Assignment 1 - Problem 53

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1 Introduction

Question 53) Suppose (X,Y) follows bivariate normal distribution with means $\mu 1\mu 2$, standard deviations $\sigma 1$,$\sigma 2$ and correlation coefficient ρ, where all parameters are un-known. Then, testing Ho: $\sigma 1 = \sigma 2$ is equivalent to testing the independence of
1.) X and Y
2.) X and X-Y
3.) X+Y and Y
4.) X+Y and X-Y

Answer: 4, X+Y and X-Y

Solution: Given X and Y are bi-variate random variables. Bivariate random variables are distribution of normal distribution to two coordinates. If any two random variables, $X_1 and X_2$ are said independent, then it follows that the co-relation co-efficient $\rho[X_1, X_2]$ is equal to 0.

$$\rho[X_1, X_2] = 0$$

We know that, for a bi-variate random variable correlation is given as follows:
$\rho[X, Y]$ =$COV[X, Y]/\sqrt{Var[X] * Var[Y]}$

For for testing independence of $[X+Y, X-Y]$, we need to see if $\rho[X+Y, X-Y]$ becomes 0. if $\rho[X+Y, X-Y] = 0$, then it follows from the bivariate random distribution that COV[X+Y,X-Y] equates to 0

We know that $COV[X+Y, Z] = COV[X, Z] + COV[Y, Z]$
$COV[X, X] = Var[X]$

Now, $COV[X+Y, X-Y]$
$= COV[(X+Y) * X] - COV[(X+Y) * Y]$
$= COV[(X, X)] + COV[X, Y] - COV[X * Y] - COV[Y * Y]$
$= Var[X] - Var[Y]$

Now testing for $\sigma_1 = \sigma_2 => COV[X+Y, X-Y] = 0 =>$
$\rho[X+Y, X-Y] = 0$
Hence testing for $\sigma_1 = \sigma_2 => X+Y, X-Y$ are independent.