ADITYA UNIVERSITY

(Formerly known as Aditya Engineering College (A))

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PART-I

S.No: 421553

Programe : B.TECH

Hall Ticket No: 22A91A3520

Name : SUNKAVILLI SAI RAJA

Examination : III SEMESTER SUPPLEMENTARY (AR20(R))

Month-Year : Nov/Dec 2025

Branch : AGRICULTURAL ENGINEERING

Course Code : 201BS3T12

Course Name : Integral Transforms and Applications of Partial Differential Equations

Date of Exam : 18-11-2025

J. Pavan.
Signature of the Controller of Examination

S. Sai Raja 18/11/25
Signature of the Student with date

18/11/25
Signature of the Invigilator with date

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INSTRUCTIONS TO STUDENTS

1. The Answer Booklet contains 36 pages Ensure all the 36 pages are in proper order.
2. Student must verify the details of particulars in the PART - 1 i.e Name, Hall Ticket No., Examination, Course Name etc.,
3. In case of any deviation in the above or if the PART-1 is torn / damaged, report to the invigilator and return the damaged booklet.
4. You are prohibited from writing on or tampering the PART-1 except affixing the signature and serial number of last page written in the space provided.
5. Student is prohibited from:
   i. Writing their Hall Ticket No. and name in any part of the answer booklet.
   ii. Addressing the examiner in any manner whatsoever in the answer booklet. If they do so, their script will not be valued.
   iii. Writing religious symbols.
   iv. Either seeking or providing any assistance to the fellow students in the examinations.
   v. Possessing a manuscript or a printed matter, in any form, in the examination hall.
   vi. Bringing Mobile Phones / Cameras/ Bluetooth Devices / Programmable calculators etc.
   vii. Violation of the above mentioned instructions will be viewed as a case of malpractice, which is punishable offence.
6. Before beginning to answer any question, the students should write the correct number of that question in the margin provided. Answers written at different places for the same question will not be valued.
7. Students should write the answers, within the margins provided on both sides of the paper and on all the lines of each page. It is not necessary to begin each answer in a fresh page. Answers must be legibly written with blue or black pen.
8. Do not write anything except Question Number in the margin.
9. No loose sheets of papers will be allowed in the examination hall. No paper must be detached from or attached to the answer booklet except graph sheet.
10. Strike off all unused pages.
11. **NO ADDITIONAL ANSWER BOOKLETS/SHEETS WILL BE SUPPLIED.**

SPACE FOR ROUGH WORK

Q.No.

Unit-IV

7b. $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$

Given that;

$$L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$$

let $f(t) = \frac{1}{s^2+4}$, $g(t) = \frac{1}{s^2+9}$

$L(f(t)) = L\left\{\frac{1}{s^2+4}\right\} = \cos 2t$, $L(g(t)) = L\left\{\frac{s^2}{s^2+9}\right\} = \cos 3t$

Using Convolution theorem.

$$L^{-1}\{\bar{f}(s) \cdot \bar{g}(s)\} = f(t) - g(t)$$

$$= \int_0^t f(x) \cdot g(t-x)\, dx$$

$$= \int_0^t \cos 2x \cdot \cos 3(t-x)\, dx$$

$$= \frac{1}{2}\int_0^t \cos(2x \cdot 3t - x)$$

$$= \frac{1}{2}\left(\frac{\sin 2t}{1} + \frac{\sin t}{5} + \frac{\sin 3t}{1} + \frac{\sin 2t}{5}\right)$$

$$= \frac{1}{2}\left(\frac{\sin 3t - \sin 2t - \sin 3t + \sin 2t}{5}\right)$$

$$= \frac{1}{2}\left(\frac{\sin t - 4\sin 2t}{5}\right) = \frac{2}{10}(\sin 3t - 2\sin 2t)$$

$$\boxed{= \frac{1}{5}(\sin 3t - 2\sin 2t)}$$

Q.No.

Unit-III

6b.

Given that,

$$f(x) = \int_0^\infty \frac{\cos 6t - \cos 4t}{t}\, dt.$$

Integral same as the

$$f(t) = \frac{\cos 6t - (-\cos 4t)}{t}.$$

$$g(t) = 2\cos 6t - \cos 4t = L\{g(t)\} = L\{\cos 6t\} - L\{\cos 4t\}$$

$$= \frac{s}{s^2+36} - \frac{s}{s^2+16} = \bar{g}(s)$$

using division by 't'

$$t\left\{\frac{\cos 6t - \cos 4t}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+36} - \frac{s}{s^2+16}\right) dt$$

$$= \frac{1}{2}\int_s^\infty \frac{2s}{s^2+36}\, ds - \frac{1}{2}\int_s^\infty \frac{2s}{s^2+16}\, ds$$

$$= \frac{1}{2}\log(s^2+36)\,\frac{1}{s} - \frac{1}{2}\log(s^2+16)\Big|_s^\infty$$

$$= \frac{1}{2}(0 - \log(s^2+36)) - \frac{1}{2}(0 - \log(s^2+16))$$

$$= \frac{1}{2}\log(s^2+16) - \frac{1}{2}\log(s^2+36)$$

$$= \log\left(\sqrt{\frac{s^2+16}{s^2+36}}\right) = \bar{f}(s)$$

Q.No.

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put $s=0$

$$\int_0^\infty \frac{\cos 6t - \cos 4t}{t} = \log\sqrt{\frac{16}{36}} = \log\frac{4}{6} = \log\left(\frac{2}{3}\right)$$

6a. Given that,

$$f(x) = L\{t^2 e^{-3t} \sin 2t\}$$

$$g(x) = \sin 2t \Rightarrow L\{g(t)\} = L(\sin 2t) = \frac{2}{s^2+4} = \bar{g}(s)$$

using multiplication by 't'

$$L\{t^2 \sin 2t\} = \frac{d^2}{ds^2} \cdot \bar{g}(s) = \frac{d^2}{2s^2}\left(\frac{2}{s^2+4}\right).$$

$$= \frac{2d}{ds} \cdot \left(\frac{-1}{(s^2+4)^2} 2s\right)$$

$$= -4\left[\frac{1(s^2+4)^2 - 2s\,(s^2+4)(2s)}{(s^2+4)^2}\right]$$

$$= -4\left[\frac{(s^2+4)^2 - 4s^2(s^2+4)}{(s^2+4)^4}\right]$$

$$= -4\left[\frac{4-3s^2}{(s^2+4)^3}\right] = \frac{12s^2-16}{(s^2+4)^3}$$

Q.No.

Unit-I

2a. Given that,

$$f(x) = e^{-x} \quad , (0, 2\pi)$$

Where, $e^{ax} = a_0 + \sum_{n=1}^{\infty} a_n - b_n$ — (1).

$$a_0 = 1/\pi \int_0^{2\pi} f(x)\,dx$$

$$a_n = 1/\pi \int_0^{2\pi} f(x) \sin nx\,dx.$$

$$b_n = 1/\pi \int_0^{2\pi} f(x) \sin nx\,dx.$$

Now,

$$a_0 = 1/\pi \int_0^{2\pi} f(x)\,dx$$

$$= 1/\pi \int_0^{2\pi} e^{-x}\,dx$$

$$= 1/\pi \left[\frac{e^{2a\pi}}{a} \quad \frac{e^0}{2} \right] = 1/\pi \left(\frac{e^{x\pi}}{a} - 1 \right] \text{ — (1)}$$

$$a_n = 1/\pi \int_0^{2\pi} f(x) \sin nx\,dx$$

$$= 1/\pi \int_0^{2\pi} e^{ax} \cos nx\,dx$$

$$= 1/\pi \left(\frac{e^{a\pi}}{a^2+n^2}(x) - \frac{1}{a^2+n^2}(a) \right]$$

$$a_n = 1/\pi \left[\frac{a}{a^2+n^2} \left[e^{2a\pi} - 1 \right) \right]$$

$$b_n = 1/\pi \int_0^{2\pi} f(x) \sin nx\,dx.$$

$$= 1/\pi \int_0^{2\pi} \frac{e^{2a\pi}}{a^2+n^2} \left(a \sin 2\pi n - n \cos 2\pi n \right)$$

Q.No.

$$= \frac{1}{a^2+n^2}(-a)\Big]$$

$$= 1/\pi\left(\frac{-\pi e^{2n\pi}}{a^2+n^2} + \frac{n}{a^2+n^2}\right]$$

$$= 1/\pi\left(\frac{n}{a^2+n^2}(-e^{2n\pi}+1)\right]$$

Substituting a_0, a_n, b_n in Eq (1), we get.

$$= e^{ax} = 1/\pi\left(\frac{e^{a\pi}a-1}{2a}\right) + \sum_{n=1}^{\infty} \cdot 1/\pi\left(\frac{-a}{a^2+n^2}(e^{2a\pi}-1)\right)^{\cos nx}$$

$$- 1/\pi\left[\frac{n}{a^2+n^2}(-e^{2n\pi}+1)\right].$$

2b. Given that,

$$f(x) = x. \quad , (0,2)$$

where,

$$a_0 = 1/2 \int_0^2 f(x)dx.$$

$$a_n = 1/2 \int_0^2 f(x)\cos n x dx.$$

$$b_n = 1/2 \int_0^2 f(x)\cos n x dx$$

Now,

$$a_0 = 1/2 \int_0^2 f(x)dx.$$

$$= 1/2 \int_0^2 x\,dx.$$

$$= 1/2\left(\frac{x\,dx}{a}\right) = 1/2\left(\frac{e\,dx}{a} - 1\right).$$

Q.No.

$$a_n = \frac{1}{2}\int_0^2 f(x)\cos\text{inex}\,dx$$

$$= \frac{1}{2}\int_0^2 x\,\text{Cosin}\,nx\,dx$$

$$= \frac{1}{2}\left(\frac{x}{a^2+n^2}\right)$$

$$a_n = \frac{1}{2}\left(\frac{x}{a^2+n^2}\right)(x-1)$$

$$b_n = \frac{1}{2}\int_0^2 f(x)\cos\text{in}\,nx\,dx$$

$$= \frac{1}{2}\int_0^n x\cos nx\,dx$$

$$= \frac{1}{2}\left(\frac{n}{a^2+n^2}\right)(-x+1)$$

Substute, a_0, a_n, b_n in Eq (1). we get

$$f(x) = \left(\frac{x\,dx}{a}\right) \sum_{n=2} \frac{1}{2}\left(\frac{x}{a^2+n^2}(x-1)\right) - \frac{1}{2}\left(\frac{n}{a^2+n^2}(-x+1)\right)$$

Q.No.

7a. Given that

$$L^{-1}\left\{\frac{2s-3}{s^2+4s+13}\right\}$$

Let $f(s) = \frac{2s-3}{s^2+4s+13}$. $g(s) = \frac{2s-3}{s^2+4s+13}$.

$L(f(s)) = L\left\{\frac{2s-3}{s^2+4s+13}\right) = \sin 3t$, $L(g(t)) = L\left(\frac{2s-3}{s^2+4s+13}\right)$

$= \cos 4t$.

Using Convension theorem,

$$L^{-1}\{f(s)\cdot g(s)\} = f(t) - g(t)$$

$$= \int_0^t f(x)\cdot g(t-x)\,dx.$$

$$= \int_0^t \sin 3t \cdot \cos 4(t-x)\,dx.$$

$$= \frac{1}{2}\int_0^t \sin(3t\cdot 4t - 4x).$$

$$= \frac{1}{2}\left(\frac{\cos t - 4\sin 2t}{5}\right)$$

$$\boxed{= \frac{1}{10}\cdot(\cos 3t - 2\cos 2t)}$$

Q.No.

Unit - V.

9a. Given that,

$$\frac{du}{dx} = 2\frac{du}{dt} + u \quad \text{— (1)}.$$

Subject to the condition $u(x,0) = 6e^{-3x}$ — (2)

Seperation of variable from eq (2).

$$u(x,t) = X(x)\,T(t) \quad \text{— (3)}.$$

Now differentiating Eq (3).

$$\left.\begin{aligned} \frac{du}{dx} &= X'(x)\,t \\ \frac{du}{dt} &= X(x)\,T'(t) \end{aligned}\right\} \text{(4)}.$$

Sub Eq (3) & (4) in Eq (1).

$$X'(x)\,T(t) = 2X(x)\,T'(t) + X(x)\,T(t)$$

$$\frac{X'(x)}{X(x)} = \frac{2T'(t) + T(t)}{T(t)} = \lambda$$

$$\frac{X'(x)}{X(x)} = \lambda$$

$$X'(x) = \lambda(x)$$

$$(0-\lambda)x = 0$$

$$m - \lambda = 0 \Rightarrow m = \lambda$$

$$\frac{2T(x + T(t)}{T(t)} = x.$$

$$2T'(t) + T(t) - \lambda T(t) = 0$$

$$2m + (1-\lambda) = 0$$

$$2m = \lambda - 1$$

$$m = \frac{\lambda - 1}{2}$$

$$CF = T(t) = C_2 e^{\left(\frac{\lambda-1}{2}\right)t}$$

Q.No.

form (3).

$$u(x,t) = X(x)\,T(t)$$

$$u(x,t) = ce^{\lambda} + \frac{a-1}{2}t \quad \text{--- (5)}$$

using eq $u(x,0) = 6e^{-3x}$.

$$6e^{-3x} = ce^{\lambda x} + \left(\frac{\lambda-1}{2}\right)(0)$$

$$6e^{-3x} = ce^{\lambda x}$$

$$c = 6 \; ; \; \lambda = -3.$$

From (5).

$$u(x,t) = 6e^{-3x} + \left(\frac{-3-1}{2}t\right)$$

$$u(x,t) = 6e^{-3x} + (-2)\,t$$

$$\boxed{u(x,t) = 6e^{-3x} - 2t}$$

9b. Given that.

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \quad \text{--- (1)}.$$

$$X''(x)\,T(t) = 2X(x)\,T'(t) + X(x)\,T(t).$$

Q.No.

Unit - II

3a. Given that.

$$f(x) = \int_0^\infty \frac{\cos \lambda x}{1+\lambda^2}\, d\lambda = \pi/2\, e^{-x}, \quad x \geq 0.$$

Q.No.

3b. Given that,

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x < 2 \\ 0 & \text{if } x \ge 2. \end{cases}$$

$$f(x) = \frac{2}{\pi}\int_0^{\infty} f(t)\sin(\lambda t)\,dt)\sin(\lambda\pi)\,d\lambda = \frac{2}{\pi}$$

$$\int_0^1 \sin(\lambda t)\,dt)$$

$$f(x)\ \frac{\sin\lambda(x)}{\lambda}\cos(\lambda x)\,dx = \frac{\pi}{2} f_{ox}(x),\ x$$

$$f(x) = 2/\pi \int_0^1 f(t)\cos(\lambda t)\,dt)\cos(\lambda\pi)\,d\lambda = \frac{2}{\pi}$$

$$\int_0^1 \cos(\lambda t)\,dt)$$

$$f(x)\ \frac{\cos\lambda(x)}{\lambda}\sin(\lambda x)\,dx = \pi/2\ f_{ox}(x),\ x)$$

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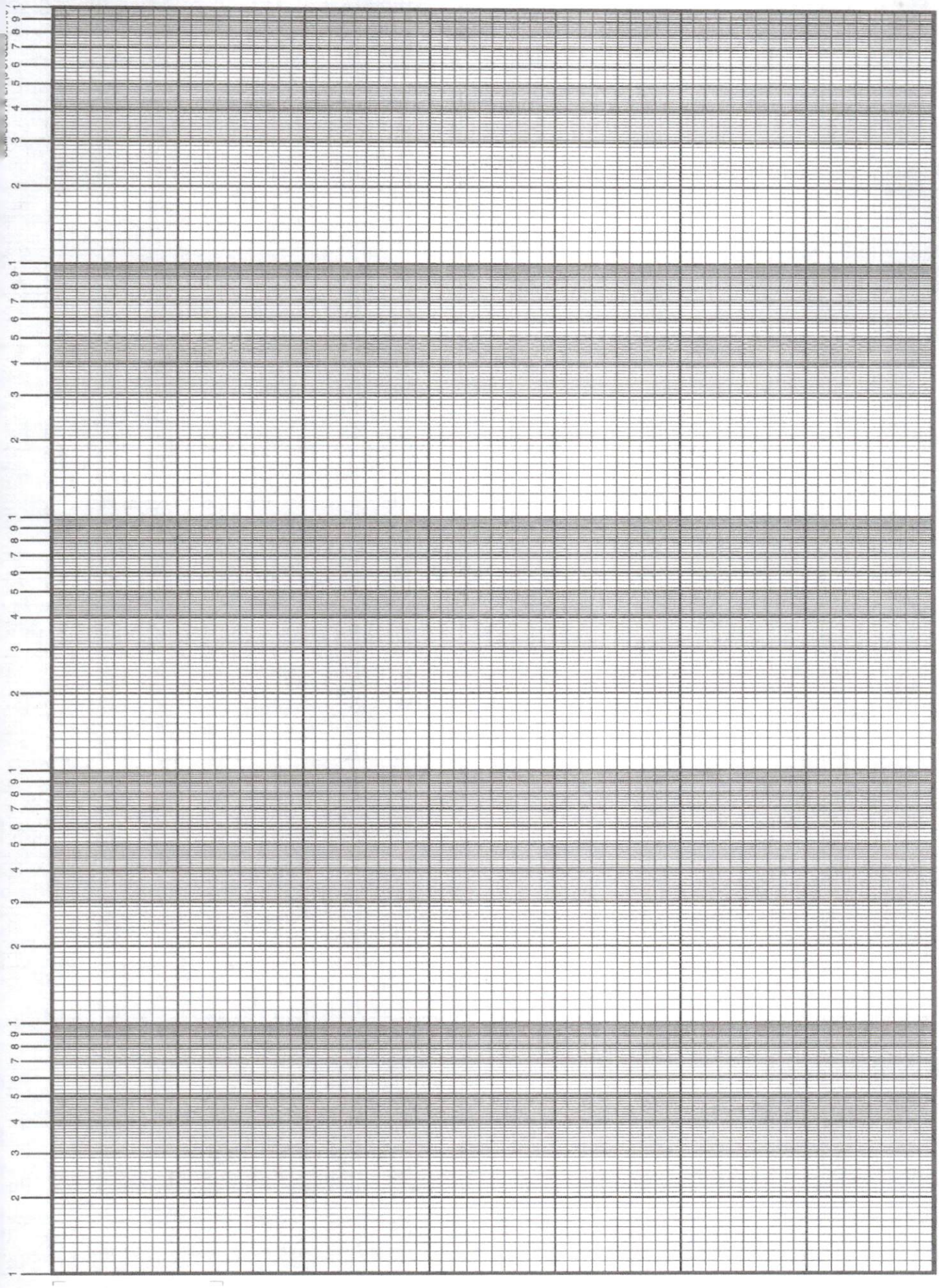
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