1 Question

49. A standard fair die is rolled until some face other than 5 or 6 turns up. Let X denote the face value of the last roll, and A = [X is even] and B = [X is at most 2]. Then.
1. $P(A \bigcap B) = 0$
2. $P(A \bigcap B) = 1/6$
3. $P(A \bigcap B) = 1/4$
4. $P(A \bigcap B) = 1/3$

Answer: 3

2 Solution

2.1 Axioms and Formulae

1. $P(X = x, Y = y) = P(X = x/Y = y) \times P(Y = y)$
2. $P(X = x) = \sum_{Y_j \in R} P(X = x)P(Y = y_j)$

2.2 Given Info

1. Given X is the face value of the dice

2. $$\begin{aligned} &P[(X \in (2,4,6), X \in (1,2))/T] \\ &= P[(X = 2)/T] \\ &= \frac{1}{6} \end{aligned}$$

2.3 Joint Probability

Joint Probability of $X = 2$ and T (i.e Trail is allowed)

$$\begin{aligned} P(X = 2, T_1) &= P(X = 2/T_1)P(T_1) = \frac{1}{6} \times 1 \\ P(X = 2, T_2) &= P(X = 2/T_2)P(T_2) \\ &= \frac{1}{6} \times P(X_1 \in \{5,6\}) = \frac{1}{6} \times \frac{1}{3} \\ P(X = 2, T_i) &= P(X = 2/T_i)P(T_i) = \frac{1}{6} \times \frac{1}{3^{i-1}} \end{aligned}$$

Probability that trail is continued till i^{th} time

$$P(T_i) = \Pi_{i=1}^{i-1} P(X_i \in \{5,6\}) = \frac{1}{3^{i-1}}$$

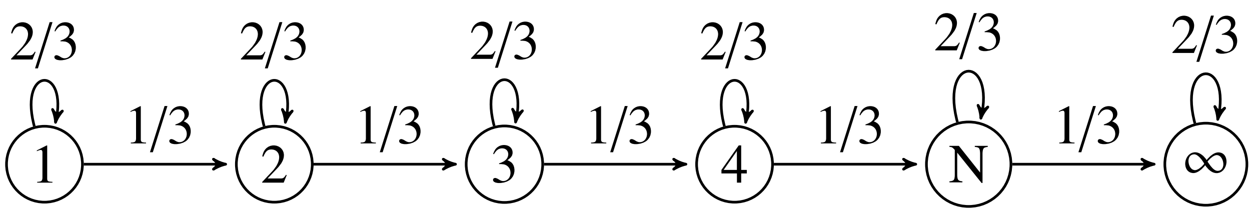
2.4 Joint Probability

Marginal probability of P(X=2)

$$
\begin{aligned}
P(X=2) &= \sum_{i=1}^{\infty} P(X=2, T_i) = \sum_{i=1}^{\infty} P(X=2/T_i) \times P(T_i) \\
&= \frac{1}{6} \times \left(1 + \frac{1}{3} + \frac{1}{3^2} + ... + \frac{1}{3^{i-1}} + ... + \infty\right) \\
&= \frac{1}{6} \times \frac{1}{1 - \frac{1}{3}} \\
&= \frac{1}{6} \times \frac{3}{2} \\
&= \frac{1}{4}
\end{aligned}
$$

2.5 Markov Chain Monte Carlo

Markov chain model for infinite number of trails



1. Since infinite number of trails are allowed, the Markov chain has infinite states
2. Also state transition matrix will have infinite dimensions

State Transition matrix

$$
\begin{bmatrix}
2/3 & 1/3 & 0 & 0 & 0 & ...... \\
0 & 2/3 & 1/3 & 0 & 0 & ...... \\
0 & 0 & 2/3 & 1/3 & 0 & ...... \\
0 & 0 & 0 & 2/3 & 1/3 & ..... \\
. & . & . & . & . & .... \\
. & . & . & . & . & .... \\
. & . & . & . & . & ....
\end{bmatrix}
$$