Computational Problems - Hints

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Problem 1: Graphical Convolution

```
close all
clear all

axis_color = [0.5 0.5 0.5]; % color of axis constant
s_int = 0.1; % sampling interval constant

t = -10:s_int:10; % interval for function 'f(t)'
f  = % definition of function 'f(t)'

t1 = -10:s_int:10; % interval for function 'go(t1)'
go = % definition of function 'go(t1)'


c = s_int * conv(f, go); % convolution of the two functions

%% Animation

g = fliplr(go); % flip 'go(t1)' for the graphical convolutions g = go(-t1)
tf = fliplr(-t1); % flipped time axis

% slide range of 'g' to discard non-ovelapping areas with 'f' in the convolution

tf = tf + ( min(t)-max(tf) );

% get the range of function 'c' which is the convolution of 'f(t)' and 'go(t1)'

tc = [ tf t(2:end)];
tc = tc + max(t1);

% start graphical output with three subplots

a_fig = figure;
set(a_fig, 'Name', 'Animated Convolution', 'unit', 'pixel', ...
             'Position', [300, 150, 600, 750]);

% plot f(t) and go(t1)

ax_1 = subplot(3,1,1);
op = plot(t,f, 'b',  t1, go, 'r');
hold on; grid on;
set(ax_1, 'XColor', axis_color, 'YColor', axis_color, 'Color', 'w', 'Fontsize', 9);
xlim( [ ( min(t)-abs(max(tf)-min(tf)) - 1 ) ( max(t)+abs(max(tf)-min(tf)) + 1 ) ] );
title('Graph of f(t) and go(t)', 'Color', axis_color );
legend({'f(t)' 'go(t)'});
```

```
% initialize animation the plot of 'g' is slided over the plot of 'f'
% plot f in the subplot number 2

ax_2 = subplot(3,1,2);
p = plot(t, f);
hold on; grid on;
title('Graphical Convolution: f(t) and g = go(-t1)', 'Color', axis_color );

% plot g in the subplot number 2

q = plot(tf, g, 'r');
xlim( [ ( min(t)-abs(max(tf)-min(tf))-1 ) ( max(t)+abs(max(tf)-min(tf))+1 ) ] );
u_ym = get(ax_2, 'ylim');

% initialize the plot the convolution result 'c'

ax_3 = subplot(3,1,3);
r = plot(tc, c);
grid on; hold on;
set(ax_3, 'XColor', axis_color, 'YColor', axis_color, 'Fontsize', 9);
xlim( [ ( min(t)-abs(max(tf)-min(tf)) - 1 ) ( max(t)+abs(max(tf)-min(tf)) + 1 ) ] )
title('Convolutional Product c(t)', 'Color', axis_color );
```

II Write an additional animation block: Use the ‘**pause**’ and ‘**drawnow**’ function to render the sliding effect as shown in the previous slide.

Problem 1: Graphical Convolution

Convolution of two rectangular functions

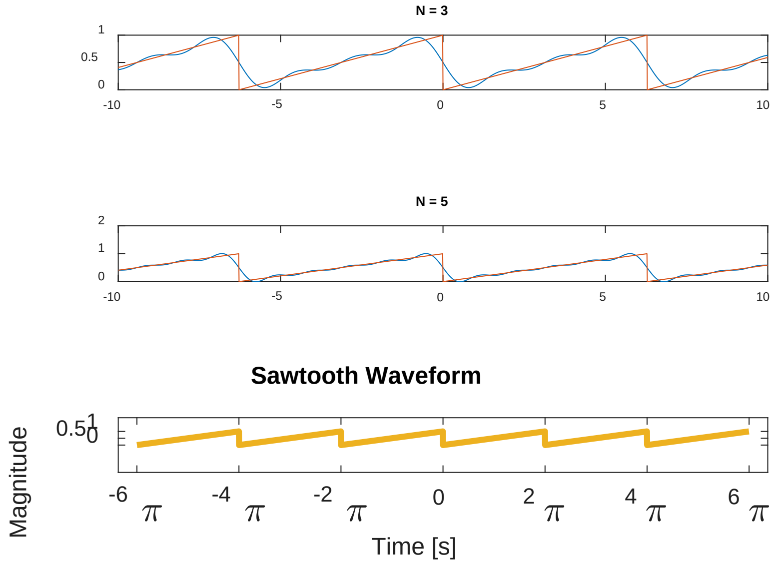
Example of final deliverable

Play Video

Problem 2: Fourier Series Expansion

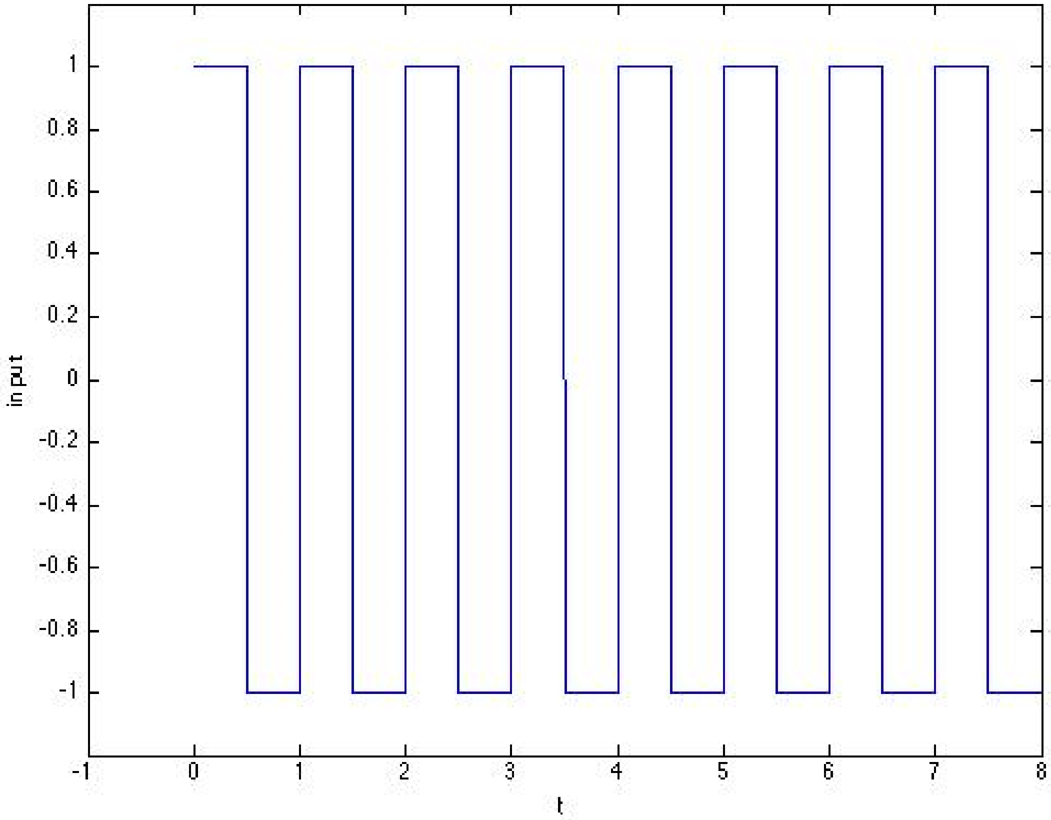
- Using ‘loop control’ and ‘conditional statements’, iteratively modify ‘’**y_prime**’ to approximate the original sawtooth waveform through Fourier Series expansion for n=3,5 and 10 terms.
- Show derivations for the Fourier Coefficients (hint: should be in terms of a DC term and sine terms). Explain why there are no cosine terms both mathematically and conceptually

```
clear all;

t = -6*pi:0.01:6*pi;

y = 0.5 * sawtooth(t) + 0.5; % Original Sawtooth Signal
y_prime = 0.5; % Signal Representing Fourier Expansion
```



Problem 3: LP Filter and Signal Distortion

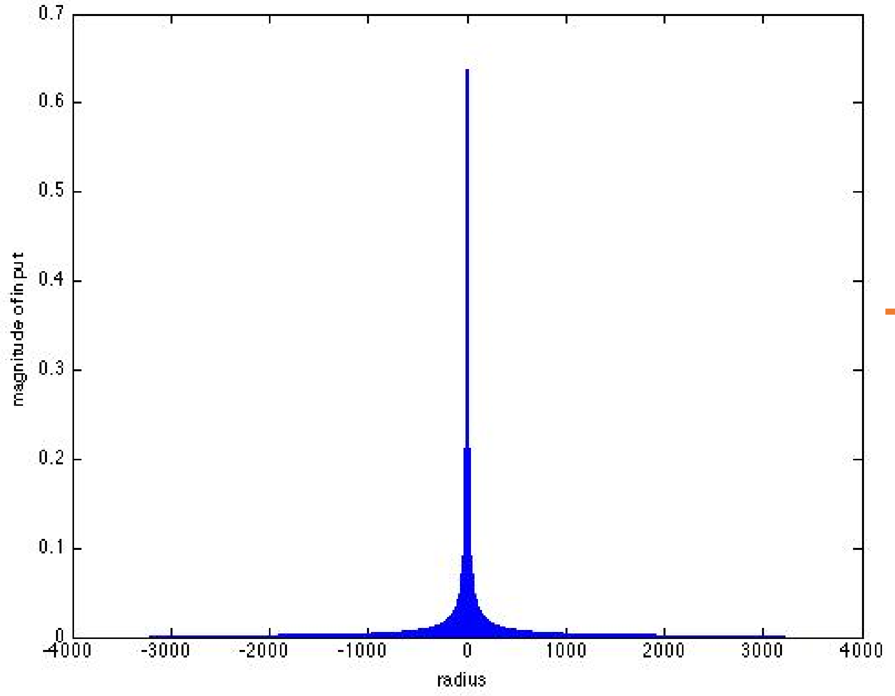
(A)



Input plot in time domain, which is a square wave with period one

Problem 3: LP Filter and Signal Distortion

(B)



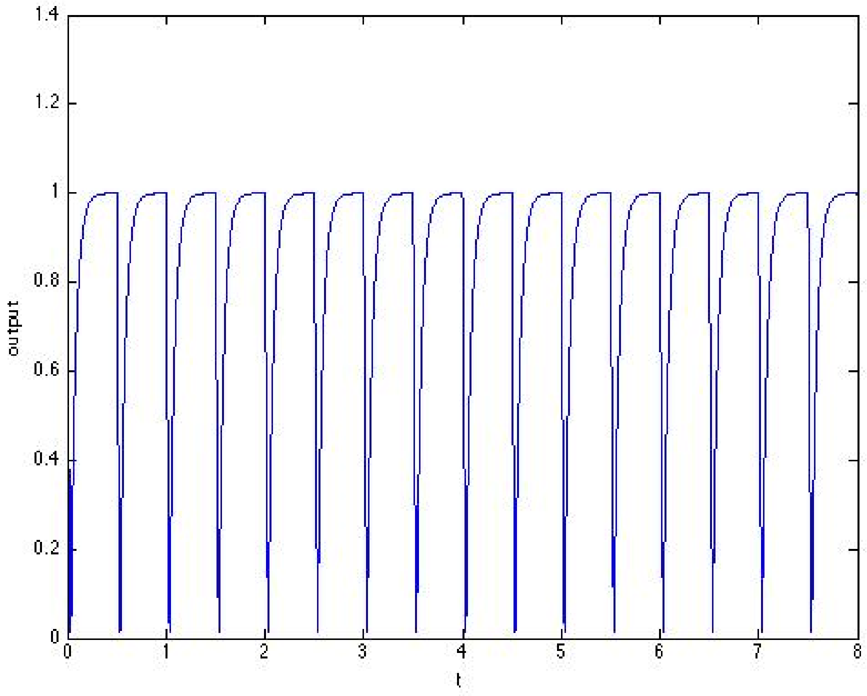
Power Spectral of the input signal

There are multiple ways to obtain the frequency spectral, such as using the built-in functions in Matlab: FFT, Fourier; or calculating the Fourier series by hand. The FFT function approximates the Fourier transform by treating the input signal as one period of a periodic signal.

This plot shows the power spectral of the square wave in one sampling frequency period, $[-\pi F_s, -\pi F_s\ ]$

Problem 3: LP Filter and Signal Distortion

(C)

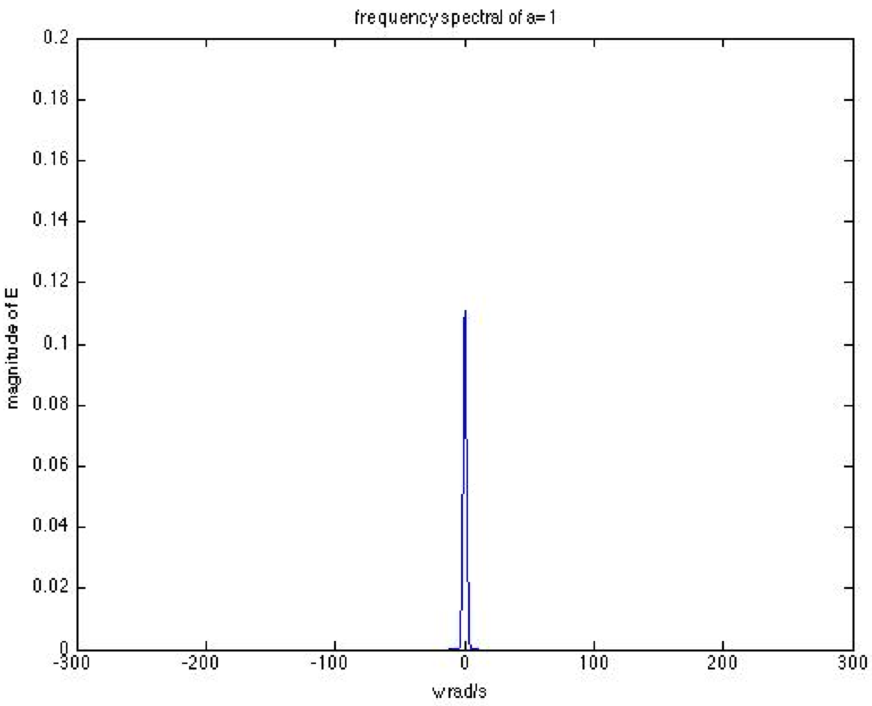


Output signal in time domain

This plot shows the magnitude of the output signal in time domain. The magnitude is a distorted version of the input signal, which results from the elimination of high frequency components by the low pass filter. You may also try to plot the phase of the output signal to observe completely that it is a distorted version of the input square wave

Problem 4: Gaussian Optical Pulses

(A)

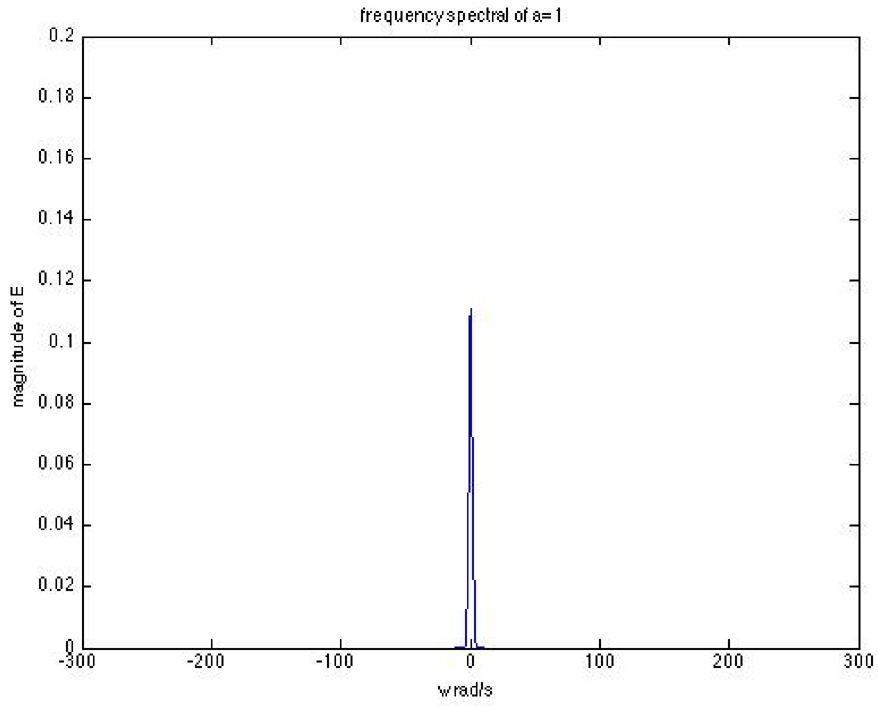


Fourier transform of e^{-t^2} for a = 1

The following plots show the Fourier transform of the three given Gaussian functions. <u>It can be observed that the Fourier transform of a Gaussian function is still Gaussian and that compression in time domain results as stretch in frequency domain</u>

Problem 4: Gaussian Optical Pulses

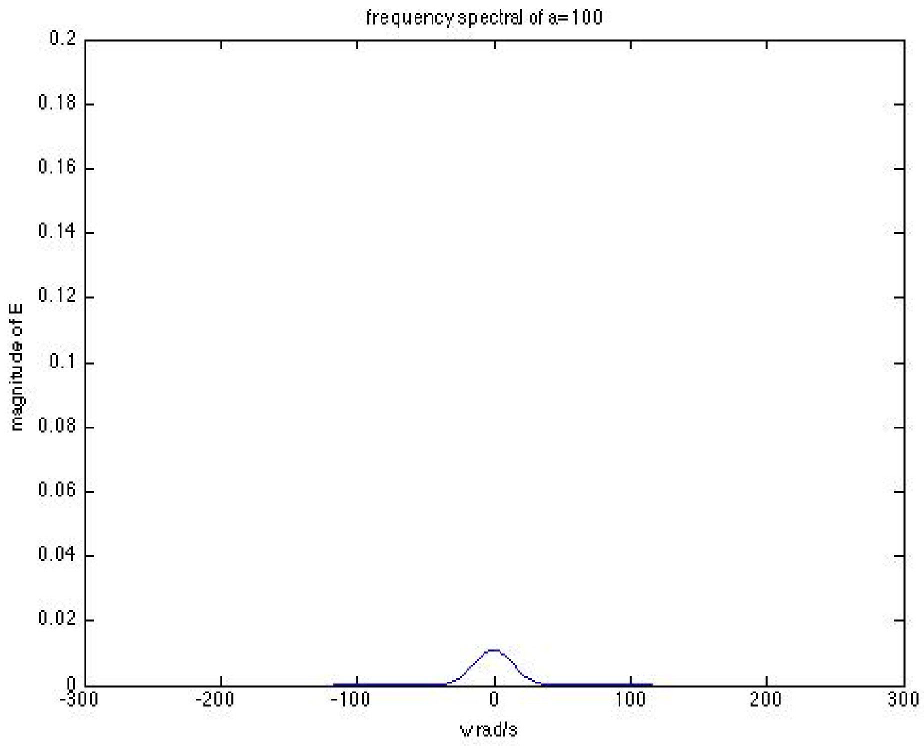
(B)



Fourier transform of e^{-10t^2} for a = 10

Problem 4: Gaussian Optical Pulses

(C)



Fourier transform of e^{-100t^2} for a = 100

Problem 5: Wagon Wheel Effect

Stroboscopic/Wagon Wheel Effect

Example of final deliverable

Just recreate the rotating spot (1st subplot) and document the behavior of the same for different sampling frequencies as viewed by the observer.

to see rotating dot