

# Assignment 4

Problem 49 UGC Math June-2018

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49. A standard fair die is rolled until some face other than 5 or 6 turns up. Let  $X$  denote the face value of the last roll, and  $A = [X \text{ is even}]$  and  $B = [X \text{ is at most } 2]$ . Then.

1.  $P(A \cap B) = 0$
2.  $P(A \cap B) = 1/6$
3.  $P(A \cap B) = 1/4$
4.  $P(A \cap B) = 1/3$

Answer: 3

1.  $P(X = x, Y = y) = P(X = x/Y = y) \times P(Y = y)$
2.  $P(X = x) = \sum_{Y_j \in R} P(X = x)P(Y = y_j)$

1. Given  $X$  is the face value of the dice
- 2.

$$\begin{aligned} &P[(X \in (2, 4, 6), X \in (1, 2))/T] \\ &= P[(X = 2)/T] \\ &= \frac{1}{6} \end{aligned}$$

Joint Probability of  $X = 2$  and  $T$  (i.e Trail is allowed )

$$P(X = 2, T_1) = P(X = 2/T_1)P(T_1) = \frac{1}{6} \times 1$$

$$\begin{aligned} P(X = 2, T_2) &= P(X = 2/T_2)P(T_2) \\ &= \frac{1}{6} \times P(X_1 \in \{5, 6\}) = \frac{1}{6} \times \frac{1}{3} \end{aligned}$$

$$P(X = 2, T_i) = P(X = 2/T_i)P(T_i) = \frac{1}{6} \times \frac{1}{3^{i-1}}$$

Probability that trail is continued till  $i^{th}$  time

$$P(T_i) = \prod_{j=1}^{i-1} P(X_j \in \{5, 6\}) = \frac{1}{3^{i-1}}$$

Marginal probability of  $P(X=2)$

$$\begin{aligned}P(X = 2) &= \sum_{i=1}^{\infty} P(X = 2, T_i) = \sum_{i=1}^{\infty} P(X = 2/T_i) \times P(T_i) \\&= \frac{1}{6} \times (1 + \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^{i-1}} + \dots + \infty) \\&= \frac{1}{6} \times \frac{1}{1 - \frac{1}{3}} \\&= \frac{1}{6} \times \frac{3}{2} \\&= \frac{1}{4}\end{aligned}$$