Date

Maths 2022 November

Q1

a) i) ① Identifying the real problem.
② Construct appropriate relations between the variables.
③ Create model that adequately describes the problem.
④ Obtain the mathematical solution.
⑤ Interpret the " "
⑥ Test the validity and effectiveness of the model compared with reality.

ii) ① Traffic Congestion System.
Provides a better framework for analyzing complex traffic systems
Also, by modelling we can simulate different secnarios and obtain best solutions
Added value is, improves overall transportation efficiency by reducing travel time, fuel, emission etc.

Supply chain optimization
Mathematics provide quantitative approach to optimize the problem.
Added value is, this is much cost saving and improve service levels. Companies can reduce extra inventory and cost.

b) $S_0 = 500$

$$\frac{ds}{dt} \propto S$$

$$\int_{S_0}^{S} \frac{1}{S}\, ds = \int_0^t k\, dt$$

$$\ln S \Big|_{S_0}^{S} = kt \Big|_0^t \qquad (k = \text{constant})$$

$$\ln S - \ln S_0 = kt$$

$$\ln S = \ln S_0 + kt \quad — ①$$

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$t=0, \quad S_0 = 500$
$t=2, \quad S = 1000$

$(1) \Rightarrow \quad \ln(1000) = \ln(500) + K(2)$

$K = 0.3466$

$K = \frac{\ln 2}{2}$

$\therefore (1) \Rightarrow \ln S = \ln S_0 + \left(\frac{\ln 2}{2}\right) t$

$\ln\left(S/S_0\right) = Kt$

$S = S_0 e^{Kt}$

$S = 500 e^{+0.3466t} \quad \text{--- } (A)$

i) when $t = 5$,

$S = 500 e^{+0.3466 \times 5}$

$= \cancel{88.377}$

$= 2828.8$

$\therefore$ number of bacteria is 2828

ii) finding t when $S = 25{,}000$

$25{,}000 = 500 e^{(\ln 2/2)t}$

$50 = e^{\frac{\ln 2}{2}t}$

$\frac{\ln 2(t)}{2} = \ln(50)$

$t = 11.288$

$\therefore$ time taken is 11.288 hours

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c) Buckingham's Pi Theorem.

i) Let 'n' be the number of parameters, and 'k' be the number of independent dimensions (ex: mass (M), time (T), length (L), temperature (θ) etc). Then the number of distinctive dimensionless groups (pi terms) is at most (n-k).

ii) variables of interest: V, μ, r, g, ρ

$$\left.\begin{aligned} V &\to LT^{-1} \\ \mu &\to ML^{-1}T^{-1} \\ r &\to L \\ g &\to LT^{-2} \\ \rho &\to ML^{-3} \end{aligned}\right\} n=5$$

$M, L, T \to k = 3$

$(kg\,m^{-3})$

$$j = n - k$$
$$= 5 - 3 = 2$$

$\therefore$ we need 2 dimensionless groups such as π_1 and π_2

$$\pi = V^a \mu^b r^c g^d \rho^e$$

$$[M^0 L^0 T^0] = [LT^{-1}]^a [ML^{-1}T^{-1}]^b [L]^c [LT^{-2}]^d (ML^{-3})^e$$

$\underline{M}$ $\quad 0 = b + e$ — (1)

$\underline{L}$ $\quad 0 = a + (-b) + c + d - 3e$ — (2)

$\underline{T}$ $\quad 0 = -a - b - 2d$ — (3)

(1) $\Rightarrow$ $\quad e = -b$ ✓

(3) $\Rightarrow$ $\quad a = -b - 2d$ ✗ don't take like this. ($\because$ a is V)

(2) $\Rightarrow$ $\quad 0 = (-b - 2d) - b + (c) + d + 3b$

$-c = b - d$

$c = d - b$

$d = -\dfrac{(a+b)}{2}$

№ 3

$$0 = a - b + c + -\frac{(a+b)}{2} + 3b$$

$$+c = -\left(\frac{a}{2} + \frac{3b}{2}\right)$$

$$(a,b,c,d,e) = \left(a, b, -\frac{a}{2} - \frac{3b}{2}, -\frac{a}{2} - \frac{b}{2}, -b\right)$$

$$= a\left(1, 0, -\frac{1}{2}, -\frac{1}{2}, 0\right) + b\left(0, 1, -\frac{3}{2}, -\frac{1}{2}, -1\right)$$

$\therefore$ Solution space $\{(1,0,-\frac{1}{2},-\frac{1}{2},0), (0,1,-3/2,-\frac{1}{2},-1)\}$ are the basis of the set of equations

$$\pi_1 \Rightarrow V^1 \mu^0 r^{-\frac{1}{2}} g^{-\frac{1}{2}} \rho^0 \Rightarrow \pi_1 = \frac{V}{\sqrt{rg}}$$

$$\pi_2 \Rightarrow V^0 \mu^1 r^{-3/2} g^{-\frac{1}{2}} \rho^{-1} \Rightarrow \pi_2 = \frac{\mu}{\rho\sqrt{r^3 g}}$$

$\therefore$ we can express the result as,

$$\pi_1 = \phi(\pi_2)$$

$$\frac{V}{\sqrt{gr}} = K\left(\frac{\mu}{\rho\sqrt{gr^3}}\right)^n$$

$$V = K\sqrt{gr}\left(\frac{\mu}{\rho\sqrt{gr^3}}\right)^n$$

$\therefore$ Terminal velocity of the raindrop is

$$V = K\sqrt{gr}\left(\frac{\mu}{\rho\sqrt{gr^3}}\right)^n$$ (where K is proportionality constant and n is a constant)

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Q2)

a) i) Advantages: ① Produces simple models for problems with limited Resources.

② Improves quality of decisions.

③ Helps to obtain optimal solution to a decision of a problem.

Disadvantages: ① Since linear programming assumes "linear" relationship of variables it doesn't match always.

② No gurantee that decision variables will set to integers.

ii)

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h) drug A

	M	N	C
drug A (90)	10	2	
drug B (120)	8	4	
drug C (min)	3	1	

Constraints:

$$10m + 2n \geq 90$$
$$8m + 4n \geq 120$$

minimize,

$$z = 3m + n$$

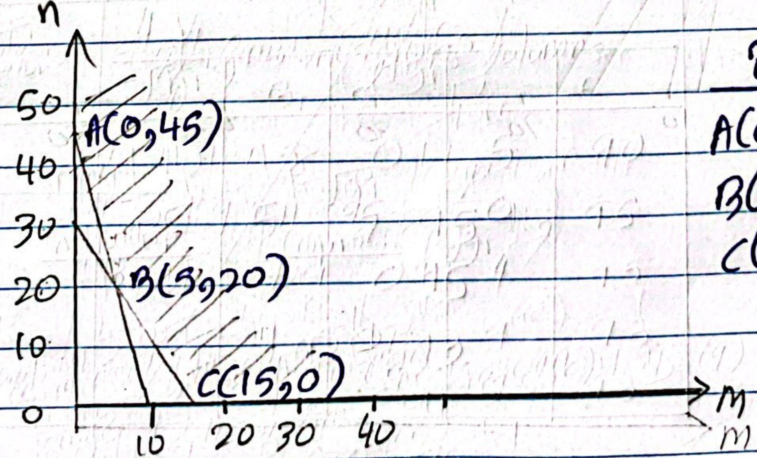
i) The objective is to minimize drug C,
objective f^n is,

$$z = 3m + n$$

Constraints are: $10m + 2n \geq 90$
$8m + 4n \geq 120$
$m, n \geq 0$

For,
$10m + 2n = 90$ — ①
$m = 0, n = 45$
$n = 0, m = 9$

$8m + 4n = 120$ — ②
$m = 0 \quad n = 30$
$n = 0 \quad m = 15$



Point	$z = 3m+n$
A(0,45)	45
B(5,20)	~~60~~35 ← min
C(15,0)	45

Point B,
Solving eq^n ① & ②,
$m = 5, n = 20$

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∴ Amount of substances that should be mixed are
5 and 20 from M & N respectively.

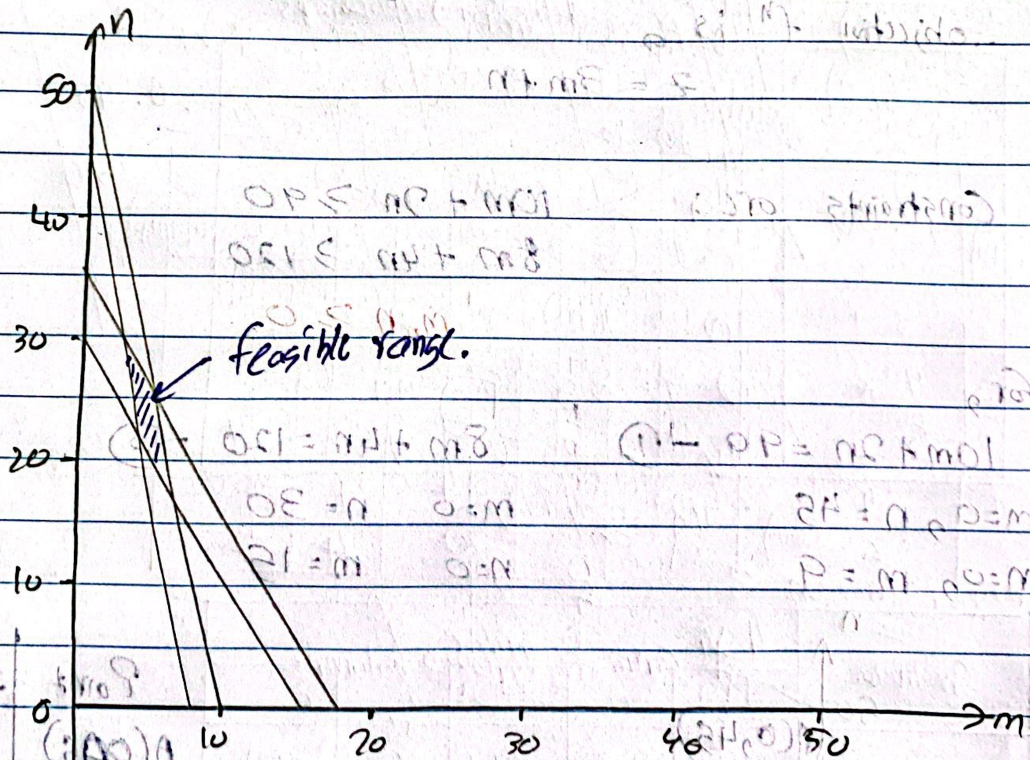
Units of undesirable drug C in the mixture are 35 units

iii) Now,

$90 \le 10m + 2n \le 100$
$120 \le 8m + 4n \le 130$

for,

$10m + 2n = 100$ & $8m + 4n = 130$
$m = 0,\ n = 50$ $\quad m = 0,\ n = 32.5$
$n = 0,\ m = 10$ $\quad n = 0,\ m = 16.25$



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i) **Balanced T.P:** If total supply equals to the total demand, the problem is said to be a balanced T.P problem.

$$\Sigma \text{supply} = \Sigma \text{demand}.$$

Unbalanced T.P: If total supply & total demands are not equals the problem is said to be an unbalanced transportation.

$$\Sigma \text{Supply} \neq \Sigma \text{Demand}$$

- If total supply exceeds total demand, we can balance the problem by adding dummy demand point

$$\Sigma \text{Supply} > \Sigma \text{demand}.$$

If total demand exceeds total supply, we can balance the problem by adding dummy supply point

h)

Unbalance T.P.
(minimize)

c)
i)

From	City 1 A	City 2 B	City 3 C	City 4 D	Supply	
1	30 [8]	– [11]	[11]	– [9]	30	1 1 3
2	20 [9]	10 [11]	20 [13]	– [7]	50	2 2 2 2 2
3	– [14]	10 [9]	. [11]	30 [5]	40	4 4 2 2 2
4 (Dummy)	– [0]	– [0]	10 [0]	– [0]	10	0 –
Demand	50	20	30	30	130	
	8	9	11	5		
	1	2	0	2		
	1	2	0			
	5	2	2			
	–	2	2			

ii) Total cost for the initial solution,

$Z = 30(8) + 20(9) + 10(11) + 20(13) + 10(9) + 30(5) + 10(0)$

$Z =$ Rs. 1030,000

Calculating stepping stone paths,

1B: 1B → 2B → 2A → 1A
$+11 - 11 + 9 - 8 = +1$

1C: 1C → 2C → 2A → 1A
$+11 - 13 + 9 - 8 = -1$

1D: 1D → 3D → 3B → 2B → 2A → 1A
$+9 - 5 + 9 - 11 + 9 - 8 = +3$

2D: 2D → 3D → 3B → 2B
$+7 - 5 + 9 - 11 = 0$

3A: 3A → 3B → 2B → 2A
$+14 - 9 + 11 - 9 = +7$

3C: $+11 - 13 + 11 - 9 = 0$

4A: $0 - 0 + 13 - 9 = +4$

4B: $0 - 0 + 13 - 11 = +2$

4D: $0 - 5 + 9 - 11 + 13 - 0 = +6$

Nº 6

cell 4C shows cost reduction;

∴ Allocating as much as possible to cell 4C,

From	A	B	C	D	Supply
1	10 [8]	– [11]	20 [11]	– [9]	30
2	40 [9]	10 [11]	– [13]	– [7]	50
3	– [14]	10 [9]	– [11]	30 [5]	40
4 ~~Dummy~~	– [0]	– [0]	10 [0]	– [0]	10
Demand	50	20	30	30	130

Calculating stepping stone paths

1B: $+11-11+9-8 = +1$

1D: $+9-5+9-11+9-8 = +3$

2C: $+13-11+8-9 = +1$

2D: $+7-5+9-11 = 0$

3A: $14-9+11-9 = +7$

3C: $+11-9+11-9+8-11 = +1$

4A: $0-0+11-8 = +3$

4B: $\cancel{0-11+8-9+11}$ $0-0+11-8+9-11 = +1$

4D: $+0-5+9-11+9-8+11-0 = +5$

There's no negative values.

∴ the solution is optimal.

Total
= optimal cost, $Z = 10(8) + 20(11) + 40(9) + 10(11) + 10(9) + 30(5)$
$+ 10(0)$

$=$ Rs. 1010,000

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Yes, there are multiple optimal solutions.

Path for cell 2D represents 0 which means that path is also optimal

Q4

a)

Instructor	Course				
	A	B	C	D	E
1	80	75	90	85	0
2	95	90	90	95	0
3	85	88	95	91	0
4	93	92	80	84	0
5	91	91	93	88	0

Instructor	Course				
	A	B	C	D	E
1	15	20	5	10	95
2	0	5	5	0	95
3	10	7	0	4	95
4	2	3	15	11	95
5	4	4	2	7	95

row reduction

Instructor	Course				
	A	B	C	D	E
1	10	15	0	5	90
2	0	5	5	0	95
3	10	7	0	4	95
4	0	1	13	9	93
5	2	2	0	5	93

Column reduction

Instructor	Course				
	A	B	C	D	E
1	10	14	0	5	0
2	0	4	5	0	5
3	10	6	0	4	5
4	0	0	13	9	3
5	2	0	0	5	3

\# lines < #rows/columns

∴ not optimal

Instructor	Course				
	A	B	C	D	E (Dummy)
1	10	14	1	5	$\boxed{0}$
2	0	4	6	$\boxed{0}$	5
3	9	5	$\boxed{0}$	3	4
4	0	0	14	9	3
5 (Dummy)	1	0	0	4	2

\# lines = #rows/columns.

Assignment

Instructor	Course	Score
1	-	
2	D	95
3	C	95
4	A	93
5	B	91

$\Sigma = 374$

$\text{average} = \frac{374}{5} = 74.8$

∴ Instructor 1 should be assigned to grade exams

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c)

i) An eq^n which expresses a value of sequence as a function of other terms in the sequence is called a difference equation.

ii) Let x^* be the fixed point of the difference equation.

$$x_{n+1} = f(x_n)$$

Stable if $-1 < f'(x^*) < 1$

unstable if $|f'(x^*)| > 1$

iii) $u(n) = 0.9u(n-1) - 3.5$

$u^* = 0.9u^* - 3.5$

$0.1u^* = -3.5$

$u^* = -35$

n	u(n)
0	-35
1	-35
2	-35
3	-35
4	⋮
⋮	⋮

∴ Equilibrium point is $\underline{\underline{-35}}$

Stability,

$f'(x^*) = 0.9$

$f'(x^*) < 1$

∴ stable.

n	u(n)	u(n)
0	-20	-40
1	-24.5	-39.5
2	-27.85	-39.05
5	-28.143	-37.952
20	-33.176	-35.608
50	-35	-35