MA 473: Computational Finance

Lab – 1

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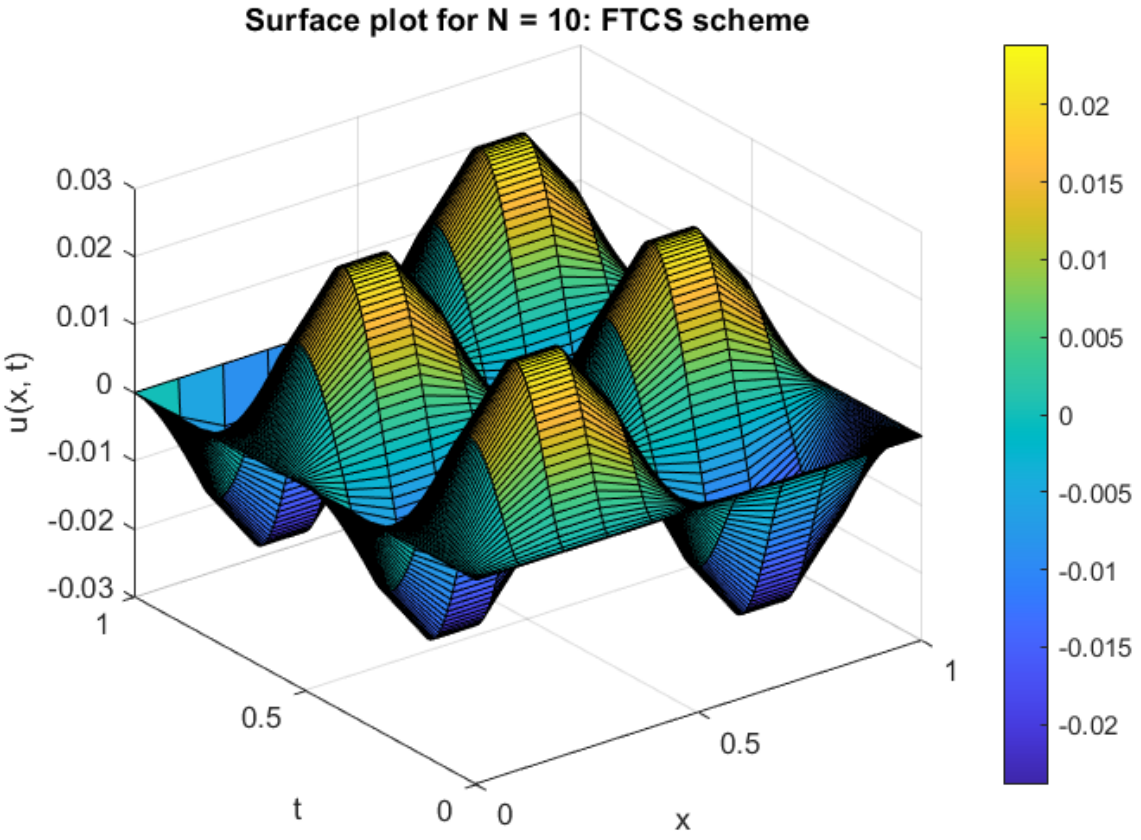
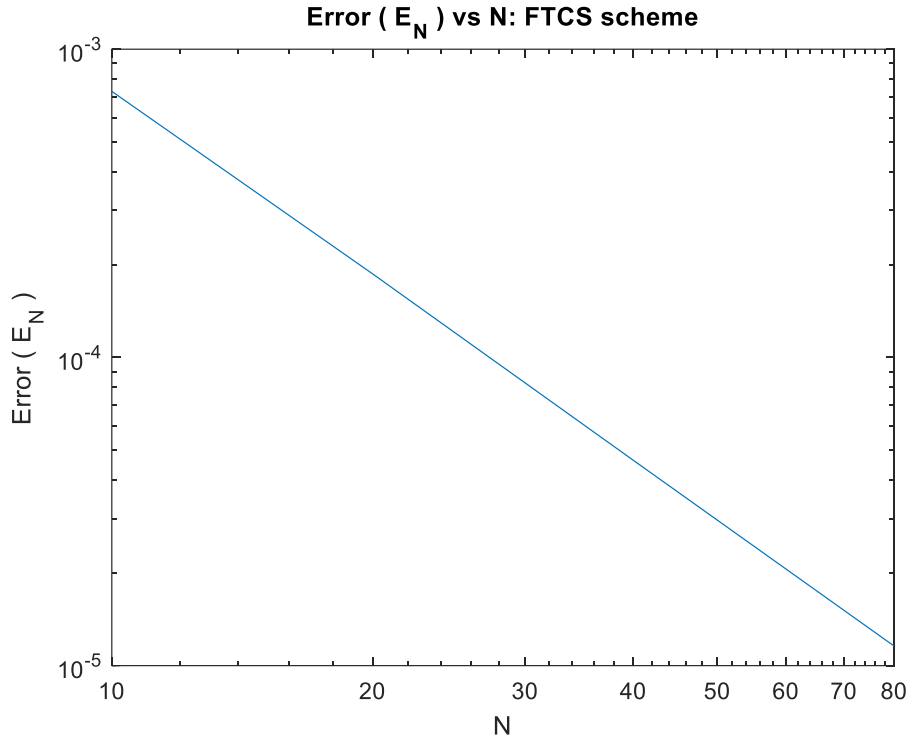
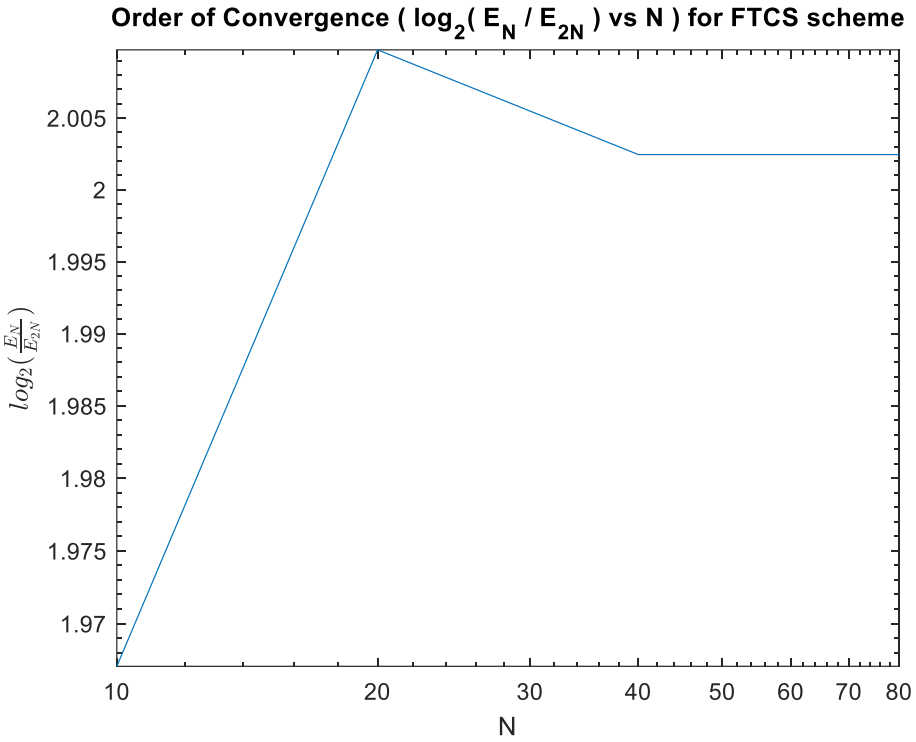
1 QUESTION - 1:

i. The output for the FTCS scheme are:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.00072986	1.9671
2	20	0.00018668	2.0098
3	40	4.6354e-05	2.0024
4	80	1.1569e-05	2.0024

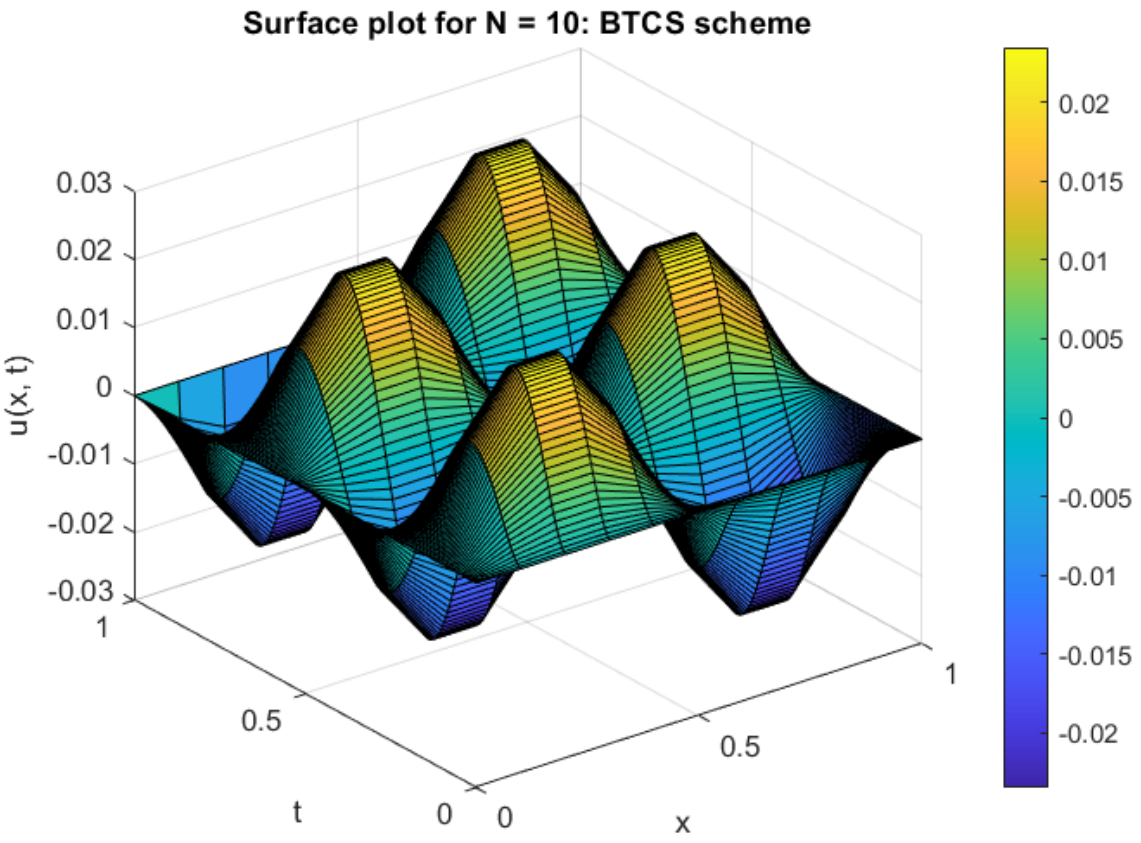


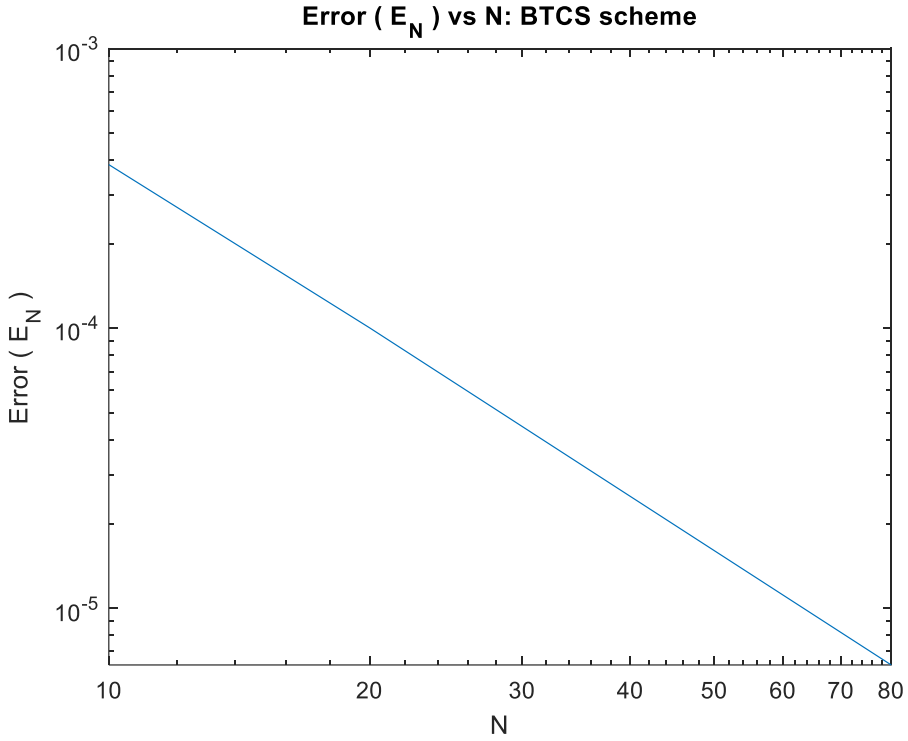


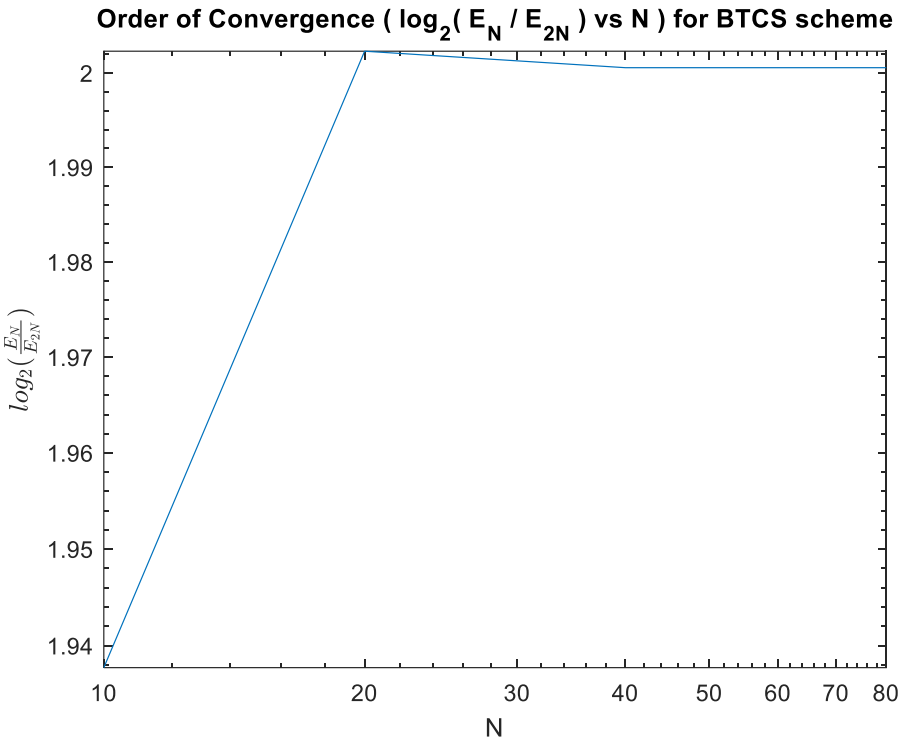


ii. The output for the BTCS scheme are:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.00038459	1.9377
2	20	0.00010039	2.0023
3	40	2.5056e-05	2.0006
4	80	6.2616e-06	2.0006

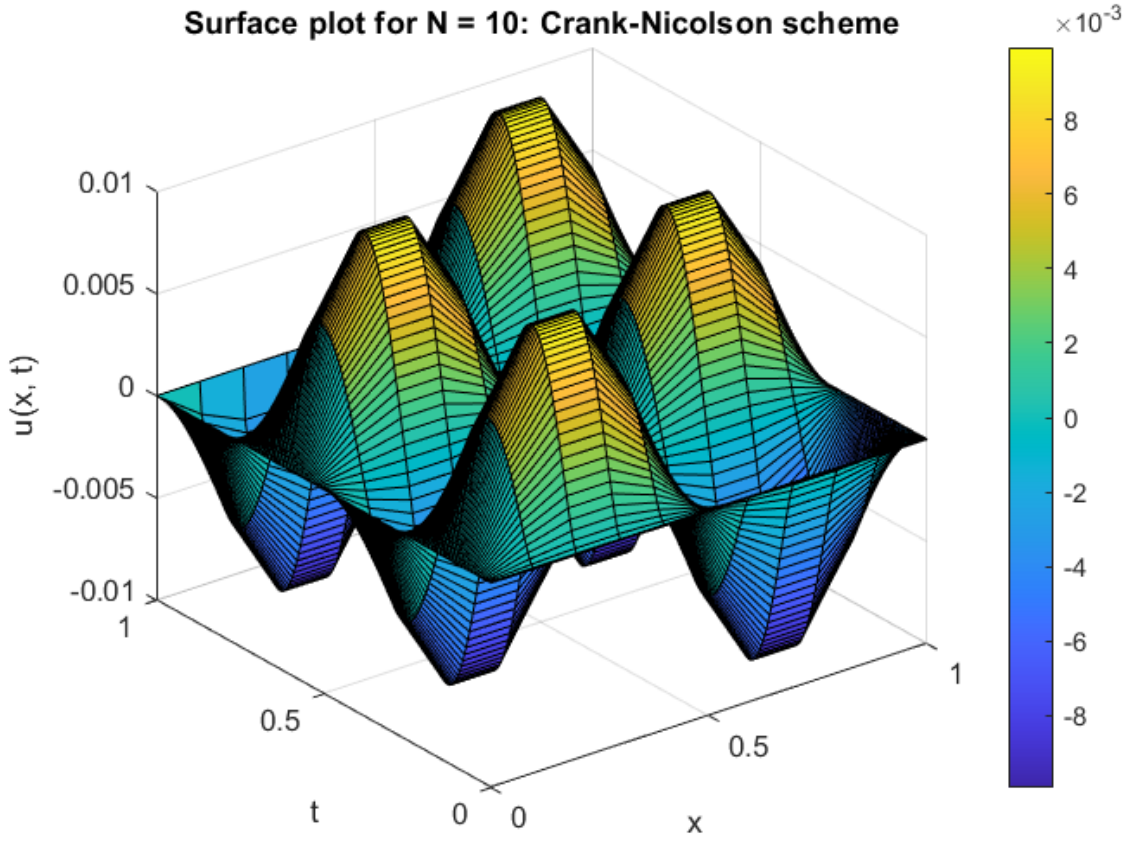

Surface plot for N = 10: BTCS scheme
u(x, t)
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1
0.5
t
0
0
0.5
1
x
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02

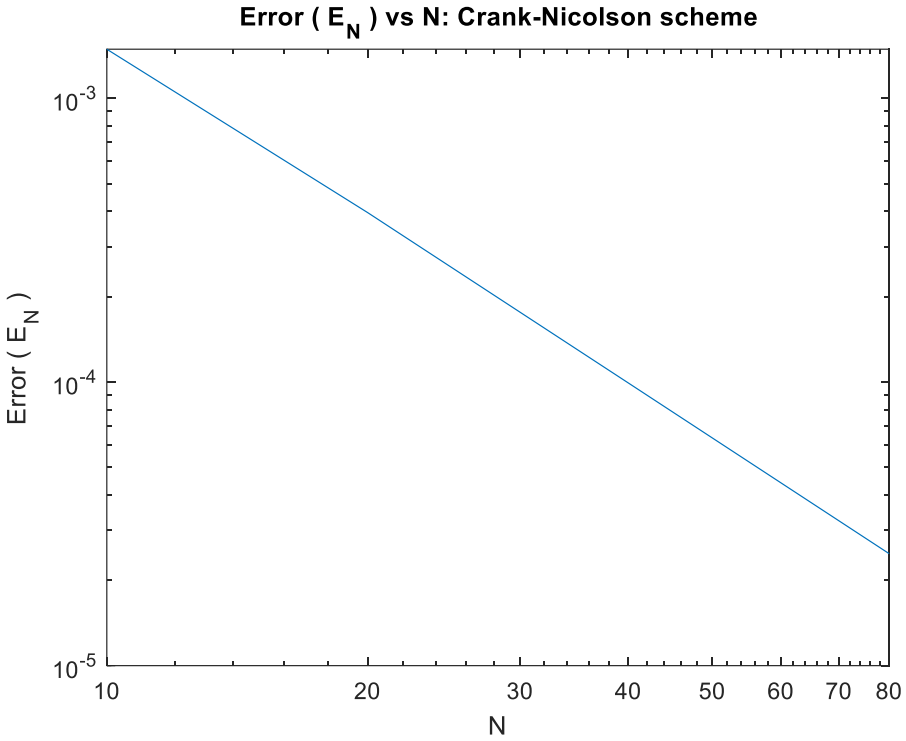

Error (E_N) vs N: BTCS scheme
Error (E_N)
10^-3
10^-4
10^-5
10
20
30
40
50
60
70
80
N

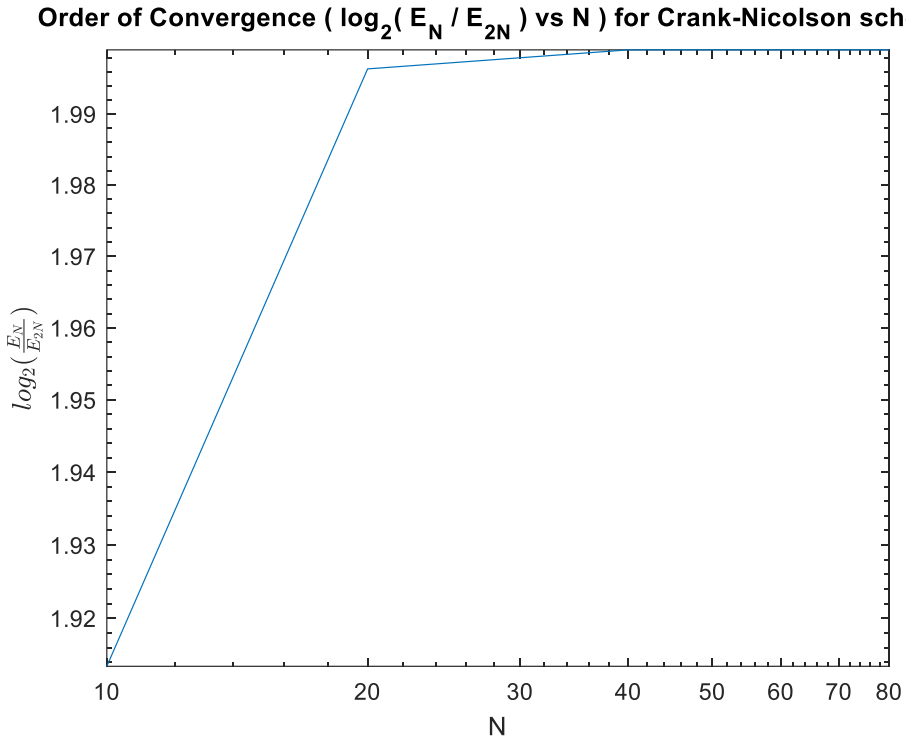


iii. The output for the Crank-Nicolson scheme are:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.0014906	1.9135
2	20	0.0003957	1.9964
3	40	9.9172e-05	1.9991
4	80	2.4809e-05	1.9991







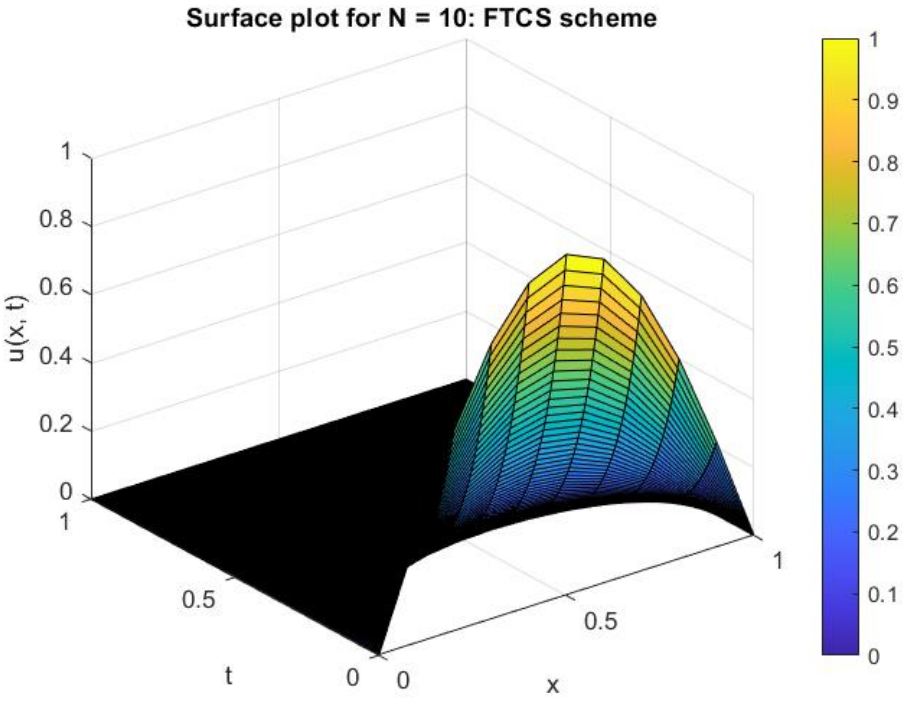
As we can observe that in all the 3 schemes the order of convergence remains almost constant around 2. And this is within the theoretical expectation since the N and M are chosen in such a way that $\lambda <= 0.5$.

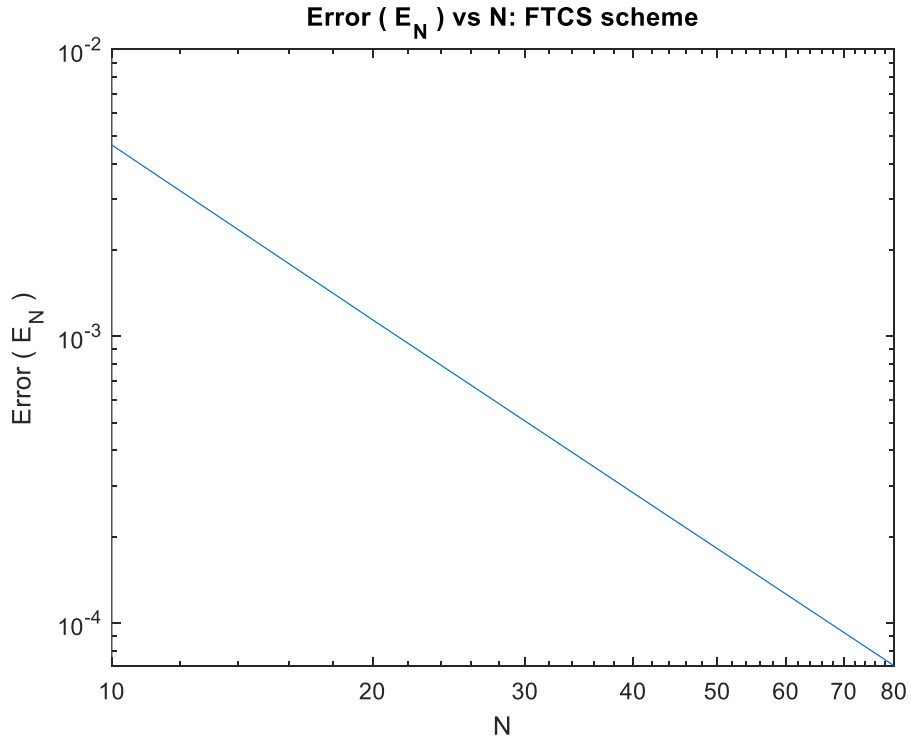
2 QUESTION - 2:

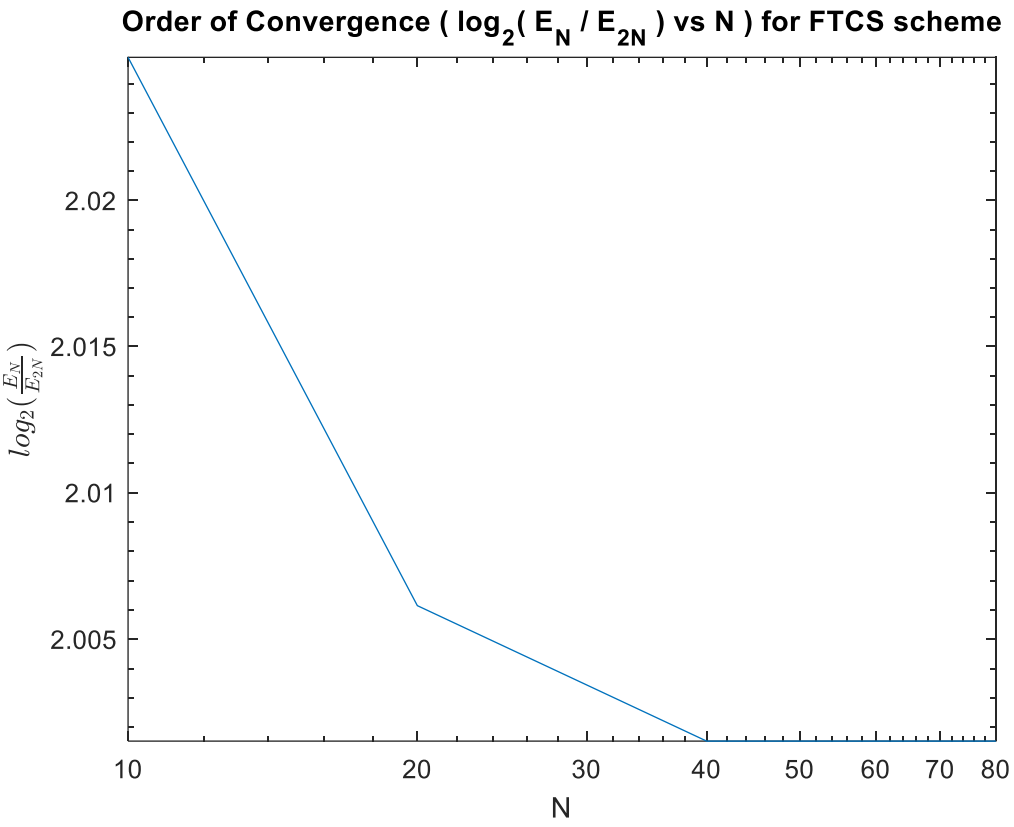
a) The outputs are:

a. For the FTCS scheme:

SI No	N	Max error (En)	Order of Convergence
1	10	0.0046439	2.0249
2	20	0.0011411	2.0061
3	40	0.00028406	2.0015
4	80	7.094e-05	2.0015

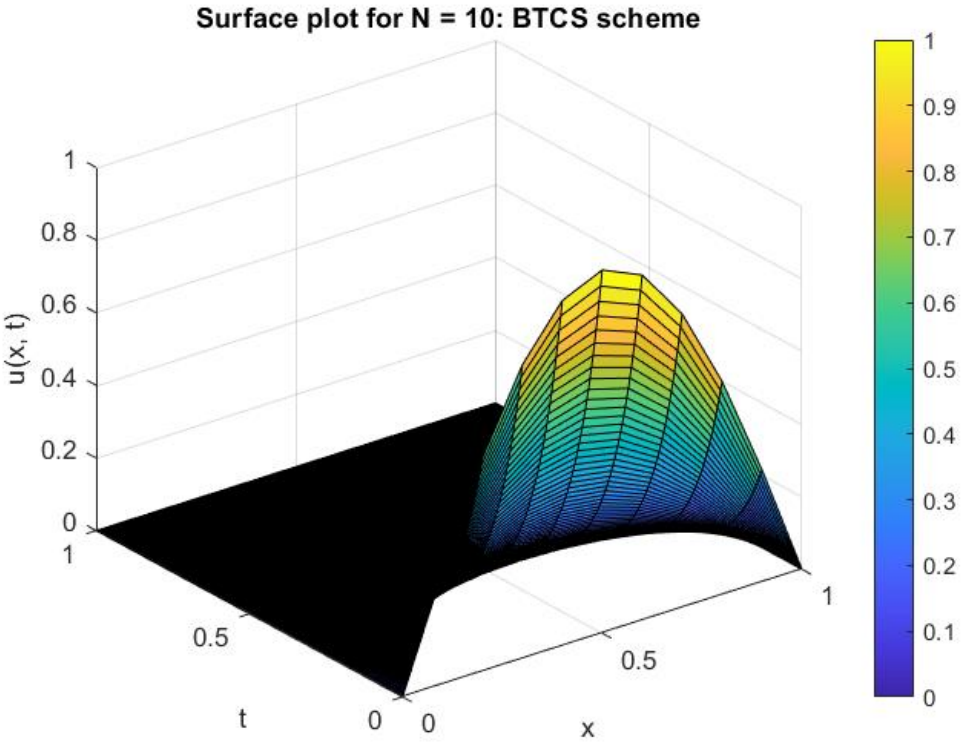


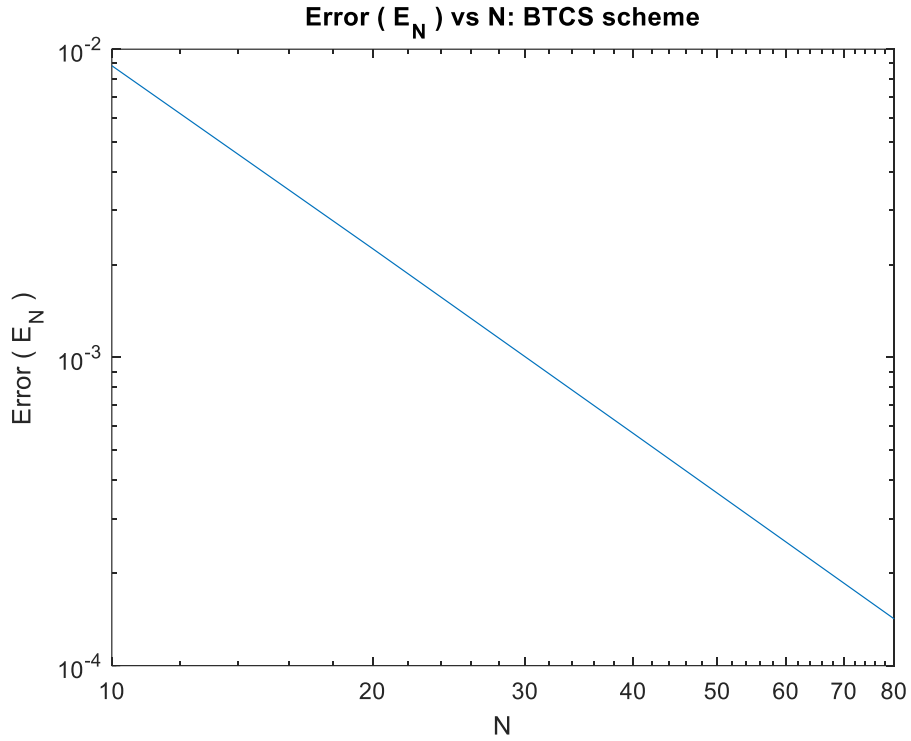


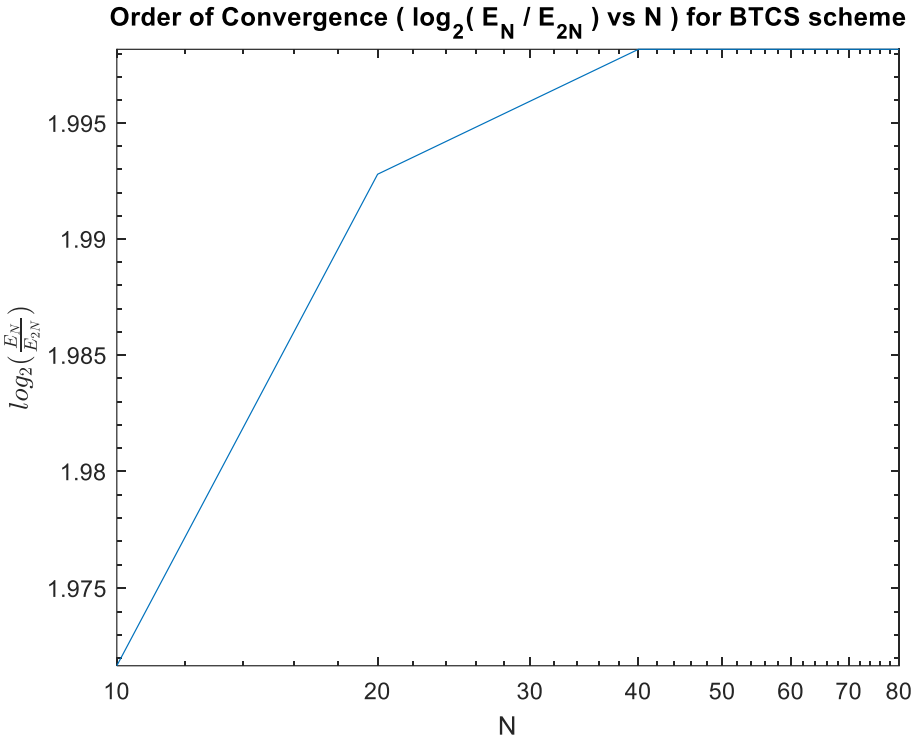


b. For the BTCS scheme:

SI No	N	Max error (En)	Order of Convergence
1	10	0.0088415	1.9717
2	20	0.0022542	1.9928
3	40	0.00056637	1.9982
4	80	0.00014177	1.9982

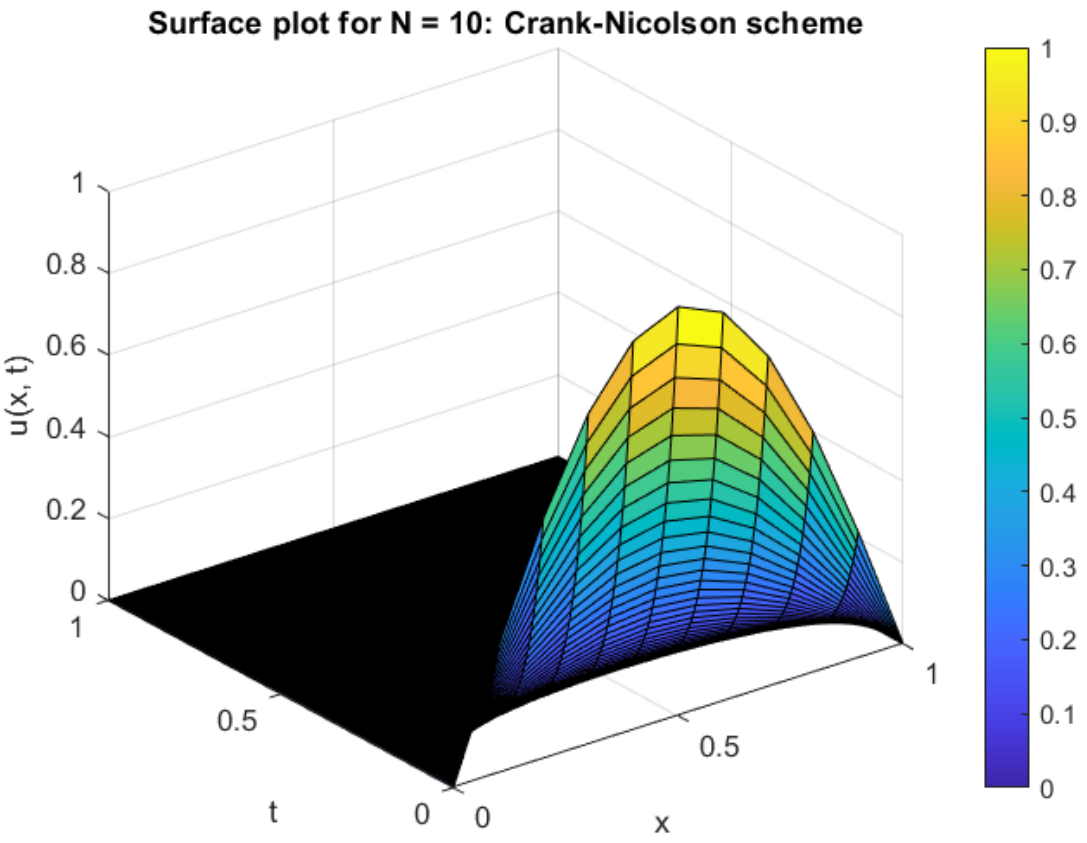


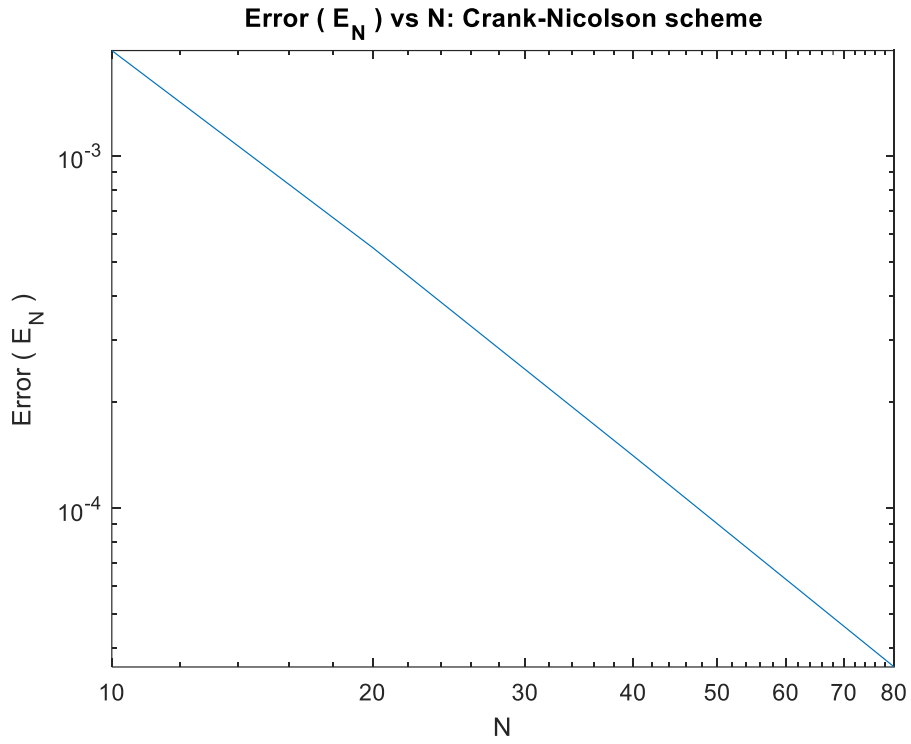


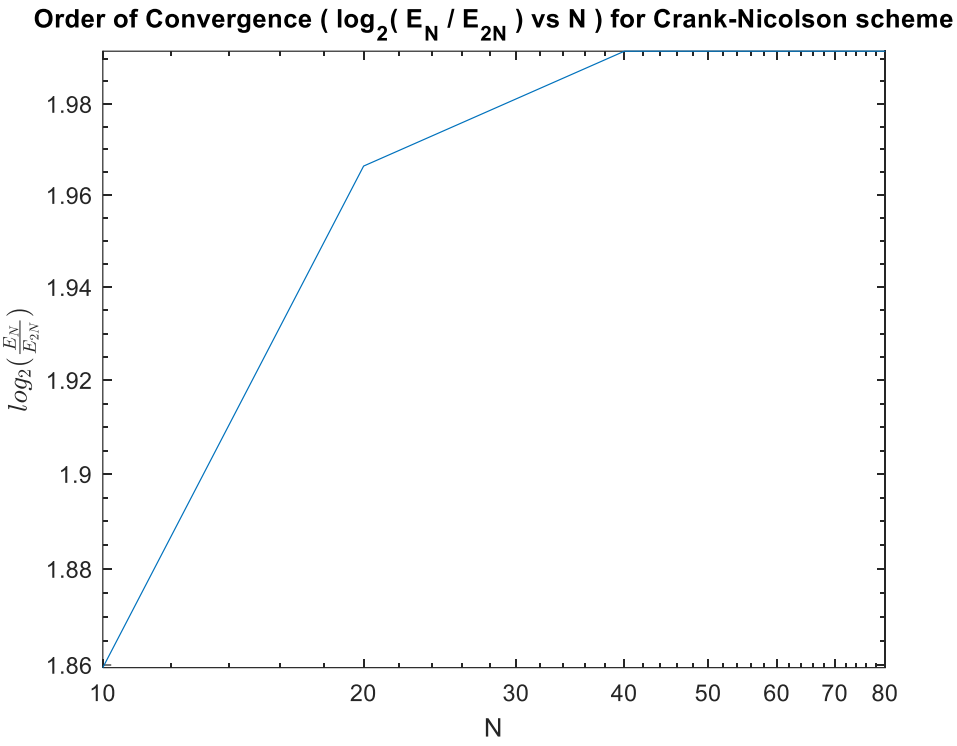


c. For the Crank-Nicolson scheme:

SI No	N	Max error (En)	Order of Convergence
1	10	0.0088415	1.9717
2	20	0.0022542	1.9928
3	40	0.00056637	1.9982
4	80	0.00014177	1.9982


Surface plot for N = 10: Crank-Nicolson scheme
u(x, t)
1
0.8
0.6
0.4
0.2
0
1
0.5
t
0
0
0.5
x
1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

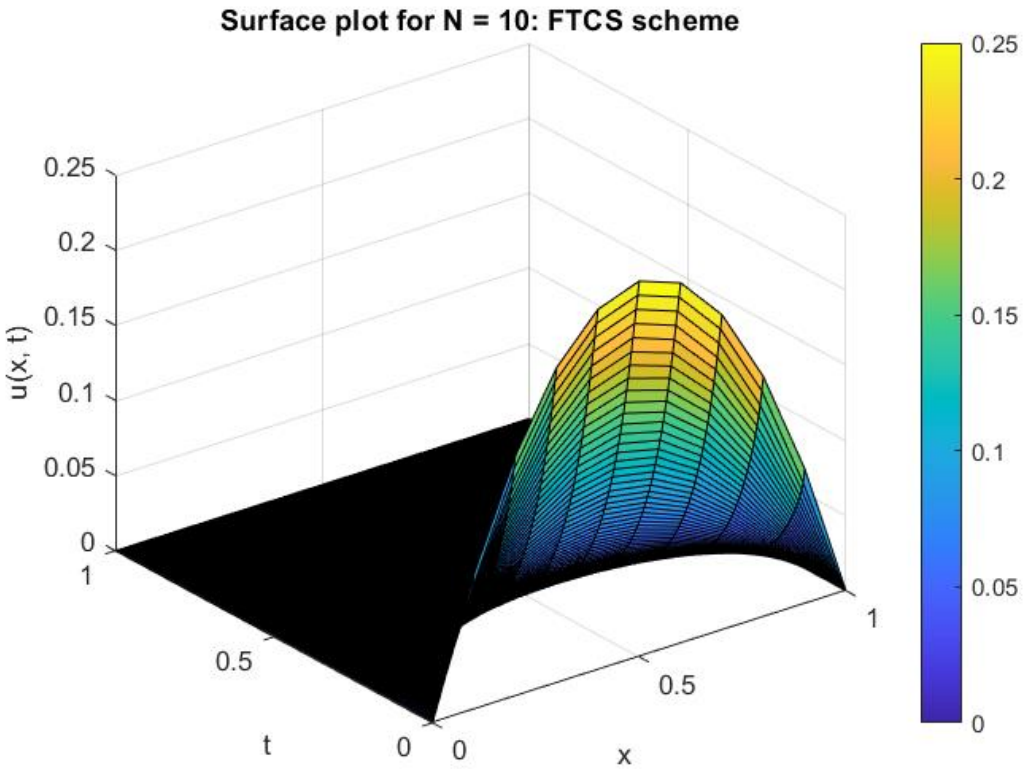

Error (E_N) vs N: Crank-Nicolson scheme
Error (E_N)
10^-3
10^-4
10
20
30
40
50
60
70
80
N

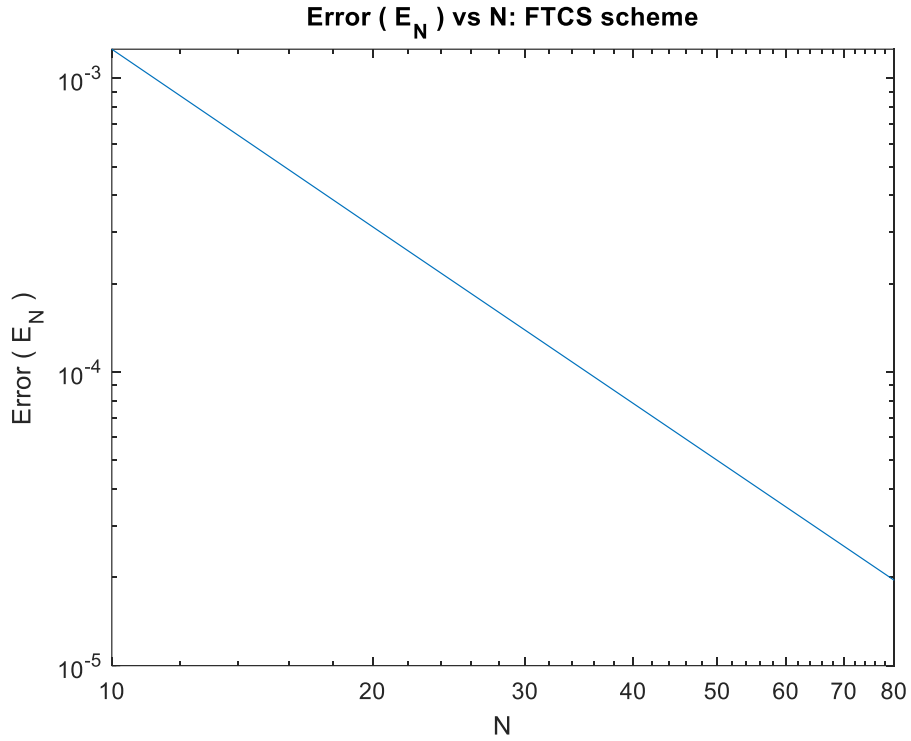


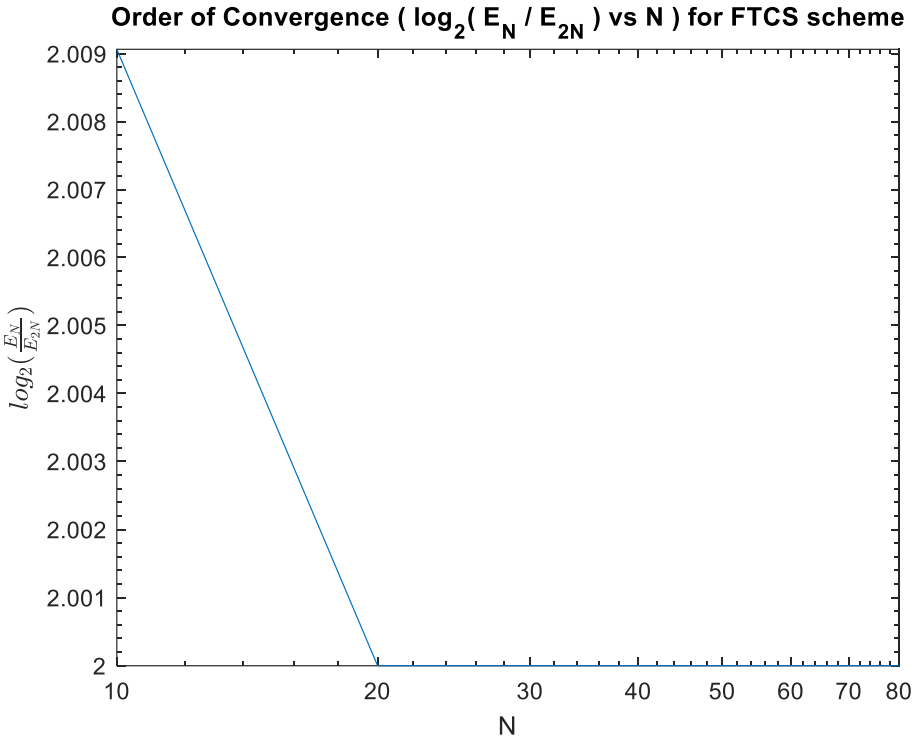
b) The outputs are:

a. For the FTCS scheme:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.0012579	2.0091
2	20	0.0003125	2
3	40	7.8125e-05	2
4	80	1.9531e-05	2

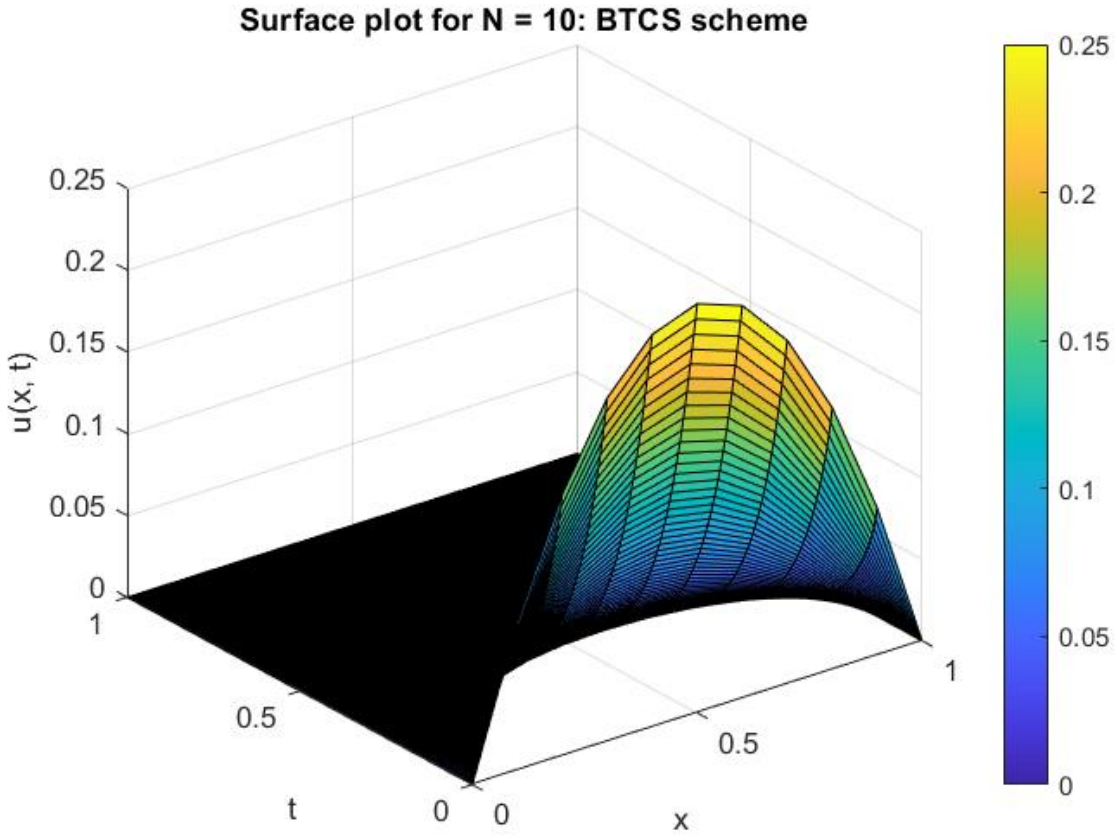


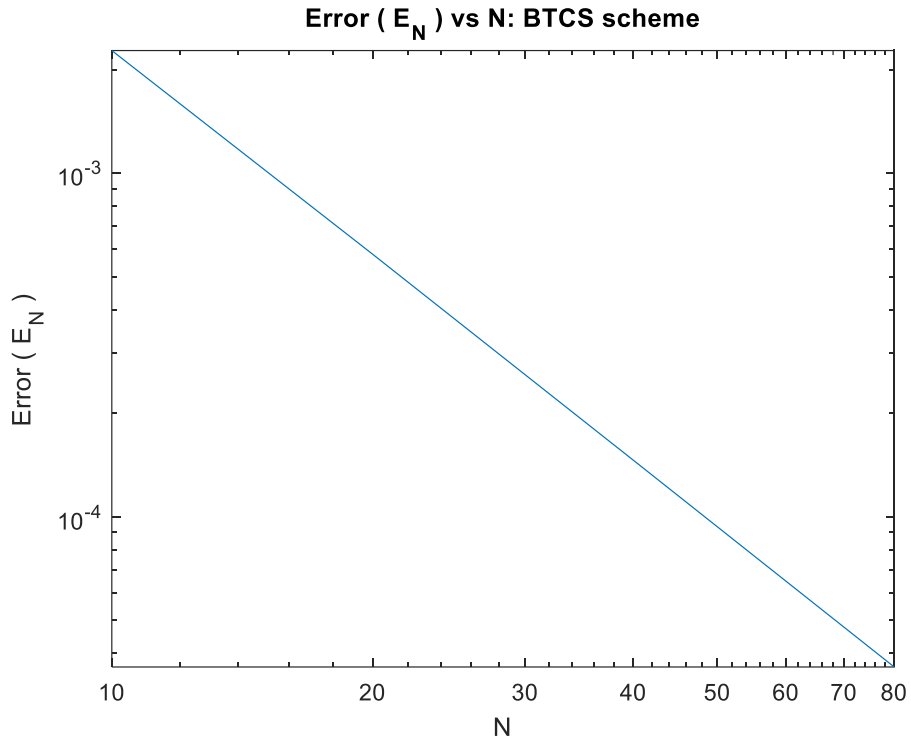


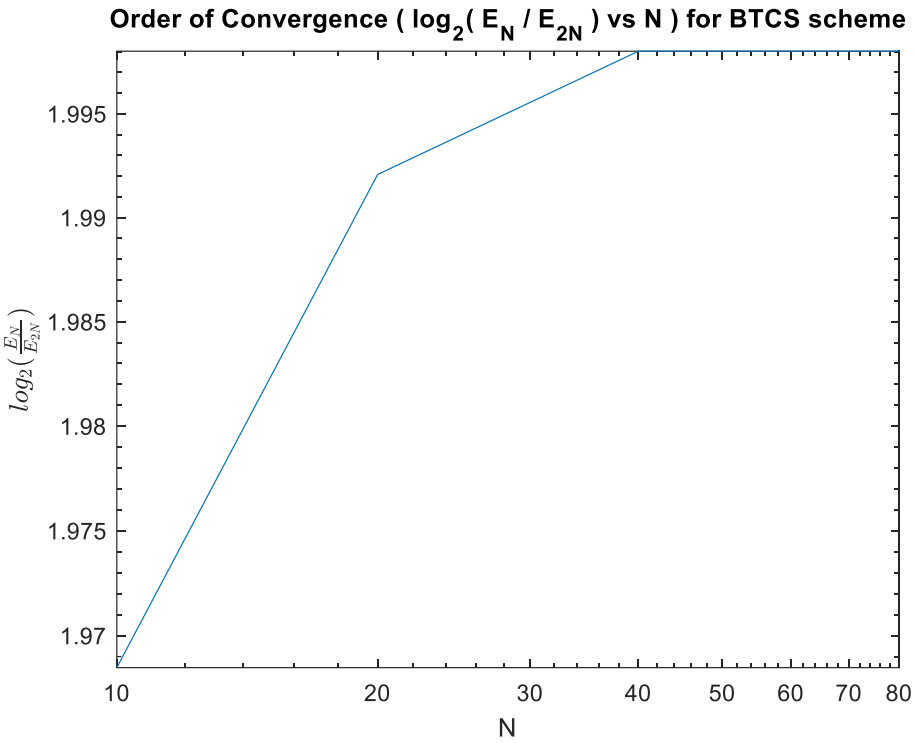


b. For the BTCS scheme:

SI No	N	Max error (En)	Order of Convergence
1	10	0.0022724	1.9685
2	20	0.00058066	1.9921
3	40	0.00014596	1.998
4	80	3.6541e-05	1.998

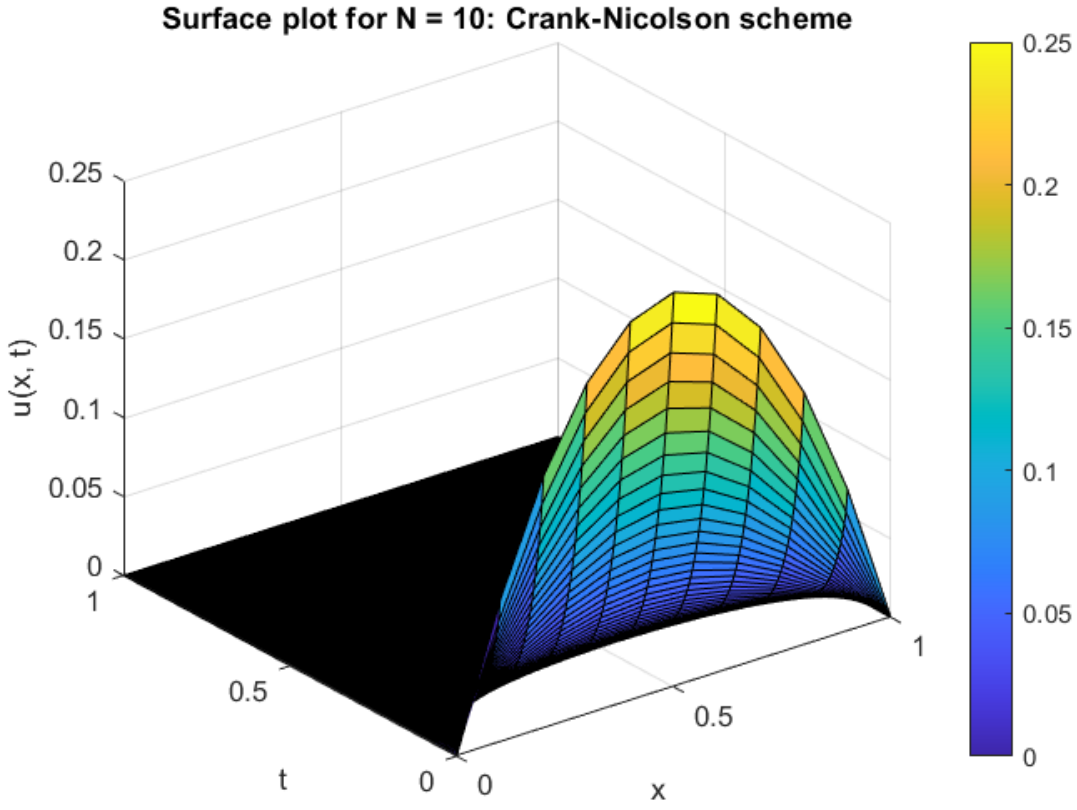

Surface plot for N = 10: BTCS scheme
u(x, t)
0.25
0.2
0.15
0.1
0.05
0
t
0
0.5
1
x
0
0.5
1
0.25
0.2
0.15
0.1
0.05
0

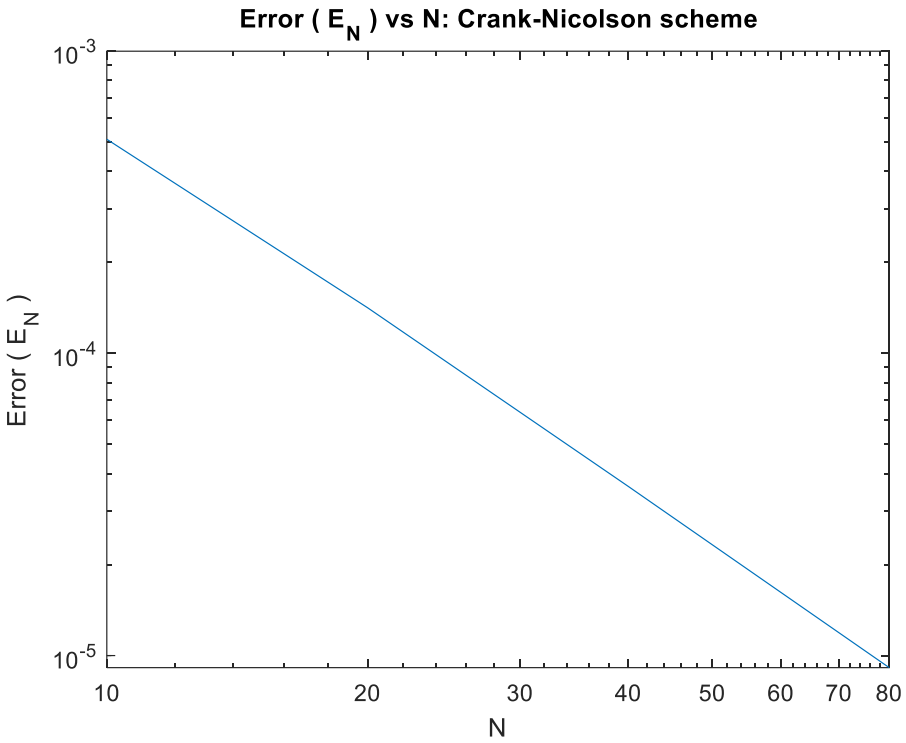

Error (E_N) vs N: BTCS scheme
Error (E_N)
10^-3
10^-4
10
20
30
40
50
60
70
80
N

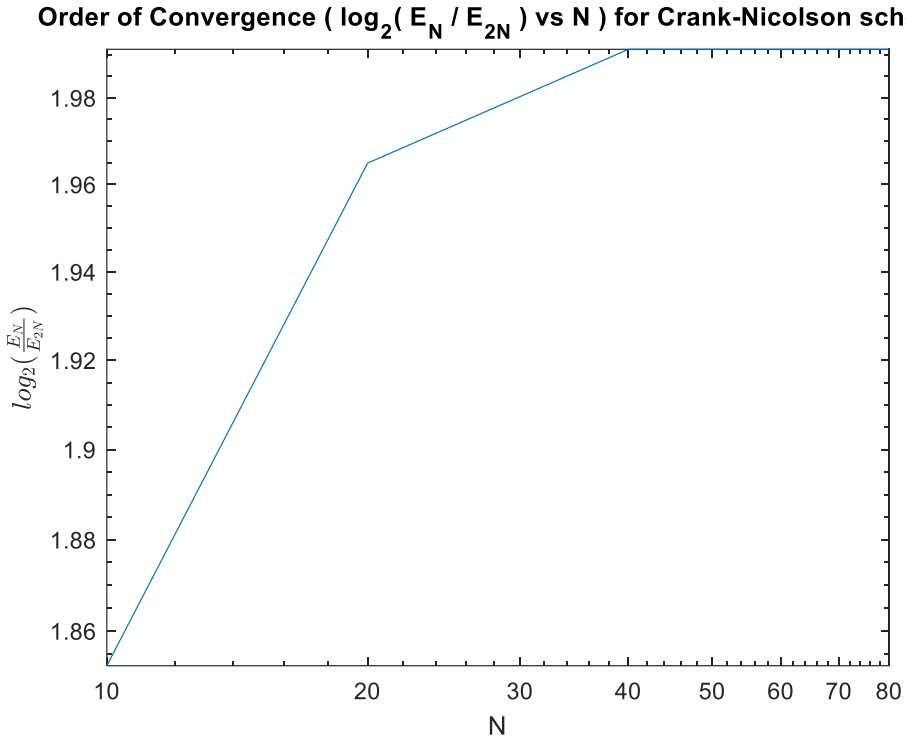


c. For the Crank-Nicolson scheme:

SI No	N	Max error (En)	Order of Convergence
1	10	0.00051125	1.8524
2	20	0.00014158	1.9649
3	40	3.6266e-05	1.9913
4	80	9.1212e-06	1.9913



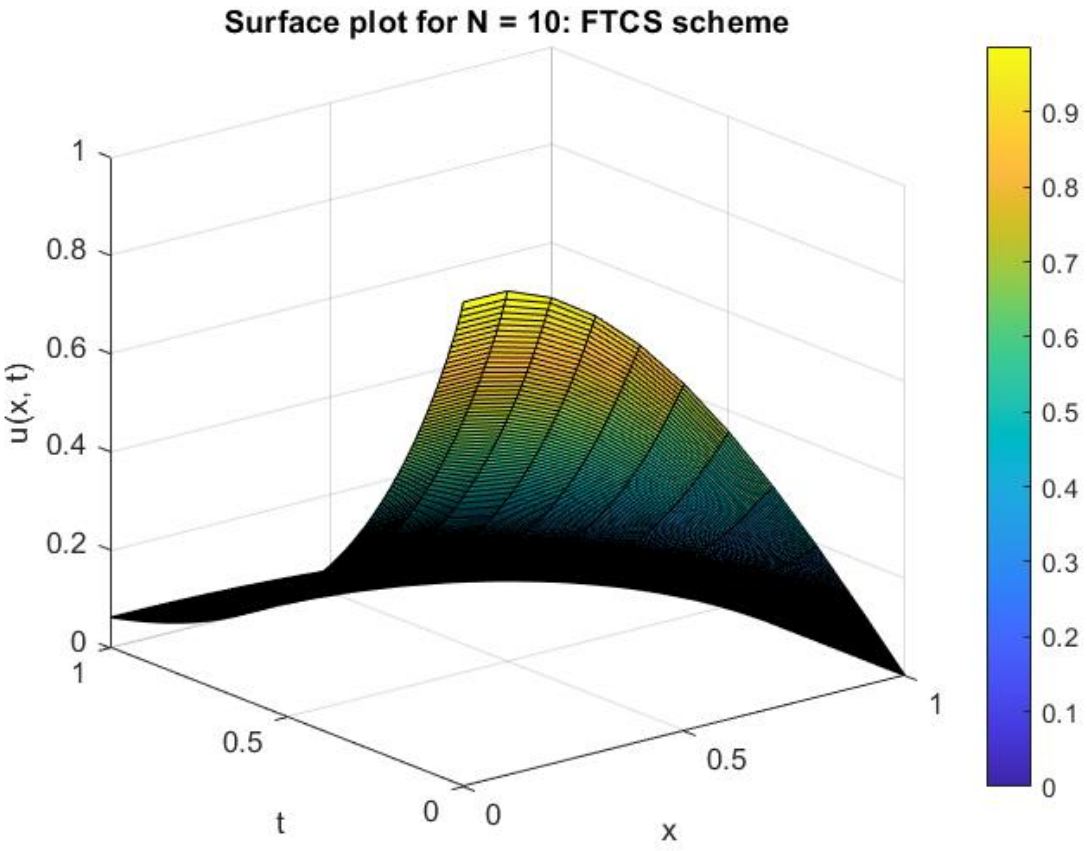


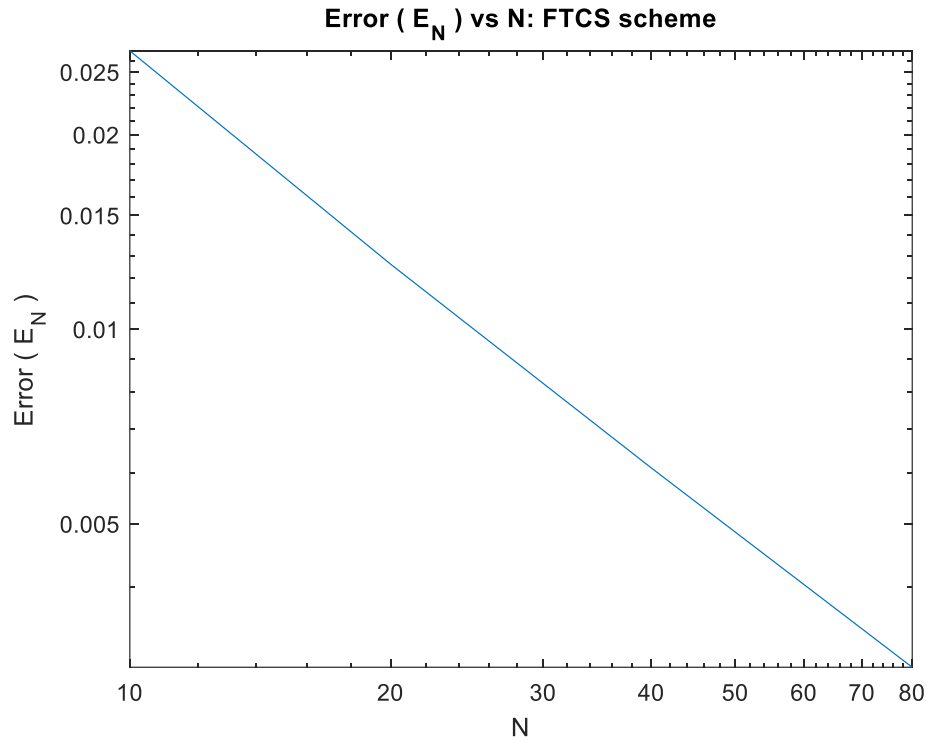


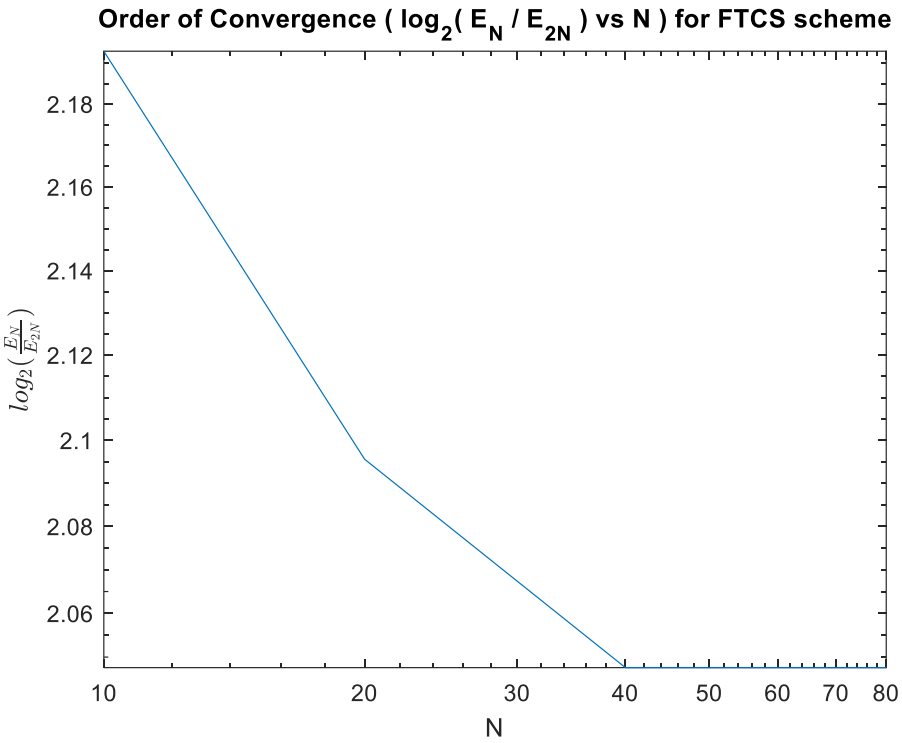
3 QUESTION - 3:

i. The output for the FTCS scheme are:

SI No	N	Max error (En)	Order of Convergence
1	10	0.026968	2.1929
2	20	0.012612	2.0956
3	40	0.0061005	2.0476
4	80	0.0030004	2.0476

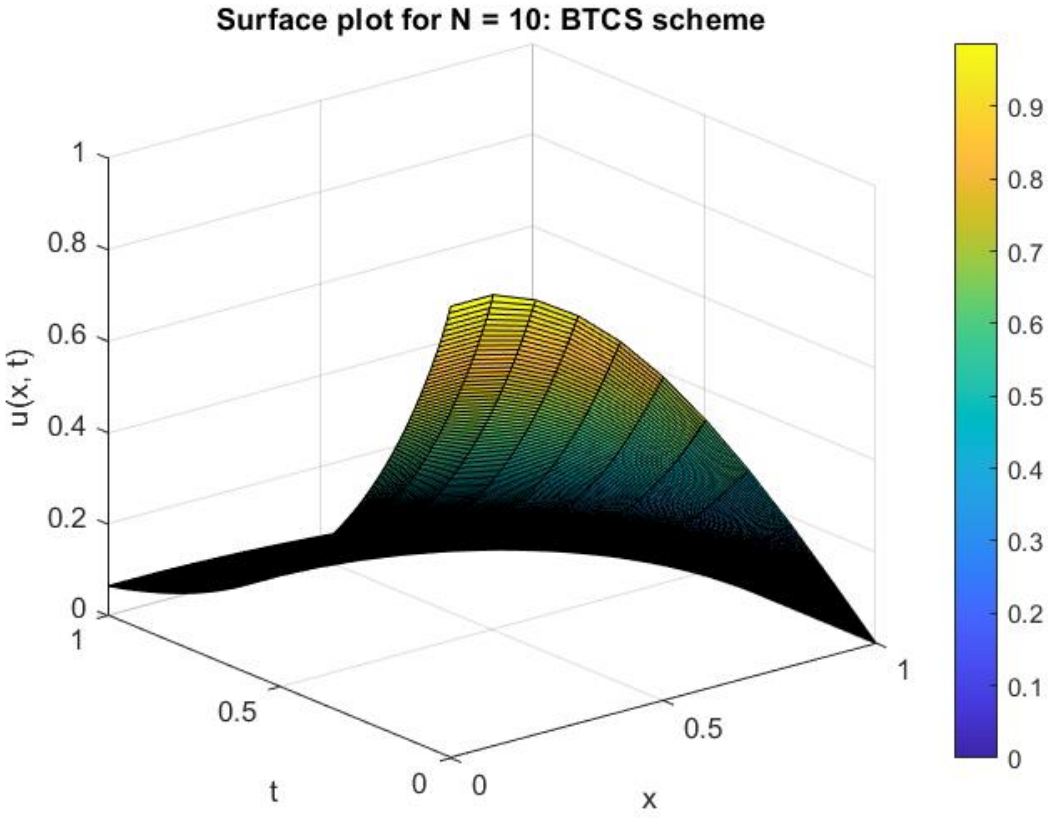

Surface plot for N = 10: FTCS scheme
u(x, t)
1
0.8
0.6
0.4
0.2
0
t
1
0.5
0
x
0
0.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

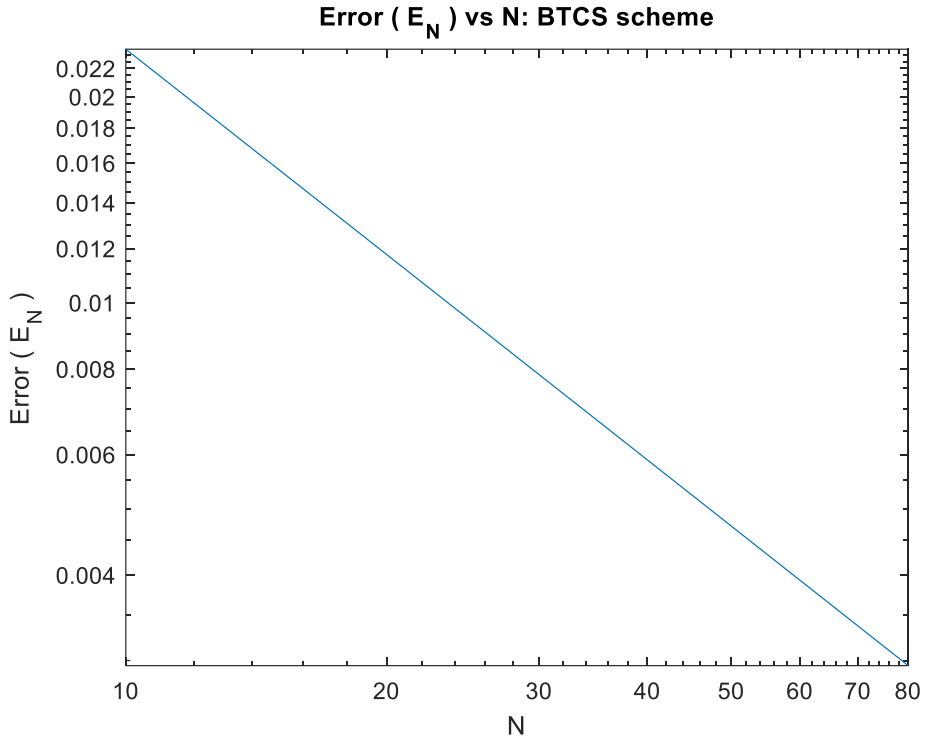

Error (E_N) vs N: FTCS scheme
Error (E_N)
0.025
0.02
0.015
0.01
0.005
10
20
30
40
50
60
70
80
N

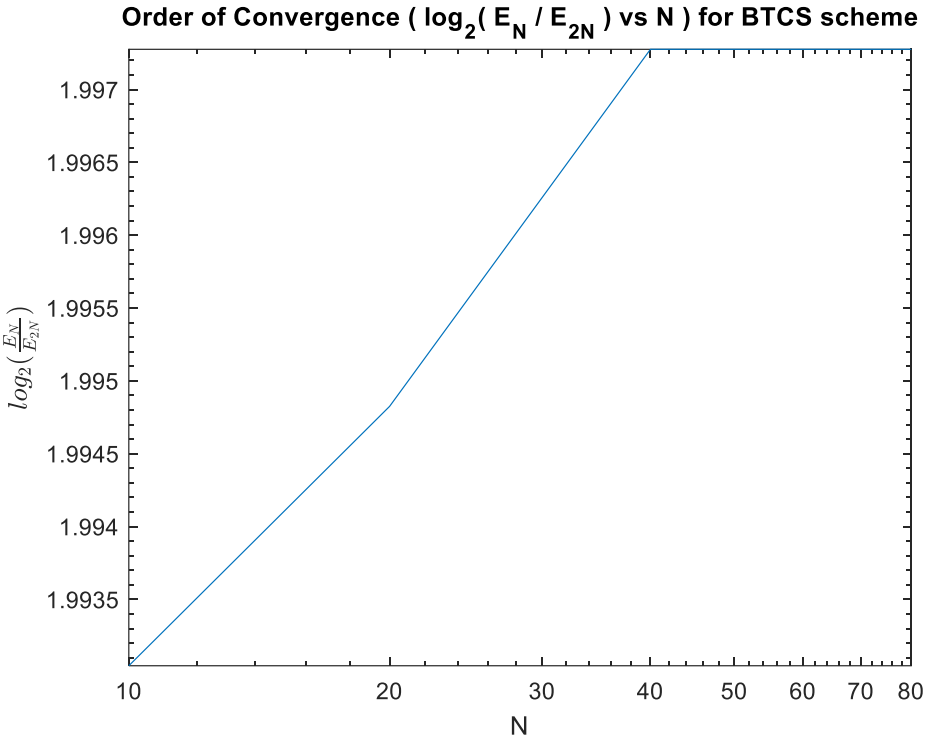


ii. The output for the BTCS scheme are:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.023468	1.993
2	20	0.011762	1.9948
3	40	0.0058916	1.9973
4	80	0.0029486	1.9973

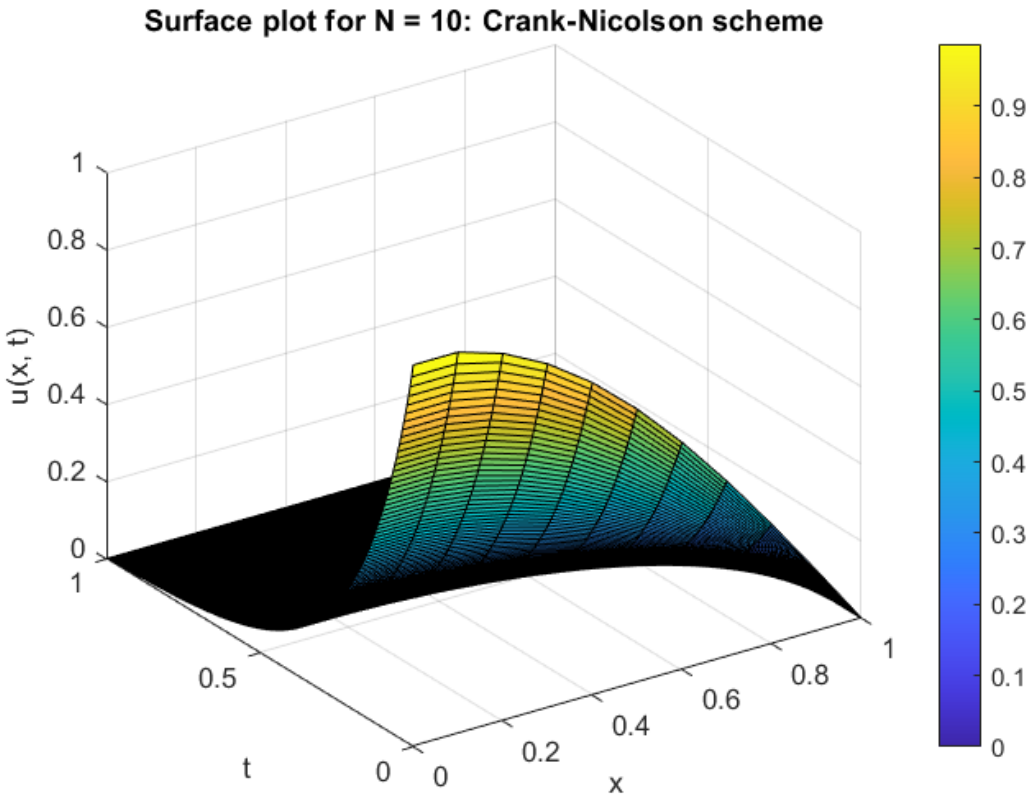


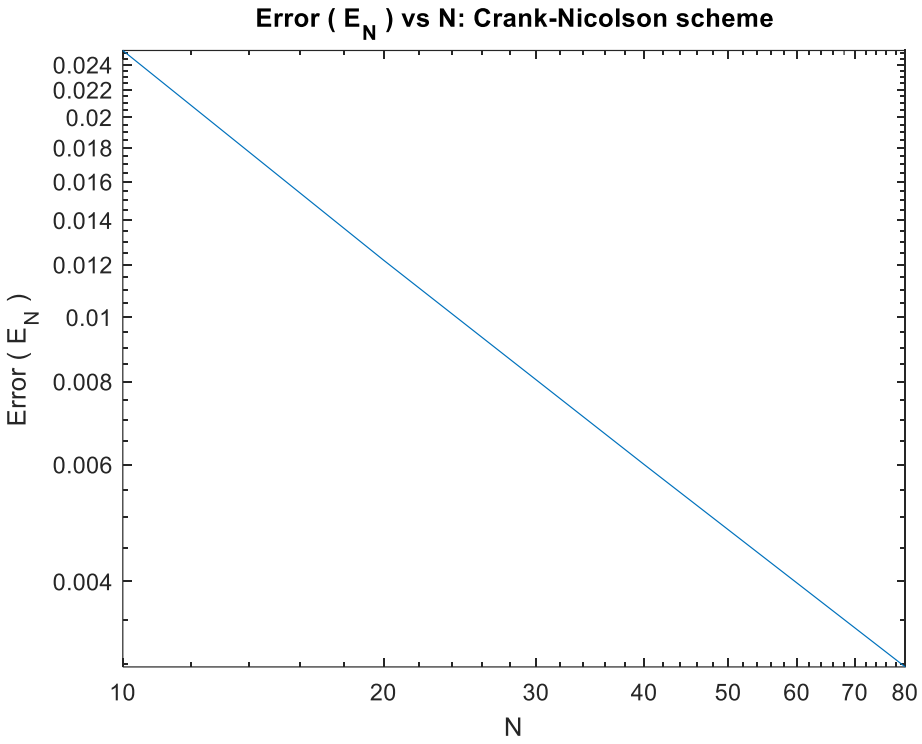


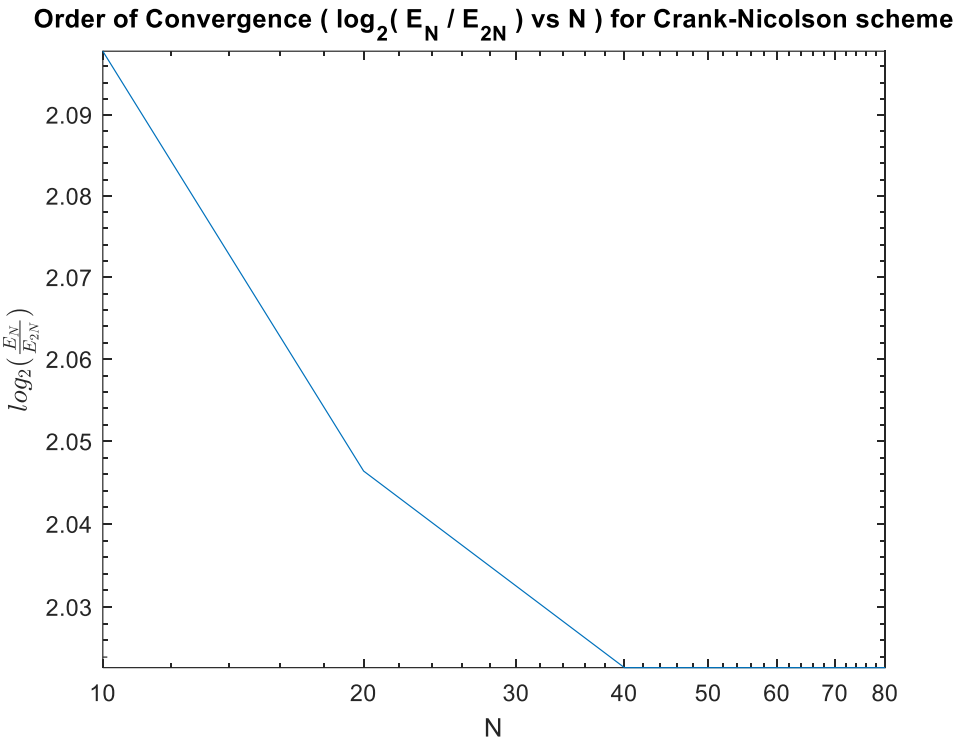


iii. The output for the Crank-Nicolson scheme are:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.025214	2.0979
2	20	0.012186	2.0464
3	40	0.005996	2.0227
4	80	0.0029745	2.0227


Surface plot for N = 10: Crank-Nicolson scheme
u(x, t)
1
0.8
0.6
0.4
0.2
0
1
0.5
t
0
0
0.2
0.4
0.6
0.8
1
x
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

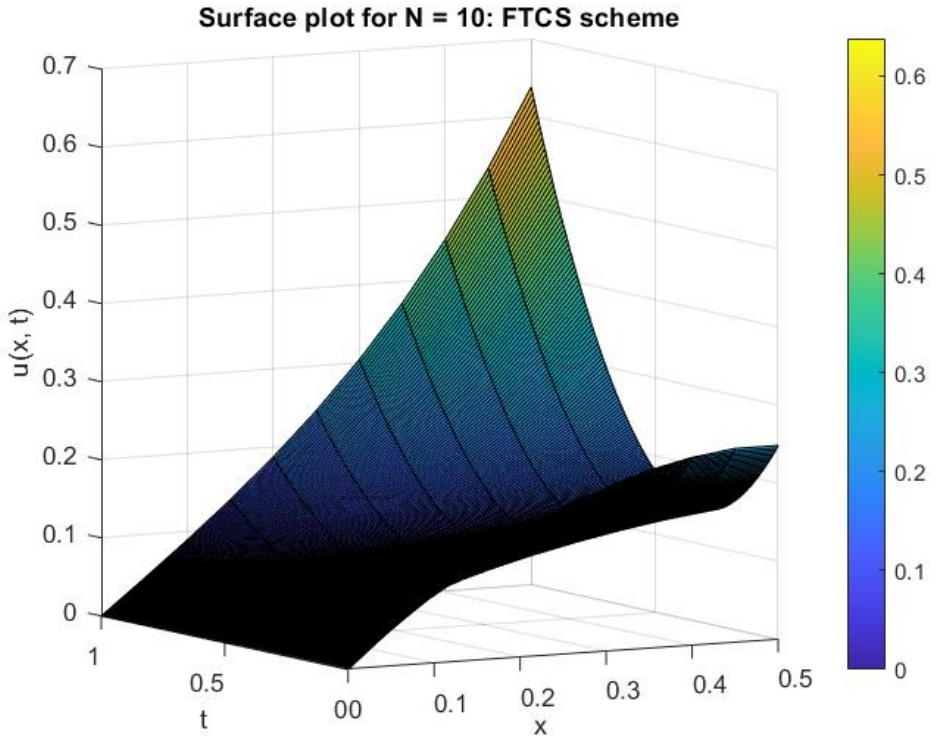

Error (E_N) vs N: Crank-Nicolson scheme
Error (E_N)
0.024
0.022
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
10
20
30
40
50
60
70
80
N

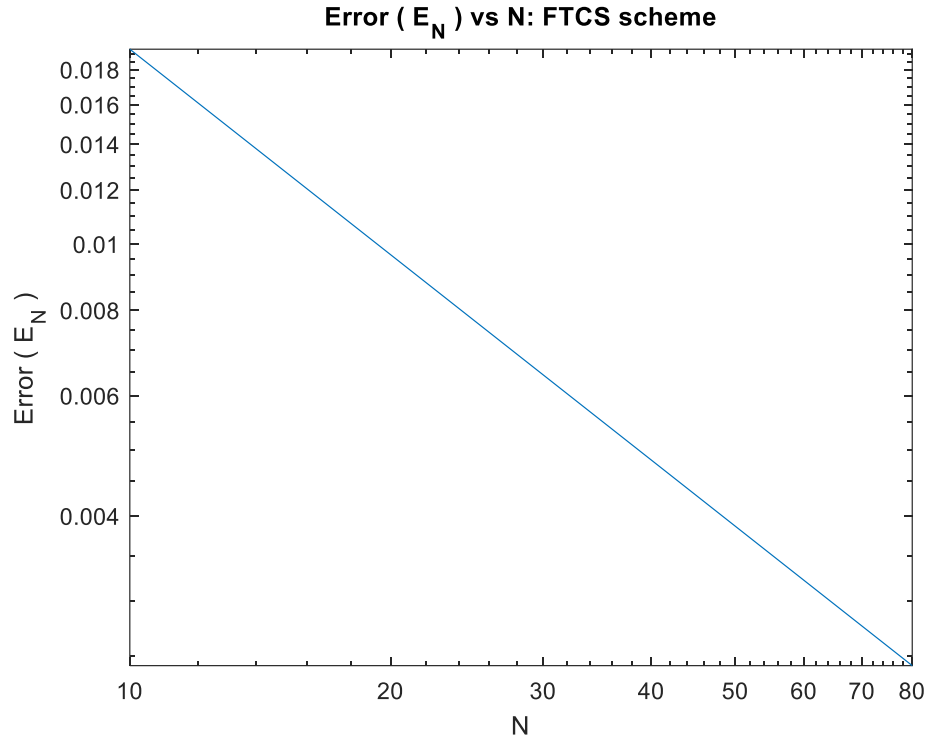


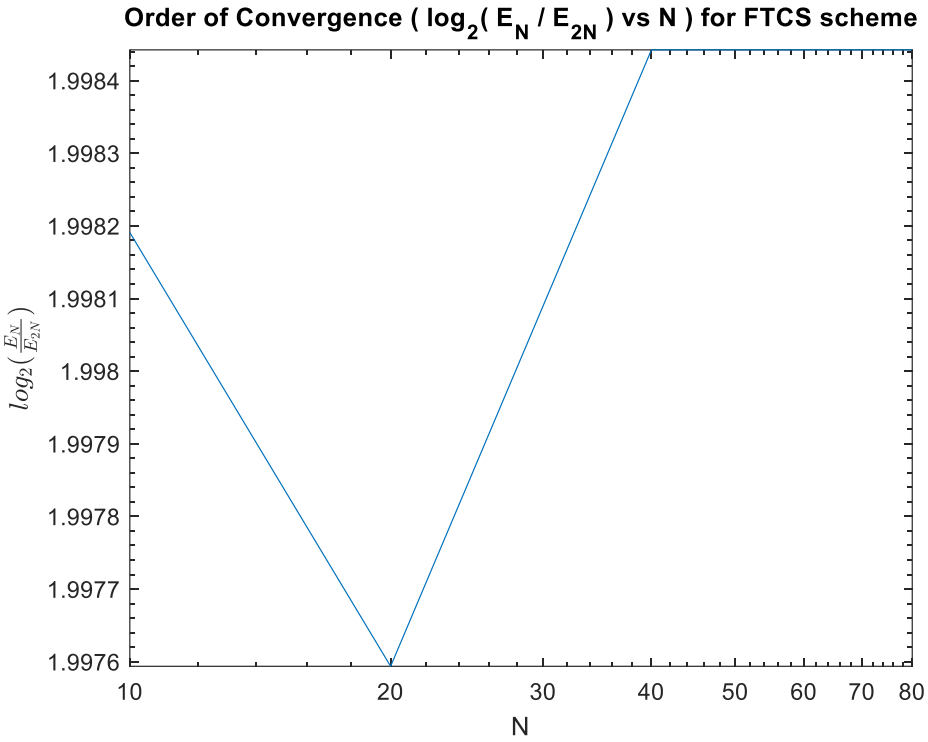
4 QUESTION - 4:

i. The output for the FTCS scheme are:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.019291	1.9982
2	20	0.0096513	1.9976
3	40	0.0048297	1.9984
4	80	0.0024161	1.9984

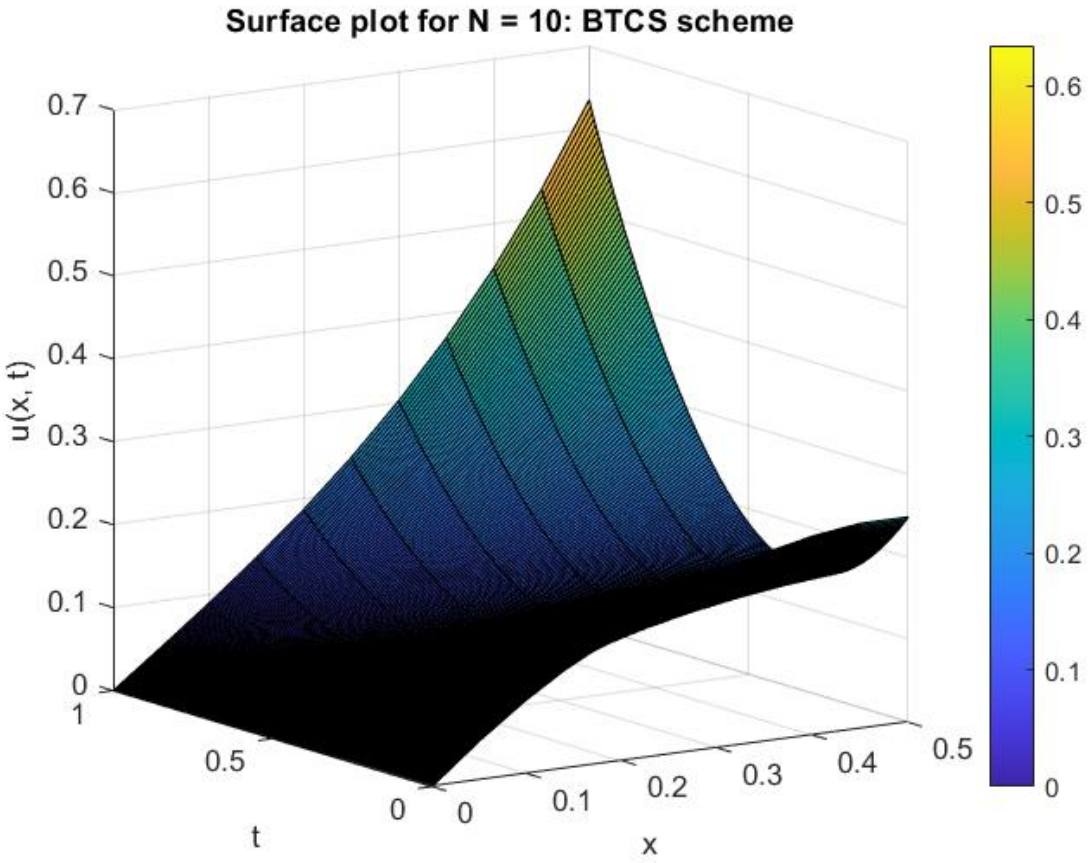


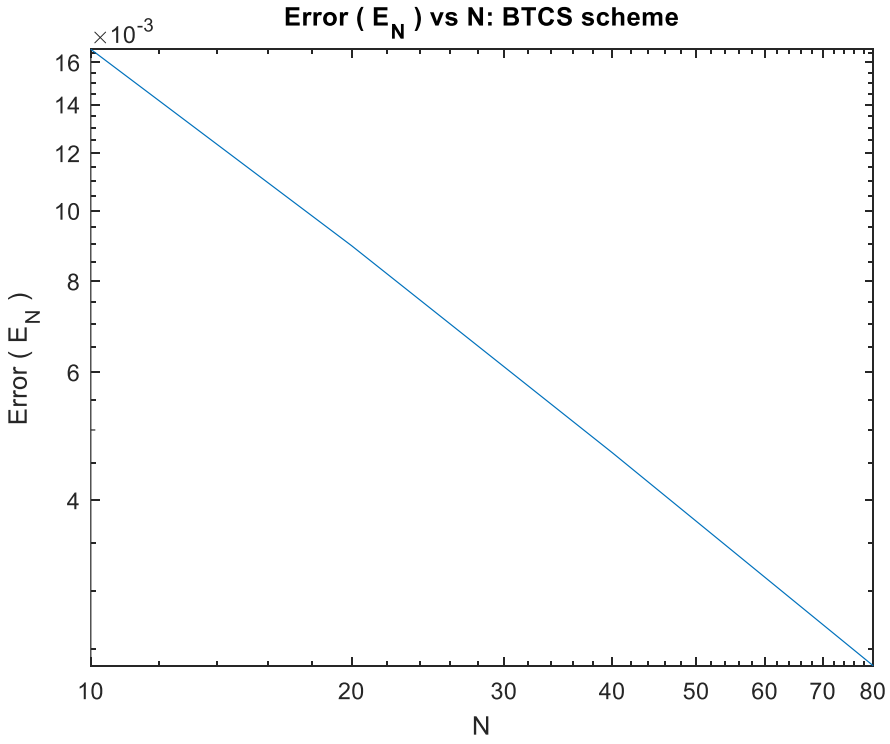


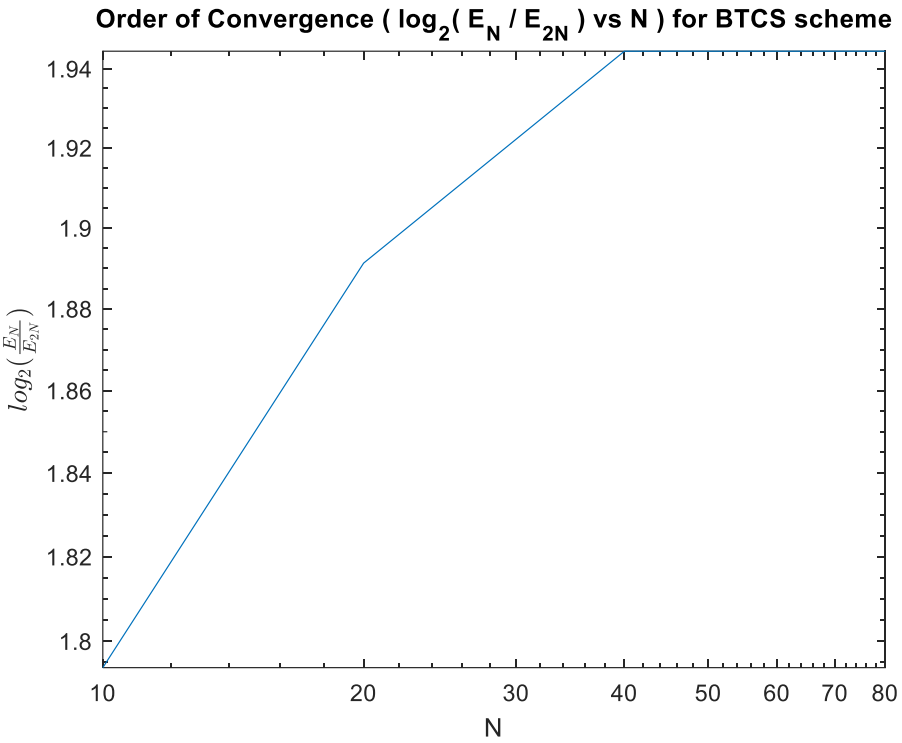


ii. The output for the BTCS scheme are:

Sl No	N	Max error (En)	Order of Convergence
1	10	0.016682	1.7938
2	20	0.0089589	1.8913
3	40	0.0046515	1.9444
4	80	0.002371	1.9444

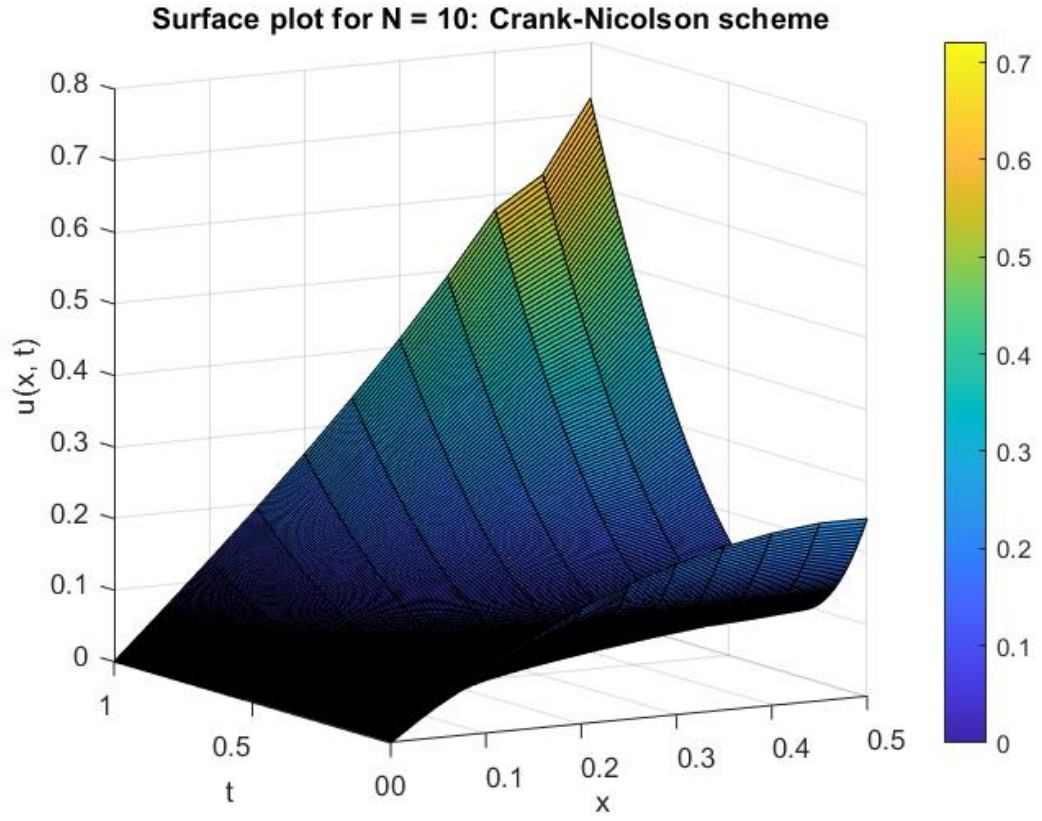

Surface plot for N = 10: BTCS scheme
u(x, t)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
0.5
0
t
0
0.1
0.2
0.3
0.4
0.5
x
0.6
0.5
0.4
0.3
0.2
0.1
0

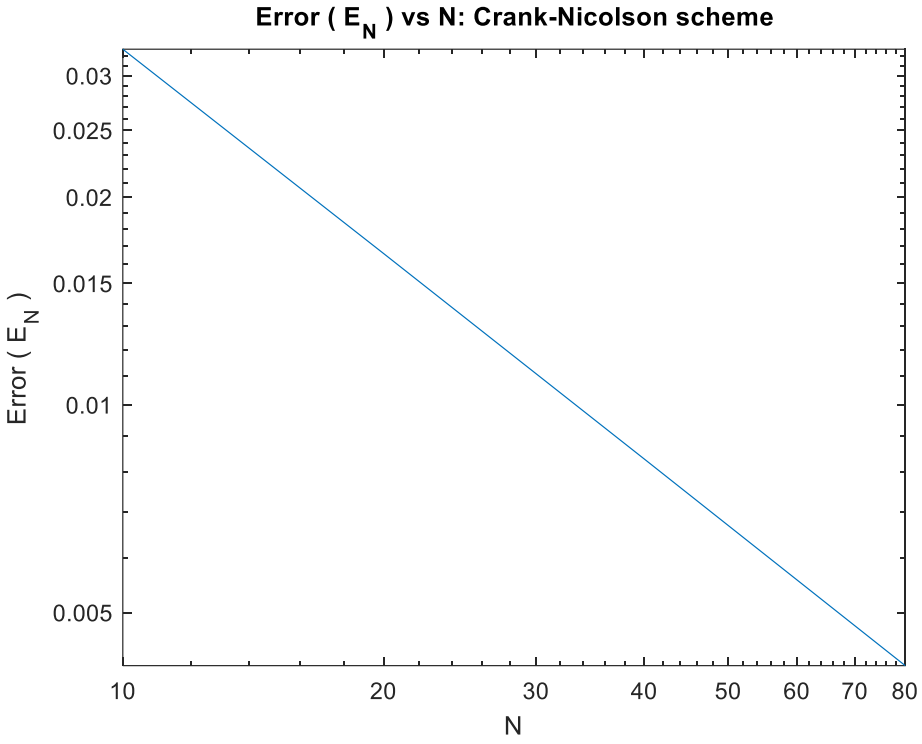

Error (E_N) vs N: BTCS scheme
×10^-3
16
14
12
10
8
6
4
Error (E_N)
10
20
30
40
50
60
70
80
N

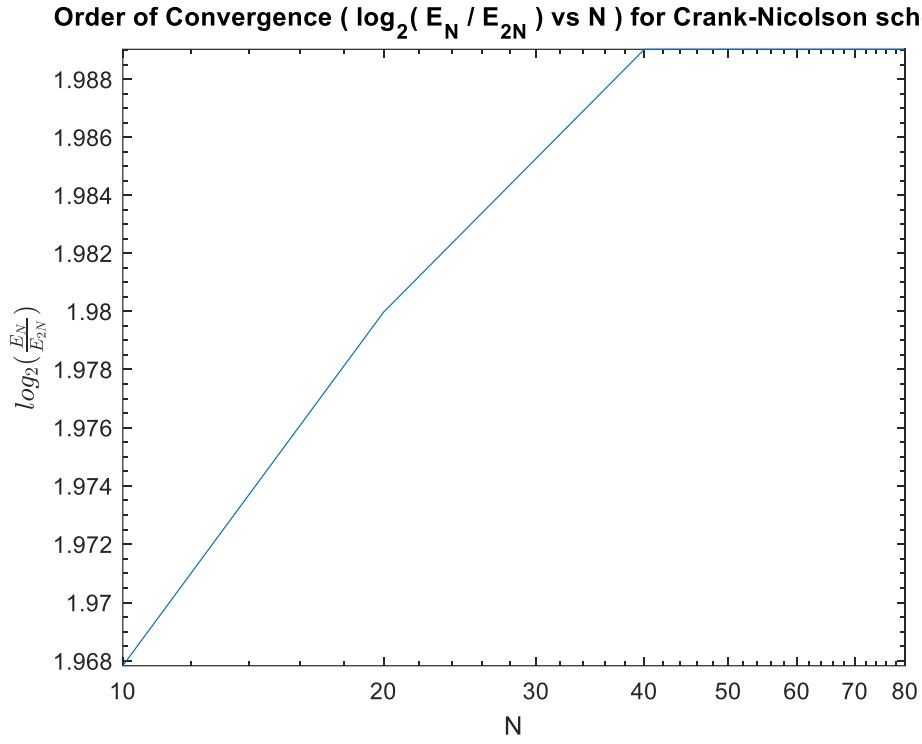


iii. The output for the Crank-Nicolson scheme are:

SI No	N	Max error (En)	Order of Convergence
1	10	0.032773	1.9678
2	20	0.01657	1.98
3	40	0.0083429	1.989
4	80	0.0041873	1.989







Observations:

- As we can observe that in all the 3 schemes the order of convergence remains almost constant around 2. And this is within the theoretical expectation since the N and M are chosen in such a way that λ <= 0.5.
- From the plot for the error analysis, we can observe that the error decreases in magnitude as we continue to make grid parameters more refined by increasing N.