# Lab Session 2

**MA-423 :** Matrix Computations July - November, 2021 S. Bora

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1. Use the MATLAB function program `[L,U] = genp(A)` to do the following:

   (a) Find the factors $L$ and $U$ of an $LU$ decomposition of $A = \begin{bmatrix} 10^{-20} & 1 \\ 1 & 1 \end{bmatrix}$. What is $A - LU$?

   (b) Solve the system of equations $Ax = b$ where $b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ by using the computed $LU$ factorization from `genp` and the programs `rowforward` and `colbackward` in Lab 1 for forward and backward substitution in the correct order. What is the difference of your answer with the correct solution in the 2-norm? (Use `x = A\b` to find the exact solution. If $y$ is the solution via `genp`, then `norm(x - y)` gives the 2-norm difference of $x$ and $y$.)

   What can you conclude about GENP from the above algorithm? Can you identify the step at which things start to go wrong?

2. Write a function program `[L,U,p] = gepp(A)` to find a unit lower triangular matrix `L`, an upper triangular matrix `U` and a column vector `p` satisfying `A(p,:) = LU` via Gaussian Elimination with Partial Pivoting (GEPP) When doing so please note the following:

   (a) Your code should make only the most minimal changes to the `genp` code written in Lab 1. In particular it should retain all important features of `genp` that ensure efficiency.

   (b) Check the outputs by tabulating the values of $\|A(p,:) - LU\|_2$ for several different random choices of $A$.

   (c) The built in Matlab function program `lu` performs GEPP and GECP to find $LU$ decompositions of appropriately permuted matrices, also giving the permutation matrices used in each case. Type `help lu` for details. Compare the output of your `gepp` code with the corresponding outputs of the `lu` program by tabulating the difference between the corresponding outputs in the 2-norm. *The comparision should be performed for several different randomly generated matrices (use* `randn` *command for this).*

3. Write a function program `x = geppsolve(A,b)` to solve a system $Ax = b$ via GEPP. Your program should call the program `[L,U,p] = gepp(A)` and the programs written in Lab 1 for solving upper and lower triangular systems. Compare your answers with that of the MATLAB command $A\backslash b$ (which uses GEPP to solve the system) for several different choices of $A$ and $b$ that are randomly generated by using the `randn` command.

4. Given $A \in \mathbb{R}^{n \times n}$, write a function program `d = mydet(A)` that uses an *efficient* version of LU factorization to compute the determinant of $A$ in $O(n^3)$ flops.

5. Write a function program `G = mychol(A)` that executes the *inner product form* of the Cholesky Decomposition for finding the Cholesky factor of an $n \times n$ positive definite matrix $A$ in $\frac{n^3}{3} + O(n^2)$.

   Record the norm difference of your output with that of the built in Matlab function program `chol` for several different choices of randomly chosen positive definite matrices generated via the commands `X = randn(n); A = X'*X;`

   Write a report of your work that records all the numerical experiments.