53. suppose (X, Y) follows bivariate normal distribution with means μ_1, μ_2, standard deviations σ_1, σ_2 and correlation coeffecient ρ, where all parameters are unknown. Then, testing $H_0: \sigma_1 = \sigma_2$ is equivalent to testing the independence of

1) X and Y
2) X and $X - Y$
3) $X + Y$ and Y
4) $X + Y$ and $X - Y$ ✓

Answer) (4) $(X + Y)$ and $(X - Y)$.

solution:-

We know that, for a bivariate random variable correlation is given as follows.

$$\rho(X,Y) = \frac{cov[X,Y]}{\sqrt{Var(X)\cdot Var(Y)}}$$

given that $\sigma_1 = \sigma_2$

now if two random variables are independent then covariance is 0.

for $(X+Y, X-Y)$ random variables,

covariance is given as

$cov[X+Y, X-Y]$

we know that

$$cov[X+Y, Z] = cov[X, Z] + cov[Y, Z]$$

~~So~~,

$$cov[X, X] = Var[X]$$

so,

$$cov[X+Y, X-Y]$$

$$= cov[(X+Y)X] + cov[-Y(X+Y)]$$

$$= cov[X, X] + cov[X, Y] - cov[X, Y) - cov(Y, Y]$$

$$= var(X] + cov(X, Y) - cov(X, Y) - var(Y).$$

now given $\sigma_1 = \sigma_2 \Rightarrow \sigma_1^2 = \sigma_2^2$

so

$$cov[X+Y, X-Y] = var(X) - var(X) + 0 = 0$$

Hence testing $\sigma_1 = \sigma_2$ implies testing independence of $X+Y$, $X-Y$.