MA 423 – Matrix Computations

Lab – 4

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1 QUESTION - 1:

The result of the driver code which evaluates the $f(x) = (x - 4)^2$ at x = 0 and x = [1 2 4]' is:

```
****** Evaluation at Scalar value *******
x =      0
Calculated using Horner formula =     16

Calculated using exact formula =     16


****** Evaluation at Vector value *******
x =      1     2     4
Calculated using Horner formula =      9     4     0

Calculated using exact formula =      9     4     0
```

2 QUESTION - 2:

The root of the polynomials for the different intervals are:

```
Computed Root for root in [1.950000, 2.050000] =      2.049971765279770e+00
Computed Root for root in [1.960000, 2.040000] =      2.023957281112672e+00
Computed Root for root in [1.970000, 2.030000] =      2.021199414730072e+00
Computed Root for root in [1.980000, 2.020000] =      2.012653608322144e+00
Computed Root for root in [1.990000, 2.010000] =      2.006189680099487e+00
Computed Root for root in [1.960000, 1.990000] =      1.972823069095611e+00
```

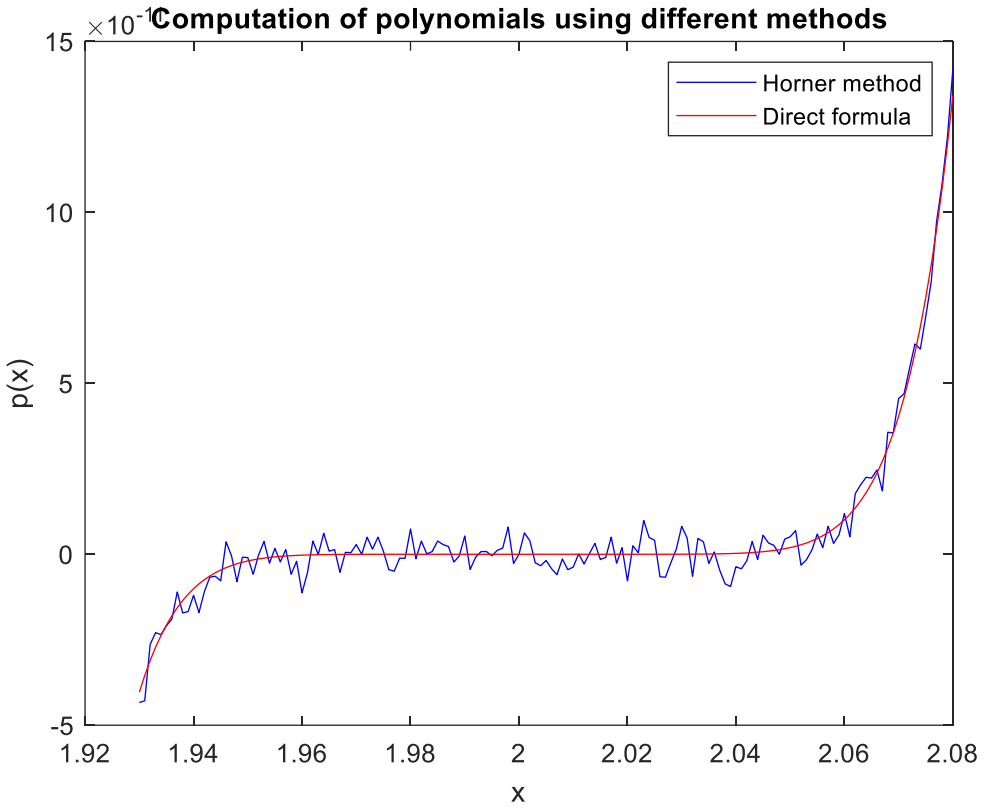
Observations:

We observe that the root of polynomial is not exactly zero, but there is a variation in it, and as the interval size is made smaller, the absolute difference between the computed root and the exact root decreases. This can be attributed to the limitations of finite precision arithmetic and the rounding errors caused while evaluation of polynomial using Horner's method. As a result, the computed roots are in the neighbourhood of 2.

Also we can see that for the last interval even though the interval does not contain 2, we are getting some solution around 2.

3 QUESTION - 3:

The plot is:



Question: Do the plots differ from one another? If yes, can you think of possible reasons? Explain the results of the previous exercise in the light of the difference in the plots.

Explanation:

- ⇨ We can observe that the plots corresponding to the Horner method and the direct formula do not exactly coincide and they differ from one another. The plot for the Horner method crosses x-axis multiple times and there is a large number of small variations around this line.
- ⇨ The possible reasons for this behaviour is the rounding errors committed while evaluation of polynomial p(x) using Horner's method and the limitations of the finite precision systems.
- ⇨ The small rounding errors are causing the values of p(x) to fluctuate around x = 2 particularly. Small negative values are pushed to negative values, while small positive values are pushed to negative numbers in the neighbourhood of 2. Due to this, the graph crosses x-axis in multiple places apart from 2 computationally. Theoretically we only have 2 as root with its multiplicity being 9, while computationally we have roots in neighbourhood of 2 (some even in intervals around 2, but not containing 2).
- ⇨ Also there are possibility of catastrophic cancellation happening in Horner's method due to the involvement of multiplication and addition at each step of the computation.