

Solve the following problems by using FTCS, BTCS and Crank-Nicolson scheme

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1. Solve the following parabolic initial-boundary-value problem:

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = \sin(2\pi x) \sin(4\pi t), & (x, t) \in (0, 1) \times (0, 1), \\ u(0, t) = u(1, t) = 0, & t \in (0, 1], \\ u(x, 0) = 0, & x \in (0, 1). \end{cases}$$

2. Solve the following parabolic PDE by using finite difference schemes:

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, & (x, t) \in (0, 1) \times (0, T), \quad T > 0 \\ u(0, t) = u(1, t) = 0, & t \in (0, T], \\ u(x, 0) = f(x), & x \in (0, 1), \end{cases}$$

for the following values of  $f$ :

- (a)  $f(x) = \sin(\pi x)$   
(b)  $f(x) = x(1 - x)$ .

3. Solve the following parabolic problem:

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, & (x, t) \in (0, 1) \times (0, t), \quad t > 0, \\ u(x, 0) = \cos(\pi x/2), & x \in (0, 1), \\ u_x(0, t) = 0, & t \geq 0, \\ u(1, t) = 0, & t \geq 0. \end{cases}$$

4. Determine the numerical solution of the following one-dimensional parabolic IBVP:

$$\begin{cases} u_t - u_{xx} = 0, & (x, t) \in (0, 1/2) \times (0, T), \quad T > 0 \\ u(0, t) = 0, \quad u_x(1/2, t) = t^2, & \forall t \in (0, T] \\ u(x, 0) = x(1 - x), & x \in (0, 1/2). \end{cases}$$

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