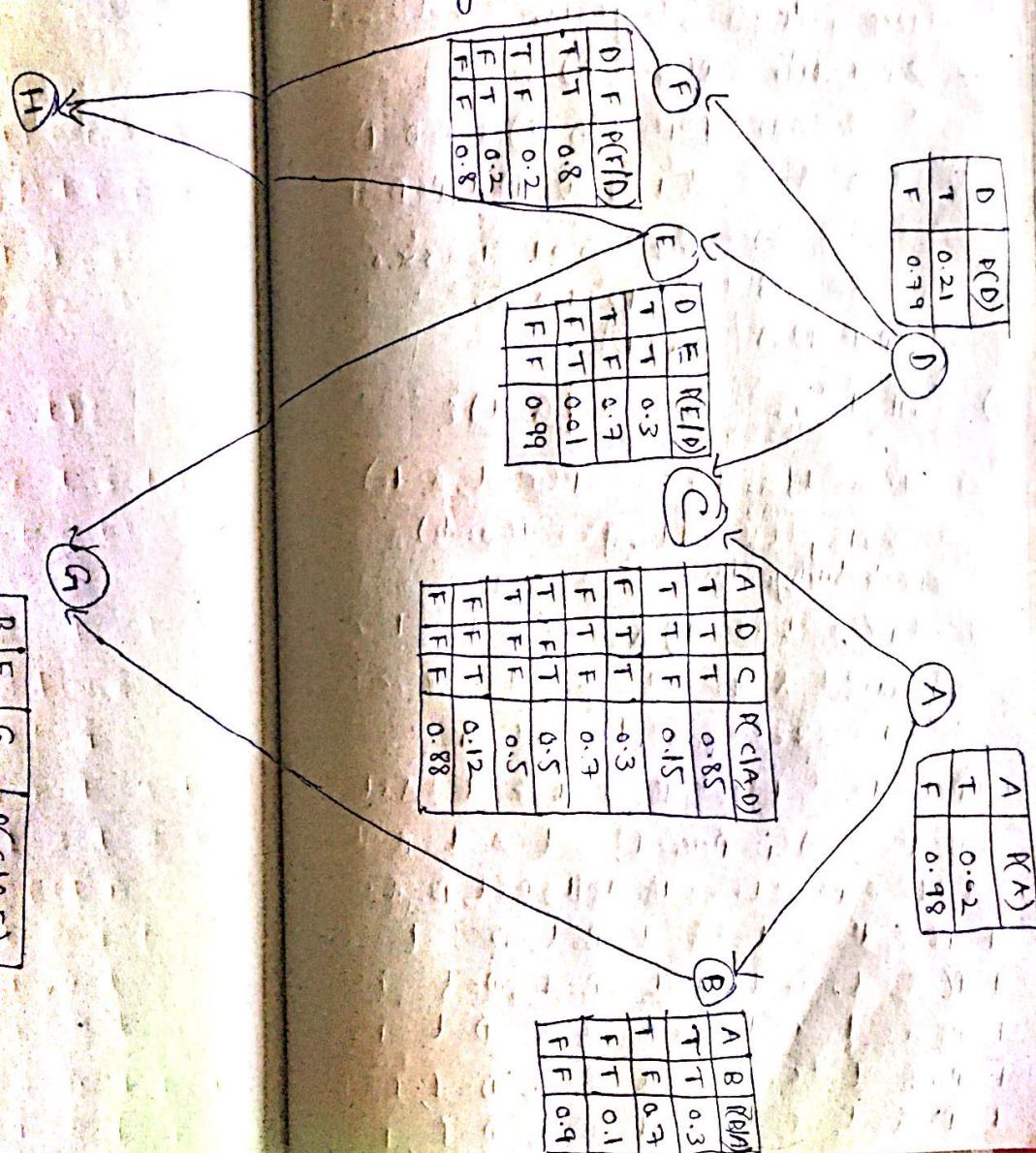


2) from the given prior & conditional probabilities below is the Bayesian network.

	E	F	H	P(H E,F)
E	T	T	T	0.71
F	T	T	F	0.29
	F	F	T	0.04
F	F	F	F	0.96
	F	T	T	0.1
F	T	F	T	0.9
	F	F	T	0.25
F	F	T	F	0.97
	F	F	F	0.75

	B	E	G	P(G B,E)
B	T	T	T	0.91
E	T	T	F	0.09
	F	T	T	0.4
F	F	T	F	0.6
	F	F	T	0.79
F	F	F	F	0.21
	F	T	F	0.97
F	F	F	F	0.03



1) What is $P(F=f)$?

Ans $P(F=f)$ is descendant of D
from above Bayesian network

$$\begin{aligned} \text{So } P(F=f) &= P(F=f, D) \\ &\Rightarrow P(F=f | D=t) * P(D=t) + \\ &\quad P(F=f | D=f) * P(D=f) \\ &\Rightarrow 0.2 \times 0.21 + 0.8 \times 0.79 \\ &\Rightarrow 0.042 + 0.632 \\ P(F=f) &\Rightarrow 0.674 \end{aligned}$$

2) What is $P(C=t | G=t)$?

Ans $P(C=t | G=t) = \frac{P(C=t, G=t)}{P(G=t)} \rightarrow ①$

(: According to conditional probability law)

Now $P(G=t, C=t) = P(G=t, C=t, A, B, D, E)$
because A, D are parents of C &
B & E are parents of G.

$$\begin{aligned} \therefore P(G=t, C=t) &= P(G=t, C=t, A=t, B=t, \\ &\quad D=t, E=t) + P(G=t, C=t, A=t, B=t, D=t, \\ &\quad E=f) + P(G=t, C=t, A=t, B=t, D=f, E=t) \\ &\quad + P(G=t, C=t, A=t, B=t, D=f, E=f) \\ &\quad + P(G=t, C=t, A=t, B=f, D=t, E=t) \\ &\quad + P(G=t, C=t, A=t, B=f, D=t, E=f) \\ &\quad + P(G=t, C=t, A=t, B=f, D=f, E=t) \\ &\quad + P(G=t, C=t, A=t, B=f, D=f, E=f) \end{aligned}$$

$$\begin{aligned}
 & + P(G=t, C=t, A=f, B=t, D=t, E=t) \\
 & + P(G=t, C=t, A=f, B=t, D=t, E=f) \\
 & + P(G=t, C=t, A=f, B=t, D=f, E=t) \\
 & + P(G=t, C=t, A=f, B=t, D=f, E=f) \\
 & + P(G=t, C=t, A=f, B=f, D=t, E=t) \\
 & + P(G=t, C=t, A=f, B=f, D=t, E=f) \\
 & + P(G=t, C=t, A=f, B=f, D=f, E=t) \\
 & + P(G=t, C=t, A=f, B=f, D=f, E=f)
 \end{aligned}$$

↓

(2)

Now expanding each term in equation (2)

$$\begin{aligned}
 a) \quad & P(G=t, C=t, A=t, B=t, D=t, E=t) \\
 \Rightarrow \quad & P(G=t | B=t, E=t) * P(C=t | A=t, D=t) \\
 * \quad & P(A=t) * P(B=t | A=t) * P(D=t) \\
 * \quad & P(E=t | D=t) \\
 \Rightarrow \quad & 0.91 * 0.85 * 0.02 * 0.3 * 0.21 \\
 * 0.3 \\
 \Rightarrow \quad & 0.00029238 \\
 b) \quad & P(G=t, C=t, A=t, B=t, D=t, E=f) \\
 \Rightarrow \quad & P(G=t | B=t, E=f) * P(C=t | A=t, D=t) \\
 * \quad & P(A=t) * P(B=t | A=t) * P(D=t) \\
 * \quad & P(E=f | D=t) \\
 \Rightarrow \quad & 0.4 * 0.85 * 0.02 * 0.3 * 0.21 * 0.7 \\
 \Rightarrow \quad & 0.00029988
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & P(G=t, C=t, A=t, B=t, D=f, E=t) \\
 \Rightarrow & P(G=t | B=t, E=t) * P(C=t | A=t, D=f) \\
 * & P(A=t) * P(B=t | A=t) * P(D=f) \\
 * & P(E=t | D=f) \\
 \Rightarrow & 0.91 * 0.5 * 0.02 * 0.3 * 0.79 * 0.01 \\
 \Rightarrow & 0.00002157
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & P(G=t, C=t, A=t, B=t, D=f, E=f) \\
 \Rightarrow & P(G=t | B=t, E=f) * P(C=t | A=t, D=f) \\
 * & P(A=t) * P(B=t | A=t) * P(D=f) \\
 * & P(E=f | D=f) \\
 \Rightarrow & 0.4 * 0.5 * 0.02 * 0.3 * 0.79 * 0.99 \\
 \Rightarrow & 0.00093852
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & P(G=t, C=t, A=t, B=f, D=t, E=t) \\
 \Rightarrow & P(G=t | B=f, E=t) * P(C=t | A=t, D=t) \\
 * & P(A=t) * P(B=f | A=t) * P(D=t) \\
 * & P(E=t | D=t) \\
 \Rightarrow & 0.79 * 0.85 * 0.02 * 0.7 * 0.21 * 0.3 \\
 \Rightarrow & 0.0005922
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & P(G=t, C=t, A=t, B=f, D=t, E=f) \\
 \Rightarrow & P(G=t | B=f, E=f) * P(C=t | A=t, D=t) \\
 * & P(A=t) * P(B=f | A=t) * P(D=t) \\
 * & P(E=f | D=t) \\
 \Rightarrow & 0.97 * 0.85 * 0.02 * 0.7 * 0.21 * \\
 & 0.7 \\
 \Rightarrow & 0.00169682
 \end{aligned}$$

g) $P(G=t, C=t, A=t, B=f, D=f, E=f)$

$$\Rightarrow P(G=t | B=f, E=f) * P(C=t | A=t, D=f)$$

$$* P(A=t) * P(B=f | A=t) * P(D=f)$$

$$* P(E=f | D=f)$$

$$\Rightarrow 0.79 * 0.5 * 0.02 * 0.7 * 0.79 *$$

$$0.01$$

$$\Rightarrow 0.00004369$$

h) $P(G=t, C=t, A=t, B=f, D=f, E=f)$

$$\Rightarrow P(G=t | B=f, E=f) * P(C=t | A=t, D=f)$$

$$* P(A=t) * P(B=f | A=t) * P(D=f) *$$

$$P(E=f | D=f)$$

$$\Rightarrow 0.97 * 0.5 * 0.02 * 0.7 * 0.79$$

$$* 0.99$$

$$\Rightarrow 0.00531046$$

i) $P(G=t, C=t, A=f, B=t, D=t, E=t)$

$$\Rightarrow P(G=t | E=t, B=t) * P(C=t | A=f, D=t)$$

$$* P(A=f) * P(D=t) * P(E=t | D=t)$$

$$* P(B=t | A=f)$$

$$\Rightarrow 0.91 * 0.3 * 0.98 * 0.21 * 0.3 * 0.1$$

$$\Rightarrow 0.001685502$$

j) $P(G=t, C=t, A=f, B=t, D=t, E=f)$

$$\Rightarrow P(G=t | E=f, B=t) * P(C=t | A=f, D=t)$$

$$* P(A=f) * P(D=t) * P(E=f | D=t)$$

$$* P(B=t | A=f)$$

$$\Rightarrow 0.4 * 0.3 * 0.98 * 0.21 * 0.7 * 0.1$$

$$\Rightarrow 0.00172872$$

$$K) P(G=t, C=t, A=f, B=t, D=f, E=f)$$

$$\Rightarrow P(G=t | B=t, E=f) * P(C=t | A=f, D=f) \\ * P(A=f) * P(B=t | A=f) * P(D=f) \\ * P(E=f | D=f)$$

$$\Rightarrow 0.91 * 0.12 * 0.98 * 0.79 * 0.01 * \\ 0.1$$

$$\Rightarrow 0.00008454264$$

$$L) P(G=t, C=t, A=f, B=t, D=f, E=f)$$

$$\Rightarrow P(G=t | B=t, E=f) * P(C=t | A=f, D=f) \\ * P(A=f) * P(B=t | A=f) * P(D=f) \\ * P(E=f | D=f)$$

$$\Rightarrow 0.4 * 0.12 * 0.98 * 0.79 * 0.99 \\ * 0.1$$

$$\Rightarrow 0.0036789984$$

$$M) P(G=t, C=t, A=f, B=f, D=t, E=t)$$

$$\Rightarrow P(G=t | E=t, B=f) * P(C=t | A=f, D=t) \\ * P(A=f) * P(D=t) * P(E=t | D=t) \\ * P(B=f | A=f)$$

$$\Rightarrow 0.79 * 0.3 * 0.98 * 0.21 * 0.3 * 0.9$$

$$\Rightarrow 0.013169142$$

$$N) P(G=t, C=t, A=f, B=f, D=t, E=f)$$

$$\Rightarrow P(G=t | E=f, B=f) * P(C=t | A=f, D=t) \\ * P(A=f) * P(D=t) * P(E=f | D=t) \\ * P(B=f | A=f)$$

$$\Rightarrow 0.97 * 0.3 * 0.98 * 0.21 * 0.7 \\ * 0.9$$

$$\Rightarrow 0.0377293$$

$$\begin{aligned}
 & p(G=t, C=t, A=f, B=f, D=f, E=t) \\
 \Rightarrow & P(G=t/E=t, B=f) * P(C=t/A=f, D=f) \\
 & * P(A=f) * P(D=f) * P(E=t/D=f) \\
 & * P(B=f/A=f) \\
 \Rightarrow & 0.79 * 0.12 * 0.98 * 0.79 * 0.01 \\
 & * 0.9 \\
 \Rightarrow & 0.00066054744
 \end{aligned}$$

$$\begin{aligned}
 & p(G=t, C=t, A=f, B=f, D=f, E=f) \\
 \Rightarrow & P(G=t/E=f, B=f) * P(C=t/A=f, D=f) \\
 & * P(A=f) * P(D=f) * P(E=f/D=f) \\
 & * P(B=f/A=f) \\
 \Rightarrow & 0.97 * 0.12 * 0.98 * 0.79 * 0.99 * \\
 & 0.9 \\
 \Rightarrow & 0.08029414008
 \end{aligned}$$

By substituting all the values in equation ②

$$\begin{aligned}
 p(G=t, C=t) = & 0.00029238 + \\
 & 0.00029988 + 0.00002157 + 0.00093852 \\
 & + 0.00059226 + 0.00169682 + 0.00004369 \\
 & + 0.00531046 + 0.001685502 + 0.00172872 \\
 & + 0.00008454264 + 0.0036789984 + 0.0131691 \\
 & - 42 + 0.0377293 + 0.00066054744 \\
 & + 0.08029414008 \Rightarrow \underline{0.1482264726}
 \end{aligned}$$

Now the denominator $P(G=t)$ in equation ①

$$P(G=t) = P(G=t, A=t, D=t, B=t, E=f)$$

because B & E are parents of G and D is parent of E & A is parent of B.

$$\begin{aligned} \therefore P(G=t, A, D, B, E) &\Rightarrow P(G=t, A=t, D=t, B=t, \\ &E=t) + P(G=t, A=t, D=t, B=t, E=f) \\ &+ P(G=t, A=t, D=t, B=f, E=t) \\ &+ P(G=t, A=t, D=t, B=f, E=f) \\ &+ P(G=t, A=t, D=f, B=t, E=t) \\ &+ P(G=t, A=t, D=f, B=t, E=f) \\ &+ P(G=t, A=t, D=f, B=f, E=t) \\ &+ P(G=t, A=t, D=f, B=f, E=f) \\ &+ P(G=t, A=f, D=t, B=t, E=t) \\ &+ P(G=t, A=f, D=t, B=t, E=f) \\ &+ P(G=t, A=f, D=t, B=f, E=t) \\ &+ P(G=t, A=f, D=t, B=f, E=f) \\ &+ P(G=t, A=f, D=f, B=t, E=t) \\ &+ P(G=t, A=f, D=f, B=t, E=f) \\ &+ P(G=t, A=f, D=f, B=f, E=t) \\ &+ P(G=t, A=f, D=f, B=f, E=f) \end{aligned}$$

→ ③

Now calculating each term in equation ③

- a) $P(G=t, A=t, D=t, B=t, E=t)$
 $\Rightarrow P(G=t/B=t, E=t) * P(B=t/A=t) *$
 $P(E=t/D=t) * P(D=t) * P(A=t)$
 $\Rightarrow 0.91 * 0.3 * 0.3 * 0.21 * 0.02$
 $\Rightarrow 0.00034398$
- b) $P(G=t, A=t, D=t, B=t, E=f)$
 $\Rightarrow P(G=t/B=t, E=f) * P(B=t/A=t)$
 $* P(E=f/D=t) * P(D=t) * P(A=t)$
 $\Rightarrow 0.4 * 0.3 * 0.7 * 0.21 * 0.02$
 $\Rightarrow 0.0003528$
- c) $P(G=t, A=t, D=t, B=f, E=t)$
 $\Rightarrow P(G=t/B=f, E=t) * P(B=f/A=t)$
 $* P(E=t/D=t) * P(D=t) * P(A=t)$
 $\Rightarrow 0.79 * 0.7 * 0.3 * 0.21 * 0.02$
 $\Rightarrow 0.00069678$
- d) $P(G=t, A=t, D=t, B=f, E=f)$
 $\Rightarrow P(G=t/B=f, E=f) * P(B=f/A=t)$
 $* P(E=f/D=t) * P(D=t) * P(A=t)$
 $\Rightarrow 0.97 * 0.7 * 0.7 * 0.21 * 0.02$
 $\Rightarrow 0.00199626$
- e) $P(G=t, A=t, D=f, B=t, E=t)$
 ~~$\Rightarrow P(G=t/B=t, E=t) * P(B=t/A=t)$~~
 $* P(E=t/D=f) * P(D=f) * P(A=t)$
 $\Rightarrow 0.91 * 0.3 * 0.01 * 0.79 * 0.02$
 $\Rightarrow 0.000043134$

$$\begin{aligned}
 f) \quad & P(G=t, A=t, D=f, B=t, E=f) \\
 \Rightarrow & P(G=t | B=t, E=f) * P(B=t | A=t) \\
 & * P(E=f | D=f) * P(D=f) * P(A=t) \\
 \Rightarrow & 0.4 * 0.3 * 0.99 * 0.79 * 0.02 \\
 \Rightarrow & 0.00187704
 \end{aligned}$$

$$\begin{aligned}
 g) \quad & P(G=t, A=t, D=f, B=f, E=t) \\
 \Rightarrow & P(G=t | B=f, E=t) * P(B=f | A=t) \\
 & * P(E=t | D=f) * P(D=f) * P(A=t) \\
 \Rightarrow & 0.79 * 0.7 * 0.01 * 0.79 * 0.02 \\
 \Rightarrow & 0.0000873744
 \end{aligned}$$

$$\begin{aligned}
 h) \quad & P(G=t, A=t, D=f, B=f, E=f) \\
 \Rightarrow & P(G=t | B=f, E=f) * P(B=f | A=t) \\
 & * P(E=f | D=f) * P(D=f) * P(A=t) \\
 \Rightarrow & 0.97 * 0.7 * 0.99 * 0.79 * 0.02 \\
 \Rightarrow & 0.010620918
 \end{aligned}$$

$$\begin{aligned}
 i) \quad & P(G=t, A=f, D=t, B=t, E=t) \\
 \Rightarrow & P(G=t | B=t, E=t) * P(B=t | A=f) \\
 & * P(E=t | D=t) * P(D=t) * P(A=f) \\
 \Rightarrow & 0.91 * 0.1 * 0.3 * 0.98 * 0.21 \\
 \Rightarrow & 0.00561834
 \end{aligned}$$

$$\begin{aligned}
 j) \quad & P(G=t, A=f, D=t, B=t, E=f) \\
 \Rightarrow & P(G=t | B=t, E=f) * P(B=t | A=f) \\
 & * P(E=f | D=t) * P(D=t) * P(A=f) \\
 \Rightarrow & 0.4 * 0.1 * 0.7 * 0.98 * 0.21 \\
 \Rightarrow & 0.0057624
 \end{aligned}$$

$$k) P(G=t, A=f, D=t, B=f, E=t)$$

$$\Rightarrow P(G=t / B=f, E=t) * P(B=f / A=f)$$

$$* P(E=t / D=t) * P(D=t) * P(A=f)$$

$$\Rightarrow 0.79 * 0.9 * 0.3 * 0.98 * 0.21$$

$$\Rightarrow 0.04389714$$

$$l) P(G=t, A=f, D=t, B=f, E=f)$$

$$\Rightarrow P(G=t / B=f, E=f) * P(B=f / A=f)$$

$$* P(E=f / D=t) * P(D=t) * P(A=f)$$

$$\Rightarrow 0.97 * 0.9 * 0.7 * 0.98 * 0.21$$

$$\Rightarrow 0.12576438$$

$$m) P(G=t, A=f, D=f, B=t, E=t)$$

$$\Rightarrow P(G=t / B=t, E=t) * P(B=t / A=f)$$

$$* P(E=t / D=f) * P(D=f) * P(A=f)$$

$$\Rightarrow 0.91 * 0.1 * 0.01 * 0.98 * 0.79$$

$$\Rightarrow 0.000704522$$

$$n) P(G=t, A=f, D=f, B=t, E=f)$$

$$\Rightarrow P(G=t / B=t, E=f) * P(B=t / A=f)$$

$$* P(E=f / D=f) * P(D=f) * P(A=f)$$

$$\Rightarrow 0.4 * 0.1 * 0.99 * 0.98 * 0.79$$

$$\Rightarrow 0.03065032$$

$$o) P(G=t, A=f, D=f, B=f, E=t)$$

$$\Rightarrow P(G=t / B=f, E=t) * P(B=f / A=f)$$

$$* P(E=t / D=f) * P(D=f) * P(A=f)$$

$$\Rightarrow 0.79 * 0.9 * 0.01 * 0.98 * 0.79$$

$$\Rightarrow 0.005504562$$

$$P(G=t, A=f, D=f, B=f, E=f)$$

$$\Rightarrow P(G=t/B=f, E=f) * P(B=f/A=f)$$

$$* P(E=f/D=f) * P(A=f) * P(D=f)$$

$$\Rightarrow 0.97 * 0.9 * 0.99 * 0.98 * 0.79$$

$$\Rightarrow 0.669117834$$

By substituting all the values
in equation ③

$$\begin{aligned}
 P(G=t) = & 0.00034398 + 0.0003528 \\
 & + 0.00069678 + 0.00199626 + 0.000043134 \\
 & + 0.00187764 + 0.0000873744 + 0.010620918 \\
 & + 0.00561834 + 0.0057624 + 0.04389714 \\
 & + 0.12576438 + 0.000704522 + \\
 & 0.03065832 + 0.005504562 + 0.669117 \\
 & 834 \Rightarrow \underline{0.9030457844}
 \end{aligned}$$

By substituting above values in
equation ②

$$P(c=t | G=t) = \frac{0.1482264726}{0.9030457844}$$

$$\boxed{P(c=t | G=t)} = \boxed{0.16414059526}$$

$$\boxed{P(c=t | G=t)} = \boxed{0.16414059526}$$

3) What is $P(E=f | A=t, B=f)$?

$$\text{Ans} \quad P(E=f | A=t, B=f) = \frac{P(E=f, A=t, B=f)}{P(A=t, B=f)}$$

(∵ According to)
Conditional law. 1

$$\text{Now } P(E=f, A=t, B=f) \Rightarrow P(E=f, A=t, B=f | D)$$

because D is parent of E.

$$\therefore P(E=f, A=t, B=f) = P(E=f, A=t, B=f | D=t) + P(E=f, A=t, B=f | D=f)$$

↓ 2

Now calculating each term in
Equation ②

a) $P(E=f, A=t, B=f, D=t) \times P(E=f | D=t)$

$$= P(E=f | D=t) * P(B=f | A=t) * P(A=t) * P(D=t)$$

$$= 0.7 * 0.7 * 0.02 * 0.21$$

$$= 0.002058$$

b) $P(E=f, A=t, B=f, D=f)$

$$= P(E=f | D=f) * P(B=f | A=t) * P(A=t) * P(D=f)$$

$$= 0.99 * 0.7 * 0.02 * 0.79$$

$$= 0.0109494$$

By substituting the above values
in equation ②

$$P(E=f, A=t, B=f) = 0.002058 + 0.0109494$$

$$\Rightarrow \underline{0.0130074}$$

Now the denominator $P(A=t, B=f)$
in equation ①

$$P(A=t, B=f) = P(B=f/A=t) \cdot P(A=t)$$

(conditional law) $\Rightarrow 0.7 \times 0.02$
 $\Rightarrow \underline{0.014}$

∴ By substituting above values
in equation ①

$$P(E=f/A=t, B=f) = \frac{0.0130074}{0.014}$$

$$\therefore \boxed{P(E=f/A=t, B=f) \Rightarrow 0.9290714286}$$

4) What is $P(C=t, G=t | H=f)$

$$P(C=t, G=t | H=f) = \frac{P(C=t, G=t, H=f)}{P(H=f)}$$

(\because According to
conditional probability
law)

$$\text{Now } P(C=t, G=t, H=f) = P(C=t, G=t, H=f, A, D, B, E, F)$$

because A, D are parents of C and
 B, E are parents of G and F is
parent of H .

$$\begin{aligned} P(C=t, G=t, H=f) &= P(C=t, G=t, H=f, \\ &A=t, D=t, B=t, E=t, F=t) + P(C=t, G=t, \\ &H=f, A=t, D=f, B=t, E=t, F=t) + \\ &P(C=t, G=t, H=f, A=f, D=t, B=t, E=t, \\ &F=t) + P(C=t, G=t, H=f, A=f, D=f, B=t, \\ &E=t, F=t) + P(C=t, G=t, H=f, A=t, \\ &D=t, B=f, E=t, F=t) + P(C=t, G=t, \\ &H=f, A=t, D=f, B=f, E=t, F=t) + \\ &P(C=t, G=t, H=f, A=f, D=t, B=f, \\ &E=t, F=t) + P(C=t, G=t, H=f, A=f, \\ &D=f, B=f, E=t, F=t) + P(C=t, G=t, \\ &H=f, A=t, D=t, B=t, E=f, F=t) + \\ &P(C=t, G=t, H=f, A=t, D=f, B=t, E=f, \\ &F=t) + P(C=t, G=t, H=f, A=f, D=t, \\ &B=t, E=f, F=t) + P(C=t, G=t, H=f) \end{aligned}$$

$$\begin{aligned}
& P(A=t, E=f, F=t) + P(c=t, G=t, H=f) \\
& P(A=t, D=t, B=f, E=f, F=t) + P(c=t, \\
& G=t, H=f, A=t, D=f, B=f, E=f, F=t) \\
& + P(c=t, G=t, H=f, A=f, D=t, B=f, E=f, \\
& F=t) + P(c=t, G=t, H=f, A=f, D=f, \\
& B=f, E=f, F=t) + P(c=t, G=t, H=f, \\
& A=t, D=t, B=t, E=t, F=f) + P(c=t, G=t, \\
& H=f, A=t, D=f, B=t, E=t, F=f) + P(c=t, \\
& G=t, H=f, A=f, D=t, B=t, E=t, F=f) \\
& + P(c=t, G=t, H=f, A=f, D=f, B=t, E=t, \\
& F=f) + P(c=t, G=t, H=f, A=t, D=t, \\
& B=f, E=t, F=f) + P(c=t, G=t, H=f, \\
& A=t, D=f, B=f, E=t, F=f) + P(c=t, G=t, \\
& H=f, A=f, D=t, B=f, E=t, F=f) + P(c=t, \\
& G=t, H=f, A=f, D=f, B=f, E=t, F=f) + \\
& P(c=t, G=t, H=f, A=t, D=t, B=t, E=f, F=f) \\
& + P(c=t, G=t, H=f, A=t, D=f, B=t, E=f, \\
& F=f) + P(c=t, G=t, H=f, A=f, D=t, B=t, \\
& E=f, F=f) + P(c=t, G=t, H=f, A=t, E=f, \\
& F=f) + P(c=t, G=t, H=f, A=t, D=t, \\
& B=f, E=f, F=f) + P(c=t, G=t, H=f, \\
& A=t, D=f, B=f, E=f, F=f) + P(c=t, G=t, \\
& H=f, A=f, D=t, B=f, E=f, F=f) + \\
& P(c=t, G=t, H=f, A=f, D=f, B=f, E=f, \\
& F=f) \rightarrow ②
\end{aligned}$$

Now calculating each term in equation ②.

(i) $P(C=t, G=t, H=f, A=t, D=t, B=t, E=t, F=t)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=t, B=t) * P(H=f | E=t, F=t) * P(A=t) * P(D=t) * P(B=t | A=t) * P(E=t | D=t) * P(F=t | D=t)$$

$$\Rightarrow 0.85 * 0.91 * 0.29 * 0.02 * 0.21 * 0.3 * 0.3 * 0.8$$

$$\Rightarrow \cancel{0.0001660735} \approx 0.000067832856$$

(ii) $P(C=t, G=t, H=f, A=t, D=t, B=t, E=t, F=f)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=t, B=t) * P(H=f | E=t, F=f) * P(A=t) * P(D=t) * P(B=t | A=t) * P(E=t | D=t) * P(F=f | D=t)$$

$$\Rightarrow 0.85 * 0.91 * 0.96 * 0.02 * 0.21 * 0.3 * 0.3 * 0.2$$

$$\Rightarrow \cancel{0.000002329} \approx 0.000056137536$$

(iii) $P(C=t, G=t, H=f, A=t, D=t, B=t, E=f, F=t)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=f, B=t) * P(H=f | E=f, F=t) * P(A=t) * P(D=t) * P(B=t | A=t) * P(E=f | D=t) * P(F=t | D=t)$$

$$\Rightarrow 0.85 * 0.4 * 0.9 * 0.02 * 0.2$$

$$* 0.3 * 0.7 * 0.8$$

$$\Rightarrow \underline{0.0000239901} = 0.0002159136$$

(iv) $P(C=t, G=t, H=f, A=t, D=t, B=t, E=f, F=f)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=f, B=t) * P(H=f | E=f, F=f) * P(A=t) * P(D=t) * P(B=t | A=t) * P(E=f | D=t) * P(F=f | D=t)$$

$$\Rightarrow 0.85 * 0.4 * 0.75 * 0.02 * 0.21$$

$$* 0.3 * 0.7 * 0.2$$

$$\Rightarrow \underline{0.0000111971} = 0.000044982$$

(v) $P(C=t, G=t, H=f, A=t, D=t, B=f, E=t, F=t)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=t, B=f) * P(H=f | E=t, F=t) * P(A=t) * P(D=t) * P(B=f | A=t) * P(E=t | D=t) * P(F=t | D=t)$$

$$\Rightarrow 0.85 * 0.79 * 0.29 * 0.02 * 0.21$$

$$* 0.7 * 0.3 * 0.8$$

$$\Rightarrow \underline{0.000336405384} = 0.000137405016$$

(vi) $P(C=t, G=t, H=f, A=t, D=t, B=f, E=t, F=f)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=t, B=f) * P(H=f | E=t, F=f)$$

$$* P(A=t) * P(D=t) * P(B=f | A=t)$$

$$* P(E=t | D=t) * P(F=f | D=t)$$

$$\Rightarrow 0.85 * 0.79 * 0.97 * 0.02 * 0.21 \\ * 0.7 * 0.3 * 0.2$$

$$\Rightarrow \cancel{0.00013574568} = 0.000113714496 \approx$$

(ii) $P(C=t, G=t, H=f, A=t, D=t, B=f, E=f, F=t)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=f, B=f) * P(H=f | E=f, F=t) * \\ P(A=t) * P(D=t) * P(B=f | A=t) \\ * P(E=f | D=t) * P(F=t | D=t)$$

$$\Rightarrow 0.85 * 0.97 * 0.9 * 0.02 * \\ 0.21 * 0.7 * 0.7 * 0.8$$

$$\Rightarrow \cancel{0.00013574568} = 0.00122171112 \approx$$

(viii) $P(C=t, G=t, H=f, A=t, D=t, B=f, E=f, F=f)$

$$\Rightarrow P(C=t | A=t, D=t) * P(G=t | E=f, B=f) * P(H=f | E=f, F=f) * \\ P(A=t) * P(D=t) * P(B=f | A=t) \\ * P(E=f | D=t) * P(F=f | D=t)$$

$$\Rightarrow 0.85 * 0.97 * 0.75 * 0.02 * 0.21 \\ * 0.7 * 0.7 * 0.2$$

$$\Rightarrow \cancel{0.000818745} = 0.00025452315 \approx$$

$$(ix) P(C=t, G=t, H=f, A=t, D=f, B=t, E=t, F=t)$$

$$\Rightarrow P(C=t | A=t, D=f) * P(G=t | E=t, B=t) * P(H=f | E=t, F=t) * P(A=t) * P(D=f) * P(B=t | A=t) * P(E=t | D=f) * P(F=t | D=f)$$

$$\Rightarrow 0.5 * 0.91 * 0.29 * 0.02 * 0.79 * 0.3 * 0.01 * 0.2$$

$$\Rightarrow \cancel{0.000003062514} = 0.000001250886$$

$$(x) P(C=t, G=t, H=f, A=t, D=f, B=t, E=t, F=f)$$

$$\Rightarrow P(C=t | A=t, D=f) * P(G=t | E=t, B=t) * P(H=f | E=t, F=f) * P(A=t) * P(D=f) * P(B=t | A=t) * P(E=t | D=f) * P(F=f | D=f)$$

$$\Rightarrow 0.5 * 0.91 * 0.95 * 0.02 * 0.79 * 0.3 * 0.01 * 0.8$$

$$\Rightarrow \cancel{0.0000069014} = 0.000016563456$$

$$(xi) P(C=t, G=t, H=f, A=t, D=f, B=t, E=f, F=t)$$

$$\Rightarrow P(C=t | A=t, D=f) * P(G=t | E=f, B=t) * P(H=f | E=f, F=t) * P(A=t) * P(D=f) * P(B=t | A=t) * P(E=f | D=f) * P(F=t | D=f)$$

$$\Rightarrow 0.5 * 0.4 * 0.9 * 0.02 * 0.79 * 0.3 * 0.99 * 0.2$$

$$\Rightarrow \cancel{0.0000187704} = 0.0001689336$$

(xii) $P(C=t, G=t, H=f, A=t, D=f, B=t, E=f, F=f)$

$$\begin{aligned} & \Rightarrow P(C=t | A=t, D=f) * P(G=t | E=f, F=f) \\ & * P(A=t) * P(D=f) * P(B=t | A=t) \\ & * P(E=f | D=f) * P(F=f | D=f) \\ & \Rightarrow 0.5 * 0.4 * 0.75 * 0.02 * 0.79 \\ & * 0.3 * 0.99 * 0.8 \end{aligned}$$

$$\Rightarrow \cancel{0.0000187704} = 0.000563112$$

(xiii) $P(C=t, G=t, H=f, A=t, D=f, B=f, E=t, F=f)$

$$\begin{aligned} & \Rightarrow P(C=t | A=t, D=f) * P(G=t | E=t, B=f) \\ & * P(H=f | E=t, F=f) * P(A=t) \\ & * P(D=f) * P(B=f | A=t) * \\ & P(E=t | D=f) * P(F=f | D=f) \\ & \Rightarrow 0.5 * 0.79 * 0.29 * 0.02 * \\ & 0.79 * 0.7 * 0.01 * 0.2 \end{aligned}$$

$$\Rightarrow \cancel{0.000006203554} = 0.00000253384$$

(xiv) $P(C=t, G=t, H=f, A=t, D=f, B=f, E=t, F=f)$

$$\begin{aligned} & \Rightarrow P(C=t | A=t, D=f) * P(G=t | E=t, B=f) \\ & * P(H=f | E=t, F=f) * P(A=t) \\ & * P(D=f) * P(B=f | A=t) * \\ & P(E=t | D=f) * P(F=f | D=f) \end{aligned}$$

$$\Rightarrow 0.5 * 0.79 * 0.97 * 0.02 * 0.79 \\ * 0.7 * 0.01 * 0.8$$

$$\Rightarrow \cancel{0.000001391984} = 0.000033551616$$

(xv) $P(C=t, G=t, H=f, A=t, D=f, \\ B=f, E=f, F=t)$

$$\Rightarrow P(C=t | A=t, D=f) * P(G=t | E=f, \\ B=f) * P(H=f | E=f, F=t) \\ * P(A=t) * P(D=f) * P(B=f | A=t) \\ * P(E=f | D=f) * P(F=t | D=f)$$

$$\Rightarrow 0.5 * 0.97 * 0.9 * 0.02 * \\ 0.79 * 0.7 * 0.99 * 0.2$$

$$\Rightarrow \cancel{0.00010620918} = 0.0009558826$$

(xvi) $P(C=t, G=t, H=f, A=t, D=f, \\ B=f, E=f, F=f)$

$$\Rightarrow P(C=t | A=t, D=f) * P(G=t | \\ E=f, B=f) * P(H=f | E=f, F=f) \\ * P(A=t) * P(D=f) * P(B=f | A=t) \\ * P(E=f | D=f) * P(F=f | D=f)$$

$$\Rightarrow 0.5 * 0.97 * 0.75 * 0.02 \\ * 0.79 * 0.7 * 0.99 * 0.8$$

$$\Rightarrow \cancel{0.0010620918} = 0.0031862754$$

(xvii) $P(C=t, G=t, H=f, A=f, D=t, \\ B=t, E=t, F=t)$

$$\Rightarrow P(C=t | A=f, D=t) * P(G=t | E=t, \\ B=t) * P(H=f | E=t, F=t)$$

$$* P(A=f) * P(D=t) * P(B=t/A=f)$$

$$* P(E=t/D=t) * P(F=t/D=t)$$

$$\Rightarrow 0.3 * 0.91 * 0.29 * 0.98 * 0.21$$

$$* 0.1 * 0.3 * 0.8$$

$$\Rightarrow \cancel{0.000957365136} 0.000391036464$$

(xviii) $P(C=t, G=t, H=f, A=f, D=t,$
 $B=t, E=t, F=f)$

$$\Rightarrow P(C=t/A=f, D=t) * P(G=t/E=t,$$

$$* P(H=f/E=t, F=f) * P(A=f) *$$

$$P(D=t) * P(B=t/A=f) *$$

$$P(E=t/D=t) * P(F=f/D=t)$$

$$\Rightarrow 0.3 * 0.91 * 0.96 * 0.98 *$$

$$0.21 * 0.1 * 0.3 * 0.2$$

$$\Rightarrow \cancel{0.000013484616} 0.000323616384$$

(ix) $P(C=t, G=t, H=f, A=f, D=t,$
 $B=t, E=f, F=t)$

$$\Rightarrow P(C=t/A=f, D=t) * P(G=t/E=f,$$

$$* P(H=f/E=f, F=t) * P(A=f)$$

$$* P(D=t) * P(B=t/A=f)$$

$$* P(E=f/D=t) * P(F=t/D=t)$$

$$\Rightarrow 0.3 * 0.4 * 0.9 * 0.98 * 0.21$$

$$* 0.1 * 0.7 * 0.8$$

$$\Rightarrow \cancel{0.0001382976} 0.0012446784$$

(xx) $P(C=t, G=t, H=f, A=f, D=t,$
 $B=t, E=f, F=f)$

$$\Rightarrow P(C=t/A=f, D=t) * P(G=t/E=f, B=t)$$

$$* P(H=f/E=f, F=f) * P(A=f)$$

$$* P(D=t) * P(B=t/A=f)$$

$$* P(E=f/D=t) * P(F=f/D=t)$$

$$\Rightarrow 0.3 * 0.4 * 0.75 * 0.98$$

$$* 0.21 * 0.1 * 0.7 * 0.2$$

$$\Rightarrow \underline{0.000086426} = 0.000259308$$

(xi) $P(C=t, G=t, H=f, A=f, D=t,$
 $B=f, E=t, F=t)$

$$\Rightarrow P(C=t/A=f, D=t) * P(G=t/E=t, B=f)$$

$$* P(H=f/E=t, F=t) * P(A=f)$$

$$* P(D=t) * P(B=f/A=f)$$

$$* P(E=t/D=t) * P(F=t/D=t)$$

$$\Rightarrow 0.3 * 0.79 * 0.29 * 0.98 * 0.21$$

$$* 0.9 * 0.3 * 0.8$$

$$\Rightarrow \underline{0.007486072656} = 0.003055240944$$

(xii) $P(C=t, G=t, H=f, A=f, D=t,$
 $B=f, E=t, F=f)$

$$\Rightarrow P(C=t/A=f, D=t) * P(G=t/E=t, B=f)$$

$$* P(H=f/E=t, F=f) * P(A=f)$$

$$* P(D=t) * P(B=f/A=f) *$$

$$P(E=t/D=t) * P(F=f/D=t)$$

$$\Rightarrow 0.3 * 0.79 * 0.97 * 0.98 *$$

$$0.21 * 0.9 * 0.3 * 0.2$$

$$\Rightarrow \underline{0.000105353136} = 0.002528475264$$

(xiii) $P(C=t, G=t, H=f, A=f, D=t,$
 $B=f, E=f, F=t)$

$$\Rightarrow P(C=t/A=f, D=t) * P(G=t/E=f,$$

 $B=f)$

$$* P(H=f/E=f, F=t) * P(A=f)$$

$$* P(D=t) * P(B=f/A=f) *$$

$$P(E=f/D=t) * P(F=t/D=t)$$

$$\Rightarrow 0.3 * 0.97 * 0.9 * 0.98 * 0.21$$

$$* 0.9 * 0.7 * 0.8.$$

$$\Rightarrow \underline{0.00301834512} = 0.02716510608$$

(xiv) $P(C=t, G=t, H=f, A=f, D=t,$
 $B=f, E=f, F=f)$

$$\Rightarrow P(C=t/A=f, D=t) * P(G=t/E=f,$$

 $B=f)$

$$* P(H=f/E=f, F=f) * P(A=f)$$

$$* P(D=t) * P(B=f/A=f)$$

$$* P(E=f/D=t) * P(F=f/D=t)$$

$$\Rightarrow 0.3 * 0.97 * 0.75 * 0.98 *$$

$$0.21 * 0.9 * 0.7 * 0.2$$

$$\Rightarrow \underline{0.0018864157} = 0.005659397$$

(xxv) $P(C=t, G=t, H=f, A=f, D=f, B=t, E=t, F=t)$

$$\Rightarrow P(C=t / A=f, D=f) * P(G=t / E=f, B=t)$$

$$* P(H=f / E=t, F=t) * P(A=f)$$

$$* P(D=f) * P(B=t / A=f)$$

$$* P(E=t / D=f) * P(F=t / D=f)$$

$$\Rightarrow 0.12 * 0.91 * 0.29 * 0.98 *$$

$$0.79 * 0.1 * 0.01 * 0.2$$

$$\Rightarrow 0.00000490347312 \approx$$

(xxvi) $P(C=t, G=t, H=f, A=f, D=f, B=t, E=t, F=f)$

$$\Rightarrow P(C=t / A=f, D=f) * P(G=t / E=t, B=t)$$

$$* P(H=f / E=t, F=f) * P(A=f)$$

$$* P(D=f) * P(B=t / A=f)$$

$$* P(E=t / D=f) * P(F=f / D=f)$$

$$\Rightarrow 0.12 * 0.91 * 0.96 * 0.98$$

$$* 0.79 * 0.1 * 0.01 * 0.8$$

$$\Rightarrow 0.00006492874752 \approx$$

(xxvii) $P(C=t, G=t, H=f, A=f, D=f, B=t, E=f, F=t)$

$$\Rightarrow P(C=t / A=f, D=f) * P(G=t / E=f, B=t)$$

$$* P(H=f / E=f, F=t) * P(A=f)$$

$$* P(D=f) * P(B=t / A=f)$$

$$* P(E=f | D=f) * P(F=t | D=f)$$

$$\Rightarrow 0.12 * 0.4 * 0.9 * 0.98 * 0.7$$
$$* 0.1 * 0.99 * 0.2$$

$$\Rightarrow 0.000662219712$$

(xviii) $P(C=t, G=t, H=f, A=f, D=f, B=t, E=f, F=f)$

$$\Rightarrow P(C=t | A=f, D=f) * P(G=t | E=f, B=t)$$

$$* P(H=f | E=f, F=f) * P(A=f)$$

$$* P(D=f) * P(B=t | A=f)$$

$$* P(E=f | D=f) * P(F=f | D=f)$$

$$\Rightarrow 0.12 * 0.4 * 0.75 * 0.98$$

$$* 0.79 * 0.1 * 0.99 * 0.8$$

$$\Rightarrow \cancel{0.00058677696} = 0.00220739904$$

(xxix) $P(C=t, G=t, H=f, A=f, D=f, B=f, E=t, F=t)$

$$\Rightarrow P(C=t | A=f, D=f) * P(G=t | E=t, B=f)$$

$$* P(H=f | E=t, F=t) * P(A=f)$$

$$* P(D=f) * P(B=f | A=f)$$

$$* P(E=t | D=f) * P(F=t | D=f)$$

$$\Rightarrow 0.12 * 0.79 * 0.29 * 0.98 *$$
$$0.79 * 0.9 * 0.01 * 0.2$$

$$\Rightarrow 0.00003831175152$$

(xx) $P(C=t, G=t, H=f, A=f, D=f, B=f, E=t, F=f)$

$$\begin{aligned} \Rightarrow & P(C=t / A=f, D=f) * \\ & P(G=t / E=t, B=f) * P(H=f / E=t, \\ & F=f) * P(A=f) * P(D=f) * \\ & P(B=f / A=f) * P(E=t / D=f) \\ & * P(F=f / D=f) \end{aligned}$$

$$\begin{aligned} \Rightarrow & 0.12 * 0.79 * 0.96 * 0.98 \\ & * 0.79 * 0.9 * 0.01 * 0.8 \end{aligned}$$

$$\Rightarrow 0.0005073004339$$

(xxxii) $P(C=t, G=t, H=f, A=f, D=f, B=f, E=f, F=t)$

$$\begin{aligned} \Rightarrow & P(C=t / A=f, D=f) * P(G=t / E=f, \\ & B=f) \\ & * P(H=f / E=f, F=t) * P(A=f) * \\ & P(D=f) * P(B=f / A=f) * P(E=f / D=f) \\ & * P(F=t / D=f) \end{aligned}$$

$$\begin{aligned} \Rightarrow & 0.12 * 0.97 * 0.9 * 0.98 * \\ & 0.79 * 0.9 * 0.99 * 0.2 \end{aligned}$$

$$\Rightarrow 0.01445294521$$

(xxii) $P(C=t, G=t, H=f, A=f, D=f, B=f, E=f, F=f)$

$$\Rightarrow P(C=t/A=f, D=f) * P(G=t/E=f, B=f)$$

$$* P(H=f/E=f, F=f) * P(A=f)$$

$$* P(D=f) * P(B=f/A=f)$$

$$* P(E=f/D=f) * P(F=f/D=f)$$

$$\Rightarrow 0.12 * 0.97 * 0.75 * 0.98 \\ * 0.79 * 0.9 * 0.99 * 0.8$$

$$\Rightarrow 0.04817648405.$$

Substituting all the above values
in equation ②

$$P(C=t, G=t, H=f) = 0.000067832856 \\ + 0.000056137536 + 0.0002159136 \\ + 0.000044982 + 0.000137405016 \\ + 0.000113714496 + 0.00122171112 \\ + 0.00025452315 + 0.000001250886 \\ + 0.000016563456 + 0.0001689336 \\ + 0.000563112 + 0.000002533846 \\ + 0.000033551616 + 0.00095588262 \\ + 0.0031862754 + 0.000391036464 \\ + 0.000323616384 + 0.0012446784 \\ + 0.000259308 + 0.003055240944$$

$$\begin{aligned} & + 0.002528475264 + \\ & 0.02716510608 + 0.0056593971 \\ & + 0.00000490347312 + 0.000064928 \\ & 74752 + 0.000662219712 \\ & + 0.00220739904 \\ & + 0.00003831175152 \\ & + 0.0005673004339 \\ & + 0.01445294521 \\ & + 0.04817648405 \\ \Rightarrow & \underline{\underline{0.1137816743}} \end{aligned}$$

Now the denominator $P(H=F)$ in
Equation ①

$$P(H=f) = P(H=f, F, E, D)$$

because E, F are parents of
 H & they both have a common
parent D .

$$\begin{aligned} \therefore P(H=f) &= P(H=f, F=t, E=t, D=t) \\ &+ P(H=f, F=t, E=t, D=f) + P(H=f, \\ &F=t, E=f, D=t) + P(H=f, F=t, E=f, \\ &D=f) + P(H=f, F=f, E=t, D=t) \\ &+ P(H=f, F=f, E=t, D=f) + \\ &P(H=f, F=f, E=f, D=t) + \\ &P(H=f, F=f, E=f, D=f) \rightarrow ③ \end{aligned}$$

Now calculating each term in
equation ③

a) $P(H=f, F=t, E=t, D=t)$

$$\Rightarrow P(H=f | F=t, E=t) * P(F=t | D=t) * P(E=t | D=t) * P(D=t)$$

$$\Rightarrow 0.29 * 0.8 * 0.3 * 0.21$$

$$\Rightarrow 0.014616$$

b) $P(H=f, F=t, E=t, D=f)$

$$\Rightarrow P(H=f | F=t, E=t) * P(F=t | D=f) * P(E=t | D=f) * P(D=f)$$

$$\Rightarrow 0.29 * 0.2 * 0.01 * 0.79$$

$$\Rightarrow 0.0004582$$

c) $P(H=f, F=t, E=f, D=t)$

$$\Rightarrow P(H=f | F=t, E=f) * P(F=t | D=t) \\ * P(E=f | D=t) * P(D=t)$$

$$\Rightarrow 0.9 * 0.8 * 0.7 * 0.21$$

$$\Rightarrow 0.10584$$

d) $P(H=f, F=t, E=f, D=f)$

$$\Rightarrow P(H=f | F=t, E=f) * P(F=t | D=f) \\ * P(E=f | D=f) * P(D=f)$$

$$\Rightarrow 0.9 * 0.2 * 0.99 * 0.79$$

$$\Rightarrow 0.140778$$

e) $P(H=f, F=f, E=t, D=t)$

$$\Rightarrow P(H=f | F=f, E=t) * P(F=f | D=t) \\ * P(E=t | D=t) * P(D=t)$$

$$\Rightarrow 0.96 * 0.2 * 0.3 * 0.21$$

$$\Rightarrow 0.012096$$

f) $P(H=f, F=f, E=t, D=f)$

$$\Rightarrow P(H=f | F=f, E=t) * P(F=f | D=f) \\ * P(E=t | D=f) * P(D=f)$$

$$\Rightarrow 0.96 * 0.8 * 0.01 * 0.79$$

$$\Rightarrow 0.0060672$$

$$g) P(H=f, F=f, E=f, D=t)$$

$$\Rightarrow P(H=f | F=f, E=f) * P(F=f | D=t)$$

$$* P(E=f | D=t) * P(D=t)$$

$$\Rightarrow 0.75 * 0.2 * 0.7 * 0.21$$

$$\Rightarrow 0.02205$$

$$h) P(H=f, F=f, E=f, D=f)$$

$$\Rightarrow P(H=f | F=f, E=f) * P(F=f | D=f)$$

$$* P(E=f | D=f) * P(D=f)$$

$$\Rightarrow 0.75 * 0.8 * 0.99 * 0.79$$

$$\Rightarrow 0.46926$$

Now substituting all the above values in equation ③

$$P(H=f) = 0.014616 + 0.0004582$$

$$+ 0.10584 + 0.140778 + 0.012096$$

$$+ 0.0060672 + 0.02205 + 0.46926$$

$$\Rightarrow \underline{0.7711654}$$

By substituting outcomes of equation ② & ③ in ①

$$P(C=t, G=t | H=f) = \frac{P(C=t, G=t, H=f)}{P(H=f)}$$

$$\Rightarrow \frac{0.1137816743}{0.7711654}$$

$$\Rightarrow P(c=t, g=t \mid H=f) = 0.1475450977$$

3) Markov Models

1) If $P(H_2 | [B, R]) = (0.3, 0.5, 0.2)$
 calculate $P(H_4 | [B, R, R, F])$. show your work.

Given $P(H_2 | [B, R]) = (0.3, 0.5, 0.2)$

$$P(H_4 | [B, R, R, F]) = P(H_4 | [B, R, R], F)$$

$$\Rightarrow \propto \cdot P(F/H_4, [B, R, R]) * P(H_4 | [B, R, R])$$

$$\Rightarrow \propto \cdot P(F/H_4) * P(H_4 | [B, R, R])$$

$$\Rightarrow \propto \cdot P(F/H_4) * \sum_{H_3} P(H_4 | H_3) * P(H_3 | [B, R, R])$$

↓
①

From equation ① we are not clear about the value of $P(H_3 | [B, R, R])$
 so finding it next

$$P(H_3 | [B, R, R]) = P(H_3 | [B, R], R)$$

$$\Rightarrow \propto \cdot P(R/H_3, [B, R]) * P(H_3 | [B, R])$$

$$\Rightarrow \propto \cdot P(R/H_3) * P(H_3 | [B, R])$$

$$\Rightarrow \propto \cdot P(R/H_3) * \sum_{H_2} P(H_3 | H_2) * P(H_2 | [B, R])$$

↓
②

Now using the given data in the question to expand ②

$$P(H_3 | (B, R, R)) = \alpha * (0.8, 0.2, 0)$$

$$\left(P(H_3 | H_2 = 0) * P(H_2 = 0 | (B, R)) \right) \\ + P(H_3 | H_2 = A) * P(H_2 = A | (B, R)) \\ + P(H_3 | H_2 = N) * P(H_2 = N | (B, R))$$

$$\Rightarrow \alpha * (0.8, 0.2, 0) \left((0.5, 0.4, 0.1) * 0.3 + \right) \\ \left((0.3, 0.5, 0.2) * 0.5 + \right) \\ \left(0, 0.4, 0.6 \right) * 0.2$$

$$\Rightarrow \alpha * (0.8, 0.2, 0) \left((0.15, 0.12, 0.3) + \right. \\ \left. (0.15, 0.25, 0.10) + \right. \\ \left. (0, 0.08, 0.12) \right)$$

$$\Rightarrow \alpha * (0.8, 0.2, 0) * (0.3, 0.45, 0.52)$$

$$\Rightarrow \alpha * (0.24, 0.09, 0)$$

$$\Rightarrow \frac{1}{0.24+0.09} * (0.24, 0.09, 0)$$

$$\Rightarrow \frac{1}{0.33} * (0.24, 0.09, 0)$$

$$\Rightarrow (0.7272727273, 0.2727272727, 0)$$

Now substituting this value to calculate outcome of equation

①

$$P(H_4 / (B, R, R, F)) = \alpha * (0.1, 0.4, 0.5)$$

$$\left(\begin{array}{l} (0.5, 0.4, 0.1) * 0.72727272 \\ + (0.3, 0.5, 0.2) * 0.272727 \\ + (0, 0.4, 0.6) * 0 \end{array} \right)$$

$$\Rightarrow \cancel{\alpha * (0.1, 0.4, 0.5)} * \cancel{(0.3636363636, 0.2909090909, 0.07272727273)} \\ + \cancel{(0.08181818181, 0.136363636364, 0.054545454545)}$$

$$\Rightarrow \alpha * (0.1, 0.4, 0.5) * \left[\begin{array}{l} (0.3636363636, 0.2909090909, 0.07272727273) \\ + \\ (0.08181818181, 0.136363636364, 0.054545454545) \end{array} \right]$$

$$\Rightarrow \alpha * (0.1, 0.4, 0.5) * \left(\begin{array}{l} (0.445454545455, 0.427272727273, 0.127272727273) \end{array} \right)$$

$$\Rightarrow \alpha * (0.044544875, 0.1709090909, 0.06363636365)$$

$$\Rightarrow \frac{1}{(0.044544875 + 0.1709090909 + 0.06363636365)} * \left(\begin{array}{l} (0.044544875, 0.1709090909, 0.06363636365) \end{array} \right)$$

$$\Rightarrow \frac{1}{0.2790903296} * \begin{pmatrix} 0.044544875 \\ 0.1709090909 \\ 0.06363636365 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0.1596073754 \\ 0.6123791216 \\ 0.2280135028 \end{pmatrix}$$

$\therefore P(H_4)(B, R, R, F) = \begin{pmatrix} 0.1596073754 \\ 0.6123791216 \\ 0.2280135028 \end{pmatrix}$

2) Calculate the most likely sequence of hidden states for the observed sequence [R, F, B, B, R]. Show your work.

Ans Using the horoscope tables lets calculate values in each layer.

Layer 1 for 'R'

$$P(R, A) = P(R/A) * P(A)$$
$$= 0.2 * 0.4 \Rightarrow 0.08$$

$$P(R, O) = P(R/O) * P(O)$$
$$= 0.8 * 0.3 \Rightarrow 0.24$$

$$P(R, N) = 0$$

for the initial layer the maximum likelihood is

$$m_{1,1}(A) = 0.08$$

$$m_{1,1}(O) = 0.24 \checkmark$$

$$m_{1,1}(N) = 0$$

among those the maximum value belongs to 'O'.

Layer 2 for 'F' for $H_0 = A$

Given ~~it's~~

$$P(F, H_0 = A) = P(F/A) * P(A/A)$$
$$\Rightarrow 0.4 * 0.5$$
$$\Rightarrow 0.2$$

$$P(F, H_1 = O) = P(F/O) * P(O/A)$$

$$\Rightarrow 0.1 * 0.3$$

$$\Rightarrow 0.03$$

$$P(F, H_1 = N) = P(F/N) * P(N/A)$$

$$\Rightarrow 0.5 * 0.2$$

$$\Rightarrow 0.1$$

for "F" & $H_0 = O$

$$P(F, H_1 = A) = P(F/A) * P(A/O)$$

$$\Rightarrow 0.4 * 0.4$$

$$\Rightarrow 0.16$$

$$P(F, H_1 = O) = P(F/O) * P(O/O)$$

$$\Rightarrow 0.1 * 0.5$$

$$\Rightarrow 0.05$$

$$P(F, H_1 = N) = P(F/N) * P(N/O)$$

$$\Rightarrow 0.5 * 0.1$$

$$\Rightarrow 0.05$$

for "F" & $H_0 = N$

$$P(F, H_1 = A) = P(F/A) * P(A/N)$$

$$\Rightarrow 0.4 * 0.4$$

$$\Rightarrow 0.16$$

$$P(F, H_1 = O) = P(F/O) * P(O/N)$$

$$\Rightarrow 0.1 * 0$$

$$\Rightarrow 0$$

$$P(F, H_1 = N) \Rightarrow P(F/N) * P(N/N)$$

$$\Rightarrow 0.5 * 0.6$$

$$\Rightarrow 0.3$$

∴ Calculating the maximum likelihood of the layer.

$$m_{1:2}(A) = \max(0.08 * 0.2, 0.24 * 0.16, 0 * 0.16)$$

$$\Rightarrow \max(0.016, 0.0384, 0)$$

$$\Rightarrow 0.0384$$

$$m_{1:2}(O) = \max(0.08 * 0.03, 0.24 * 0.09, 0)$$

$$\Rightarrow \max(0.012, 0.0024, 0)$$

$$\Rightarrow 0.012$$

$$m_{1:2}(N) = \max(0.08 * 0.1, 0.24 * 0.05, 0)$$

$$\Rightarrow \max(0.0008, 0.012, 0)$$

$$\Rightarrow 0.012$$

among the above the maximum value belongs to 'A'.

Layer 3 for 'B' & $H_1 = A$

$$P(B, \cancel{H_1} A) \Leftrightarrow P(B/A \neq A)$$

$$P(B/A) * P(A/A)$$

$$\Rightarrow 0.4 * 0.5 = 0.2$$

$$P(B, \cancel{H_1=O}) \Rightarrow P(B/O) * P(O/A)$$
$$\Rightarrow 0.1 * 0.3$$
$$\Rightarrow 0.03$$

$$P(B, \cancel{H_1=N}) \Rightarrow P(B/N) * P(N/A)$$
$$\Rightarrow 0.5 * 0.2$$
$$\Rightarrow 0.1$$

for 'B' & $H_1 = O$

$$P(B, A) = P(B/A) * P(A/O) = 0.4 * 0.4$$
$$\Rightarrow 0.16$$

$$P(B, O) \Rightarrow P(B/O) * P(O/O) = 0.1 * 0.5$$
$$\Rightarrow 0.05$$

$$P(B, N) \Rightarrow P(B/N) * P(N/O) \Rightarrow 0.5 * 0.1$$
$$\Rightarrow 0.05$$

for 'B' & $H_1 = N$

$$P(B, A) = P(B/A) * P(A/N) \Rightarrow 0.4 * 0.4$$
$$\Rightarrow 0.16$$

$$P(B, O) = P(B/O) * P(O/N) = 0.1 * 0$$
$$\Rightarrow 0$$

$$P(B, N) \Rightarrow P(B/N) * P(N/N) \Rightarrow 0.5 * 0.6$$
$$\Rightarrow 0.3$$

$$m_{1:3}(A) \Rightarrow \max(0.2 * 0.0384, \\ 0.16 * 0.012, \\ 0.012 * 0.16)$$

$$\Rightarrow \max(0.00768, 0.00192, \\ 0.00192)$$

$$\Rightarrow 0.00768 \checkmark$$

$$m_{1:3}(B) \Rightarrow \max(0.012 * 0.05, 0.0384 \\ * 0.03, 0)$$

$$\Rightarrow \max(0.0006, 0.001152)$$

$$\Rightarrow 0.001152$$

$$m_{1:3}(N) \Rightarrow \max(0.0384 * 0.1, 0.012 \\ * 0.05, 0.012 * 0.3)$$

$$\Rightarrow \max(0.00384, 0.0006, \\ 0.0036)$$

$$\Rightarrow 0.00384$$

∴ among the values above
the maximum belongs to 'A'.

Layer 4

For ' B ' & $H_2 = 'A'$

$$P(B, A) \Rightarrow P(B/A) * P(A/A) = 0.4 * 0.5 \\ \Rightarrow 0.2$$

$$P(B, o) \Rightarrow P(B/o) * P(o/A) \Rightarrow 0.1 * 0.3 \\ \Rightarrow 0.03$$

$$P(B, N) = P(B|N) * P(N/A)$$
$$\Rightarrow 0.5 * 0.2$$
$$\Rightarrow 0.1$$

for $'B'$ & $H_2 = 'O'$

$$P(B, A) = P(B|A) * P(A/O)$$
$$\Rightarrow 0.4 * 0.4$$
$$\Rightarrow 0.16$$

$$P(B, O) \Rightarrow P(B/O) * P(O/O)$$
$$\Rightarrow 0.1 * 0.5$$
$$\Rightarrow 0.05$$

$$P(B, N) \Rightarrow P(B/N) * P(N/O)$$
$$\Rightarrow 0.5 * 0.1$$
$$\Rightarrow 0.05$$

for $'B'$ & $H_2 = 'N'$

$$P(B, A) \Rightarrow P(B|A) * P(A/N)$$
$$\Rightarrow 0.4 * 0.4$$
$$\Rightarrow 0.16$$

$$P(B, O) \Rightarrow P(B/O) * P(O/N)$$
$$\Rightarrow 0.1 * 0$$
$$\Rightarrow 0$$

$$P(B, N) \Rightarrow P(B/N) * P(N/N)$$
$$\Rightarrow 0.5 * 0.6$$
$$\Rightarrow 0.3$$

$$m_{1:4}(A) = \max(0.00768 * 0.2, \\ 0.001152 * 0.16, \\ 0.00384 * 0.16) \\ \Rightarrow \max(0.001536, 0.00064, \\ 0.00018432) \\ \Rightarrow 0.001536 \checkmark$$

$$m_{1:4}(O) = \max(0.00768 * 0.03, \\ 0.001152 * 0.05, 0) \\ \Rightarrow \max(0.0002304, 0, 0.0002304) \\ \Rightarrow 0.0002304 \checkmark$$

$$m_{1:4}(N) = \max(0.3 * 0.00384, 0.05 * \\ 0.001152, \\ 0.1 * 0.00768) \\ \Rightarrow \max(0.000768, 0.000576, \\ 0.001152) \\ \Rightarrow 0.001152 \checkmark$$

among the values above
the maximum value belongs
to 'A'.

Layer 5

for 'R' & $H_3 = 'A'$

$$P(R, A) = P(R/A) * P(A/A) \\ \Rightarrow 0.2 * 0.5 = 0.1$$

$$P(R, O) = P(R|O) * P(O|A)$$
$$\Rightarrow 0.8 * 0.3$$
$$\Rightarrow 0.24$$

$$P(R, N) = P(R|N) * P(N|A)$$
$$\Rightarrow 0 * 0.2$$
$$\Rightarrow 0.$$

for 'R' & $H_3 = O$

$$P(R, A) = P(R|A) * P(A|O)$$
$$\Rightarrow 0.2 * 0.4$$
$$\Rightarrow 0.08$$

$$P(R, O) = P(R|O) * P(O|O)$$
$$\Rightarrow 0.8 * 0.5$$
$$\Rightarrow 0.4$$

$$P(R, N) \Rightarrow P(R|N) * P(N|O)$$
$$\Rightarrow 0.$$

for 'R' & $H_3 = N$

$$P(R, A) = P(R|A) * P(A|N)$$
$$\Rightarrow 0.2 * 0.4$$
$$\Rightarrow 0.08$$

$$P(R, O) = P(R|O) * P(O|N)$$
$$\Rightarrow 0$$

$$P(R, N) = P(R|N) * P(N|N)$$
$$\Rightarrow 0$$

$$m_{1:5}(A) = \max(0.1 * 0.001536, \\ 0.0002304 * 0.08, \\ 0.08 * 0.001152)$$

$$\Rightarrow \max(0.0001536, 0.00001152, \\ 0.00009216)$$

$$m_{1:5}(0) = \max(0.24 * 0.001536, \\ 0.0002304 * 0.4, 0)$$

$$\Rightarrow \max(0.00009216, 0.00036864, 0)$$

$$\Rightarrow 0.00036864$$

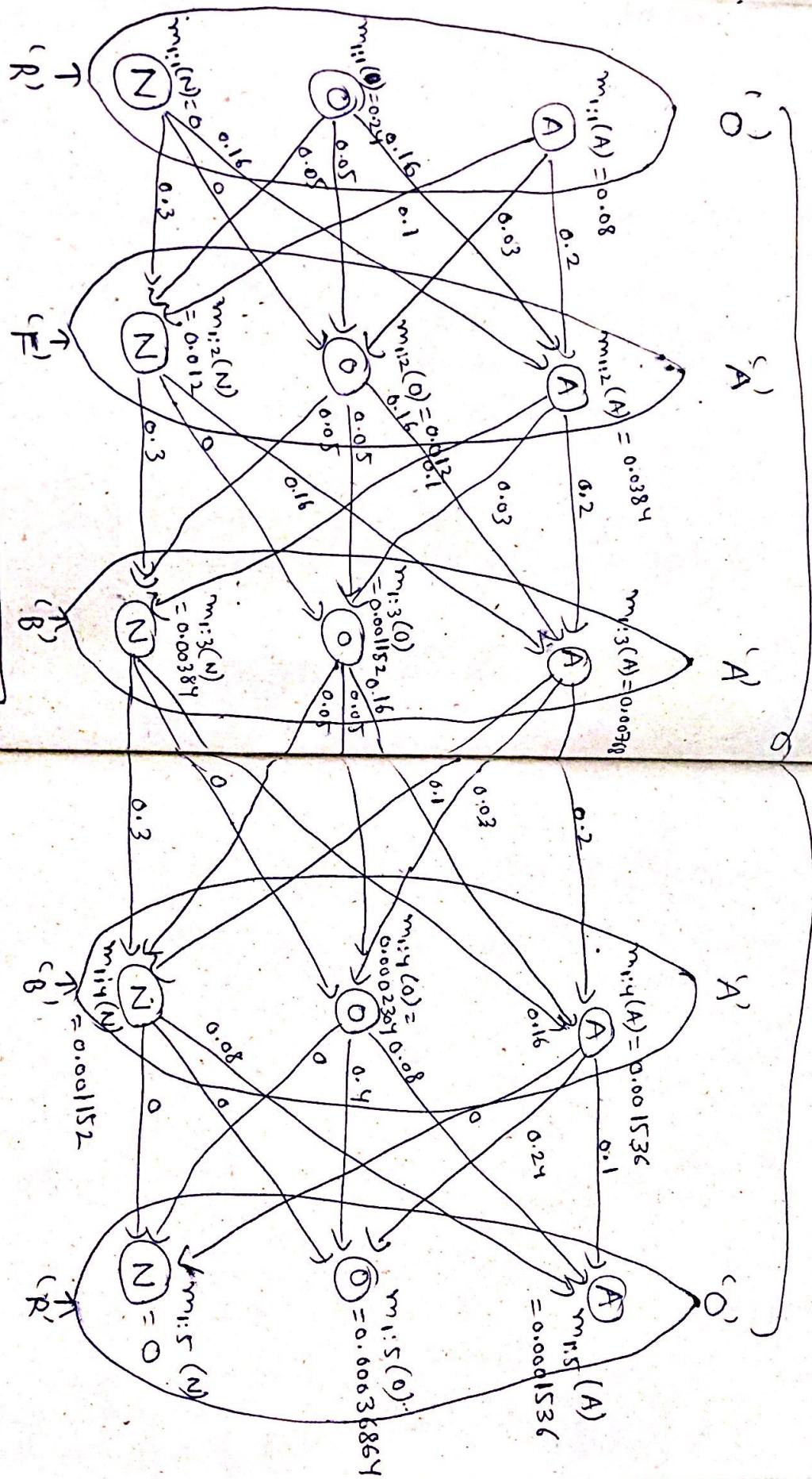
$$m_{1:5}(N) = \max(0, 0)$$

$$\Rightarrow 0$$

\therefore Among the values above the maximum belong to '0'.

Most likely sequence = "0AAAO"

Maximum selection in each layer



∴ Maximum likelihood Sequence
= "O AAO"