Research Statement

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During my first postdoc with Prof. Dibyendu Roy at the Raman Research Institute, my research has been focused on low-dimensional heat transport using techniques such as the quantum Langevin equations and non-equilibrium Green's functions. Low-dimensional heat transport has been an active area of research due to the presence of *anomalous* transport. It is called anomalous because most of the one dimensional systems or lattice models that conserve linear momentum violate Fourier's law of heat diffusion, which is the simplest model for the transport of thermal energy. One general conclusion is that normal diffusive transport in one dimension can occur in the presence of nonlinear inter-site potentials as well as momentum non-conserving (pinning) terms, while anomalous transport (of which the simplest possibility is ballistic) occurs when the potential is harmonic.

In [1], we investigated heat transport in a scalar field theory that undergoes a stability-to-instability transition as a dimensionless coupling constant is increased. This one dimensional bosonic model features two massive scalar fields interacting via a bilinear derivative interaction with the previously mentioned dimensionless coupling. This model is interesting in part due to the presence of an effective non-Hermitian but PT-symmetric dynamical matrix, which generates time evolution. Here, the stability-to-instability transition is associated to the breaking of the PT symmetry. In this case, the transport is anomalous. In fact, we find that the heat current is system size independent (ballistic) as expected because of the quadratic interactions.

To understand heat transport in this model, we employed the framework of quantum Langevin equations and Green's functions (LEGF) by coupling the system to a pair of heat baths at each end of the chain with different system-bath couplings to the two scalar fields. The baths at any one end have a common temperature, which is different from that at the other end (T_L and T_R). We used an effective low-energy discretized lattice description of this field theory via a network of oscillators. When the coupling between the scalar fields exceeds a critical value, the ordinary oscillatory modes become inverted oscillator modes leading to the instability. For general system-bath couplings at the boundaries of this network, these inverted oscillator modes prevent the establishment of a non-equilibrium steady-state. The unbounded nature of these inverted oscillator modes prevent us from studying the heat conduction for general system-bath couplings. To avoid this problem, we engineered a special system-bath coupling, where at each edge of the network, a single bath is coupled to both the scalar fields with a common strength. This allowed the development of a steady-state heat current, which receives contributions only from the regular oscillator modes. The absence of contributions from inverted oscillator modes allows the heat current to be bounded in time for any value of the coupling between the fields. For more general system-bath couplings, the heat current remains bounded only upto a critical value of coupling between the fields. In both cases, we obtained analytical expressions for high-temperature (large $T = (T_L + T_R)/2$) classical heat currents as a function of the coupling strength between the fields. Even when T is not very large, we could find the temperature dependences of the quantum heat current for both system-bath couplings. Furthermore, we compared the nature of heat currents when the baths have different spectral properties, such as Ohmic as well as Rubin. We believe that this work could shed light on heat transport in systems with a non-Hermitian dynamical matrix that could involve novel topological phases and phase transitions. Investigating these aspects is one of our future goals.

We have also been exploring the properties of the heat current in a nonlinear version of the Kane-Lubensky (KL) model, which is a one-dimensional mechanical model of coupled rotors and springs. In this project, we computed the steady state heat current in both linear and nonlinear regimes of the KL model using the LEGF method. The linear KL model is interesting due to the close resemblance of its phonon spectrum with the electronic spectrum of the Su-Schrieffer-Heeger (SSH) model. One of the main questions we are trying to answer in this project is the connection between topology and heat transport in both linear and nonlinear regimes of the KL model. In the nonlinear regime, the KL model exhibits different topologies depending on the values of the radius of the rotors r and the lattice spacing a. Depending on the ratio r/a, the continuum limit of the nonlinear KL model can be expressed as either a ϕ^4 or a sine-Gordon type model. We have been trying to understand the crossover between different kinds of transport (such as ballistic to more general anomalous or anomalous to diffusive) depending on the topological phases of the nonlinear KL model.

In another project, we are exploring the topology of a non-Hermitian SSH model. We can view the non-Hermitian hamiltonian of the SSH model as the dynamical matrix of an associated Hermitian quadratic bosonic model. The primary objective is to understand the properties of the non-trivial topological phases of the non-Hermitian SSH model by analyzing the properties of the associated Hermitian bosonic Hamiltonian. We would like to explore the thermodynamic properties of the bosonic model such as the heat capacity and thermal conductivity and analyse their behaviour in various topological phases. This work is in collaboration with Prof. Dibyendu Roy and graduate student Kiran Babasaheb Estake.

I have also been part of another project titled "Spectral solutions for the Schrödinger equation with a regular singularity" [2], conducted in collaboration with Pushkar Mohile, Ayaz Ahmed and Prof. Pichai Ramadevi. In this project, we tried to understand the exact quantization conditions (EQC) for quantum periods associated with potentials that are singular at the origin such as $V(x) = |x| + a/|x| + b/|x|^2$. We validated our EQC proposal by showing that our computed Voros spectrum in the limit $a, b \to 0$ matches the known spectrum for the $|x|$ potential.

My PhD research was in mathematical physics with a focus on dynamical systems and their integrability. In particular, my thesis problem concerned the dynamics and integrability of a mechanical system called the Rajeev-Ranken model, which describes a class of nonlinear wave solutions of a scalar field theory dual to the 1+1 dimensional SU(2) Principal chiral model. These screw-type nonlinear waves could play a role similar to solitons in other field theories. This scalar field theory is strongly coupled in the UV and could serve as a toy model to study nonperturbative features of theories with a perturbative Landau pole. Classically, we showed that the RR model which is based on a quadratic Hamiltonian and a nilpotent/Euclidean Poisson algebra is Liouville integrable. In the process, I became familiar with topics in the field of integrable systems, nonlinear PDEs and dynamics that arise in various physical contexts. For instance, these include Lax pairs, r-matrices, zero-curvature condition, Yang-Baxter equation, inverse-scattering transfrom and Bethe ansatz.

In an extension of my PhD work, in collaboration with Prof. Govind Krishnaswami, I have written an article titled "Screwon spectral statistics and dispersion relation in the quantum Rajeev-Ranken model"[3]. In this work, we examined the spectral statistics and dispersion relation of quantized screwons via numerical diagonalization validated by variational and perturbative approximations. We also derived a semiclassical estimate for the cumulative level distribution which compares favorably with the one from numerical diagonalization. The spectrum shows level crossings typical of an integrable system. There are some interesting directions for future research. Although we have recovered the expected universal behaviour of number variance and spectral rigidity at small lengths, they both display system-dependent saturation and oscillations for larger lengths. We would like to understand this nonuniversal behaviour using Gutzwiller's trace formula and Berry's semi-classical theory and our exact solutions of the classical RR model [4, 5, 6].

In addition, we have written an expository article on Lax pairs and the zero curvature representation [7, 8]. The idea of realizing a nonlinear evolution equation as a compatibility condition between a pair of linear equations is explained by considering the examples of the harmonic oscillator, Toda chain, Eulerian rigid body, Rajeev-Ranken model, KdV equation and the nonlinear Schrödinger equation.

Aside from the above research directions, I am also interested in pursuing other research directions in mathematical and theoretical physics.

References

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