

ECE 4960 Spring 2018: Computational and Software Engineering
Reading 2: Differentiation in Local Analysis

Deposit a pdf file of the two tables below to your Git directory before 11:59pm of 2/11

Document your programming environment: Language; development platform; operating system

Prob. 1. (Quadratic function to observe the tradeoffs between the truncation error and round-off error): For $f(x) = x^2$, we know the exact $f'(x=1) = 2$.

- 1.1 Use Eq. (1) below to estimate $f'(x=1)$ varying the value of h from 0.1 to 10^{-18} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
- 1.2 Repeat your calculation with $f(x) = x^2 + 10^8$. Add your results to the same table.
- 1.3 Repeat the above two procedures by using Eq. (2). Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h) \quad (1)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \quad (2)$$

h	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
10^{-1}	0.1	0.0999999	4.44089e-16	-4.47035e-08
10^{-2}	0.01	0.00999975	1.77636e-15	-4.17233e-07
10^{-3}	0.001	0.00100243	-1.64757e-13	4.05312e-06
10^{-4}	0.0001	3.38554e-05	-7.7538e-13	-4.06504e-05
10^{-5}	1e-05	-0.000264168	2.00018e-12	-0.000264168
10^{-6}	9.99924e-07	-0.0032444	1.99973e-12	-0.0032444
10^{-7}	1.01088e-07	-0.062849	5.75109e-11	-0.062849
10^{-8}	-1.21549e-08	-0.509884	-6.60383e-09	-0.509884
10^{-9}	1.65481e-07	-2	5.44584e-08	-2
10^{-10}	1.65481e-07	-2	1.65481e-07	-2
10^{-11}	1.65481e-07	-2	1.65481e-07	-2
10^{-12}	0.000177801	-2	6.67789e-05	-2
10^{-13}	-0.00159856	-2	-0.000488333	-2
10^{-14}	-0.00159856	-2	-0.00159856	-2
10^{-15}	0.220446	-2	0.109424	-2
10^{-16}	-2	-2	-0.889777	-2
10^{-17}	-2	-2	-2	-2
10^{-18}	-2	-2	-2	-2

Prob. 2. (Cubic function to observe the Richardson error estimation): For $f(x) = x^3$, we know the exact value of $f'(x=1) = 3$.

- 2.1 Use Eqs. (3) – (5) below to estimate $f'(x=1)$ varying the value of h from 2^{-4} to 2^{-40} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
- 2.2 Estimate η from Eqs. (6) and (7) for each choice of h . Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + E(h); \quad E(h) = O(h) = \frac{1}{2} h f''(x) + O(h^2) \quad (3)$$

$$f'(x) = \frac{f(x+2h) - f(x)}{2h} + E(2h); \quad E(2h) = O(h) = \frac{1}{2} 2h f''(x) + O(h^2) \quad (4)$$

$$f'(x) = \frac{-1}{2h} f(x+2h) - \frac{3}{2h} f(x) + \frac{2}{h} f(x+h) + O(h^2) \quad (5)$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \quad (6)$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \quad (7)$$

h	Error in $f'(x=l)$ by Eq. (3)	Error in $f'(x=l)$ by Eq. (4)	Error in $f'(x=l)$ by Eq. (5)	η by Eq. (6)	η by Eq. (7)
2^{-4}	0.191406	0.390625	-0.0078125	0.48999936	
2^{-5}	0.0187891	0.0376562	-7.8125e-05	0.498964314	
2^{-6}	0.00187539	0.00375156	-7.81249e-07	0.499896043	
2^{-7}	0.000187504	0.000375016	-7.80346e-09	0.499989334	
2^{-8}	1.875e-05	3.75001e-05	-1.16415e-10	0.499998667	
2^{-9}	1.87501e-06	3.74995e-06	0	0.500009333	
2^{-10}	1.91378e-07	3.72566e-07	7.45058e-09	0.513675429	
2^{-11}	7.05854e-08	1.72947e-08	1.19209e-07	4.081331275	
2^{-12}	2.48221e-07	2.48221e-07	4.76837e-07	1	
2^{-13}	2.48221e-07	2.48221e-07	0	1	
2^{-14}	-5.30425e-05	2.48221e-07	-6.10352e-05	-213.6906225	
2^{-15}	0.000266702	-0.000266205	0.000488281	-1.001866982	
2^{-16}	-0.00506237	0.000266702	-0.0117188	-18.98137247	
2^{-17}	-0.0157205	-0.0157205	0	1	
2^{-18}	0.197442	0.197442	0.5	1	
2^{-19}	-3	2.32907	-11	-1.28806777	
2^{-20}	-3	-3	-3	1	
2^{-21}	-3	-3	-3	1	
2^{-22}	-3	-3	-4099	1	
2^{-23}	-3	-3	-3	1	
2^{-24}	-3	-3	-524291	1	
2^{-25}	-3	-3	-3	1	
2^{-26}	-3	-3	-3	1	
2^{-27}	-3	-3	-3	1	
2^{-28}	-3	-3	-3	1	
2^{-29}	-3	-3	-3	1	
2^{-30}	-3	-3	-3	1	
2^{-31}	-3	-3	-3	1	
2^{-32}	-3	-3	-3	1	
2^{-33}	-3	-3	-3	1	
2^{-34}	-3	-3	-3	1	
2^{-35}	-3	-3	-3	1	
2^{-36}	-3	-3	-3	1	
2^{-37}	-3	-3	-3	1	
2^{-38}	-3	-3	-3	1	
2^{-39}	-3	-3	-3	1	
2^{-40}	-3	-3	-3	1	