Lab Session 5

MA423: Matrix Computations July-November 2021 S. Bora

Instructions: This lab assignment is based on Lecture 23. Please listen to it before starting your work. The report for question 4 is to be written clearly in a pdf file.

1. Write a MATLAB function program $[\texttt{Q},\texttt{R}] = \texttt{cgs}(\texttt{V})$ to orthonormalize the columns of an $n \times m$ matrix V, $(n \geq m)$ by the Classical Gram Schmidt procedure so that Q is an isometry satisfying

$$\begin{aligned} \text{span}\{Q(:,1)\} &= \text{span}\{V(:,1)\} \\ \text{span}\{Q(:,1), Q(:,2)\} &= \text{span}\{V(:,1), V(:,2)\}, \\ &\vdots \\ \text{span}\{Q(:,1), Q(:,2), \ldots, Q(:,m)\} &= \text{span}\{V(:,1), V(:,2), \ldots, V(:,m)\} \end{aligned}$$

   and R is an upper triangular matrix such that $R(i,j) = \langle V(:,j), Q(:,i)\rangle$.

2. A slight modification of the above program leads to the Modified Gram Schmidt procedure for orthonormalizing the columns of V. Perform this modification to obtain another function program $[\texttt{Q},\texttt{R}] = \texttt{mgs}(\texttt{V})$.

3. Write a function program $[\texttt{Q},\texttt{R}] = \texttt{cgsrep}(\texttt{V})$ that performs Classified Gram Schmidt with reorthogonalization by making appropriate changes to your function program `cgs`.

   Take care to replace *for loops* by matrix-vector multiplications as far as possible in each of the above programs.

4. Given $kappa(\geq 1) \in \mathbb{R}$, a random $n \times n$ positive definite matrix A with $\kappa(A) = kappa$, may be formed by generating a random orthogonal matrix U and a diagonal matrix D with $D(i,i) = kappa^{(i-1)/(n-1)}, i = 1 : n$, and setting $A = U * D * U'$. The following command will generate U:

   $>> \texttt{X} = \texttt{randn}(\texttt{n},\texttt{n}); [\texttt{U},\texttt{R}] = \texttt{qr}(\texttt{X});$

   Use the above information to write a function program $\texttt{A} = \texttt{condmat}(\texttt{n},\texttt{kappa})$ that generates a random $n \times n$ positive definite matrix A with given condition number *kappa*. Use the program to generate a random 50×50 positive definite matrix A with given condition number 10^5. Store the matrix in a workspace file `Q4-1.mat` and run the `cgs`, `mgs` and `cgsrep` codes written above on the matrix. Finally also find a condensed QR decomposition of the matrix using the appropriate MATLAB command. (Find out what this is!)

   Generate two more 50×50 positive definite matrices with condition numbers 10^7 and 10^{11} and store them in workspace files `Q4-2.mat` and `Q4-3.mat` respectively. Repeat the above process on those matrices also.

   Record the departure from orthonormality for the isometries obtained from each of the processes and record them in a properly labeled table that also mentions the condition numbers.