

MA 423 – Matrix Computations

Lab – 7

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1 QUESTION - 1:

The output is as follows:

***** Method 1 *****

Condition number of coeff matrix = 5.634850960284134e+13

r1 norm = 8.184730188469400e-13

***** Method 2 *****

Condition number of coeff matrix = 4.959020767557438e+26

r2 norm = 1.440260069424386e-09

***** Method 3 *****

Condition number of coeff matrix = 1.961760846966694e+14

r3 norm = 8.184403893900406e-13

Observations:

1. All the coefficient matrices are ill-conditioned, but the matrix in method 2 (normal equation) is the most ill-conditioned.
2. The methods 1 and 3 have comparable values of $\|r\|_2$ and the value of $\|r\|_2$ is less than that of method - 2. So both the methods 1 and 3 provide relatively better fit than the method – 2.

2 QUESTION - 2:

The tabular data for various choices of k are:

k	Compression ratio	Relative Error
1	0.008125	0.25276
11	0.089375	0.067909
21	0.17062	0.038911
31	0.25188	0.026793
41	0.33313	0.021712
51	0.41437	0.018855
61	0.49562	0.016387
71	0.57688	0.014447
81	0.65812	0.012996
91	0.73938	0.011606
101	0.82063	0.010394
111	0.90187	0.0093099
121	0.98313	0.0081764
131	1.0644	0.0071563
141	1.1456	0.0063134
151	1.2269	0.0054592
161	1.3081	0.0045929
171	1.3894	0.0038446
181	1.4706	0.0029322
191	1.5519	0.0022403

Observations:

As the value of k increases, the relative error decreases and the compression ratio increases.

3 QUESTION - 3:

The output is as follows:

***** Part (i) *****

```
sigma_1    =    8.7893e+00
sigma_89    =    2.3842e-03
sigma_90    =    3.9606e-15
Rank of A  =    89
```

***** Part (ii) - Perturbed Matrix *****

```
sigma_1    =    8.7893e+00
sigma_89    =    2.3842e-03
sigma_90    =    3.9607e-15
Rank of A  =    89
```

***** Part (iii) - QR decomposition with col pivoting *****

```
|| I - E || =    0
R(90, 90)   =    1.9039e-03
```

Observations:

$R(90, 90)$ is not roughly equal to the u (unit-round off) and also it is not very much less than 1. Since we have used rank-revealing QR decomposition to obtain Q and R , it implies that the computed rank of the matrix is 90, which is evidently not true.

The correct rank of A as computed by Matlab is 89 which is also reflected by SVD decomposition.

(since σ_{90} is comparable to u and $\sigma_{90} \ll 1 \Rightarrow$ Numerical rank of A via SVD = 89)

Hence, Rank Revealing QR Decomposition is less efficient than the SVD method when detecting numerical rank deficiency.