower suspension link must be in equilibrium under the action of the four forces as under:

(i) The weight of the ball 'w'

(ii) Radially outwards acting centrifugal force $F = \frac{w}{g}\omega^2 r$

(iii) Tension T_1 in the upper arm

(iv) Tension T_2 in the lower suspension link.

These four forces must form a closed polygon as shown in fig-3.5 (b).

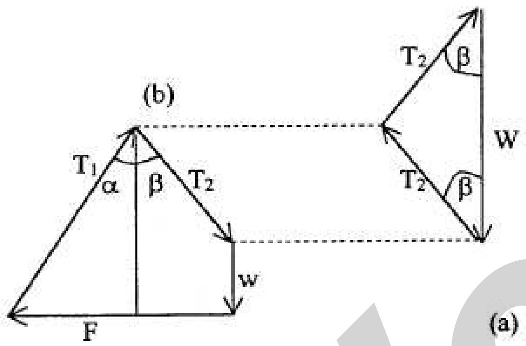


Fig-3.5

From force triangle for the sleeve, we get

$W = 2\,T_2 \cos\beta$ (or) $T_2 = \dfrac{W}{2\cos\beta}$ (iii)

From the polygon of forces on the ball, we have

$T_1 \cos\alpha = T_2 \cos\beta + w$ (resolving vertically) (iv)

Resolving horizontally,

$F = T_1 \sin\alpha + T_2 \sin\beta$ (v)

From equation (iv)

$$T_1 = \frac{\frac{W}{2} + w}{\cos\alpha}$$

When the value of T_1 and T_2 are substituted in the equation (v),

$$F = \left(\frac{W}{2} + w\right)\tan\alpha + \frac{W}{2}\tan\beta$$

$$= \left[\frac{W}{2}(1+k) + w\right]\tan\alpha \qquad \text{(v)}$$

where $k = \dfrac{\tan\beta}{\tan\alpha}$.

By substituting the value of $\tan\alpha$ and F, equation (i), which is derived earlier, can be done.

(c) Proell Governor

Fig-3.1(c) shows a type of Proell governor. This governor is similar to the Porter governor except that the revolving balls are attached to the extensions of the lower links. This has the effect of reducing the change of speed necessary for a given sleeve movement. In other words the governor is made more sensitive.

The action of this governor is again similar to that of the other governors described earlier. The analysis of the Proell governor can be done by considering the equilibrium of the lower arm, which is referred fig-3.8.

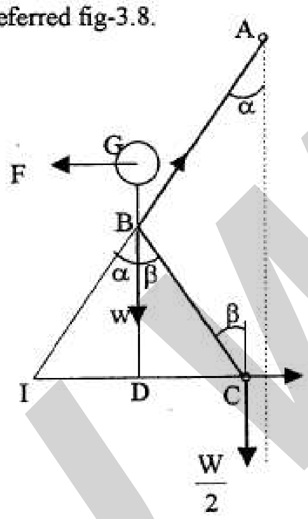


Fig-3.8

There are five forces acting on the lower link:

(i) The centrifugal force F, acting radially outwards, through the center of the gravity of the ball

(ii) The weight 'w', acting vertically downwards through the center of gravity of the ball

(iii) The pull $\frac{W}{2}$ at C acting vertically downwards

(iv) The tension T_1 along the length of the link AB

(v) Reaction at C along a line at right angles to the axis of the governor spindle.

The instantaneous center of the lower suspension link BC lies at the point of intersection of AB produced and a line drawn through C perpendicular to the axis of the governor spindle. It is assumed that the extension BG of the lower suspension link BC is vertical for the given configuration.

Take moment about I, the instantaneous center of the lower suspension link. The tension T_1 and the reaction at C give no moment. Therefore,

$$F \times DG = w \times ID + \frac{W}{2} \times (ID + DC) \quad \text{(i)}$$

Dividing both sides by BD,

$$F \times \frac{DG}{BD} = w \times \frac{ID}{BD} + \frac{W}{2}\left[\frac{ID}{BD} + \frac{DC}{BD}\right]$$

$$= w \tan\alpha + \frac{W}{2}[\tan\alpha + \tan\beta]$$

$$= \left[w + \frac{W}{2}\right]\tan\alpha + \frac{W}{2}\tan\beta$$

$$\therefore F = \frac{BD}{DG}\left\{\left[w + \frac{W}{2}\right]\tan\alpha + \frac{W}{2}\tan\beta\right\} \quad \text{(ii)}$$

Let $\dfrac{\tan\beta}{\tan\alpha} = k$

$$\therefore F = \frac{BD}{DG}\left\{\frac{W}{2}(1+k) + w\right\}\tan\alpha \quad \text{(iii)}$$

But, $\quad \tan\alpha = \dfrac{r}{h} \quad$ and $\quad F = \dfrac{w}{g}\omega^2 r \quad$ (iv)

Substituting the values given by equation (iv) in equation (iii),

$$\frac{w}{g}\omega^2 r = \frac{BD}{DG}\left\{\frac{W}{2}(1+k) + w\right\}\frac{r}{h}$$

$$\omega^2 = \frac{g}{h} \times \frac{BD}{DG}\left\{\frac{\frac{W}{2}(1+k) + w}{w}\right\} \quad \text{(v)}$$

Thus, the effect of placing the ball at G, instead of at the pin joint B is to reduce the equilibrium speed for given values of the height of the governor, the weight of the ball and the weight of the sleeve. Hence in order to give the same equilibrium speed for the given height and the weight of the sleeve, the smaller ball is required in Proell governor than that in Porter governor.

3.5 Spring Loaded Controlled Governors

In spring loaded controlled governors the control of speed is affected either wholly or in part by means of springs. Some of the representative of spring loaded controlled governors are shown in fig-3.2.

The spring loaded controlled governors posses the following advantages over the gravity loaded controlled governors.

(i) The spring loaded controlled governors may be operated at very high speeds.

(ii) With proper proportioning the spring loaded controlled governors can be made both powerful and capable of very closed regulation.

(iii) It can be much smaller in over all size.

(iv) As it does not depend on gravity for its action, it may revolve about a horizontal, vertical or inclined axis.

In spring loaded controlled governors the spring may be placed upon the axis of rotation or they may be transverse as shown in fig-3.2.

(a) Spring loaded Controlled Governor of the Hartnell Type

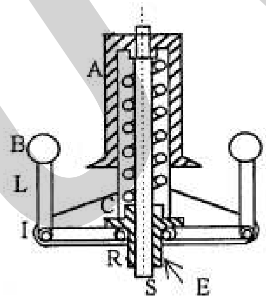


Fig-3.10

Fig-3.10 shows spring loaded controlled governor of Hartnell type. Two bell crank levers L are mounted on pins I, carried by the frame A, which is attached to the rotating spindle S. Each lever carries a ball B at the end of one arm and a roller R the end of the other. The centrifugal forces of the balls cause the rollers R to press against the collar C on the sleeve E. The upward pressure of the rollers on the collar of the sleeve is balanced by the downward thrust of the helical spring, which is in compression. The angle of the bell crank lever is usually 90° but in practice it may be grater.

Let w be the weight of each ball, S the spring force exerted on the sleeve, k the stiffness of the spring, ω the speed of rotation, r the radius of rotation, a and b the lengths of the vertical and horizontal arms of the bell crank lever and F the centrifugal force on the ball.

By taking moment about the fulcrum of the lever, neglecting the effect of pull of gravity on the governor balls and arms,

$$F \times a = \frac{S}{2} \times b$$

or $$S = 2F\frac{a}{b} \qquad \text{(i)}$$

It is assumed that the arms are mutually perpendicular and the lines of action of forces are at right angles to the arm.

Let the suffixes 1 and 2 denote the values of maximum and minimum radii respectively. Then at maximum radius

$$S_1 = 2F_1\frac{a}{b} \qquad \text{(ii)}$$

At minimum radius, $$S_2 = 2F_2\frac{a}{b} \qquad \text{(iii)}$$

$$\therefore S_1 - S_2 = 2\frac{a}{b}(F_1 - F_2)$$

Let θ be the angular movement of the bell crank lever from the position of minimum radius to the position of the maximum radius, then

$$(r_1 - r_2) = a\theta \qquad \text{(iv)}$$

If h be the lift of the sleeve, then

$$h = b\theta \qquad \text{(v)}$$

Dividing equation (v) by (iv),

$$\frac{h}{r_r - r_2} = \frac{b}{a} \qquad \text{(or)}$$

$$h = \frac{b}{a}(r_1 - r_2) \qquad \text{(vi)}$$

The difference in the forces exerted by the compressed spring in the two positions is $S_1 - S_2$; therefore, the force per unit compression is known as the stiffness of the spring. The stiffness of the spring is denoted by k.

UNIT – VI
BALANCING

Introduction

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses

Balancing of Rotating Masses

We have already discussed, that whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotating masses.***

The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.

2. Balancing of a single rotating mass by two masses rotating in different planes.

3. Balancing of different masses rotating in the same plane.

4. Balancing of different masses rotating in different planes.

We shall now discuss these cases, in detail, in the following pages.

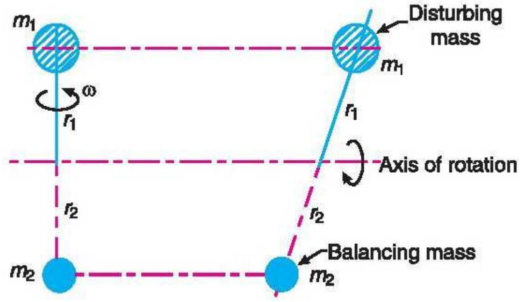
Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass m_1 attached to a shaft rotating at ⌉ rad/s as shown in Fig. Let r_1 be the radius of rotation of the mass m_1 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.



Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

$\therefore$ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 . \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. In other words, the two forces are equal in magnitude and opposite in direction. But this type of arrange- ment for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing*.**

2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

The conditions **(1)** and **(2)** together give ***dynamic balancing*.** The following two possibili- ties may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and

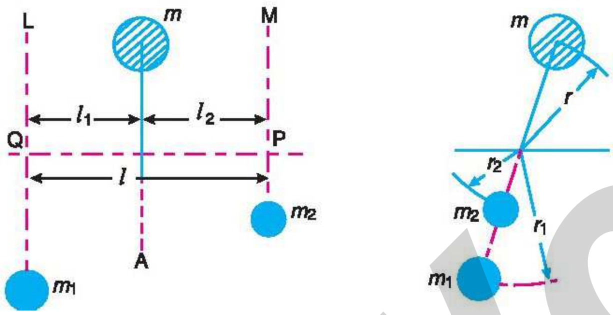
2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

We shall now discuss both the above cases one by one.

1. *When the plane of the disturbing mass lies in between the planes of the two balancing masses*

Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes L and M as shown in Fig. 21.2. Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, L and M respectively

Let l_1 = Distance between the planes A and L,
l_2 = Distance between the planes A and M, an
l = Distance between the planes L and M.



We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1 . \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore \quad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \quad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \quad \ldots (ii)$$

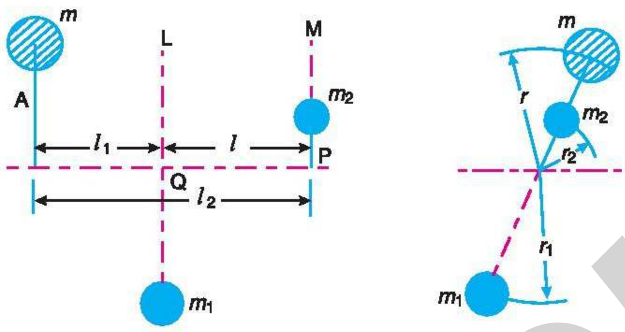
Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\therefore \quad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \quad \ldots (iii)$$

It may be noted that equation ***(i)*** represents the condition for static balance, but in order to achieve dynamic balance, equations ***(ii)*** or ***(iii)*** must also be satisfied.

2. *When the plane of the disturbing mass lies on one end of the planes of the balancing masses*



plane of single rotating mass lies at one end of the planes of balancing masses.

In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, as shown in Fig. 21.3. As discussed above, the following conditions must be satisfied in order to balance the system, *i.e.*

$$F_C + F_{C2} = F_{C1} \quad \text{or} \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 . r_1 \qquad \text{. . . (iv)}$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \text{. . . (v)}$$

. . . [Same as equation ***(ii)***]

Similarly, to find the balancing force in the plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

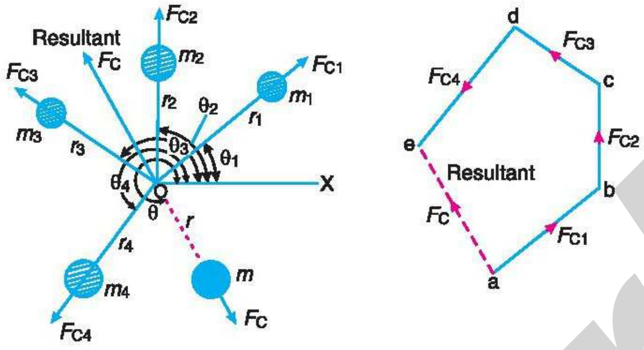
$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \text{. . . (vi)}$$

. . . [Same as equation ***(iii)***]

Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let θ_1, θ_2, θ_3 and θ_4 be the angles of these masses with the horizontal line OX, as shown in Fig. 21.4 (*a*). Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below



(*a*) Space diagram. (*b*) Vector diagram.

1. *Analytical method*

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ©*H* and ©*V* . We know that

Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction.***

6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and

r = Its radius of rotation.

2. *Graphical method*

The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. (*a*).

2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.

3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).

4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig.(*b*).

5. The balancing force is, then, equal to the resultant force, but in ***opposite direction.***

6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$
$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

Balancing of Several Masses Rotating in Different Planes

When several masses revolve in different planes, they may be transferred to a ***reference plane*** (briefly written as ***R.P.***), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.

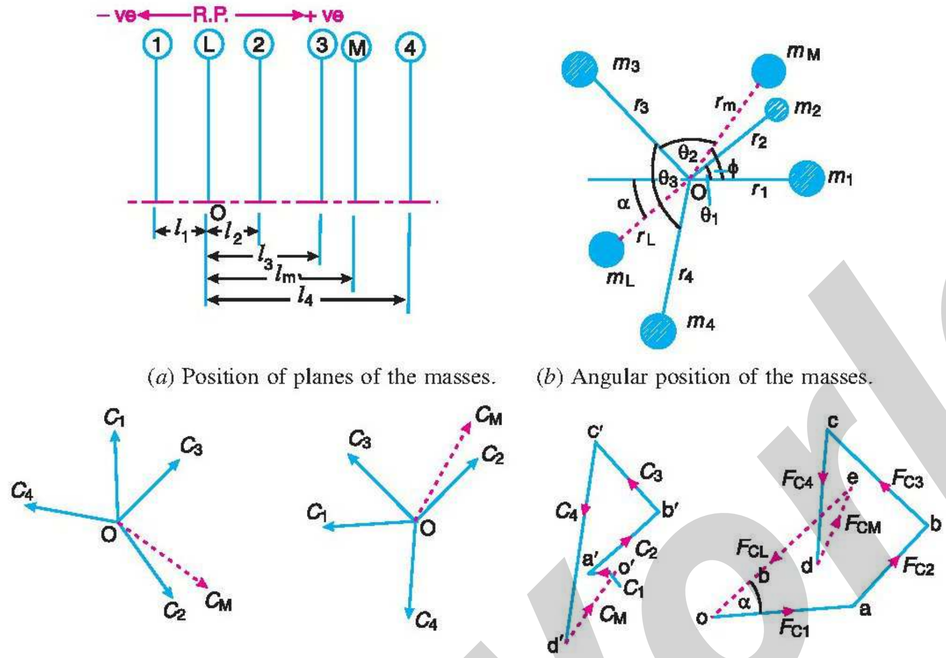
2. The couples about the reference plane must balance, *i.e.* the resultant couple must be zero.

Let us now consider four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. (*a*). The relative angular positions of these masses are shown in the end view [Fig. (*b*)]. The magnitude of the balancing masses m_L and m_M in planes *L* and *M* may be obtained as discussed below :

1. Take one of the planes, say *L* as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative,*** and those to the right as ***positive.***

2. Tabulate the data as shown in Table 21.1. The planes are tabulated in the same order in which they occur, reading from left to right.

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



(*a*) Position of planes of the masses. (*b*) Angular position of the masses.

(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is proportional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. 21.7 (*c*). Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (*d*). We see that their relative positions remains unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in *opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***

5. Now draw the couple polygon as shown in Fig. 21.7 (*e*). The vector $d2$ $o2$ represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$m_L \cdot r_L = \text{vector } eo \qquad \text{or} \qquad m_L = \frac{\text{vector } eo}{r_L}$$

From this expression, the value of the balancing mass m_L in the plane L may be obtained and the angle of inclination 〈 of this mass with the horizontal may be measured from Fig.(b).