

MA 374 – Financial Engineering Lab

Lab – 2

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Roll No - 180123053

1 QUESTION - 1:

The initial option prices for the European Call Option and European Put Option are:

i. Set – 1:

European Call Option = 12.085380013710187

European Put Option = 4.397014652374166

ii. Set – 2:

European Call Option = 12.12304707401251

European Put Option = 4.434681712676494

Binomial Pricing Algorithm:

- At time $t = t_i$ ($= i \cdot \delta t$), there are $i + 1$ possible asset prices, i.e,

$$S_n^i = d^{i-n} u^n S_0, \quad 0 \leq n \leq i$$

- Since continuous compounding convention is used, gross return is $R = e^{r \cdot \delta t}$.

- The probability (p) of an upward return in price is $\frac{R-d}{u-d}$.

- At expiry, i.e, $t = T$, we calculate the price of the option using the respective payoff function for both the call and put option, i.e,

$$C_n^M = \max(S_n^M - K, 0), \quad 0 \leq n \leq M$$

$$P_n^M = \max(K - S_n^M, 0), \quad 0 \leq n \leq M$$

where, C_n^M is the nth possible price of the call option for the Mth interval, and

P_n^M is the nth possible price of the put option for the Mth interval

- Now, we continuously apply **Backward Induction** to find out the option price at $t = 0$ by using following relation:

$$C_n^i = (1 - p) \cdot C_{n+1}^{i+1} + p \cdot C_n^{i+1}, \quad 0 \leq n \leq i \quad \& \quad 0 \leq i \leq M - 1$$

$$P_n^i = (1 - p) \cdot P_{n+1}^{i+1} + p \cdot P_n^{i+1}, \quad 0 \leq n \leq i \quad \& \quad 0 \leq i \leq M - 1$$

- C_0^0 and P_0^0 are the required values, i.e, initial option prices.

Sensitivity Analysis:

Put Call Parity –

$$C^E - P^E = S(0) - Ke^{-rT}$$

where, C^E = European Call Option

P^E = European Put Option

K = Exercise Price

T = Exercise Time

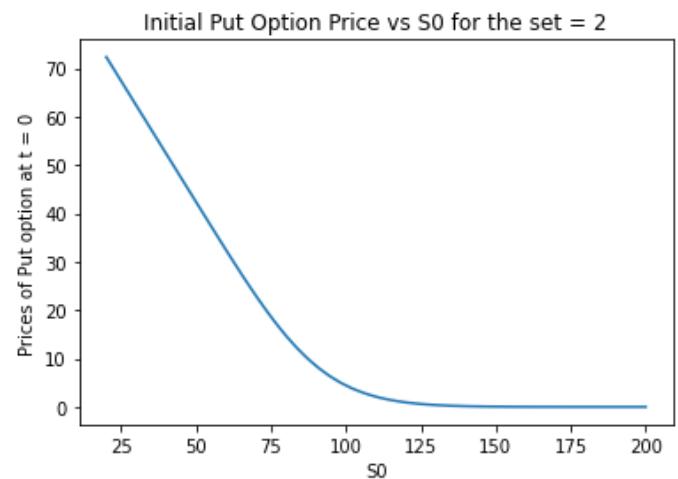
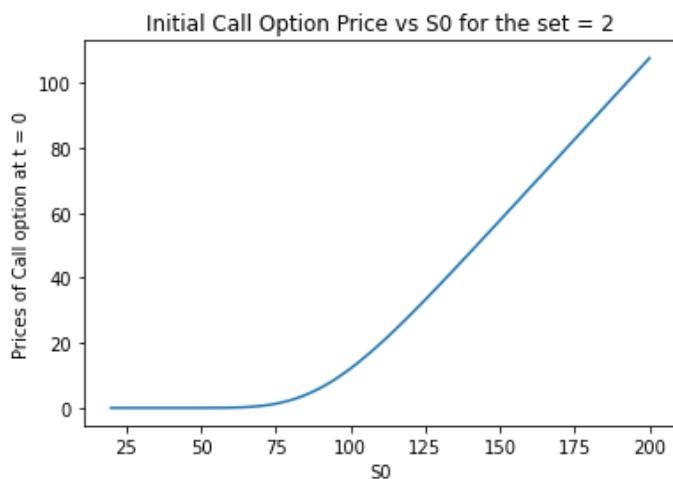
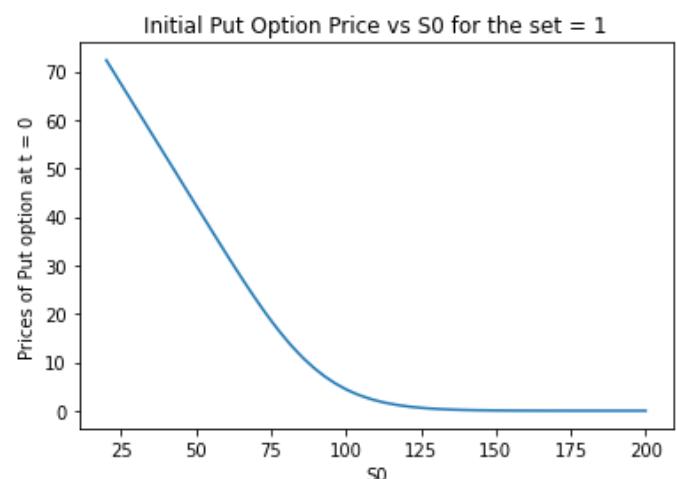
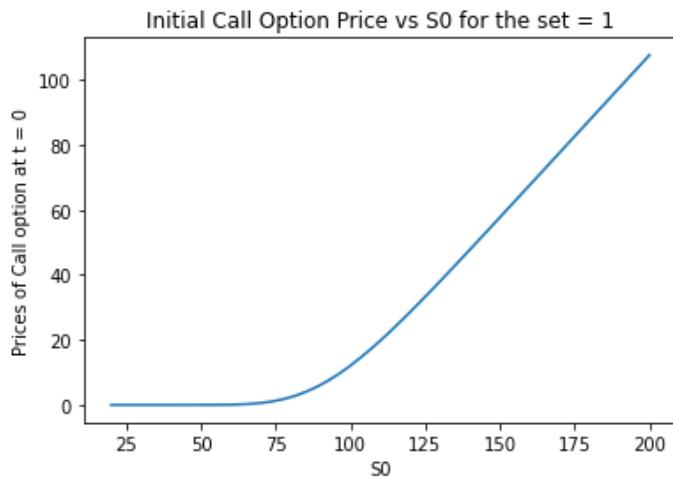
Cox – Ross – Rubeinstein Formula –

$$C^E = \frac{1}{(1+R)^M} \sum_{k=0}^M \binom{M}{k} p'^k (1-p')^{M-k} (S(0) \cdot (1+U)^M \cdot (1+D)^{M-k} - K)^+$$

where, $p' = p \frac{1+U}{1+R}$

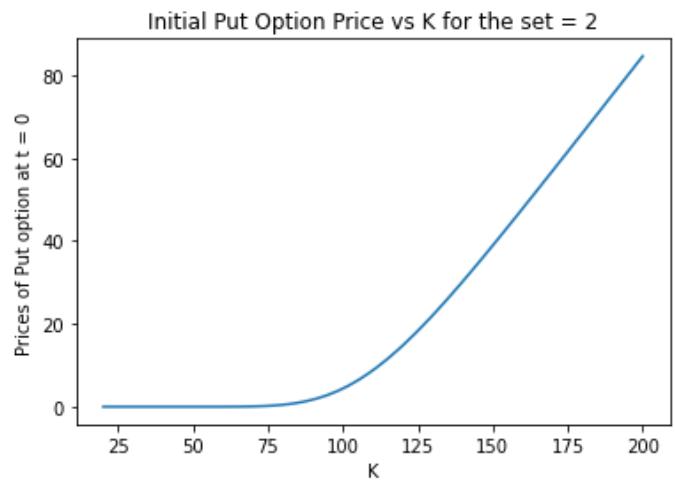
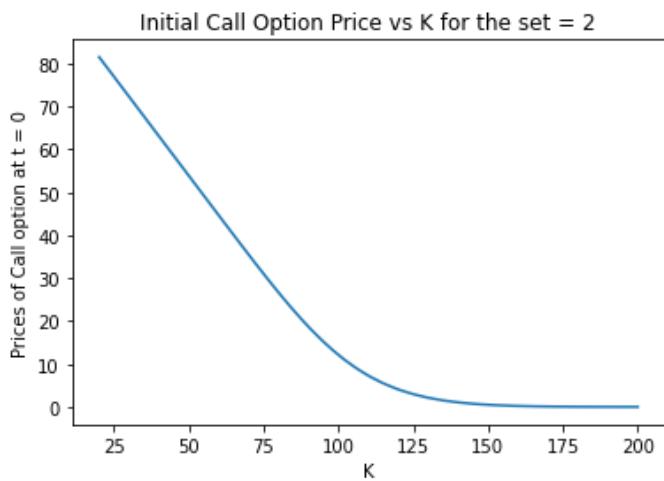
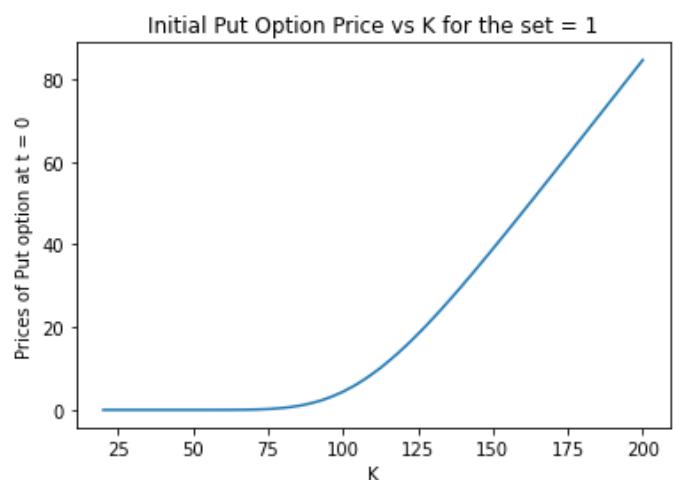
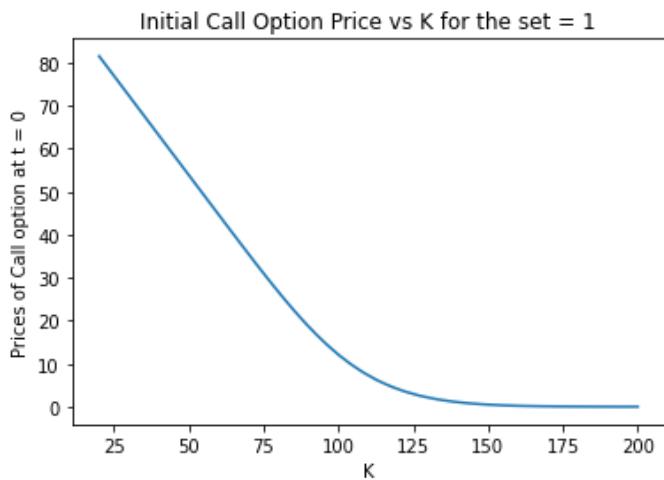
C^E = European Call Option

1. Variation with $S(0)$ –



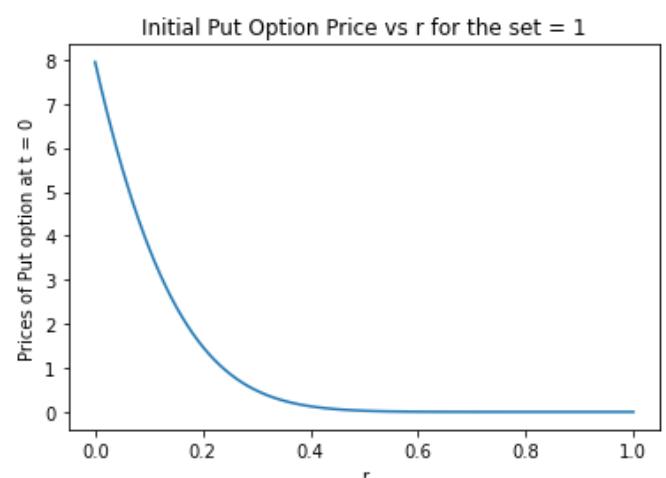
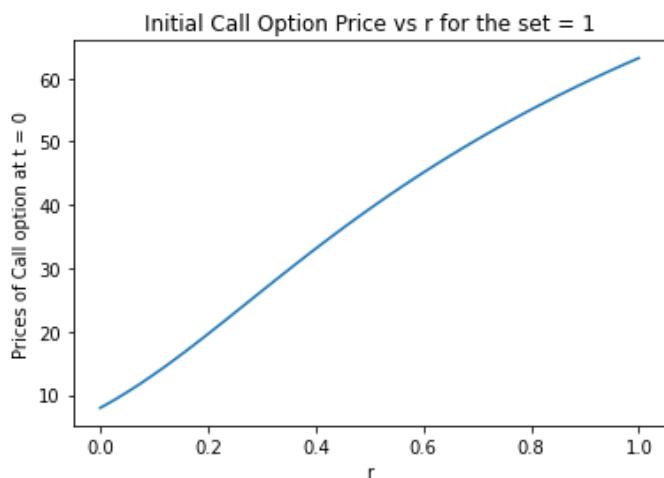
- Using Cox-Rubeinstein formula, it is evident that the Call Option price has positive dependence on $S(0)$, and then using Put-Call Parity shows that the Put Option price has negative dependence on $S(0)$.

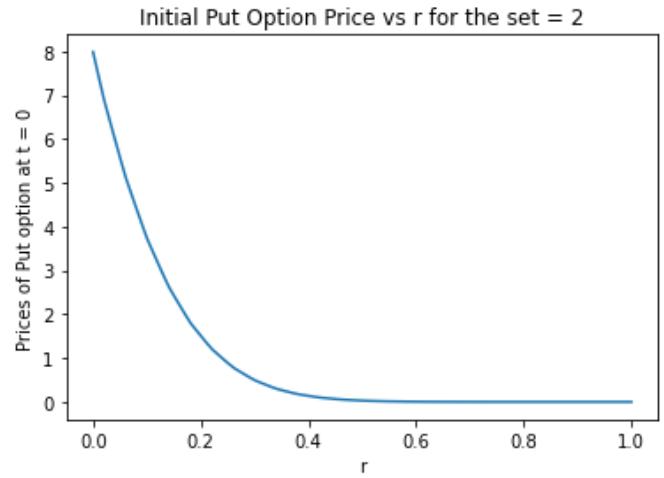
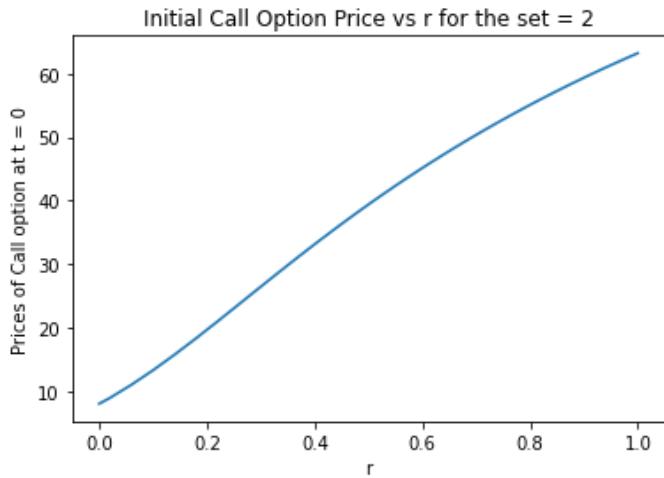
2. Variation with K -



- Using Cos-Rubeinstein formula, it is evident that the Call Option price has negative dependence on K, and then using Put-Call Parity shows that the Put Option price has positive dependence on K.

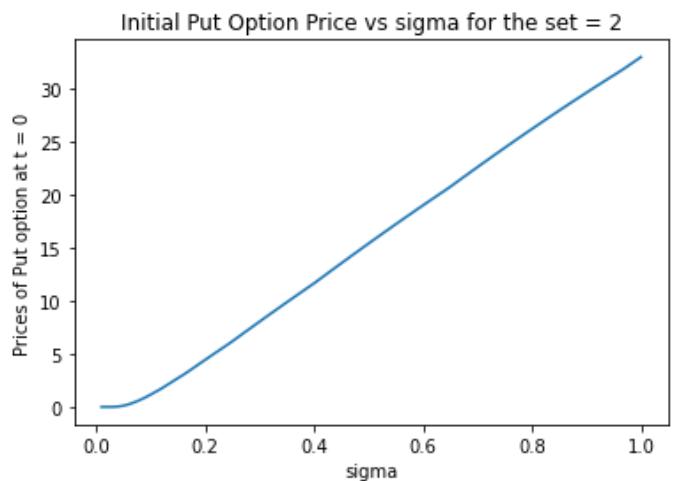
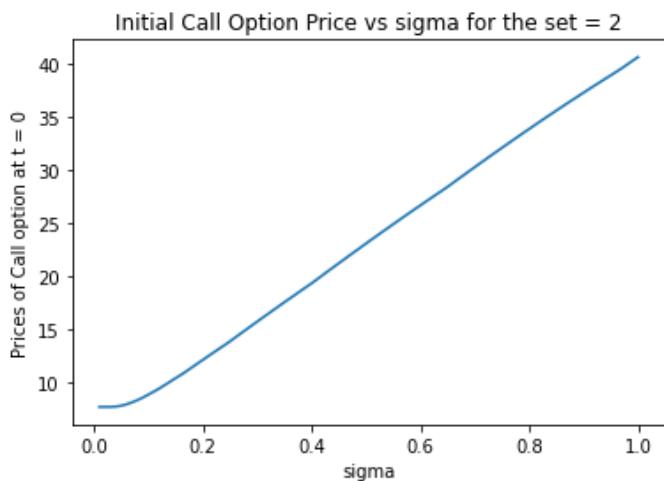
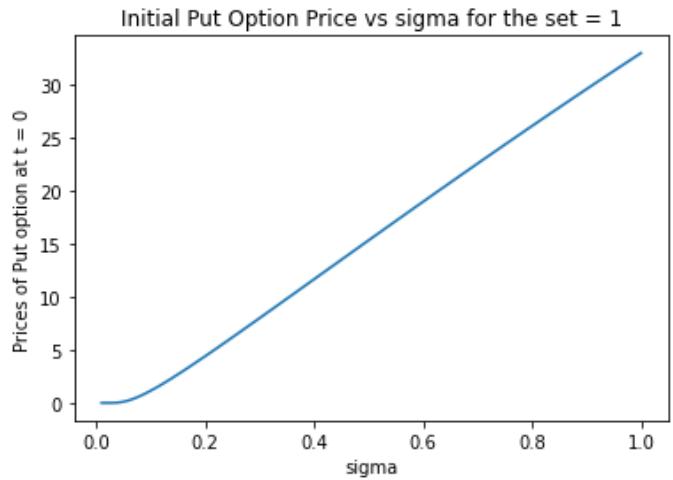
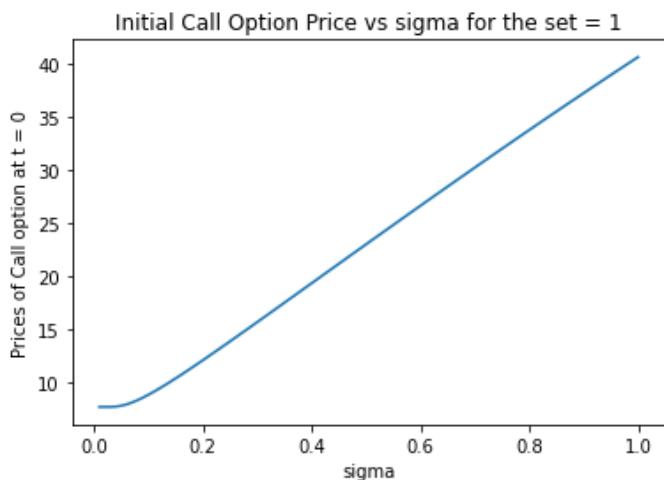
3. Variation with r -





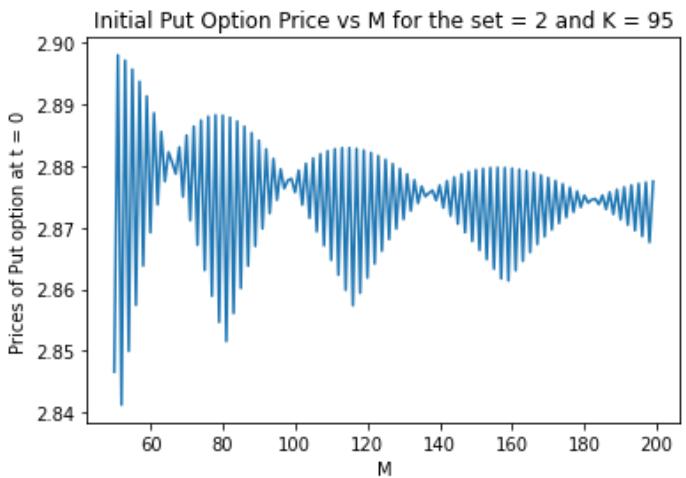
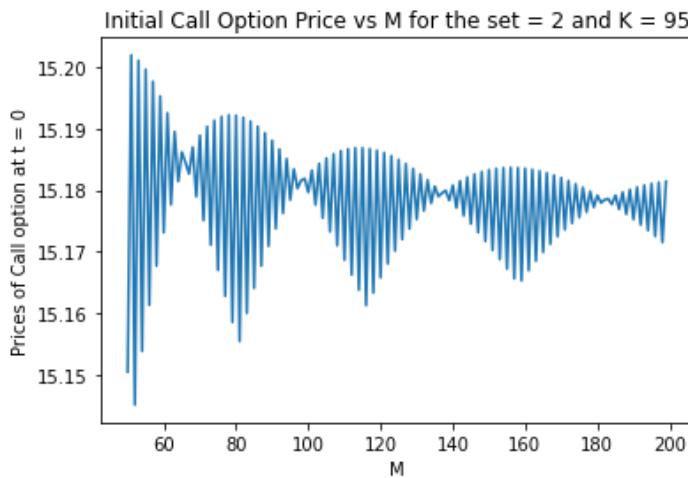
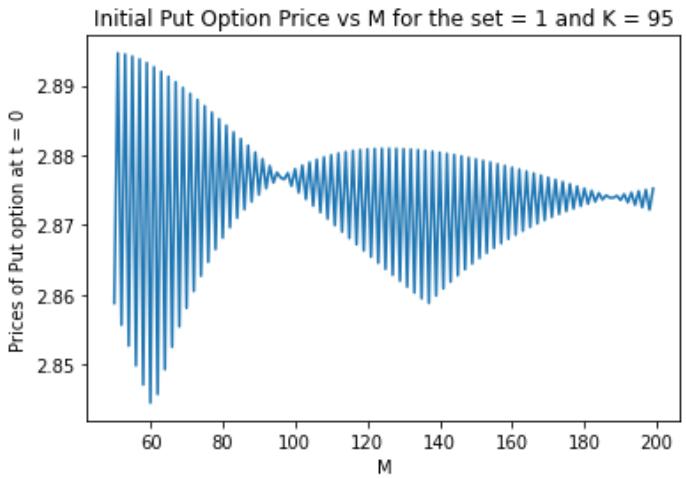
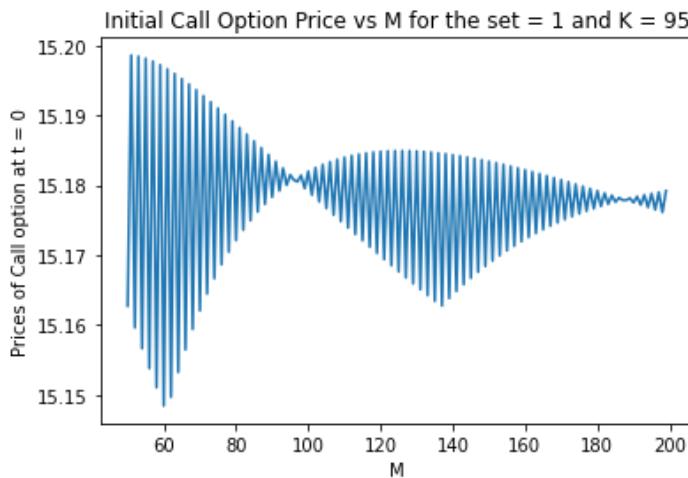
- Using Cos-Rubeinstein formula, it is evident that the Call Option price has positive dependence on r , and then using Put-Call Parity shows that the Put Option price has negative dependence on r .

4. Variation with σ -

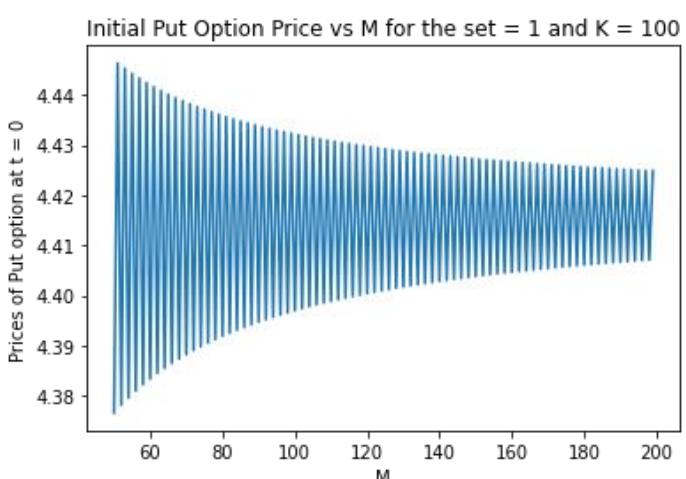
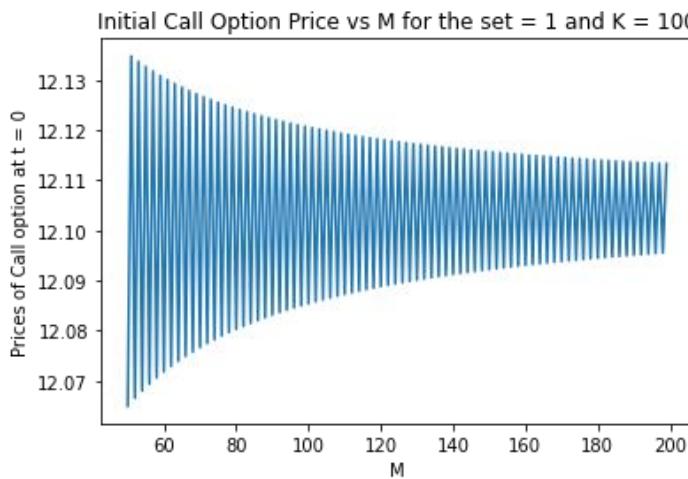


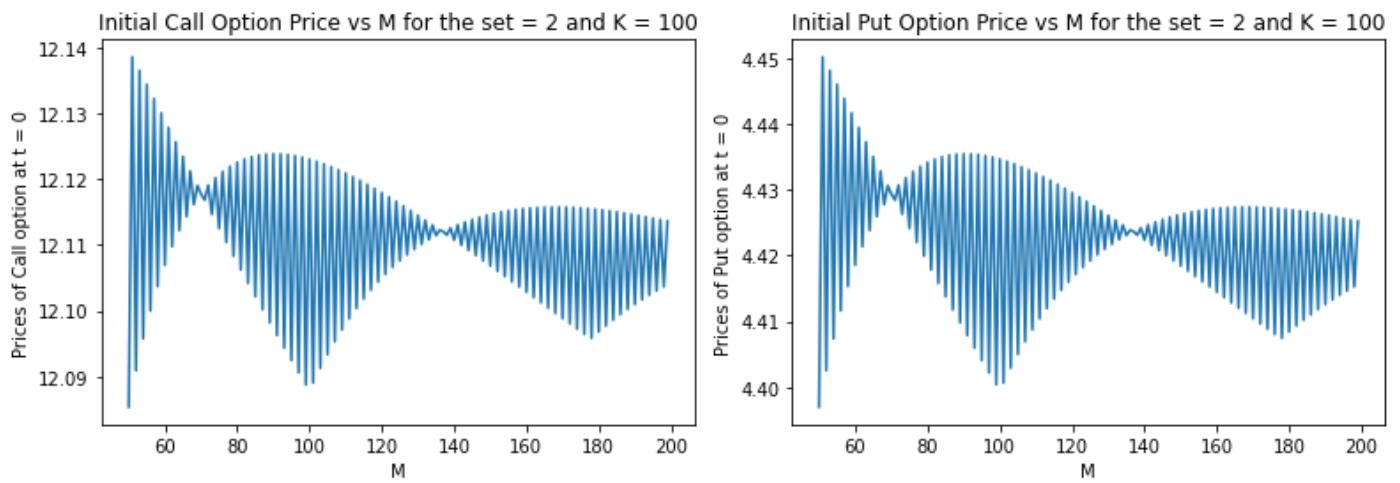
5. Variation with M –

Case (i): K = 95

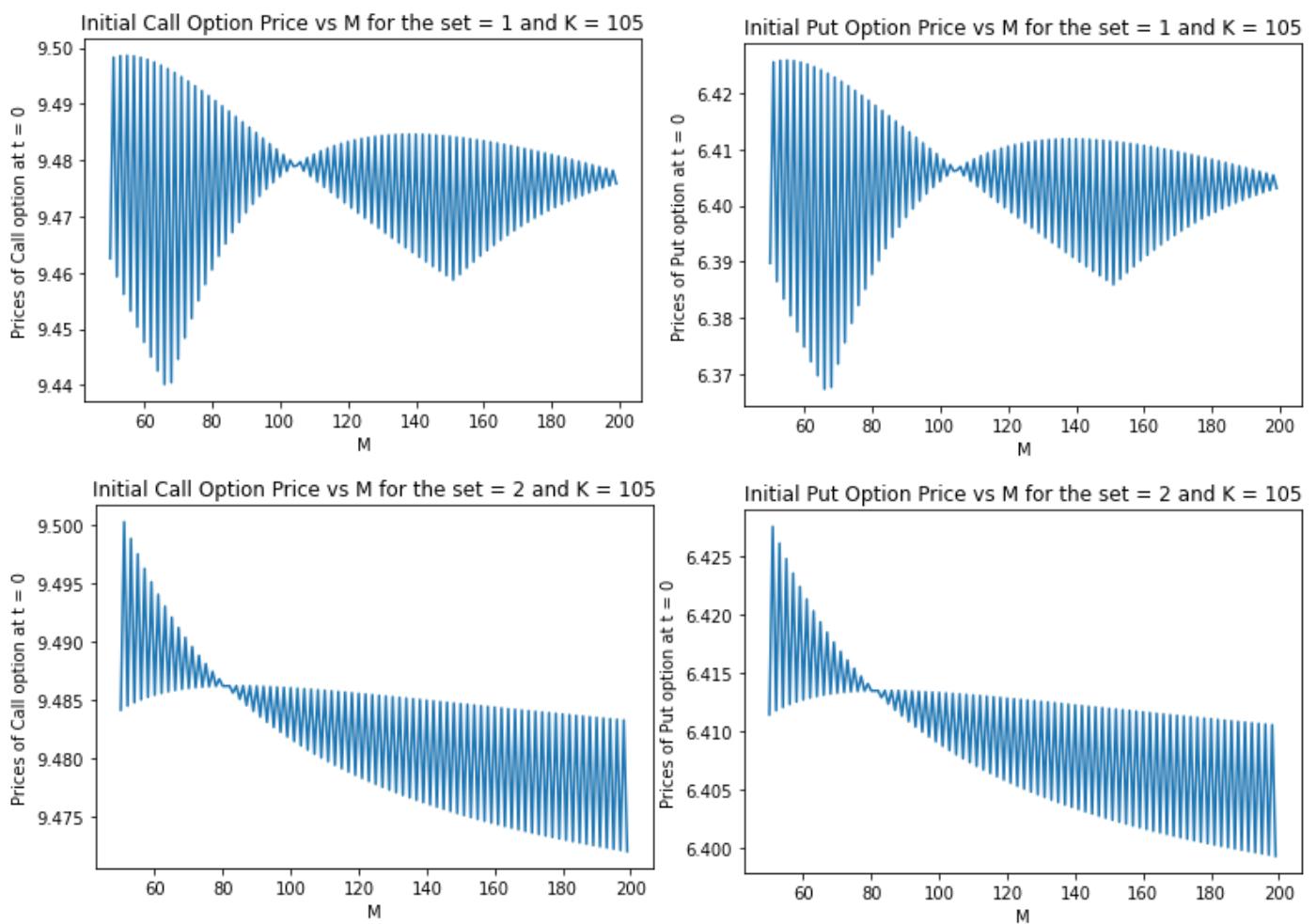


Case (ii): K = 100



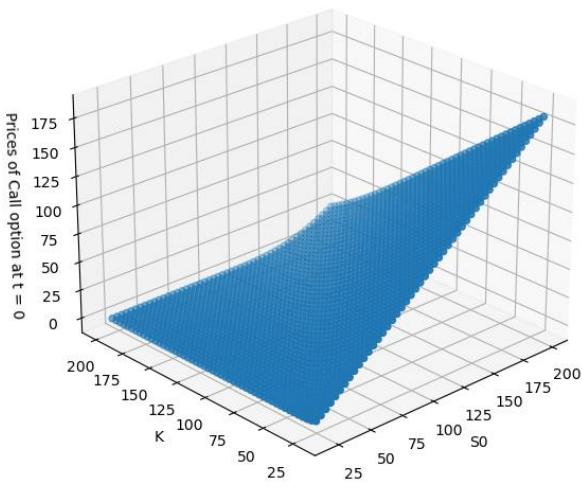


Case (iii): K = 105

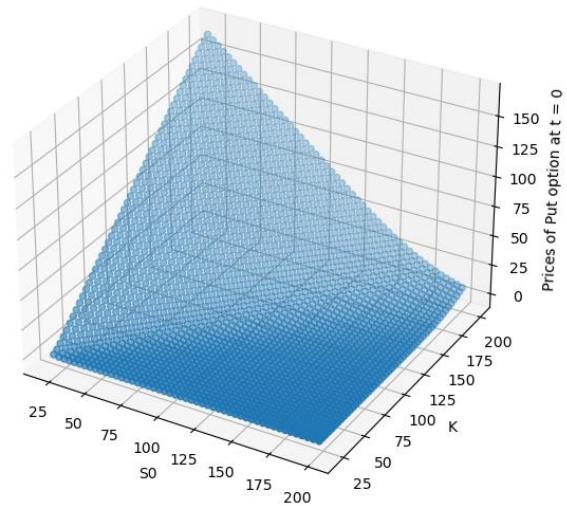


6. Variation with $S(0)$ and K –

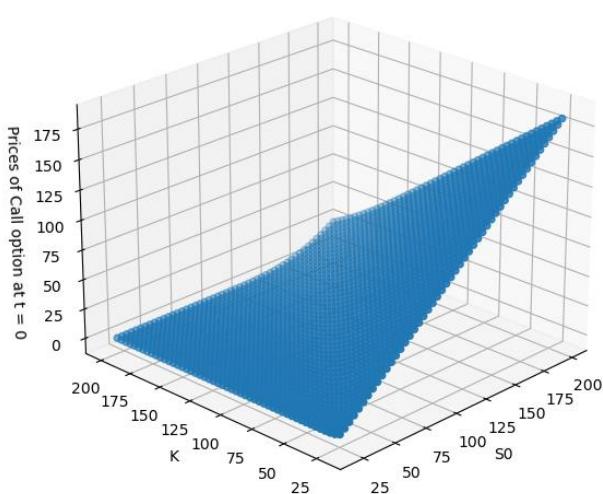
Initial Call Option Price vs S_0 and K for the set = 1



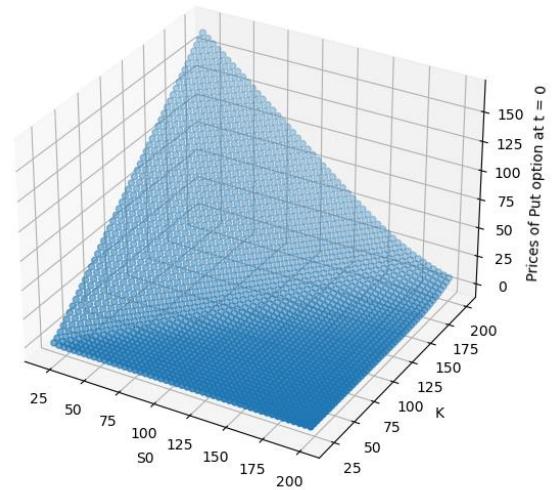
Initial Put Option Price vs S_0 and K for the set = 1



Initial Call Option Price vs S_0 and K for the set = 2

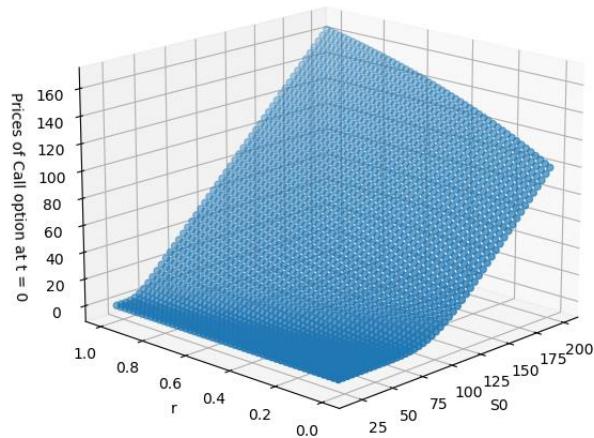


Initial Put Option Price vs S_0 and K for the set = 2

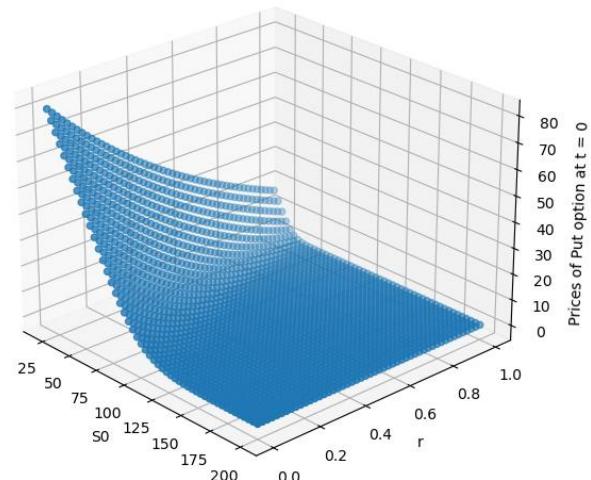


7. Variation with $S(0)$ and r –

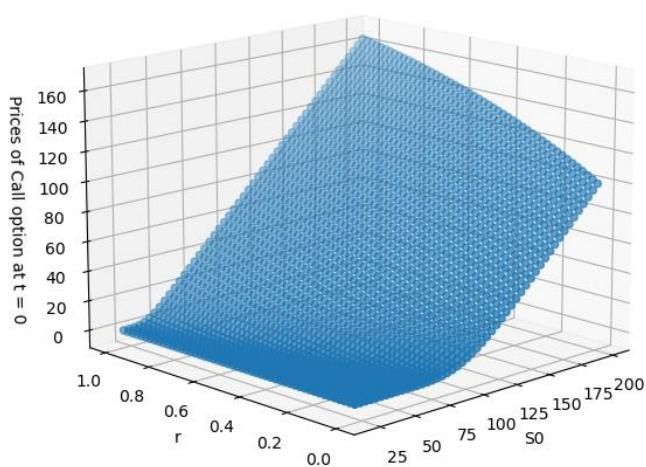
Initial Call Option Price vs S_0 and r for the set = 1



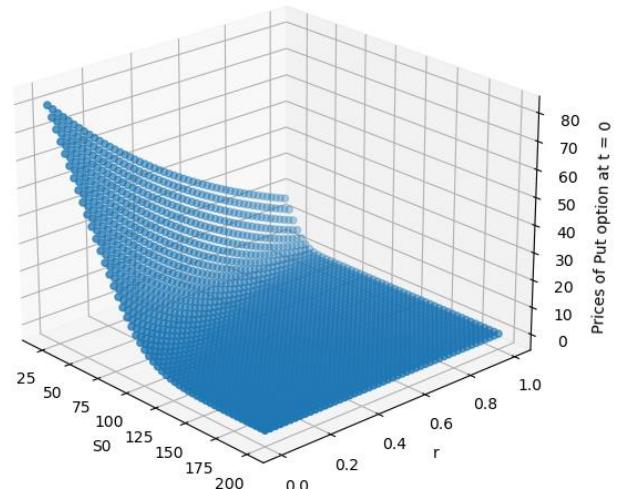
Initial Put Option Price vs S_0 and r for the set = 1



Initial Call Option Price vs S0 and r for the set = 2

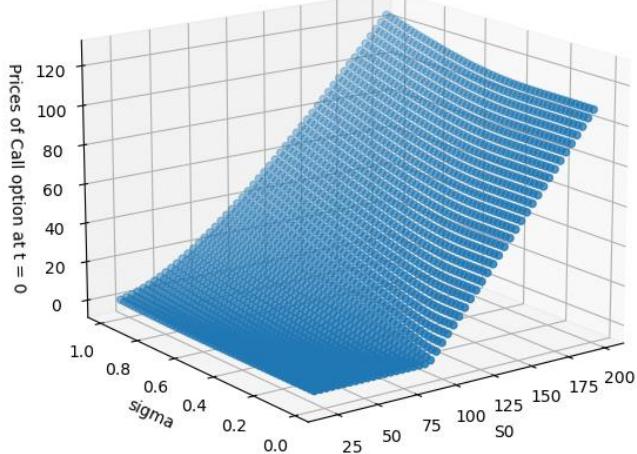


Initial Put Option Price vs S0 and r for the set = 2

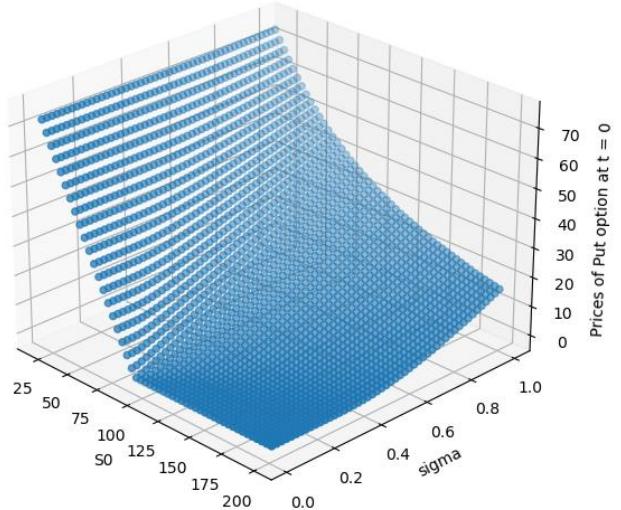


8. Variation with $S(0)$ and σ –

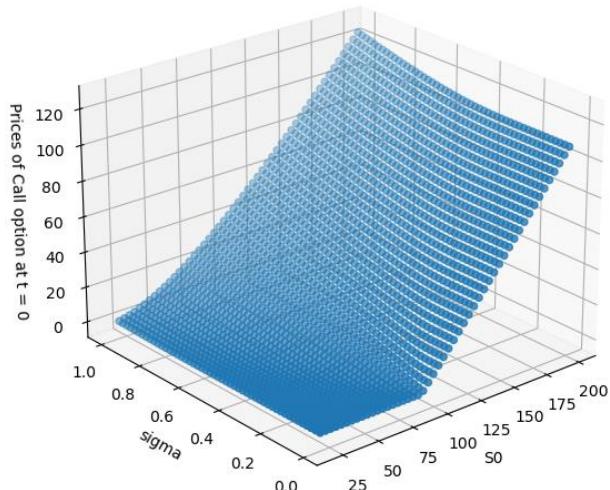
Initial Call Option Price vs S0 and sigma for the set = 1



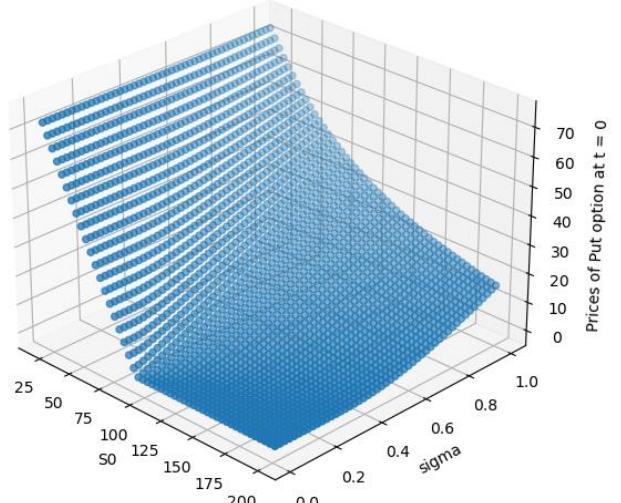
Initial Put Option Price vs S0 and sigma for the set = 1



Initial Call Option Price vs S0 and sigma for the set = 2

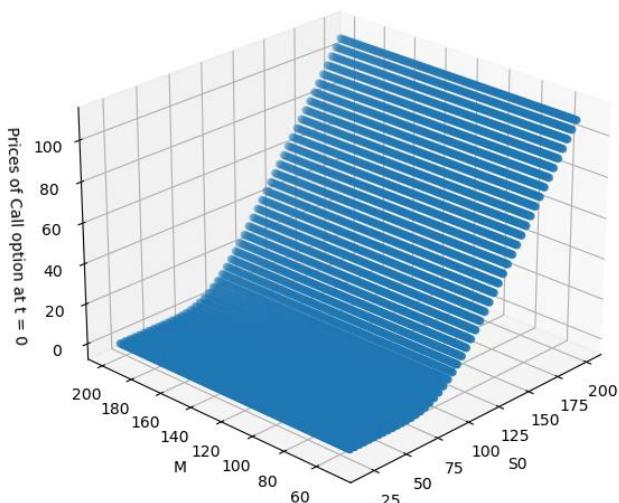


Initial Put Option Price vs S0 and sigma for the set = 2

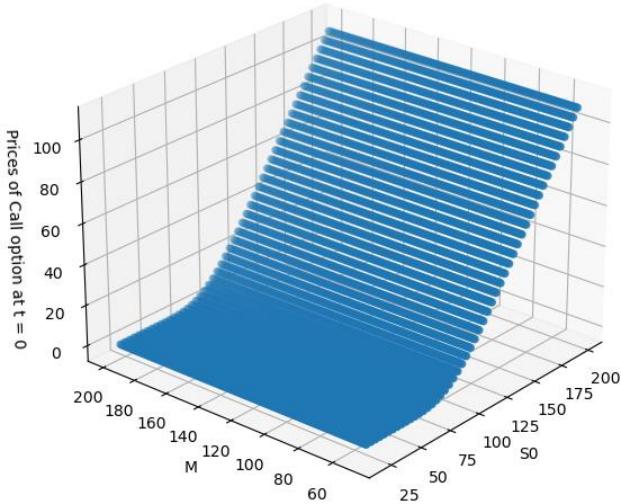


9. Variation with $S(0)$ and M –

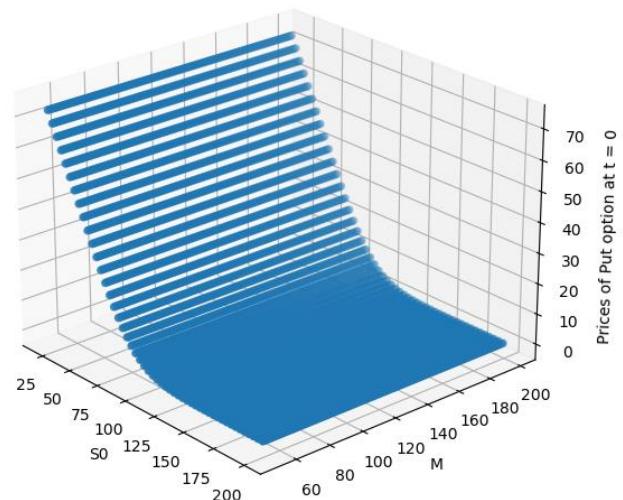
Initial Call Option Price vs S_0 and M for the set = 1



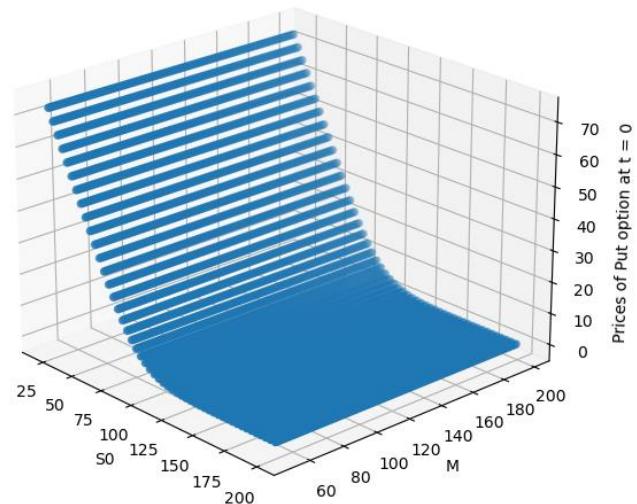
Initial Call Option Price vs S_0 and M for the set = 2



Initial Put Option Price vs S_0 and M for the set = 1

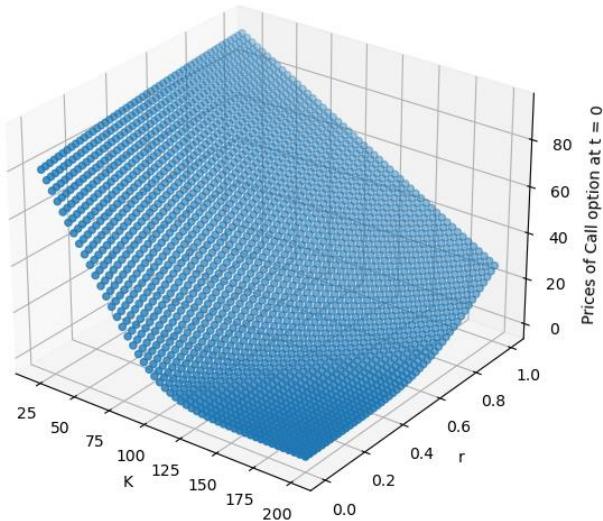


Initial Put Option Price vs S_0 and M for the set = 2

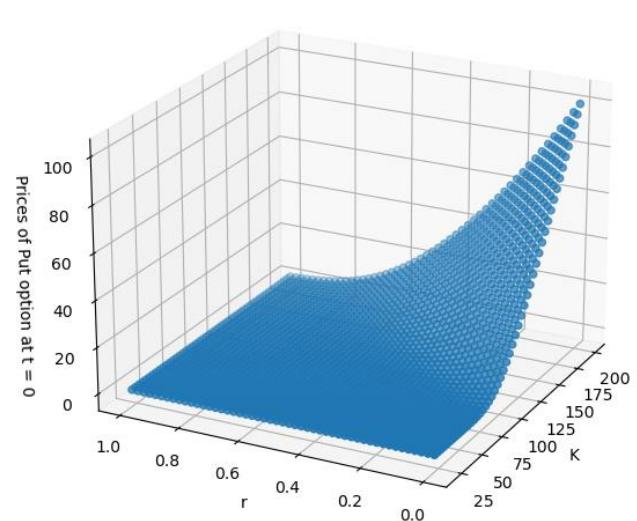


10. Variation with K and r –

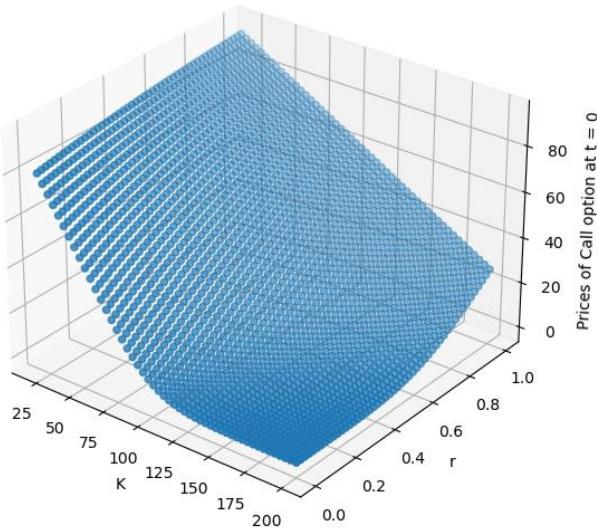
Initial Call Option Price vs K and r for the set = 1



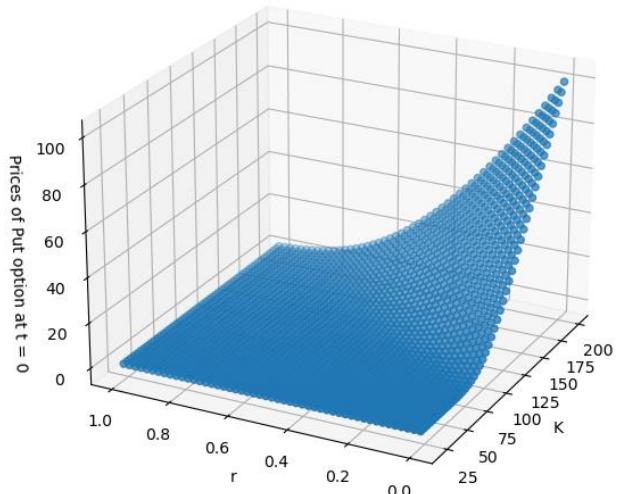
Initial Put Option Price vs K and r for the set = 1



Initial Call Option Price vs K and r for the set = 2



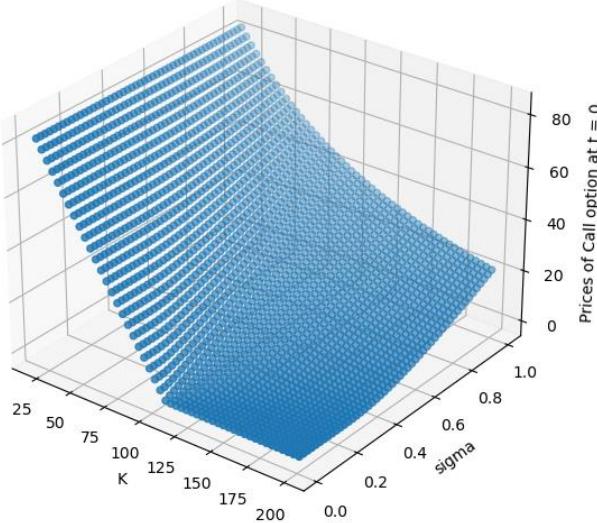
Initial Put Option Price vs K and r for the set = 2



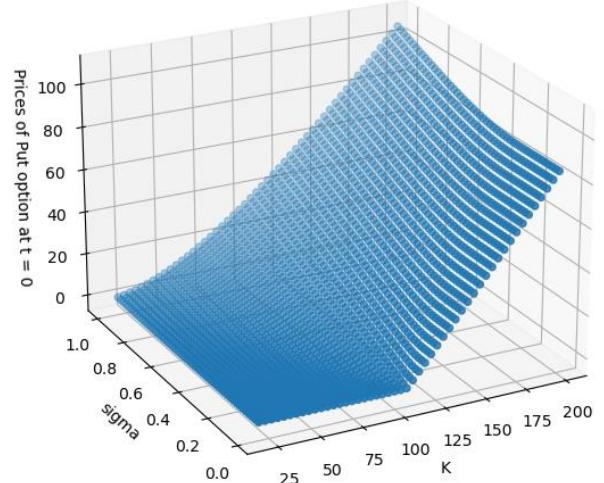
11. Variation with K and σ –

Initial Call Option Price vs K and sigma for the set = 1

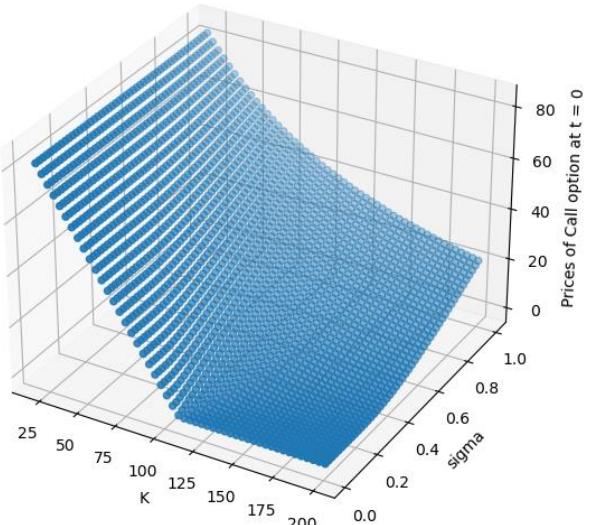
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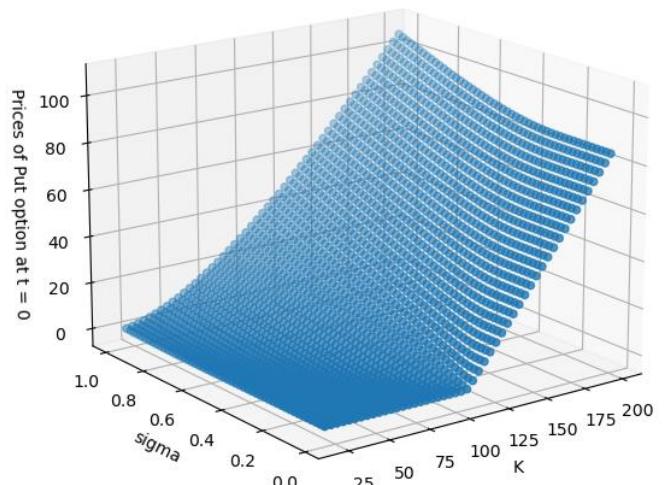
Initial Put Option Price vs K and sigma for the set = 1



Initial Call Option Price vs K and sigma for the set = 2

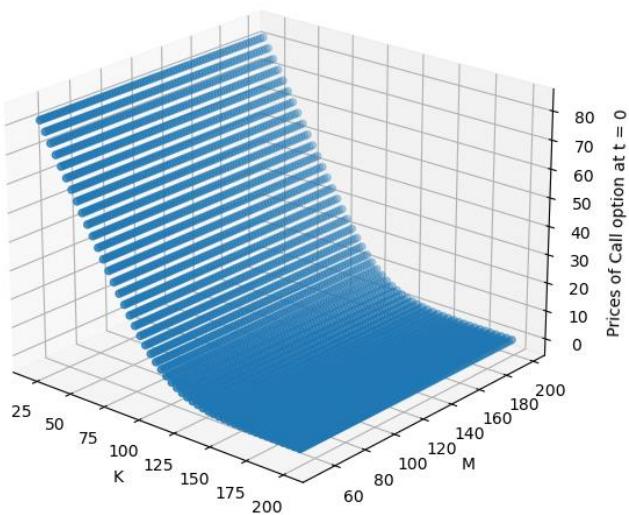


Initial Put Option Price vs K and sigma for the set = 2

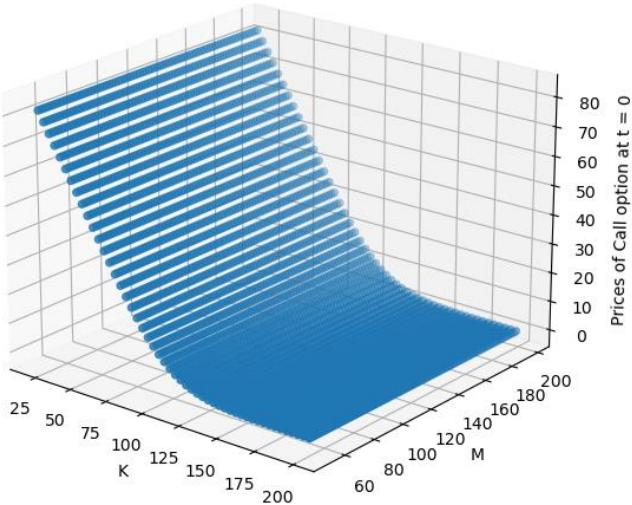


12. Variation with K and M -

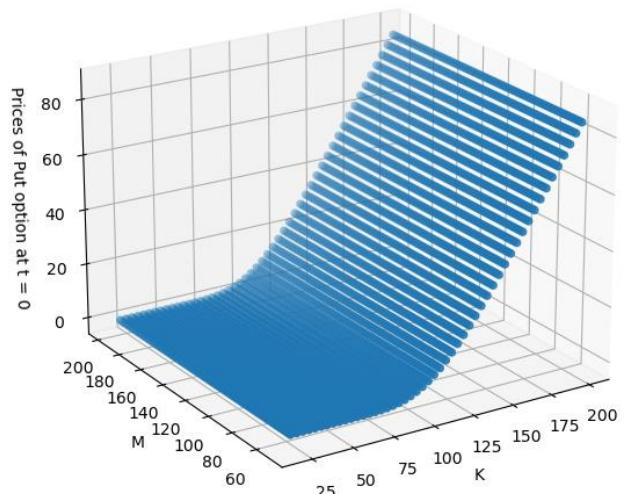
Initial Call Option Price vs K and M for the set = 1



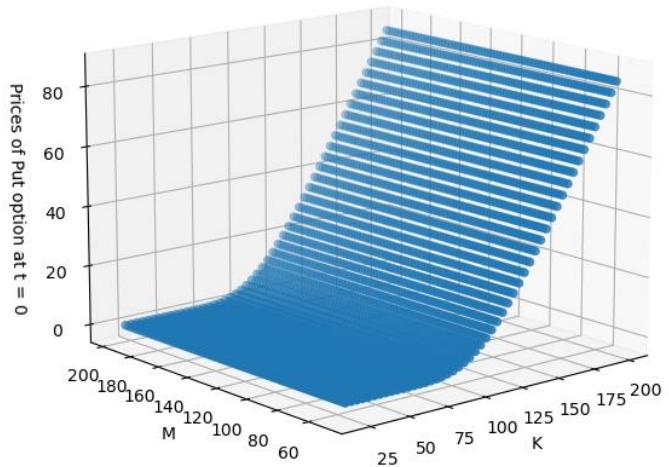
Initial Call Option Price vs K and M for the set = 2



Initial Put Option Price vs K and M for the set = 1

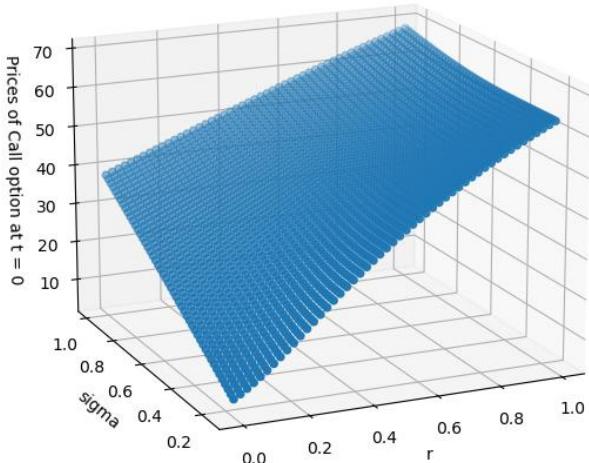


Initial Put Option Price vs K and M for the set = 2

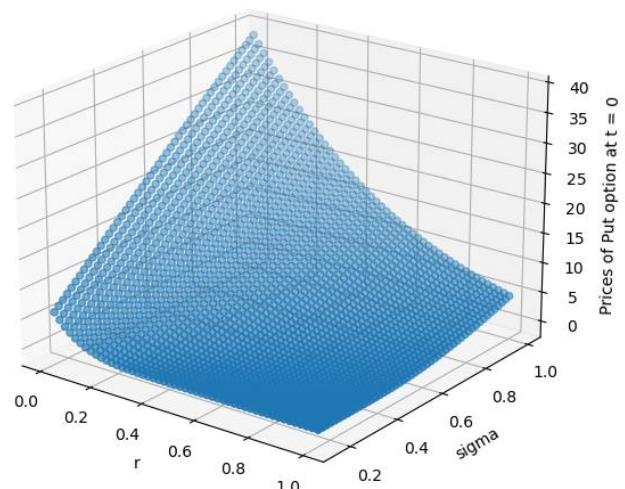


13. Variation with r and σ -

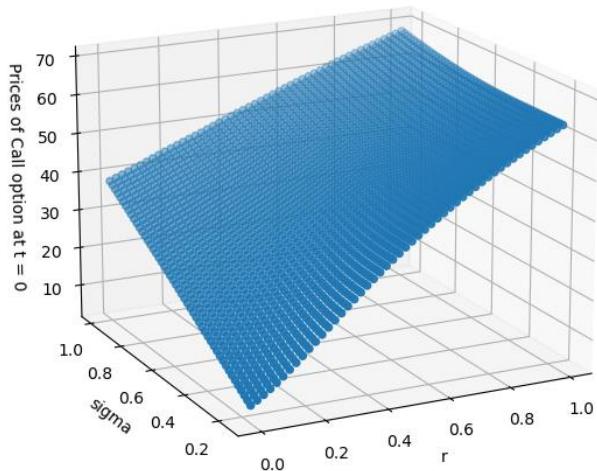
Initial Call Option Price vs r and sigma for the set = 1



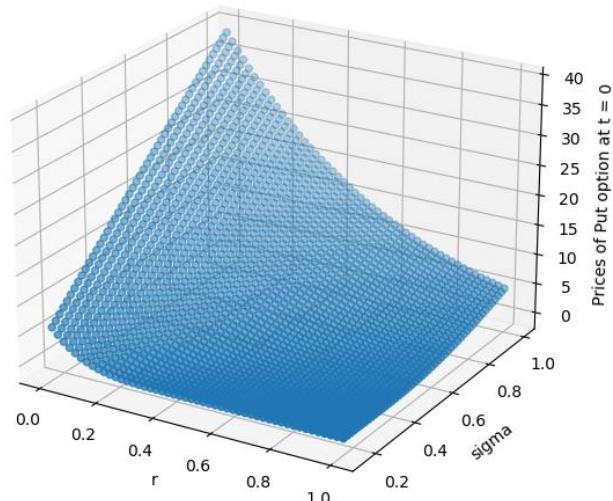
Initial Put Option Price vs r and sigma for the set = 1



Initial Call Option Price vs r and sigma for the set = 2

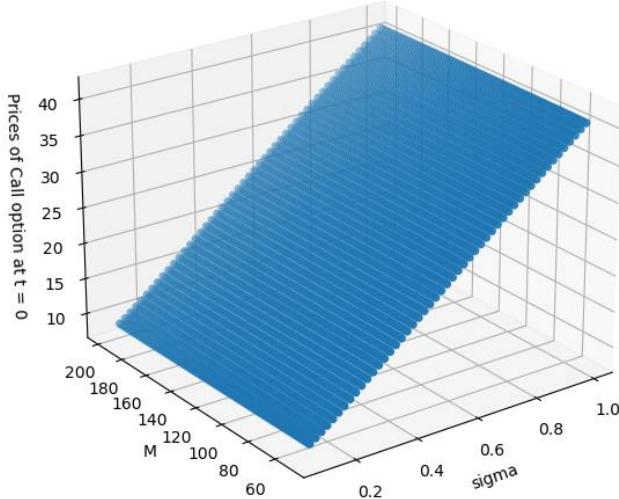


Initial Put Option Price vs r and sigma for the set = 2

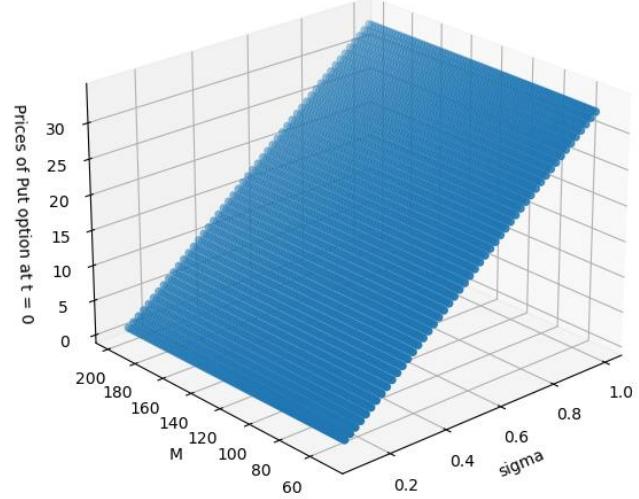


14. Variation with σ and M –

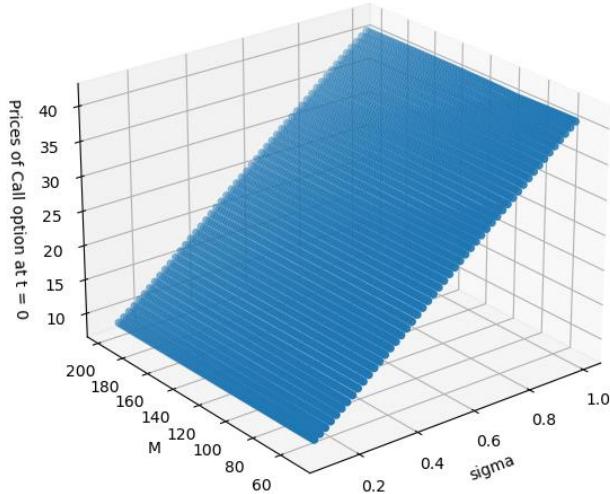
Initial Call Option Price vs sigma and M for the set = 1



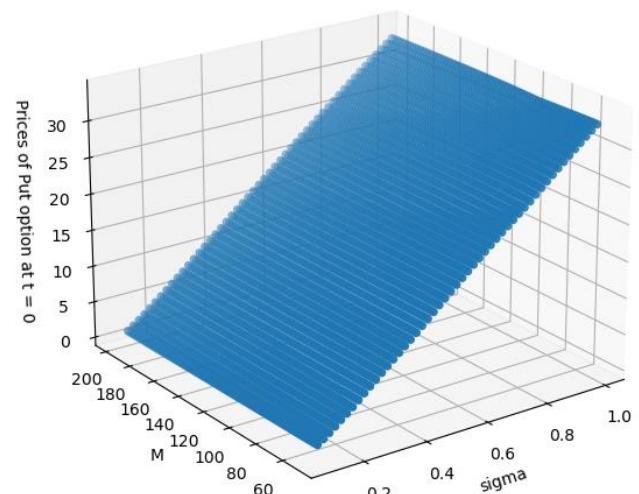
Initial Put Option Price vs sigma and M for the set = 1



Initial Call Option Price vs sigma and M for the set = 2

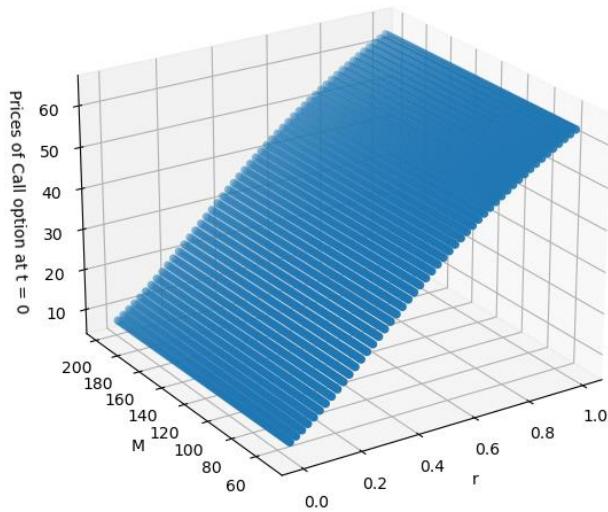


Initial Put Option Price vs sigma and M for the set = 2

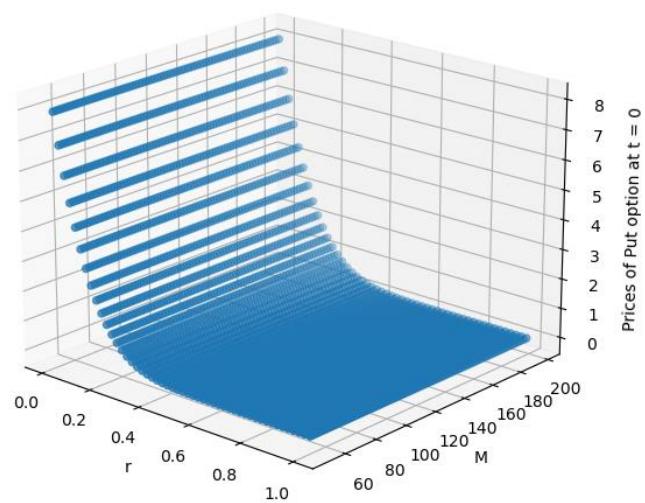


15. Variation with r and M –

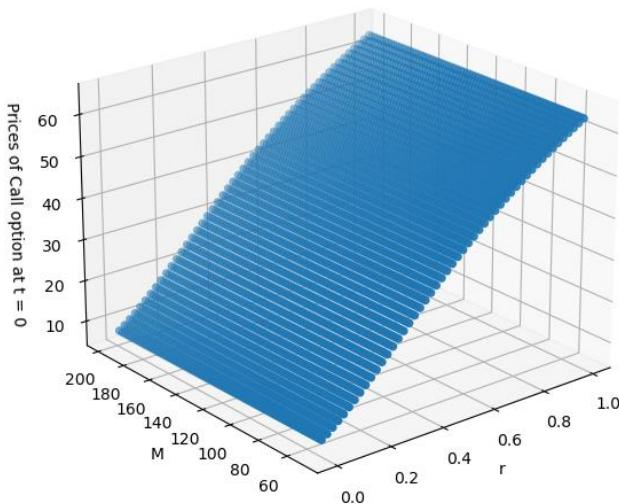
Initial Call Option Price vs r and M for the set = 1



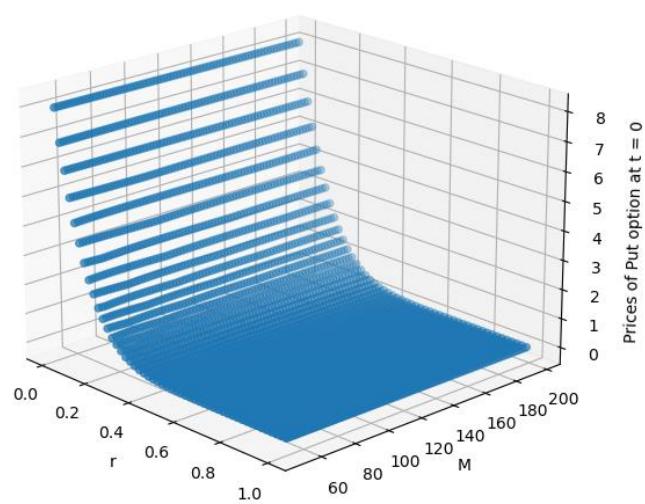
Initial Put Option Price vs r and M for the set = 1



Initial Call Option Price vs r and M for the set = 2



Initial Put Option Price vs r and M for the set = 2



2 QUESTION - 2:

The path-dependent derivative chosen is **Asian Option**.

The initial option prices for the Asian Call Option and Asian Put Option for **M = 10** are:

i. **Set – 1:**

Asian Call Option = 6.476003047446573

Asian Put Option = 2.6779455899442355

ii. **Set – 2:**

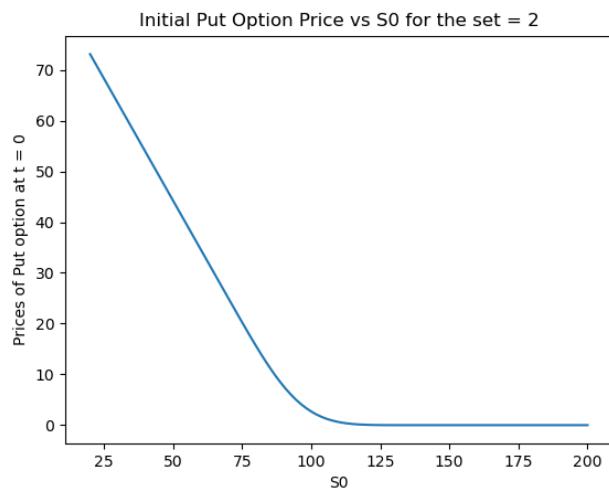
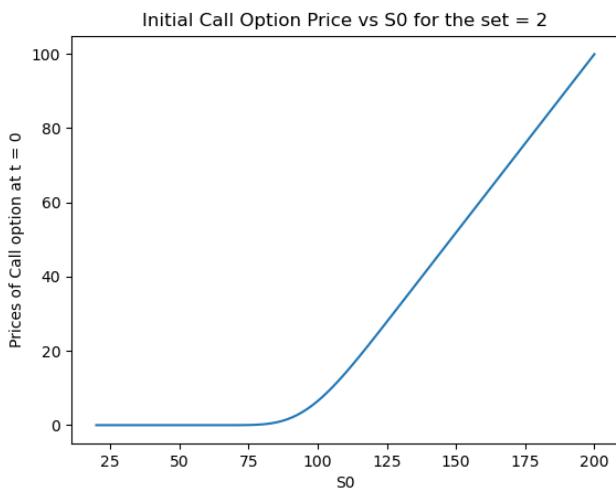
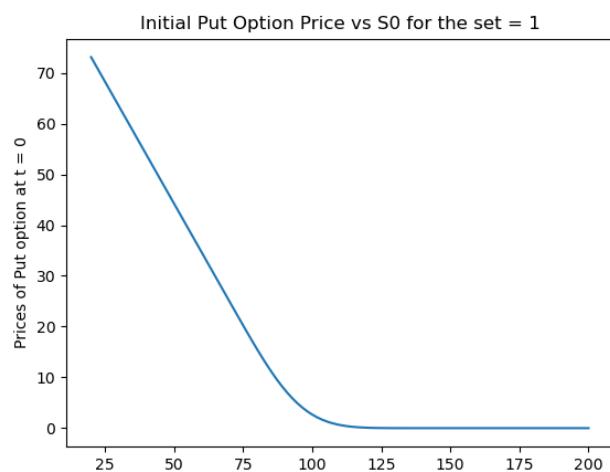
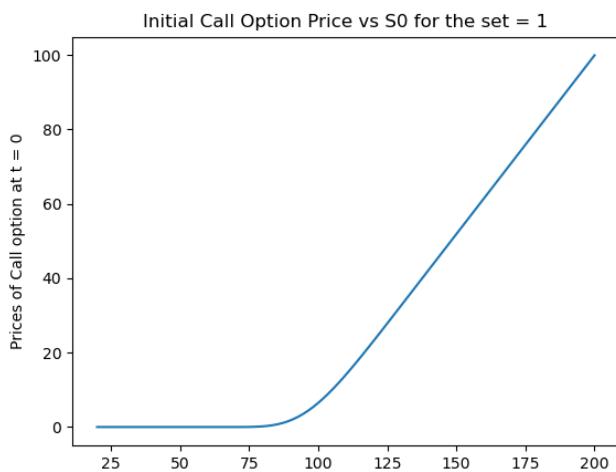
Asian Call Option = 6.490029377643386

Asian Put Option = 2.6919719201410475

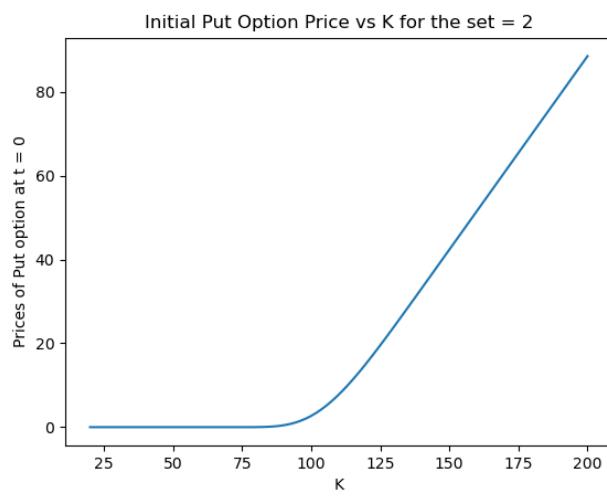
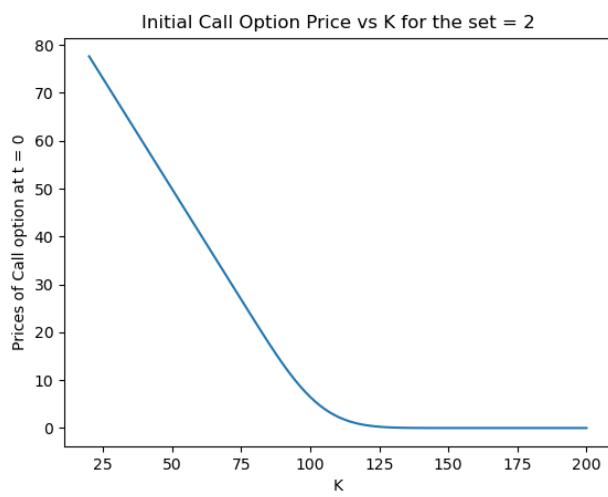
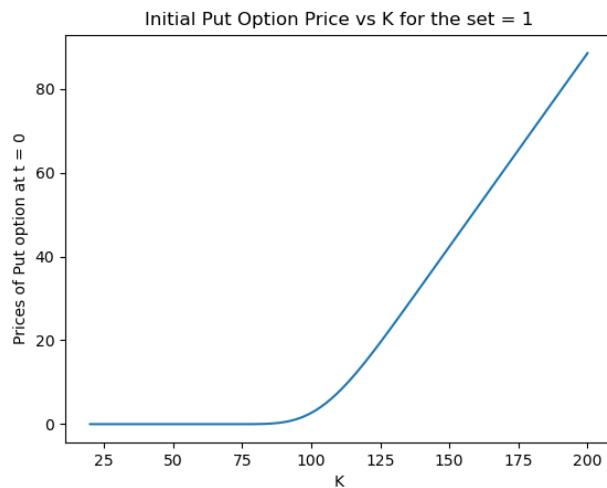
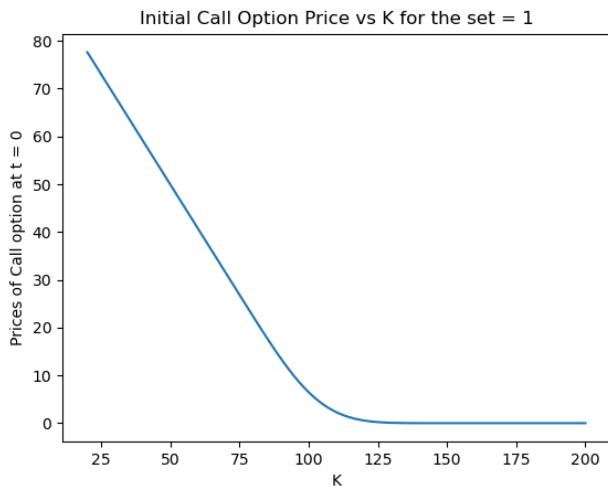
The **payoff for Asian Call Option** at expiry T is $\max\left(\frac{1}{N} \sum_{i=1}^N S(t_i) - K, 0\right)$, while the same **payoff for Asian Put Option** is $\max\left(K - \frac{1}{N} \sum_{i=1}^N S(t_i), 0\right)$.

Sensitivity Analysis:

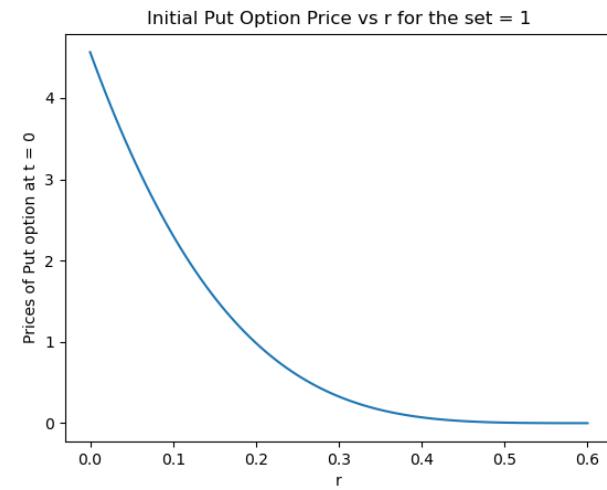
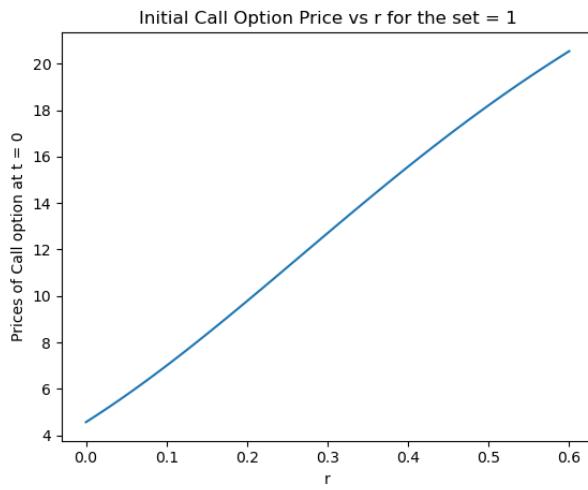
1. Variation with S(0) –

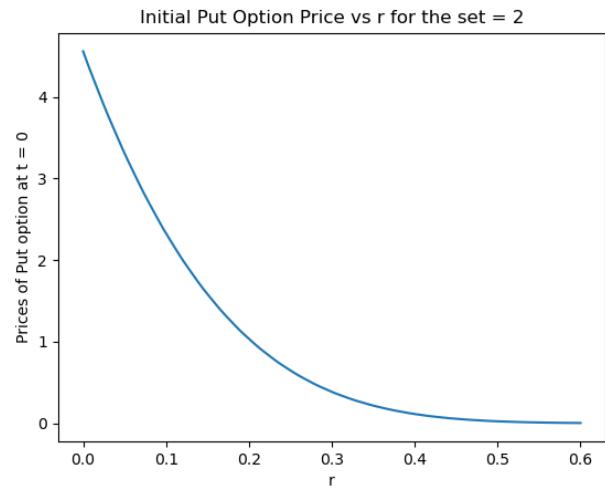
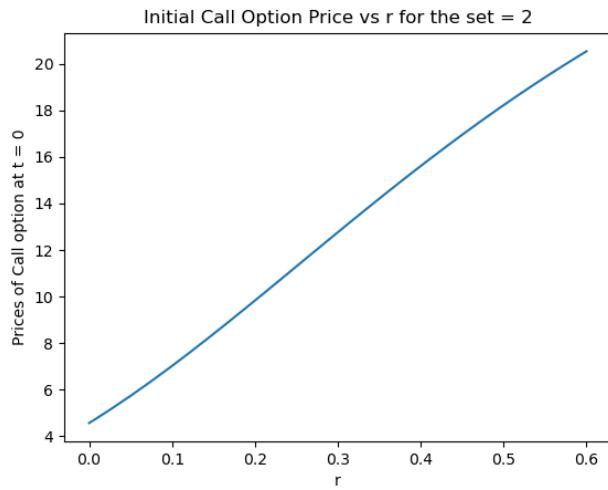


2. Variation with K –

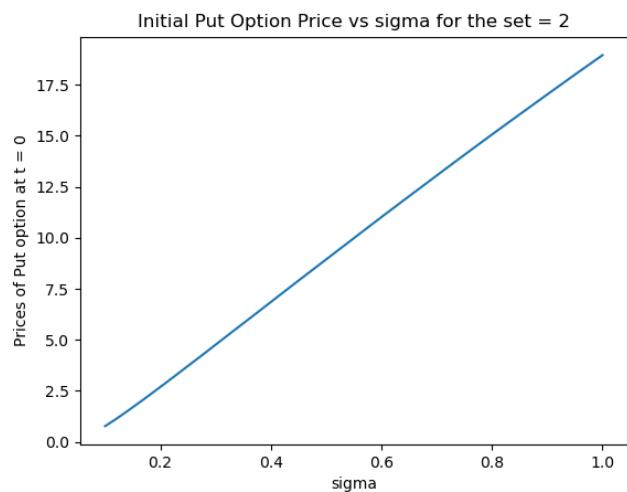
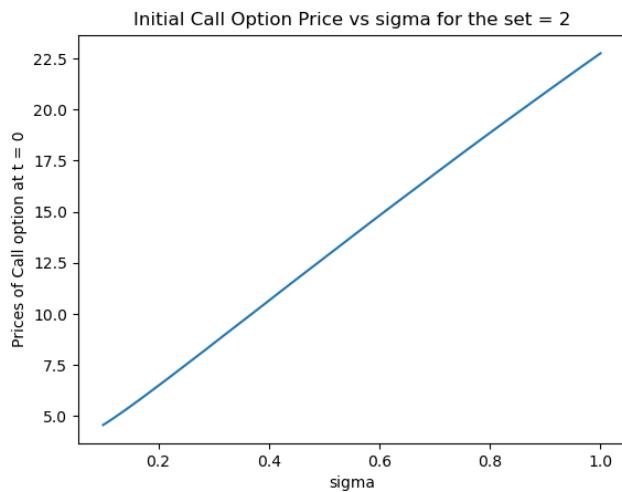
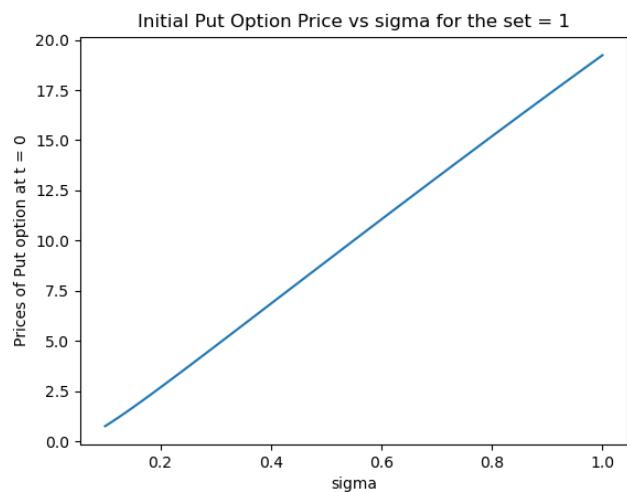
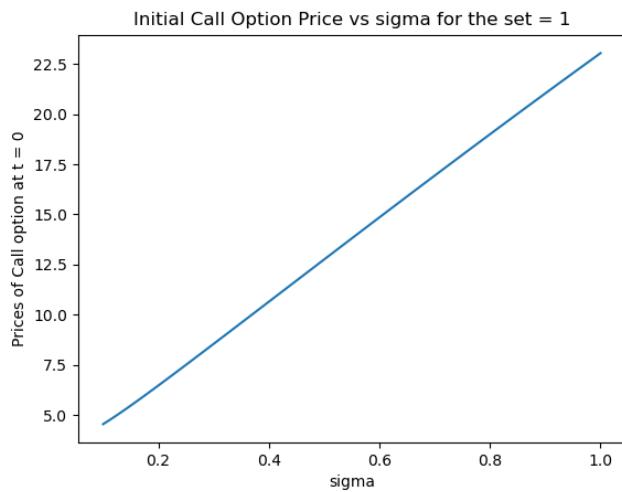


3. Variation with r –



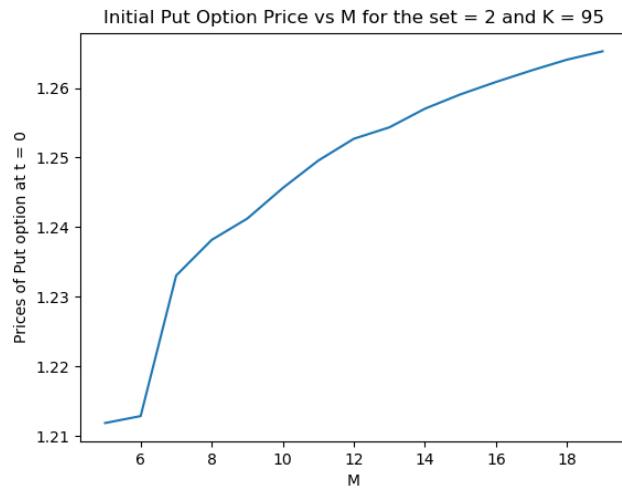
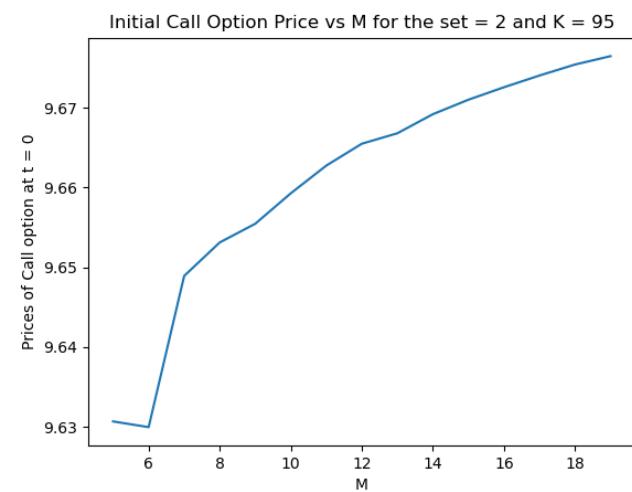
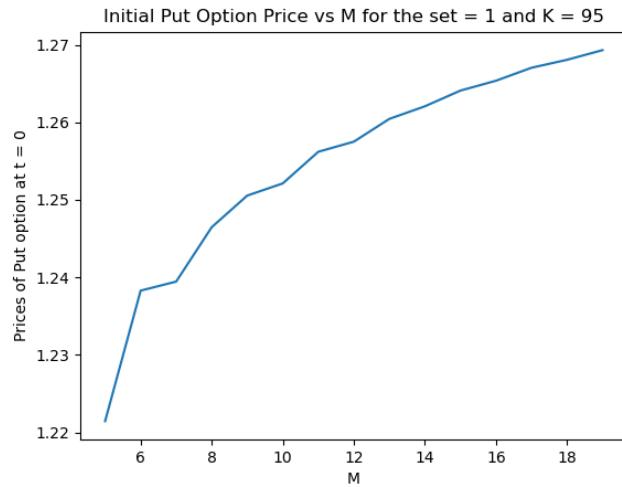
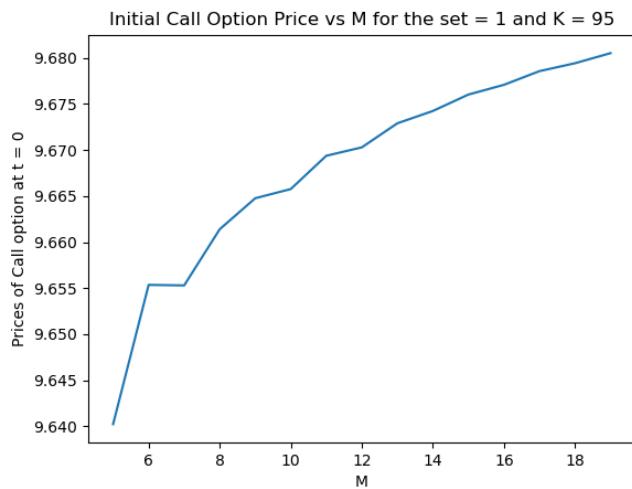


4. Variation with σ –

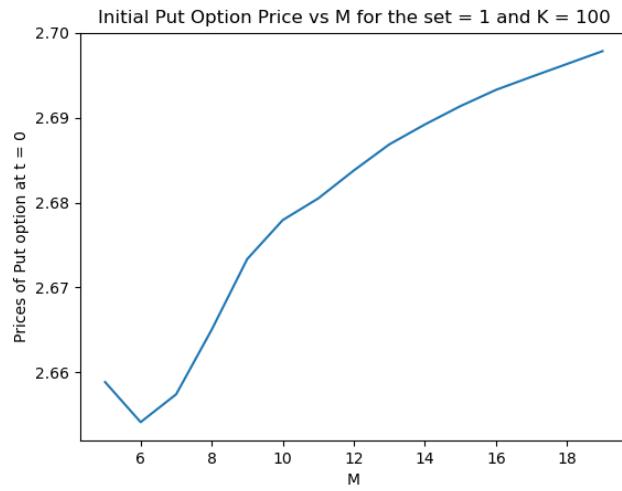
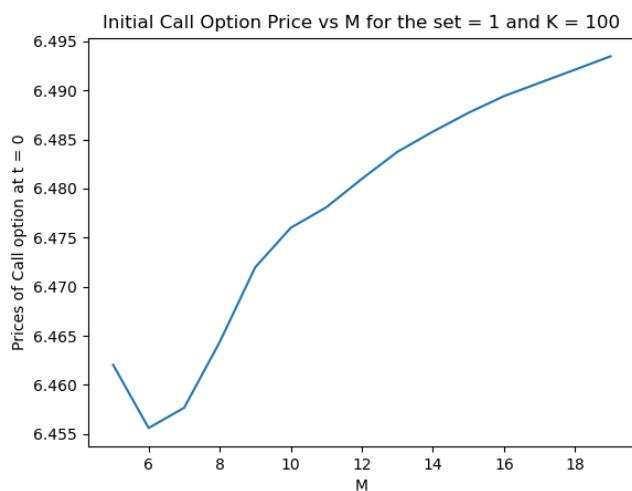


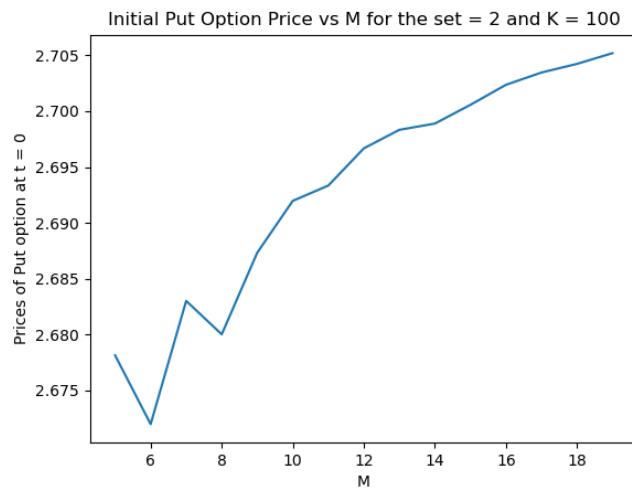
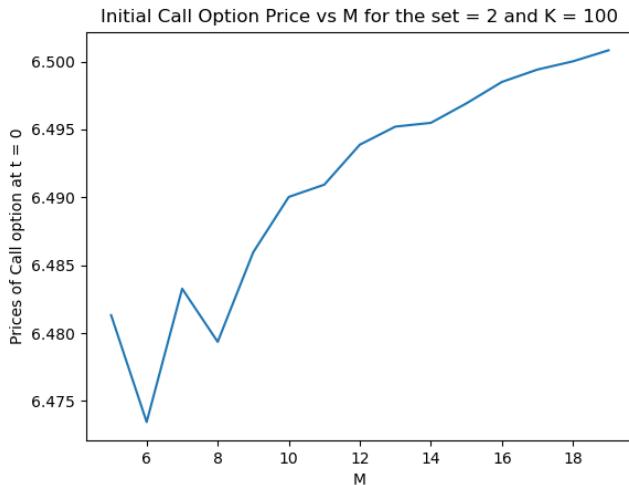
5. Variation with M –

Case (i): K = 95

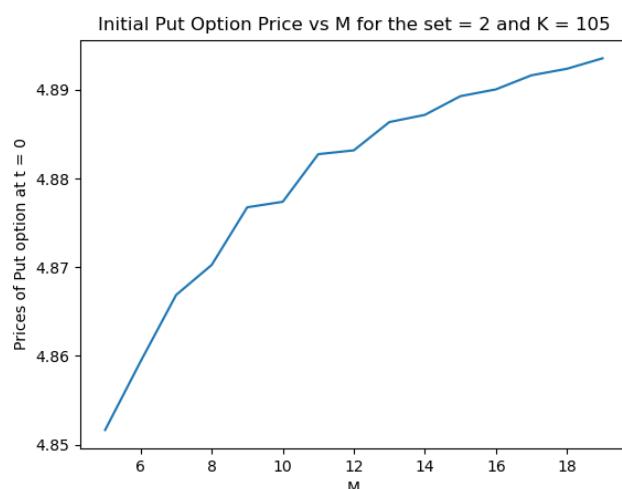
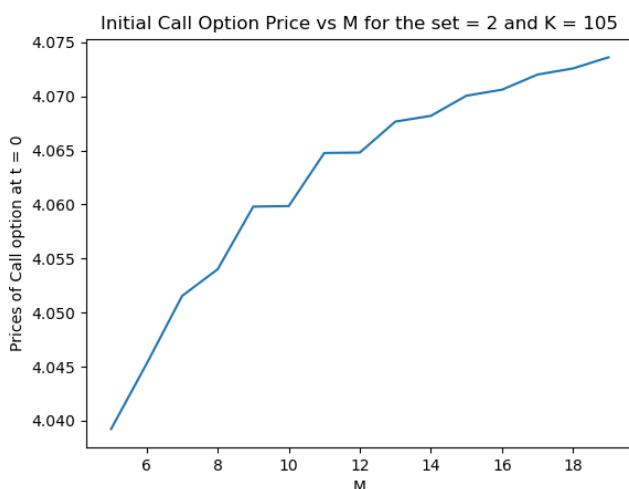
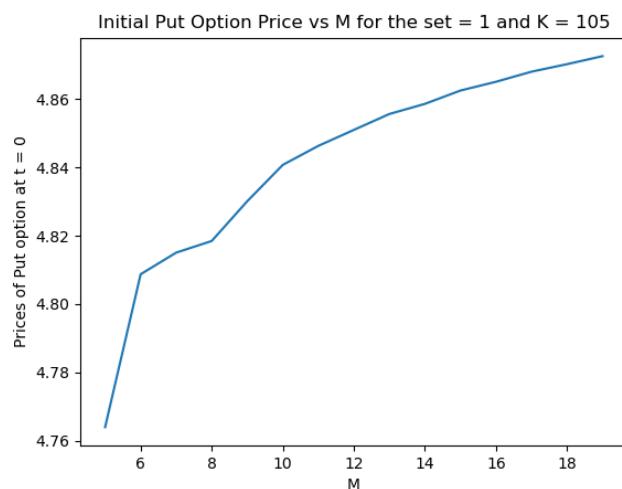
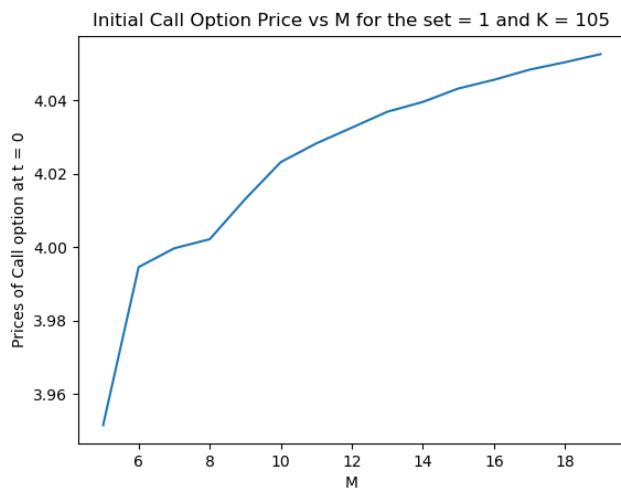


Case (ii): K = 100



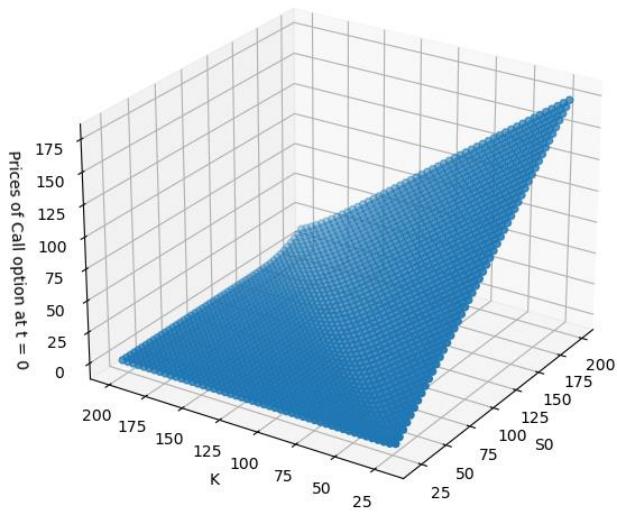


Case (iii): K = 105

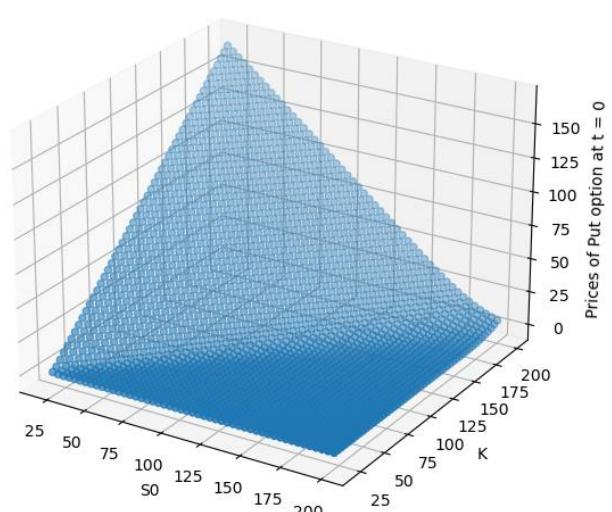


6. Variation with $S(0)$ and K –

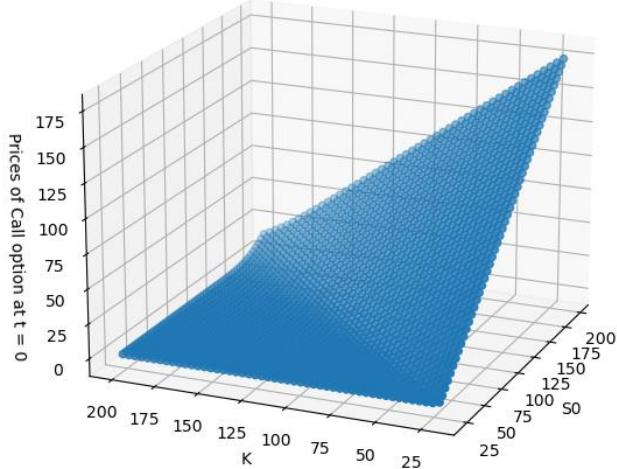
Initial Call Option Price vs $S(0)$ for the set = 1



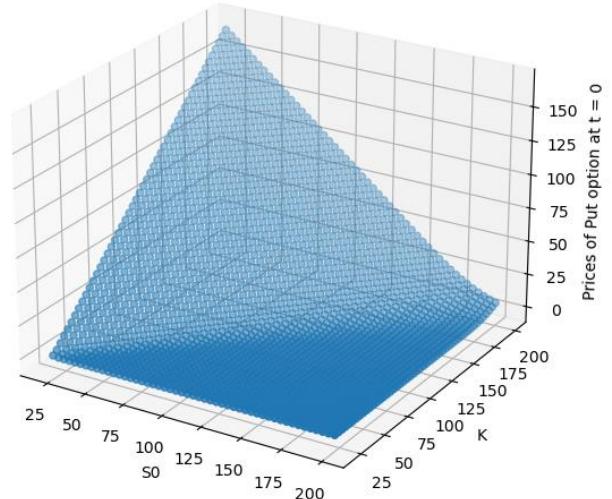
Initial Put Option Price vs $S(0)$ for the set = 1



Initial Call Option Price vs $S(0)$ for the set = 2

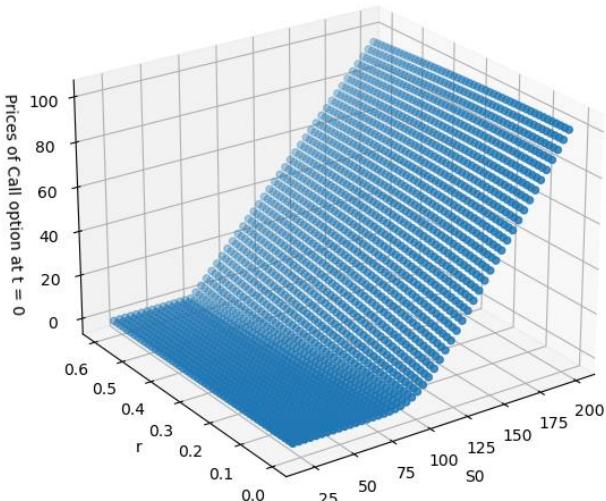


Initial Put Option Price vs $S(0)$ for the set = 2

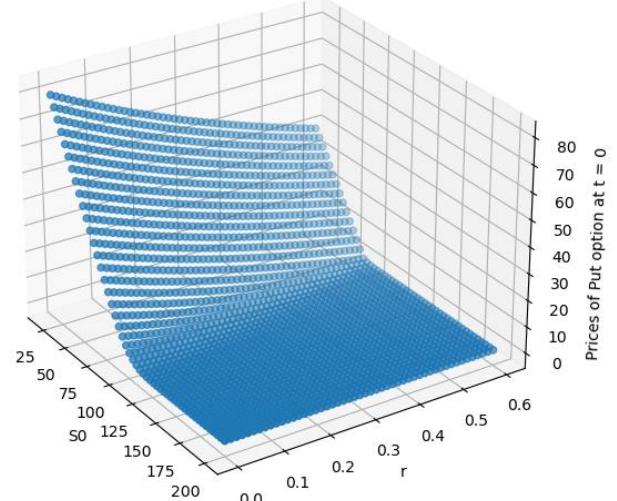


7. Variation with $S(0)$ and r –

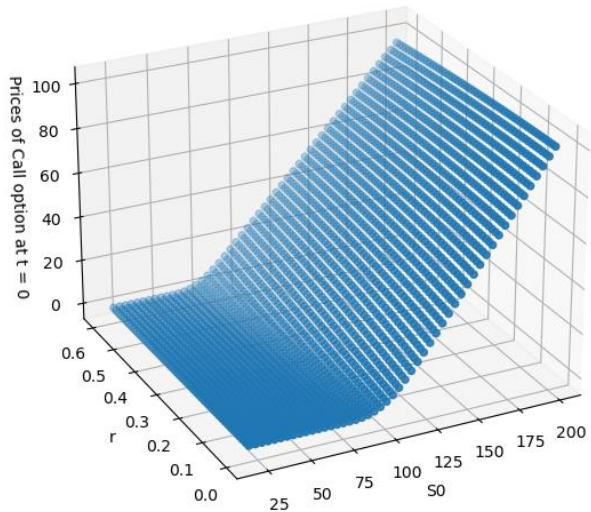
Initial Call Option Price vs $S(0)$ and r for the set = 1



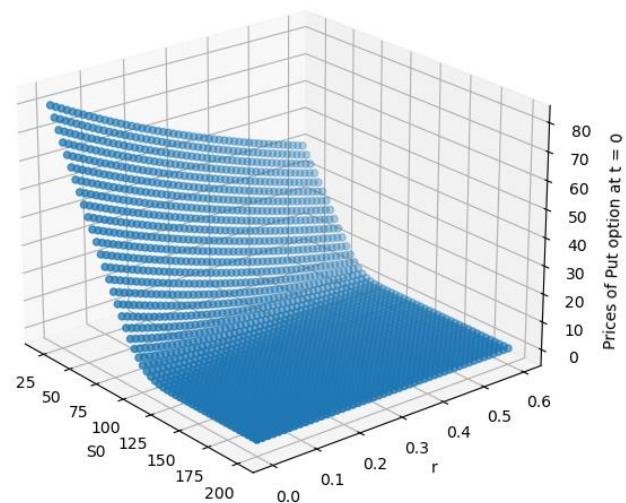
Initial Put Option Price vs $S(0)$ and r for the set = 1



Initial Call Option Price vs S_0 and r for the set = 2

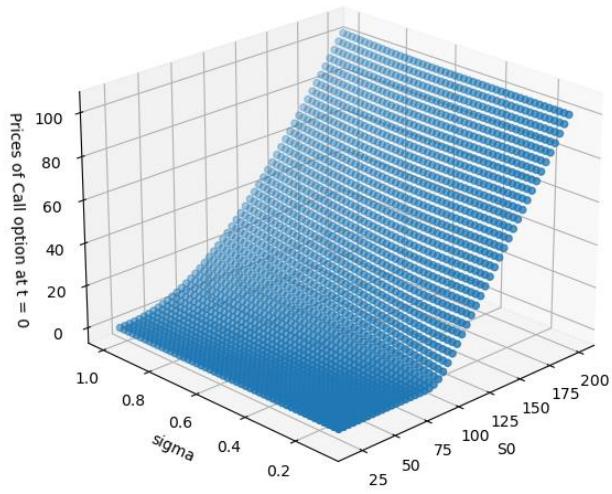


Initial Put Option Price vs S_0 and r for the set = 2

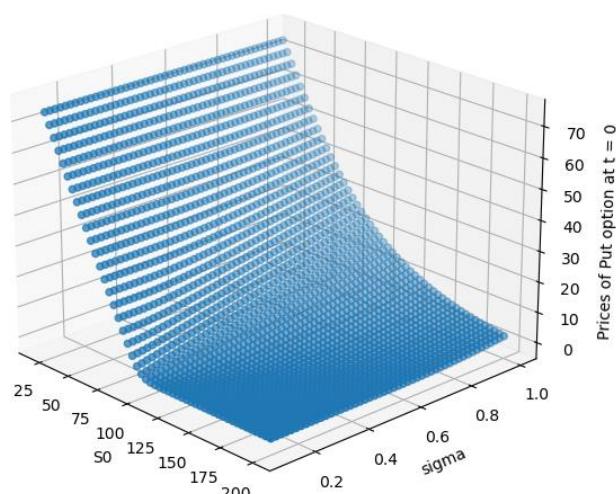


8. Variation with $S(0)$ and σ –

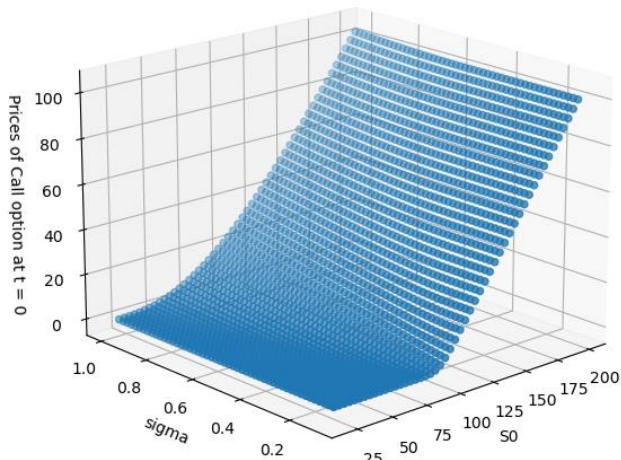
Initial Call Option Price vs S_0 and sigma for the set = 1



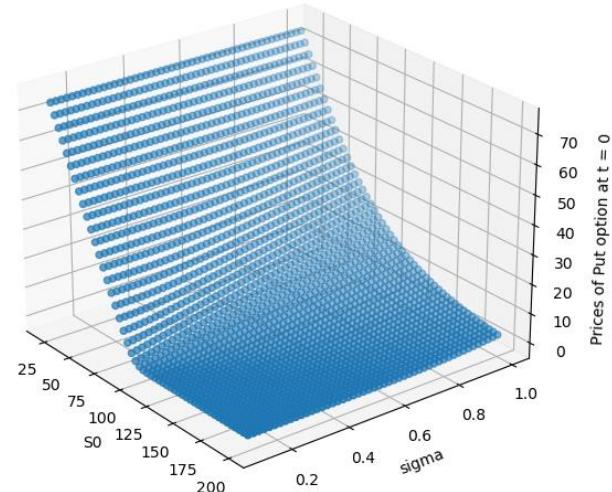
Initial Put Option Price vs S_0 and sigma for the set = 1



Initial Call Option Price vs S_0 and sigma for the set = 2

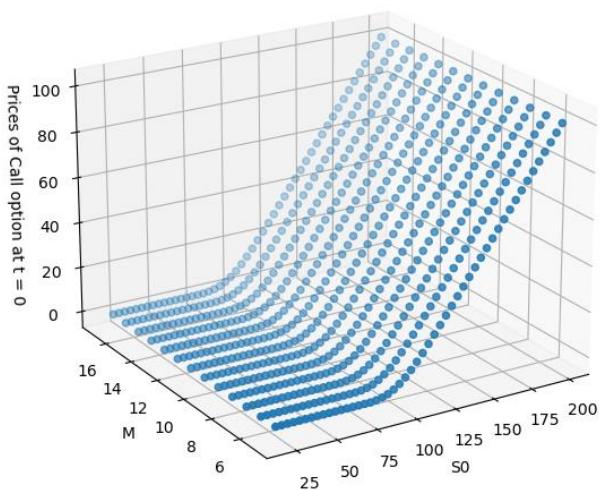


Initial Put Option Price vs S_0 and sigma for the set = 2

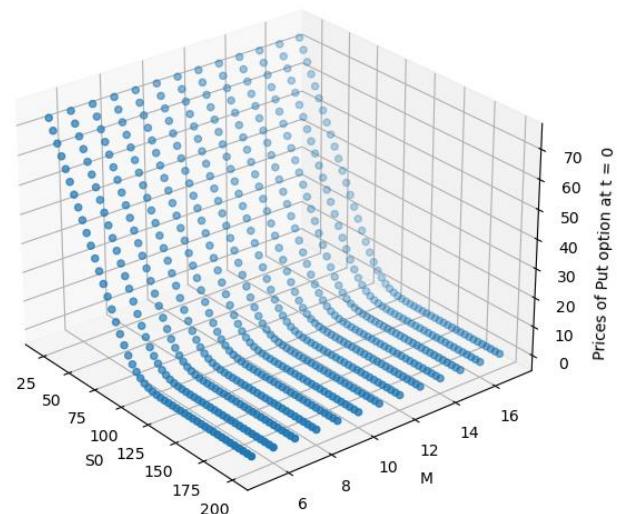


9. Variation with $S(0)$ and M –

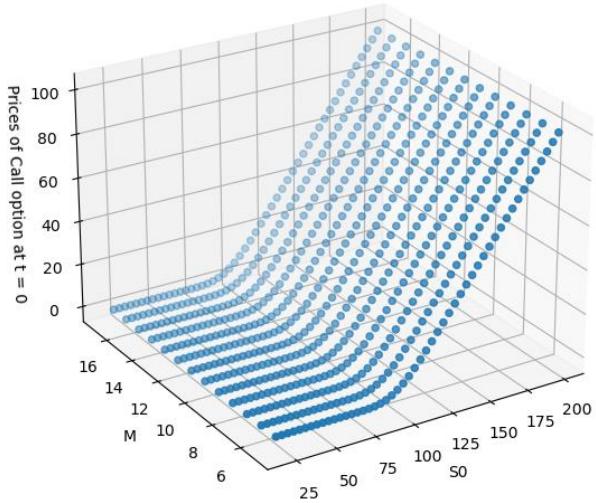
Initial Call Option Price vs S_0 and M for the set = 1



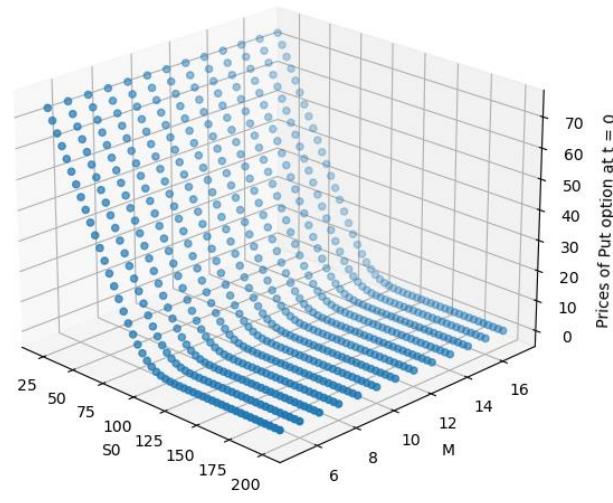
Initial Put Option Price vs S_0 and M for the set = 1



Initial Call Option Price vs S_0 and M for the set = 2



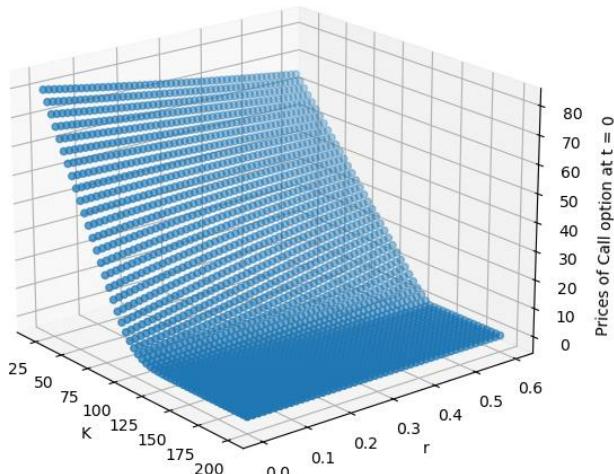
Initial Put Option Price vs S_0 and M for the set = 2



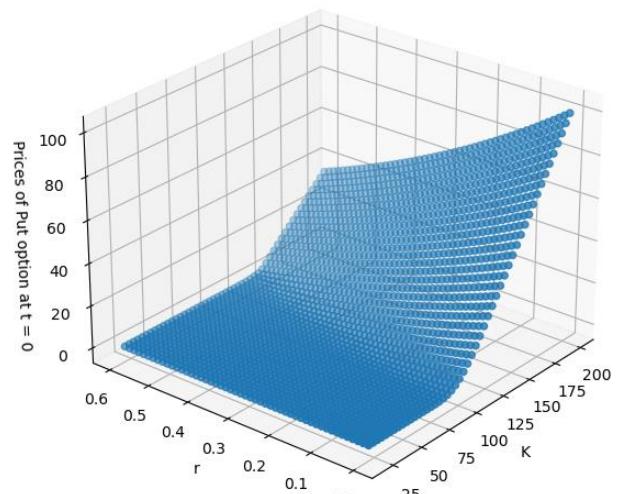
10.

Variation with K and r –

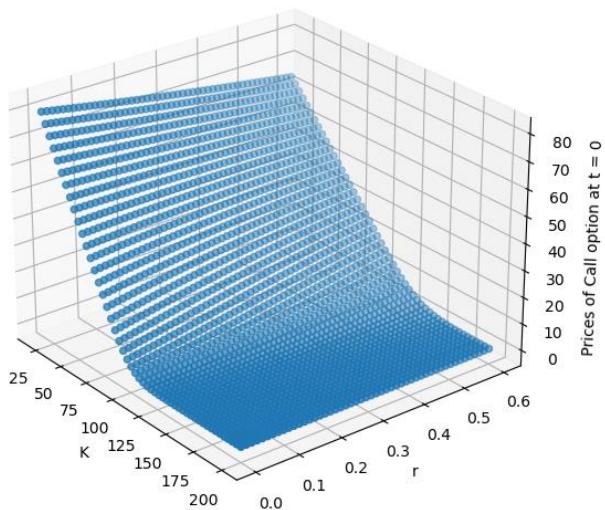
Initial Call Option Price vs K and r for the set = 1



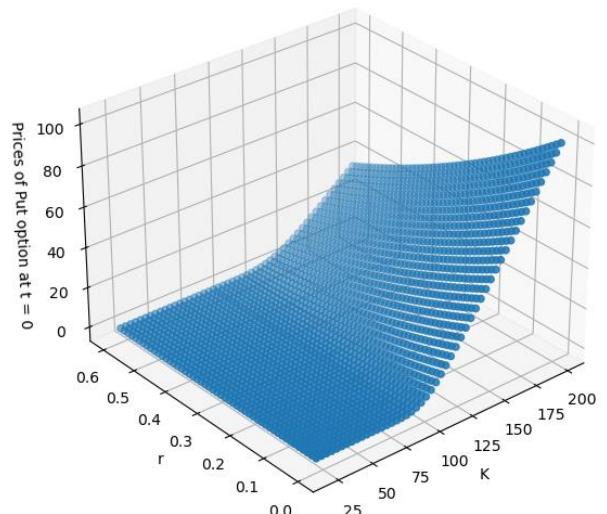
Initial Put Option Price vs K and r for the set = 1



Initial Call Option Price vs K and r for the set = 2

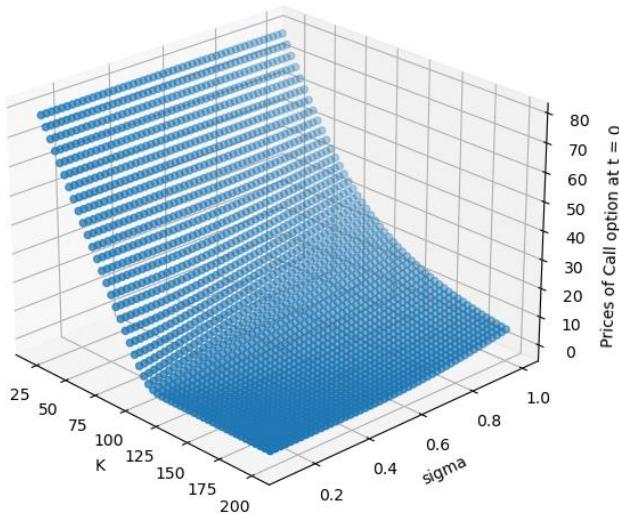


Initial Put Option Price vs K and r for the set = 2

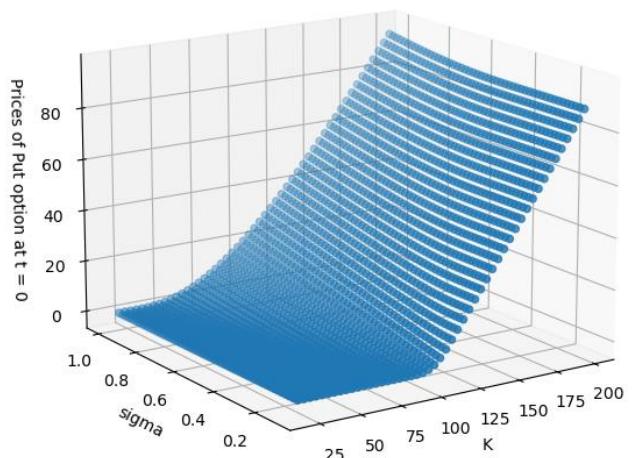


11. Variation with K and σ –

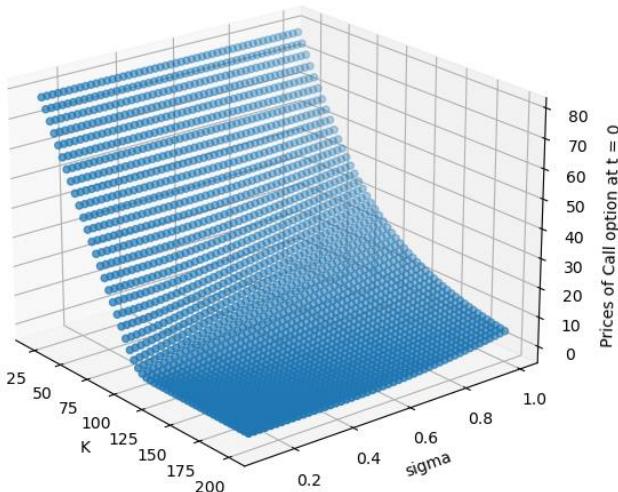
Initial Call Option Price vs K and sigma for the set = 1



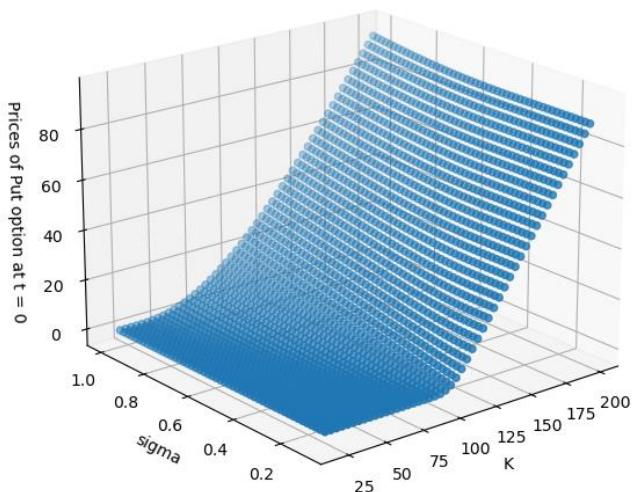
Initial Put Option Price vs K and sigma for the set = 1



Initial Call Option Price vs K and sigma for the set = 2

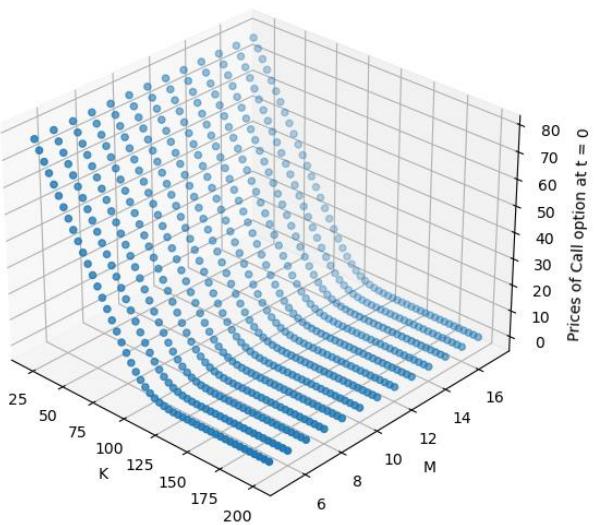


Initial Put Option Price vs K and sigma for the set = 2

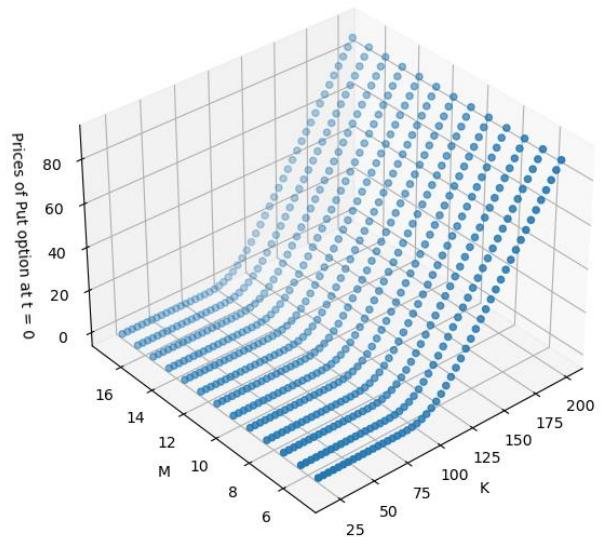


12. Variation with K and M -

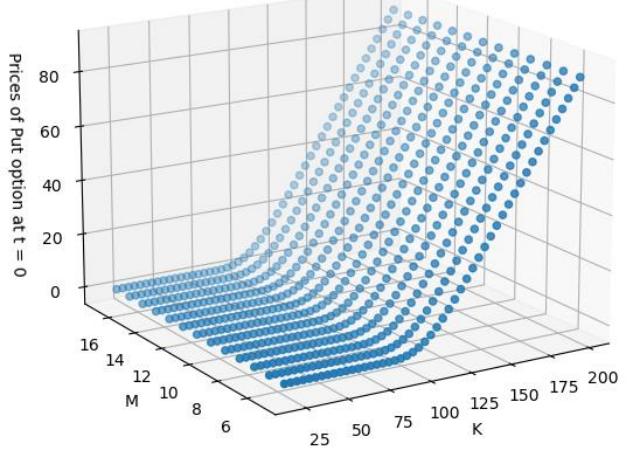
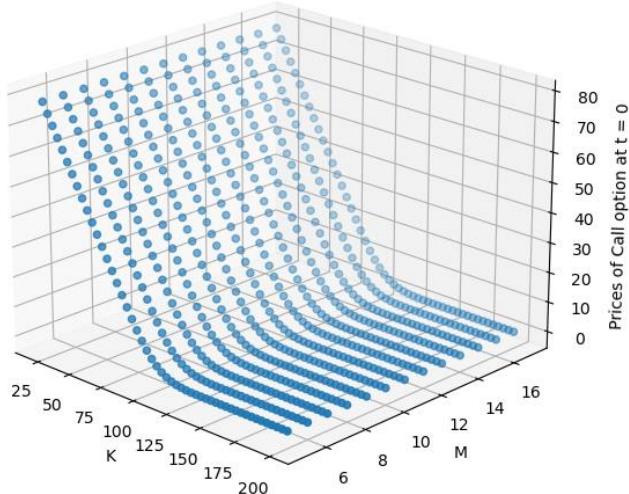
Initial Call Option Price vs K and M for the set = 1



Initial Put Option Price vs K and M for the set = 1



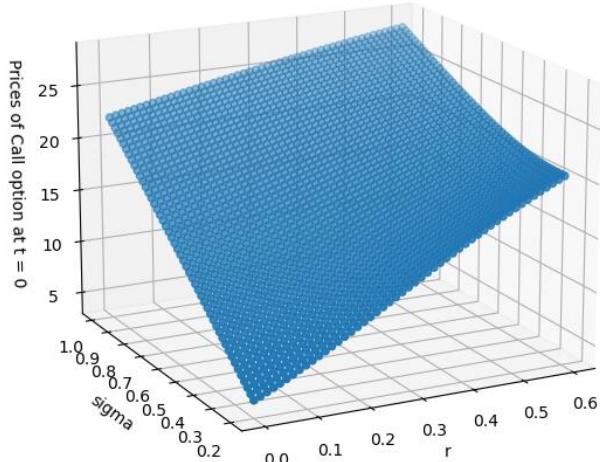
Initial Call Option Price vs K and M for the set = 2



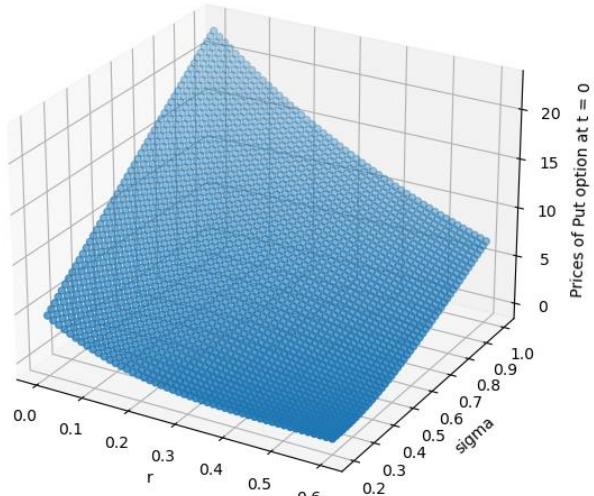
13.

Variation with r and σ –

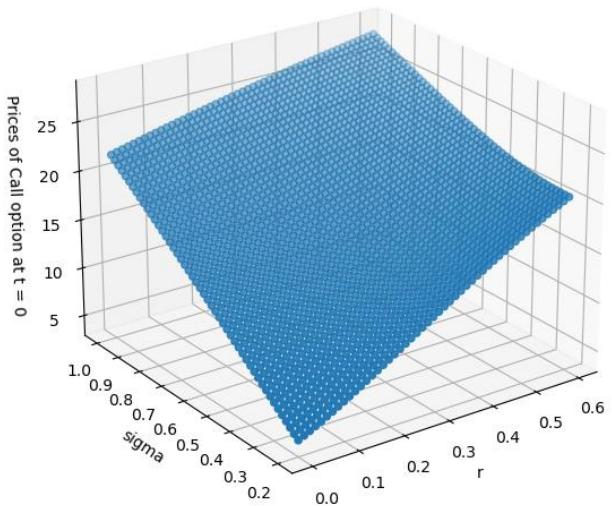
Initial Call Option Price vs r and sigma for the set = 1



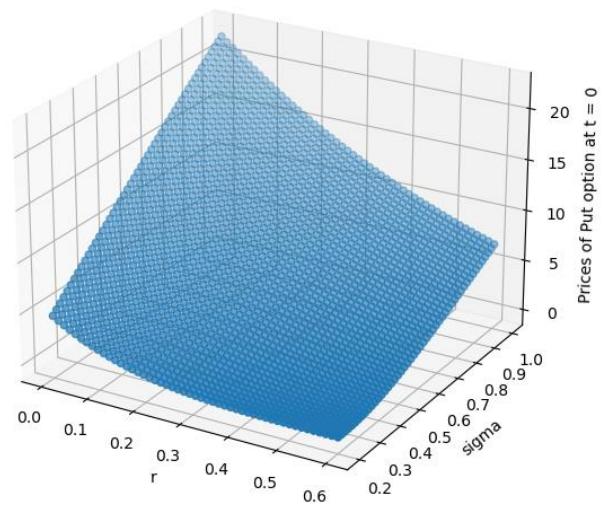
Initial Put Option Price vs r and sigma for the set = 1



Initial Call Option Price vs r and sigma for the set = 2

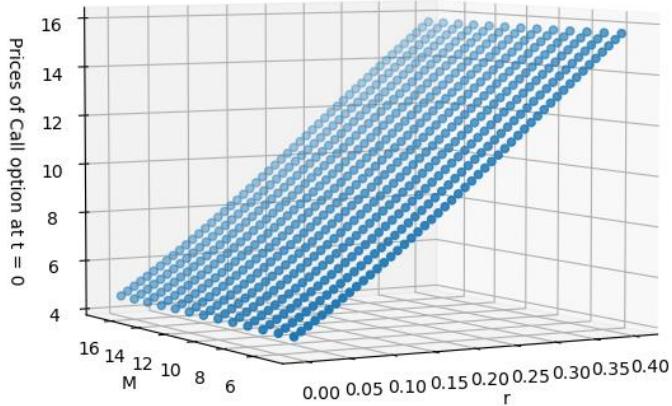


Initial Put Option Price vs r and sigma for the set = 2

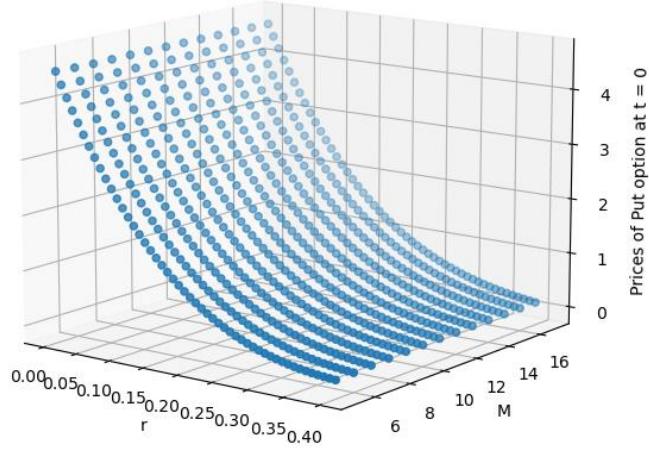


14. Variation with r and M –

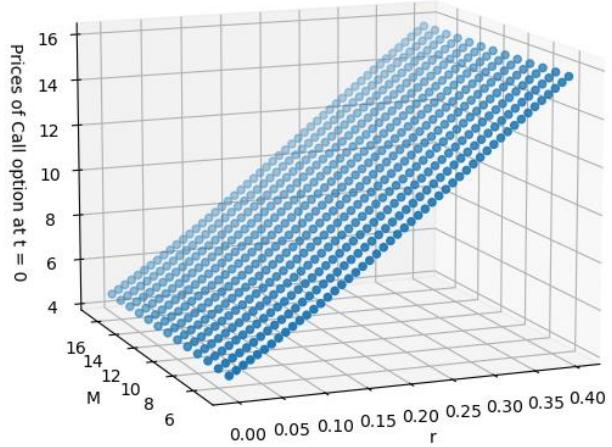
Initial Call Option Price vs r and M for the set = 1



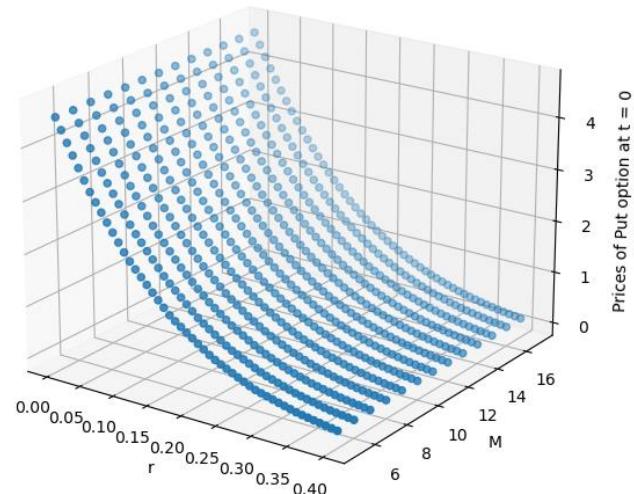
Initial Put Option Price vs r and M for the set = 1



Initial Call Option Price vs r and M for the set = 2



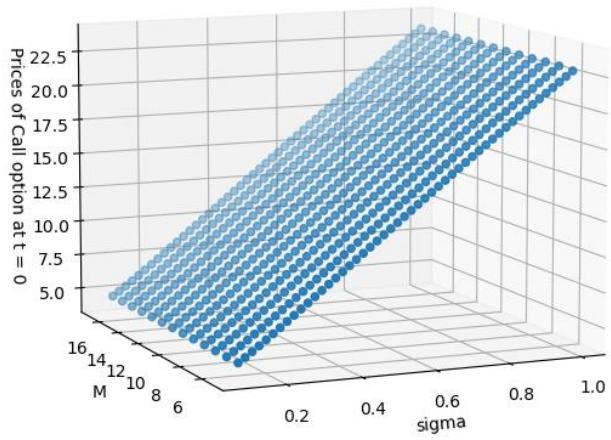
Initial Put Option Price vs r and M for the set = 2



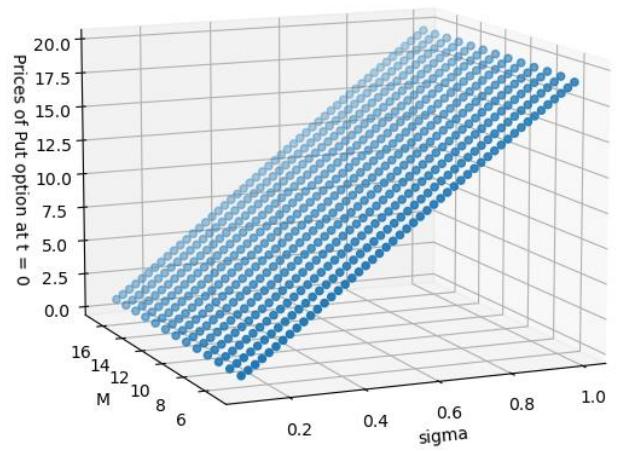
15.

Variation with σ and M –

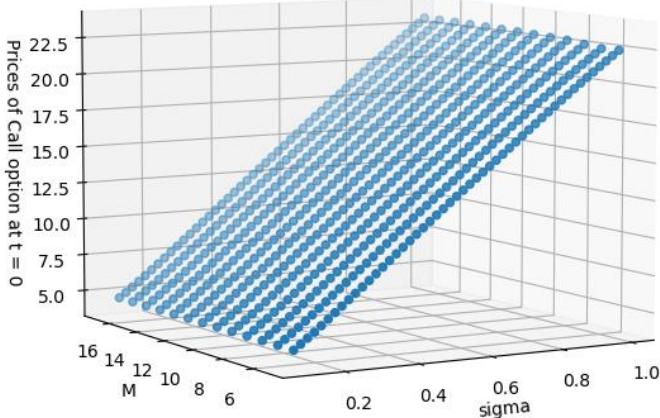
Initial Call Option Price vs sigma and M for the set = 1



Initial Put Option Price vs sigma and M for the set = 1



Initial Call Option Price vs sigma and M for the set = 2



Initial Put Option Price vs sigma and M for the set = 2

