Project 2

ENPM662
Introduction to Robot Modeling

Vishnu Mandala - 119452608
Rohit Sai Suresh - 119283684

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

Introduction

This project centers on the development and analysis of a cutting-edge robotic system. The robot in focus is a mobile unit that is uniquely integrated with a UR10 manipulator, a widely recognized robotic arm known for its precision and versatility. The UR10 is mounted atop the mobile platform, combining mobility with advanced manipulation capabilities. This integration significantly enhances the robot's operational scope, enabling it to perform a wide range of tasks in various environments.

Aim

The primary aim of this project is to create an autonomous robotic system capable of picking and handling tools from a conveyor belt. The system consists of a UR10 manipulator mounted on a mobile chassis with four wheels. The integration of these components was intended to facilitate the autonomous identification and manipulation of objects. However, due to challenges in integrating a vision system, the robot currently operates based on direct commands rather than autonomous decision-making.

Robot Model

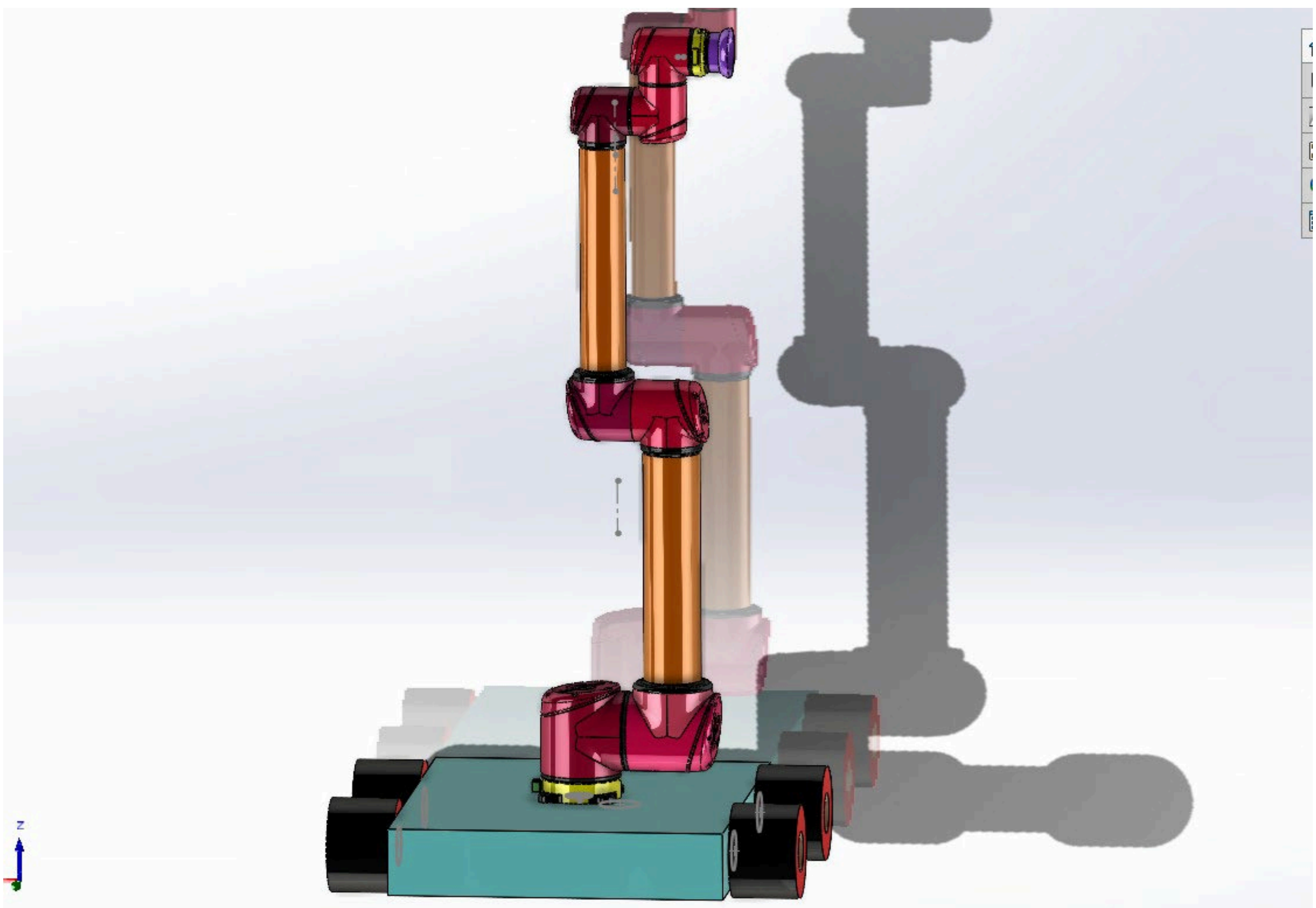

Figure 1. UR10 Manipulator on a 4 wheeled Chassis

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

Robot Specifications

The robotic system is a hybrid of advanced manipulation and mobility. It consists of two primary components:

- **UR10 Manipulator**: The UR10 is a highly versatile robotic arm known for its precision and flexibility. It features six degrees of freedom (DOF), allowing complex and nuanced movements. Each joint of the UR10 contributes to its ability to reach various positions and orientations in three-dimensional space.

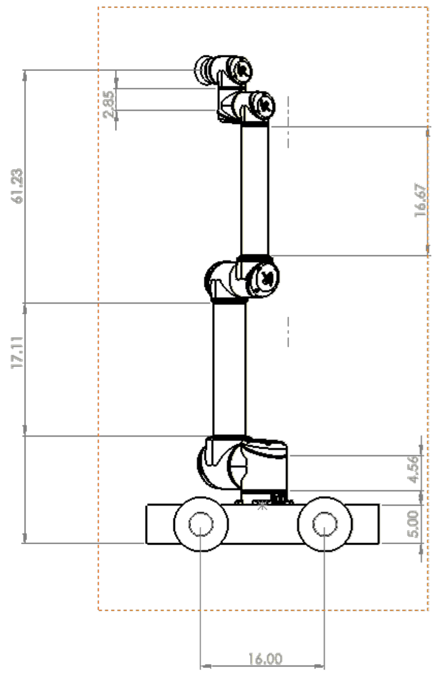

Figure 2. Dimensions of the Manipulator

  - **Mobile Chassis**: Supporting the UR10 is a mobile chassis equipped with four wheels. This chassis provides an additional three degrees of freedom - two translational and one rotational. This mobility enhances the robot's operational range, enabling it to maneuver to different positions as required by the task.

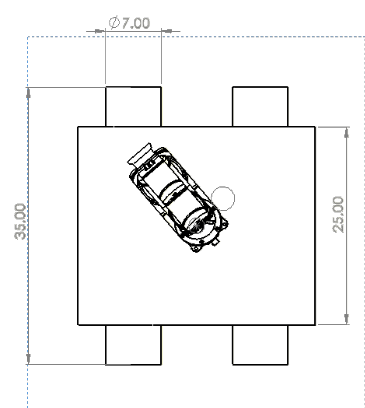

Figure 3. Dimensions of the Chassis

[1] [Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)]

Forward Kinematics using Spong Convention

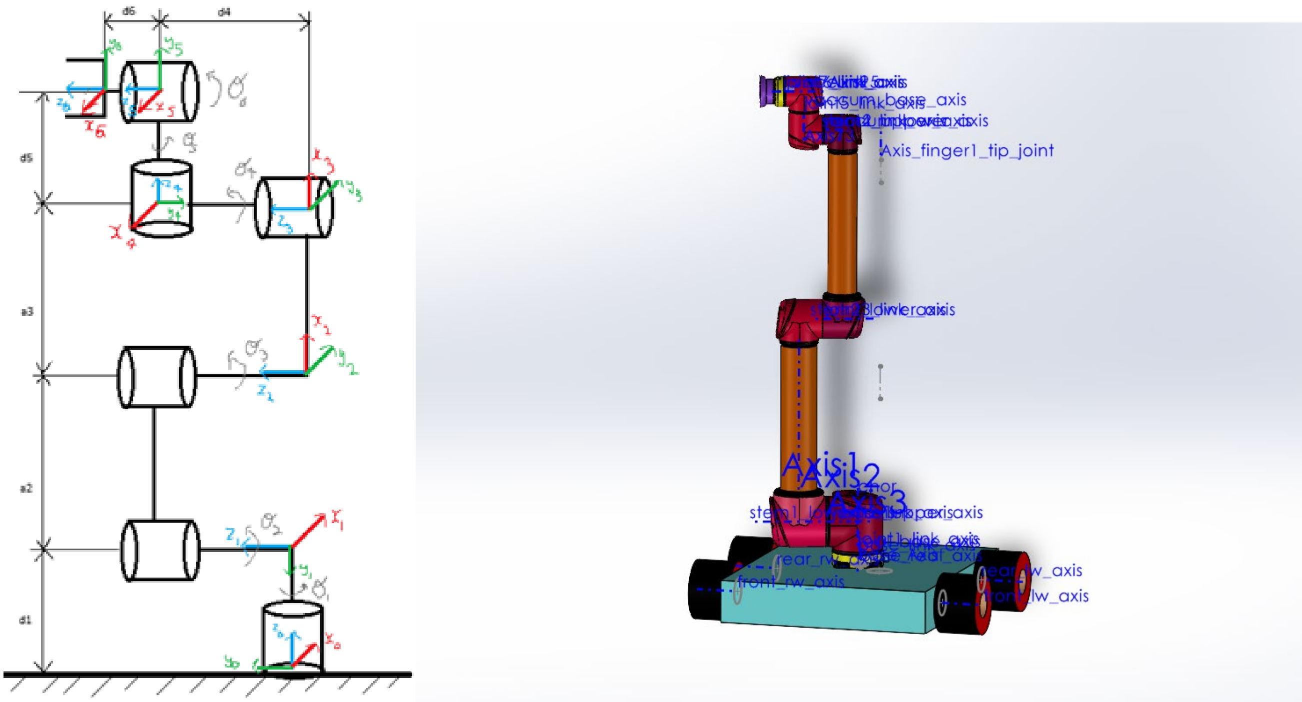

Figure 4. UR10 Robot Frame Assignment

The UR10 Robot has 6 joints with joint axes in the configuration depicted above. Denavit–Hartenberg parameters (also called DH parameters) are the four parameters associated with a particular convention for attaching reference frames to the links of a spatial kinematic chain, or robot manipulator. They are –

- d: offset along previous z to the common normal
- θ: angle about previous z, from old x to new x
- a: length of the common normal. Assuming a revolute joint, this is the radius about previous z.
- α: angle about common normal, from old z axis to new z axis

We draw the axes based on the following conditions –

- the x_n-axis is perpendicular to both the z_{n-1} and z_n axes
- the x_n-axis intersects both z_{n-1} and z_n axes
- the origin of joint n is at the intersection of x_n and z_n
- y_n completes a right-handed reference frame based on x_n and z_n

The transformation matrix for each frame is given by –

$$^{n-1}T_n = \mathrm{Trans}(z_{n-1}, d_n)\,\mathrm{Rot}(z_{n-1}, \theta_n)\,\mathrm{Trans}(x_n, a_n)\,\mathrm{Rot}(x_n, \alpha_n)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n) & 0 & 0 \\ \sin(\theta_n) & \cos(\theta_n) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_n) & -\sin(\alpha_n) & 0 \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{n-1}T_n = \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n)\cos(\alpha_n) & \sin(\theta_n)\sin(\alpha_n) & a_n\cos(\theta_n) \\ \sin(\theta_n) & \cos(\theta_n)\cos(\alpha_n) & -\cos(\theta_n)\sin(\alpha_n) & a_n\sin(\theta_n) \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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The D-H Parameter table (values taken from [1]) for the robot is –

Frames	**a** (m)	α (deg)	**d** (m)	θ (deg)
0 (Base Frame) → 1	0	-90^0	$d_1 = 0.1273$	$\theta_1 = 0^0$
1 → 2	$a_2 = 0.612$	0	0	$\theta_2 - 90^0 = -90^0$
2 → 3	$a_3 = 0.5723$	0	0	$\theta_3 = 0^0$
3 → 4	0	-90^0	$d_4 = 0.163941$	$\theta_4 - 90^0 = -90^0$
4 → 5	0	90^0	$d_5 = 0.1157$	$\theta_5 = 0^0$
5 → 6=n (Final Frame)	0	0	$d_6 = 0.0922$	$\theta_6 = 0^0$

Transformation Matrices –

$$^0T_1=\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1)\cos(-90) & \sin(\theta_1)\sin(-90) & 0\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1)\cos(-90) & -\cos(\theta_1)\sin(-90) & 0\sin(\theta_1) \\ 0 & \sin(-90) & \cos(-90) & 0.128 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & \cos(\theta_1) & 0 \\ 0 & -1 & 0 & 0.128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2=\begin{bmatrix} \cos(\theta_2-90) & -\sin(\theta_2-90)\cos(0) & \sin(\theta_2-90)\sin(0) & 0.6127\cos(\theta_2-90) \\ \sin(\theta_2-90) & \cos(\theta_2-90)\cos(0) & -\cos(\theta_2-90)\sin(0) & 0.6127\sin(\theta_2-90) \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sin(\theta_2) & \cos(\theta_2) & 0 & 0.6127\sin(\theta_2) \\ -\cos(\theta_2) & \sin(\theta_2) & 0 & -0.6127\cos(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3=\begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3)\cos(0) & \sin(\theta_3)\sin(0) & 0.5716\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3)\cos(0) & -\cos(\theta_3)\sin(0) & 0.5716\sin(\theta_3) \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0.5716\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0.5716\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3T_4=\begin{bmatrix} \cos(\theta_4-90) & -\sin(\theta_4-90)\cos(-90) & \sin(\theta_4-90)\sin(-90) & 0\cos(\theta_4-90) \\ \sin(\theta_4-90) & \cos(\theta_4-90)\cos(-90) & -\cos(\theta_4-90)\sin(-90) & 0\sin(\theta_4-90) \\ 0 & \sin(-90) & \cos(-90) & 0.1639 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ -\cos(\theta_4) & 0 & \sin(\theta_4) & 0 \\ 0 & -1 & 0 & 0.1639 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^4T_5=\begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5)\cos(90) & \sin(\theta_5)\sin(90) & 0\cos(\theta_5) \\ \sin(\theta_5) & \cos(\theta_5)\cos(90) & -\cos(\theta_5)\sin(90) & 0\sin(\theta_5) \\ 0 & \sin(90) & \cos(90) & 0.1157 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0.1157 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^5T_6=\begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6)\cos(0) & \sin(\theta_6)\sin(0) & 0\cos(\theta_6) \\ \sin(\theta_6) & \cos(\theta_6)\cos(0) & -\cos(\theta_6)\sin(0) & 0\sin(\theta_6) \\ 0 & \sin(0) & \cos(0) & 0.0922 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 0.0922 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Transformation Matrix ($^0T_6$) = $^0T_1$ x $^1T_2$ x $^2T_3$ x $^3T_4$ x $^4T_5$ x $^5T_6$

Final Transformation Matrix Elements -

- **r_{11}:**
  $((-(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\cos(\theta_5) + \sin(\theta_1)\cdot\sin(\theta_5))\cdot\cos(\theta_6) + ((-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_6)$
- **r_{12}:**
  $-((-(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\cos(\theta_5) + \sin(\theta_1)\cdot\sin(\theta_5))\cdot\sin(\theta_6) + ((-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\cos(\theta_6)$
- **r_{13}:**
  $(-(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) - \sin(\theta_1)\cdot\cos(\theta_5)$
- **r_{14}:**
  $0.0922\cdot(-(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) + 0.1157\cdot(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) +$

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$0.1157\cdot(\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\cos(\theta_4) - 0.0922\cdot\sin(\theta_1)\cdot\cos(\theta_5) - 0.1639\cdot\sin(\theta_1) + 0.5716\cdot\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) + 0.6127\cdot\sin(\theta_2)\cdot\cos(\theta_1) + 0.5716\cdot\sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2)$

- **r_{21}:** $((-(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\cos(\theta_5) - \sin(\theta_5)\cdot\cos(\theta_1))\cdot\cos(\theta_6) + ((-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_6)$
- **r_{22}:** $-((-(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\cos(\theta_5) - \sin(\theta_5)\cdot\cos(\theta_1))\cdot\sin(\theta_6) + ((-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\cos(\theta_6)$
- **r_{23}:** $(-(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) + \cos(\theta_1)\cdot\cos(\theta_5)$
- **r_{24}:** $0.0922\cdot(-(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) + 0.1157\cdot(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + 0.1157\cdot(\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4) + 0.5716\cdot\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) + 0.6127\cdot\sin(\theta_1)\cdot\sin(\theta_2) + 0.5716\cdot\sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2) + 0.0922\cdot\cos(\theta_1)\cdot\cos(\theta_5) + 0.1639\cdot\cos(\theta_1)$
- **r_{31}:** $((-\sin(\theta_2)\cdot\sin(\theta_3) + \cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) - (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\cos(\theta_5)\cdot\cos(\theta_6) + ((-\sin(\theta_2)\cdot\sin(\theta_3) + \cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_6)$
- **r_{32}:** $-((-\sin(\theta_2)\cdot\sin(\theta_3) + \cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) - (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_6)\cdot\cos(\theta_5) + ((-\sin(\theta_2)\cdot\sin(\theta_3) + \cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\cos(\theta_6)$
- **r_{33}:** $((-\sin(\theta_2)\cdot\sin(\theta_3) + \cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) - (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5)$
- **r_{34}:** $0.0922\cdot((-\sin(\theta_2)\cdot\sin(\theta_3) + \cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) - (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5) + 0.1157\cdot(-\sin(\theta_2)\cdot\sin(\theta_3) + \cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + 0.1157\cdot(-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4) - 0.5716\cdot\sin(\theta_2)\cdot\sin(\theta_3) + 0.5716\cdot\cos(\theta_2)\cdot\cos(\theta_3) + 0.6127\cdot\cos(\theta_2) + 0.128$
- **r_{41}:** 0
- **r_{42}:** 0
- **r_{43}:** 0
- **r_{44}:** 1

The r_{14}, r_{24} and r_{34} elements give the position of the end effector in x, y and z directions respectively.

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Forward Kinematics Validation

Peter Corke's Robotics Toolbox for MATLAB is a renowned software package in robotics education and research. It provides a comprehensive set of tools for simulating and analyzing robotic systems. In this project, the toolbox was employed to validate the forward kinematics developed using the Spong convention. The validation process is crucial to ensure that the theoretical models accurately represent the robot's physical movements.

Procedure for Validation -

- Modeling the Robot in MATLAB: The first step involved modeling the UR10 manipulator and the mobile chassis in MATLAB using the Robotics Toolbox. This model incorporated the robot's physical parameters, such as link lengths and joint limits.
- Setting Joint Angles: For the validation process, a specific scenario was considered where one of the UR10 manipulator's joint angles (theta) was set to 90 degrees, and the remaining angles were set to 0 degrees. This configuration presents a critical test case for assessing the accuracy of the forward kinematics model.
- Computing Forward Kinematics: Using the Robotics Toolbox, the forward kinematics of the robot were computed. This involved calculating the position and orientation of the end effector based on the given joint angles.
- Comparison and Validation: The results obtained from the Robotics Toolbox were then compared with the theoretical outcomes derived from the Spong convention. This comparison was essential to validate the accuracy of the kinematic model.

Implications of the Validation -

The validation process using Peter Corke's Robotics Toolbox served a dual purpose. Firstly, it ensured the reliability of the forward kinematics model developed for the robot. Secondly, it provided a practical framework for testing various joint configurations, which is pivotal in understanding the robot's capabilities and limitations in real-world scenarios.

For Theta2 = 90^0 –

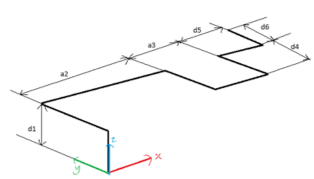
Geometric Validation:

Configuration [θ_2=90°]:

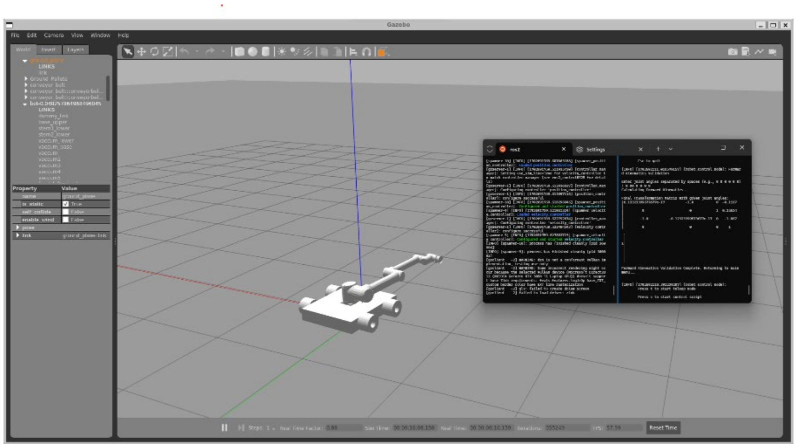
$x = a_2 + a_3 = 0.6127 + 0.5716 = 1.3m$

$y = d_4 + d_6 = 0.1639 + 0.0922 = 0.2561m$

$z = d_1 = 0.128m$



Gazebo Validation:



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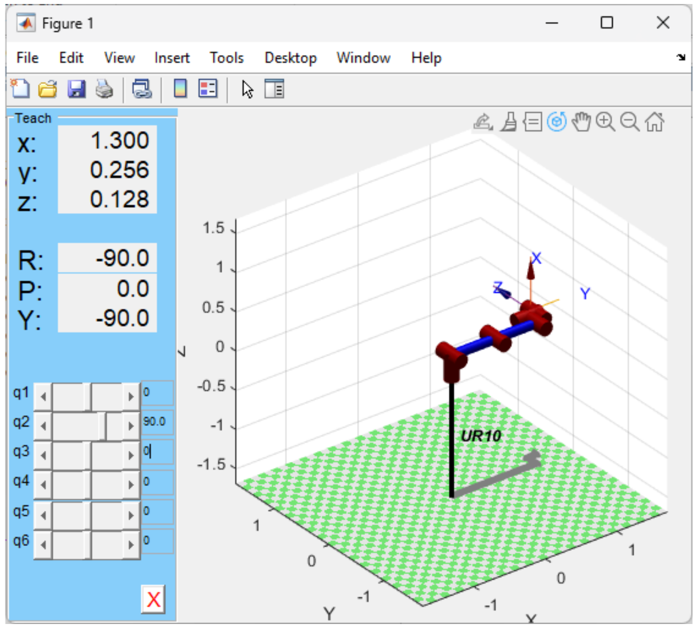
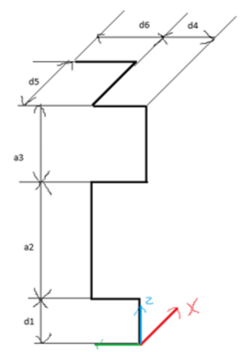
Peter Corke's Validation:



For Theta4 = 90^0 –

Geometric Validation:



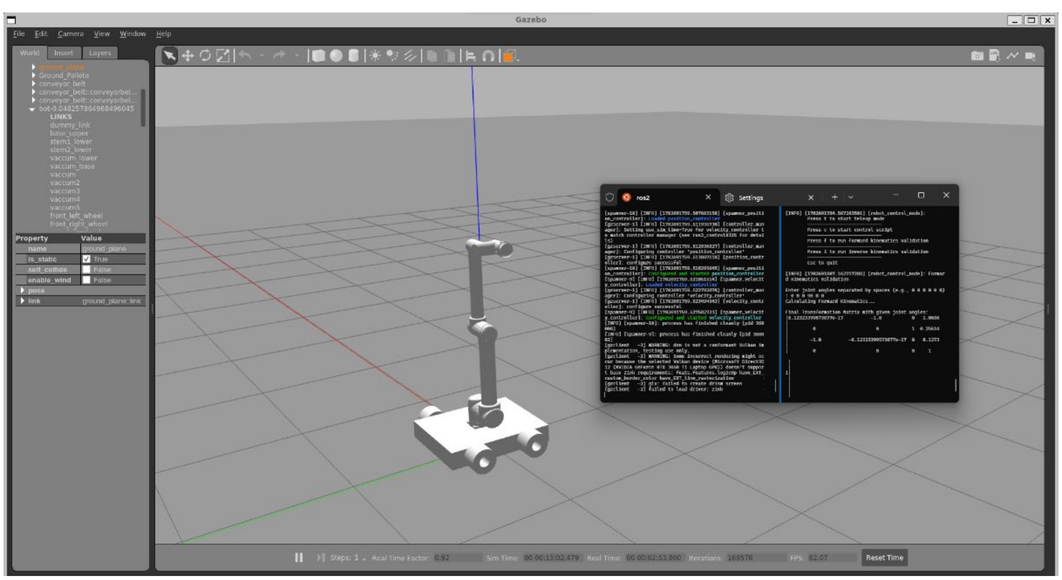
Configuration [θ_4=90°]:

$x = d_5 = 0.1157\text{m}$

$y = d_4 + d_6 = 0.1639 + 0.0922 = 0.2561\text{m}$

$z = d_1 + a_2 + a_3 = 0.128 + 0.6127 + 0.5716 = 1.3123\text{m}$

Gazebo Validation:



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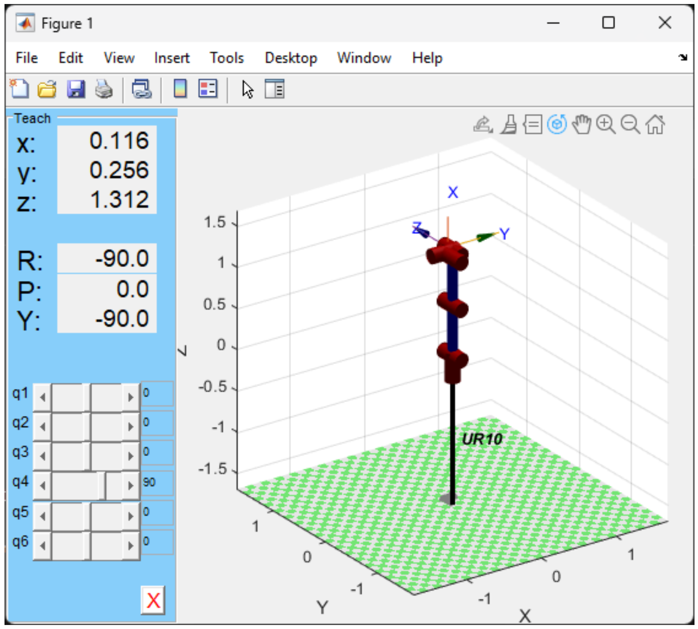
Peter Corke’s Validation:



For Theta5 = 90^0 –

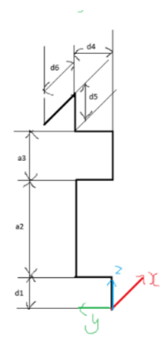
Geometric Validation:

Configuration [θ_5=90°]:

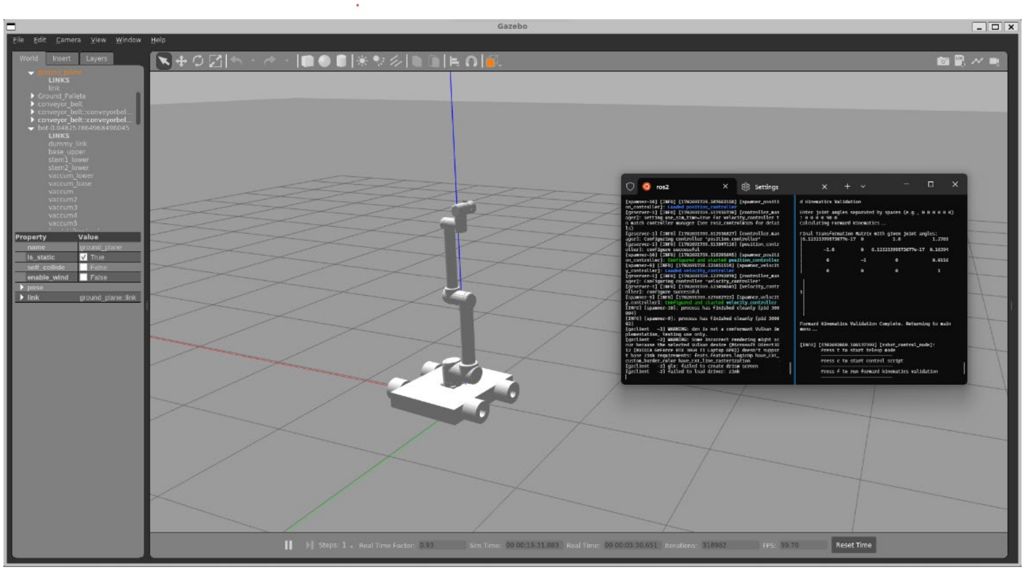
x = - d_6 = - 0.0922m

y = d_4 = 0.1639m

z = $d_1 + a_2 + a_3 + d_5$ = 0.128 + 0.6127 + 0.5716 +0.1157 = 1.428m



Gazebo Validation:



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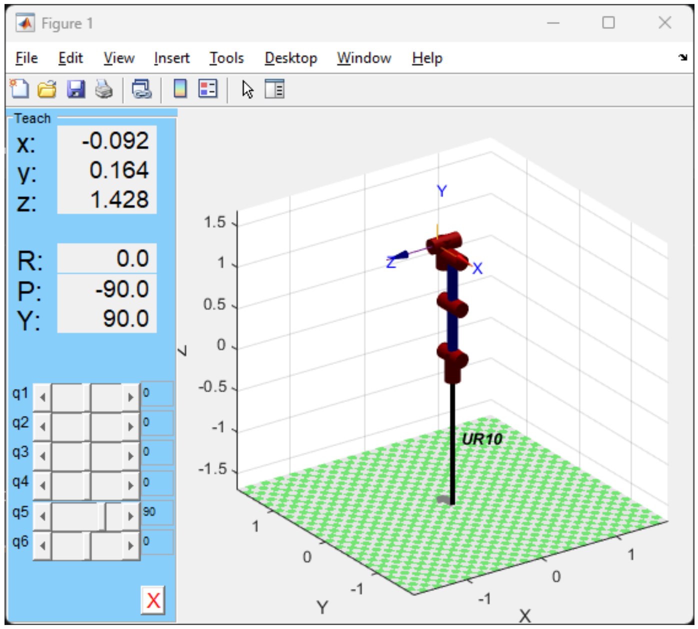
Peter Corke’s Validation:



Inverse Kinematics

The Jacobian matrix is crucial for understanding the relationship between joint velocities and end-effector velocities. It is defined as the matrix of all first-order partial derivatives of the end-effector's position and orientation with respect to the joint variables. For a robot with n joints, the Jacobian J is:

$$J = \begin{bmatrix} \frac{\partial X_p}{\partial \theta_1} & \frac{\partial X_p}{\partial \theta_2} & \frac{\partial X_p}{\partial \theta_3} & \frac{\partial X_p}{\partial \theta_4} & \frac{\partial X_p}{\partial \theta_5} & \frac{\partial X_p}{\partial \theta_6} \\ z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \end{bmatrix}$$

where, X_p is a vector representing the end-effector's position and orientation (pen in this case) with respect to the origin frame, obtained from the fourth column of the final transformation matrix

θ_i are the joint angles or displacements.

Each element $\frac{\partial X_p}{\partial \theta_i}$ represents how the end-effector's position and orientation change with a small change in the ith joint variable.

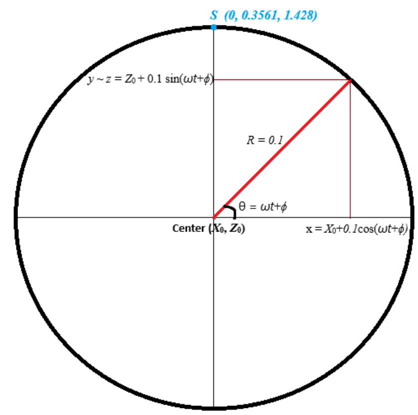
z_i are the rotation axes of the joints obtained from the 3rd column of ith transformation matrix.

The Jacobian is typically divided into two parts: the linear part and the angular part.

- Linear Part: This part relates the joint velocities to the linear velocity of the end-effector. It is derived from the derivatives of the end-effector's position with respect to the joint variables. (Rows 1-3)
- Angular Part: This part links the joint velocities to the angular velocity of the end-effector. It is constructed from the rotational axes of the joints. (Rows 4-6)

Final Jacobian Matrix Obtained from calculations, $J = \begin{bmatrix} -0.356141 & 1.3 & 0.688 & 0.1157 & -0.0922 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

[1] [Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)]



The Jacobian is used to calculate the joint velocities required to achieve a desired end-effector velocity, particularly useful in trajectory planning. The desired trajectory for the end-effector is a circle in the XZ-plane. In the general convention, the equations are calculated in parametric form in XY plane. As we performed a transformation of frames, the final equations for X and Y planes correspond to X and Z planes respectively.

A typical equation for such a circle with radius R centered at (X_0, Z_0) is:

$$x(t) = X_0 + R\cos(\omega t+\phi)$$

$$z(t) = Z_0 + R\sin(\omega t+\phi)$$

where, ω is the angular frequency of the motion $= \frac{2\pi}{Total\ Time\ (T)} = \frac{2\pi}{20} = \frac{\pi}{10}$

ϕ is the phase angle $= \frac{\pi}{2}$

t is time.

T is total time = 20 seconds

R is Radius = 0.0m

The velocity of the end-effector $\dot{x}$ along this circular path can be derived by differentiating the position equations with respect to time:

$v_x(t) = -R\omega\sin(\omega t+\phi) = 0.5 \text{ x } \frac{\pi}{100}\sin(\frac{\pi}{10}t + \frac{\pi}{2})$ [Negative sign was not considered to obtain the required trajectory]

$$v_z(t) = R\omega\cos(\omega t+\phi) = 0.5 \text{ x } \frac{\pi}{100}\cos(\frac{\pi}{10}t + \frac{\pi}{2})$$

Thus, the velocity vector $\dot{x}$ is:

$$\dot{x} = \begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \\ \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \end{bmatrix} = \begin{bmatrix} 0.5 \text{ x } \frac{\pi}{100}\ \sin(\frac{\pi}{10}t\ +\ \frac{\pi}{2}) \\ 0 \\ 0.5 \text{ x } \frac{\pi}{100}\ \cos(\frac{\pi}{10}t\ +\ \frac{\pi}{2}) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The last three components are zero, assuming no angular velocity in our case.

To move the end-effector along the desired path, we need to compute the corresponding joint velocities $\dot{q}$. This is achieved using the Jacobian matrix J, which relates joint velocities to end-effector velocities:

$$\dot{q} = J^{-1}.\dot{x}$$

Here, J^{-1} is the inverse (or pseudo-inverse in case of non-square or singular matrices) of the Jacobian.

The joint angles are updated for the next time step using $q_{new} = q_{old} + \dot{q} \cdot dt$

where, q is initialized as a vector of initial angles with offset = $[0.0001 \quad -0.0002 \quad 0.0003 \quad -0.0004 \quad 0.0001 \quad 0.0002]^T$

dt is discrete timestep to update joint angles $= \frac{Total\ Time\ (T)}{Number\ of\ Steps\ (N)} = \frac{20}{200}$

The trajectory was observed and the time for which the manipulator has to move to reach the table has been calculated to be 3.3 seconds. After this, the grippers at the end effector of the robot get switched on, grabbing the box closest to them. The linkAttacher plugin is activated, which creates a joint between the end effector link and the box, holding it steady.

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

```
Jacobian :

Symbolic:
⎡ 0.1922·((sin(θ1)·sin(θ2)·sin(θ3) - sin(θ1)·cos(θ2)·cos(θ3))·cos(θ4) + (sin(
⎢
⎢0.1922·((-sin(θ2)·sin(θ3)·cos(θ1) + cos(θ1)·cos(θ2)·cos(θ3))·cos(θ4) + (-sin(
⎢
⎢
⎢
⎢
⎢
⎢
⎣

θ1)·sin(θ2)·cos(θ3) + sin(θ1)·sin(θ3)·cos(θ2))·sin(θ4))·sin(θ5) - 0.1157·(sin(

θ2)·cos(θ1)·cos(θ3) - sin(θ3)·cos(θ1)·cos(θ2))·sin(θ4))·sin(θ5) - 0.1157·(-sin

θ1)·sin(θ2)·sin(θ3) - sin(θ1)·cos(θ2)·cos(θ3))·sin(θ4) + 0.1157·(sin(θ1)·sin(θ

(θ2)·sin(θ3)·cos(θ1) + cos(θ1)·cos(θ2)·cos(θ3))·sin(θ4) + 0.1157·(-sin(θ2)·cos

                                        0

                                     -sin(θ1)

                                      cos(θ1)

                                        0

2)·cos(θ3) + sin(θ1)·sin(θ3)·cos(θ2))·cos(θ4) + 0.5716·sin(θ1)·sin(θ2)·sin(θ3)

(θ1)·cos(θ3) - sin(θ3)·cos(θ1)·cos(θ2))·cos(θ4) - 0.1922·sin(θ1)·cos(θ5) - 0.1

 - 0.5716·sin(θ1)·cos(θ2)·cos(θ3) - 0.6127·sin(θ1)·cos(θ2) - 0.1922·cos(θ1)·co

639·sin(θ1) - 0.5716·sin(θ2)·sin(θ3)·cos(θ1) + 0.5716·cos(θ1)·cos(θ2)·cos(θ3)

s(θ5) - 0.1639·cos(θ1)    0.1922·((sin(θ2)·sin(θ3)·cos(θ1) - cos(θ1)·cos(θ2)·c

+ 0.6127·cos(θ1)·cos(θ2)  0.1922·((sin(θ1)·sin(θ2)·sin(θ3) - sin(θ1)·cos(θ2)·c
```

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

$0.1922\cdot$

$os(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\cos(\theta_4)$

$os(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4)$

$((\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos($

$)\cdot\sin(\theta_5) + 0.1157\cdot(\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) - \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4)$

$)\cdot\sin(\theta_5) + 0.1157\cdot(\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) - \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4)$

$\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) - 0.1157\cdot(\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + 0$

$-\sin(\theta_1)$

$\cos(\theta_1)$

0

$- 0.1157\cdot(-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4) - 0.571$

$- 0.1157\cdot(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4) - 0.571$

$.1157\cdot(\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4) + 0.5716\cdot\sin(\theta_2)\cdot\sin(\theta_3) - 0$

$6\cdot\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - 0.6127\cdot\sin(\theta_2)\cdot\cos(\theta_1) - 0.5716\cdot\sin(\theta_3)\cdot\cos(\theta_1)\cdot co$

$6\cdot\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - 0.6127\cdot\sin(\theta_1)\cdot\sin(\theta_2) - 0.5716\cdot\sin(\theta_1)\cdot\sin(\theta_3)\cdot co$

$.5716\cdot\cos(\theta_2)\cdot\cos(\theta_3) - 0.6127\cdot\cos(\theta_2)$

$s(\theta_2)\quad 0.1922\cdot((\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) - \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + ($

$s(\theta_2)\quad 0.1922\cdot((\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) - \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + ($

$0.1922\cdot((\sin(\theta_2)\cdot\sin(\theta_3) - \cos$

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

$-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5) + 0.1157\cdot$

$-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5) + 0.1157\cdot$

$(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) + (\sin(\theta_2)\cdot\cos(\theta_3) + \sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) -$

$(\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) - \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) - 0.1157\cdot(-\sin(\theta_2)$

$(\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) - \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) - 0.1157\cdot(-\sin(\theta_1)$

$0.1157\cdot(\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + 0.1157\cdot(\sin(\theta_2)\cdot\cos(\theta_3)$

$-\sin(\theta_1)$

$\cos(\theta_1)$

0

$\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4) - 0.5716\cdot\sin(\theta_2)\cdot\cos(\theta_1)\cdot c$

$\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4) - 0.5716\cdot\sin(\theta_1)\cdot\sin(\theta_2)\cdot c$

$+ \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4) + 0.5716\cdot\sin(\theta_2)\cdot\sin(\theta_3) - 0.5716\cdot\cos(\theta_2)\cdot\cos(\theta_3)$

$os(\theta_3) - 0.5716\cdot\sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2)\quad 0.1922\cdot(-(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) +$

$os(\theta_3) - 0.5716\cdot\sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2)\quad 0.1922\cdot(-(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) +$

0.19

$\cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)$

$\sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)$

$22\cdot((\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) - (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot$

$-(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot$

$-(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot$

$(\sin(\theta_2)\cdot\sin(\theta_3) - c$

$\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5) - 0.1157\cdot(-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2$

$\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5) - 0.1157\cdot(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2$

$\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) - 0.1157\cdot(\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4)$

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

$\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2)$

$\cos(\theta_2)\cdot\cos(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2)$

$os(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta_4) - (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4)$

$)\cdot\cos(\theta_3))\cdot\cos(\theta_4) - 0.1157\cdot(-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2$

$)\cdot\cos(\theta_3))\cdot\cos(\theta_4) - 0.1157\cdot(-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2$

$- 0.1157\cdot(-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4)$

$)\cdot\cos(\theta_4)$

$)\cdot\cos(\theta_4)$

$))\cdot\sin(\theta_4)\quad 0.1922\cdot((-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta$

$))\cdot\sin(\theta_4)\quad 0.1922\cdot((-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta$

$0.1922\cdot((\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot co$

$((-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta$

$((-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot\cos(\theta$

$((\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot\cos(\theta_3$

$_4) + (-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\cos(\theta_5) + 0$

$_4) + (-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\cos(\theta_5) - 0$

$s(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\cos(\theta_5)$

$_4) + (-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) - s$

$_4) + (-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5) + c$

$))\cdot\sin(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5)$

$.1922\cdot\sin(\theta_1)\cdot\sin(\theta_5)$

$.1922\cdot\sin(\theta_5)\cdot\cos(\theta_1)$

$in(\theta_1)\cdot\cos(\theta_5)\qquad ((-\sin(\theta_2)\cdot\sin(\theta_3)\cdot\cos(\theta_1) + \cos(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot c$

$os(\theta_1)\cdot\cos(\theta_5)\qquad ((-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\sin(\theta_3) + \sin(\theta_1)\cdot\cos(\theta_2)\cdot\cos(\theta_3))\cdot c$

$((\sin(\theta_2)\cdot\sin(\theta_3) - \cos(\theta_2)\cdot co$

0

0

0

$os(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_1)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_1)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5)$

$os(\theta_4) + (-\sin(\theta_1)\cdot\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_1)\cdot\sin(\theta_3)\cdot\cos(\theta_2))\cdot\sin(\theta_4))\cdot\sin(\theta_5)$

$s(\theta_3))\cdot\sin(\theta_4) + (-\sin(\theta_2)\cdot\cos(\theta_3) - \sin(\theta_3)\cdot\cos(\theta_2))\cdot\cos(\theta_4))\cdot\sin(\theta_5)$

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

```
            ⎤
            ⎥
            ⎥
            ⎥
            ⎥
            ⎥
- sin(θ₁)·cos(θ₅)⎥
            ⎥
+ cos(θ₁)·cos(θ₅)⎥
            ⎥
            ⎦
```

Inverse Kinematics Validation

The purpose of validating the inverse kinematics was to ensure that the theoretical calculations for the robot's joint angles accurately correspond to desired end effector positions and trajectories in real-world scenarios. Inverse kinematics is particularly challenging as it often involves solving complex equations to find joint angles that achieve specific end effector positions.

1. Script Development: A Python script was developed to simulate the robot's inverse kinematics. The script incorporates the physical parameters of the robot, including its dimensions and joint limits.
2. Trajectory Planning: The key focus was to plot an arc trajectory, a common and illustrative task in robotics. The arc trajectory was selected to demonstrate the robot's ability to follow curved paths, which is essential in tasks like object manipulation and assembly.
3. 3D Visualization: The Python script includes a 3D visualization tool. This tool plots the calculated trajectory of the robot's end effector in a three-dimensional space, allowing for an intuitive understanding of the robot's movement.
4. Inverse Kinematics Calculation: The script calculates the required joint angles for each position along the arc. This involves applying the inverse kinematics algorithms to determine the angles that would position the end effector along the desired arc.

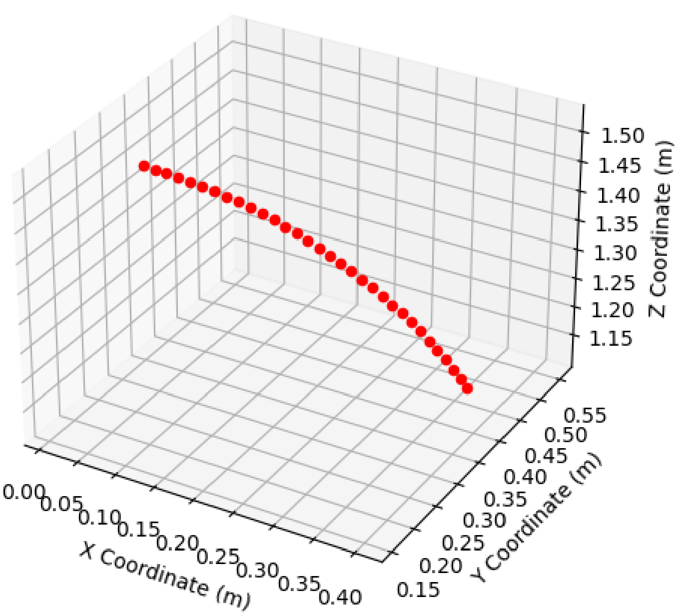


Figure 5. Trajectory of the end-effector

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

The visualization of the arc trajectory in a 3D frame provided clear insights into the effectiveness of the inverse kinematics calculations. By observing how the robot's end effector moved along the plotted arc, we could assess the precision and accuracy of the inverse kinematics model.

Control Methods

1. **Open Loop Control:** Our robotic system employs open loop control as one of its primary operational methods. In this approach, the robot follows a set of pre-programmed commands without requiring feedback or sensor inputs for adjustments. This control method is particularly advantageous for executing straightforward, repetitive tasks where environmental variables remain constant.
2. **Teleoperation:** Complementing the open loop control, teleoperation serves as a vital control method, allowing human operators to remotely control the robot. This approach brings in the flexibility and adaptability of human judgment, crucial for managing complex tasks or navigating unpredictable scenarios.

Integration of Specific GitHub Plugins

To enhance the functionality and operational efficiency of our robotic system, we have integrated two specific plugins from GitHub repositories: IFRA_ConveyorBelt and IFRA_LinkAttacher.

1. IFRA_ConveyorBelt Plugin:

- **Function and Purpose:** The IFRA_ConveyorBelt plugin is instrumental in simulating a conveyor belt's motion within our system's environment. Its integration is crucial for creating a realistic and dynamic setting, essential for testing and operating our robot in conditions that closely mimic its intended real-world application.

- **Operational Impact:** By accurately simulating the conveyor belt dynamics, this plugin plays a key role in assessing the robot's capability to interact with moving objects, particularly in the context of picking and handling tools on a conveyor belt.

2. IFRA_LinkAttacher Plugin:

- **Function and Purpose:** The IFRA_LinkAttacher plugin is employed to create a temporary joint between two links. This functionality becomes crucial when the robot utilizes its vacuum gripper to secure objects.

- **Application in Object Manipulation:** Post gripping an object, the plugin is activated to establish a joint between the gripper and the object, ensuring a stable and secure hold. This feature is particularly important for reliable handling of objects, which is a core aspect of our robot's operational objectives.

Gazebo Visualization

Though we mentioned we will use a camera in the Project Proposal, we did not use it finally. There is no RViz Visualization.

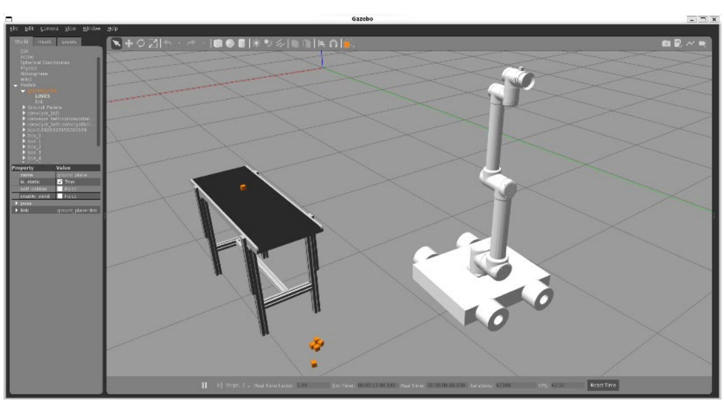


Figure 6. Gazebo Visualization of the Robot along with Conveyor Belt

[1] [Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)](universal-robots.com)

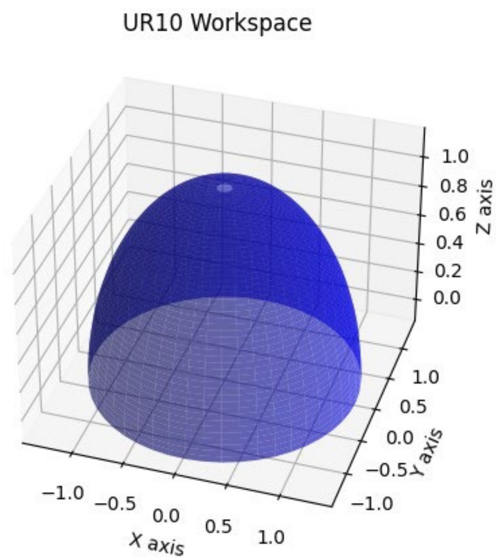
Workspace Study

The workspace of the UR10 robotic manipulator refers to the three-dimensional space within which the end effector (the tool or 'hand' of the robot) can reach and operate. This space is defined by the mechanical structure and the range of motion of the robot's joints. Key characteristics of the UR10's workspace include:

1. **Reachable Volume**: The UR10 has an extensive reach, allowing it to cover a large working area. This is especially beneficial for tasks requiring a wide range of movement.

2. **Dexterity**: Due to its 6 degrees of freedom, the UR10 can maneuver its end effector in various orientations, making it highly dexterous within its workspace.

3. **Shape of Workspace**: The workspace typically resembles a complex, irregular shape due to the varying lengths of the robot's arms and the constraints of its joints. In the following figure, the outer boundary of the workspace was plotted.



Assumptions

1. **Uniform Mass Distribution:**
   - The robot's components, particularly the manipulator arms and chassis, are assumed to have a uniform mass distribution. This simplification aids in the ease of calculating dynamics and kinematics without complex mass distribution models.

2. **Constant Environmental Conditions:**
   - The operational environment of the robot is assumed to have constant and predictable conditions. This includes stable temperature, humidity, and absence of external disturbances like wind or vibrations, which could affect the robot's performance.

3. **Idealized Sensors and Actuators:**
   - Sensors and actuators in the robot are considered to be ideal, with no delays, noise, or errors in measurement and execution. This assumption simplifies control algorithms and response analyses.

[1] [Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)](universal-robots.com)

4. **Reliable Communication for Teleoperation:**

    - In the case of teleoperation, it is assumed that there is always a reliable and uninterrupted communication link between the operator and the robot, with no latency or data loss.

5. **Constant Load Characteristics:**

    - The objects interacted with by the robot, especially on the conveyor belt, are assumed to have consistent and known characteristics such as weight, size, and surface texture. This assumption is crucial for pre-programming gripping and handling procedures.

6. **Absence of External Forces:**

    - External forces, other than gravity, such as magnetic or electrostatic forces, are assumed to be negligible and do not affect the robot's operation.

7. **Singularities in Robotic Manipulation:**

    - While the current design and operational model of the robotic system do not account for singularities, it is important to acknowledge their potential impact. Singularities occur when the robot's manipulator loses degrees of freedom in certain positions, which could lead to control and precision issues. Recognizing this, it is assumed that the robot's operational range avoids these singularity positions, ensuring smooth and uninterrupted movement.

8. **Friction Between Tires and Ground:**

    - As previously mentioned, the robot is designed to maintain a no-slip condition due to sufficient friction between its tires and the ground. This assumption is pivotal for ensuring stable movement, especially when the robot is maneuvering with a load or changing directions. The tires are assumed to have a consistent coefficient of friction with the ground surface, which allows for predictable traction and braking.

9. **Neglecting Ground Irregularities:**

    - The interaction between the robot's tires and the ground surface is assumed to occur on a uniformly flat and smooth surface. This simplification negates the need to account for ground irregularities like bumps, slopes, or varied textures, which would otherwise require more complex suspension and traction control systems.

10. **Assumed Environmental Conditions Affecting Ground Friction:**

    - Environmental conditions that could affect ground friction, such as wet or icy surfaces, are assumed to be non-existent in the robot's operating environment. This ensures the consistency of the frictional assumptions throughout the robot's operation.

Future Work

1. **Enhancing Sensory and Perception Capabilities**:

    - Future enhancements should focus on integrating advanced sensors, potentially allowing the robot to perform tasks with greater awareness and responsiveness to its environment.

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

2. **Addressing Mechanical Design and Singularity Issues**:
   - Resolving issues related to singularities within the robot's mechanical design is critical. This will lead to more precise and reliable functionality, especially in tasks demanding high maneuverability and accuracy.
3. **Improving Gripper Design and Attachment**:
   - Enhancing the design of the gripper to accommodate a wider variety of shapes and sizes, along with optimizing its attachment point for greater stability and control, is essential.
4. **Developing Closed-Loop Control Systems**:
   - While the current system operates on open-loop control, transitioning to a closed-loop control system could offer substantial benefits in terms of feedback-driven adjustments and accuracy.
5. **Advancing Kinematic Models and Control Algorithms**:
   - Further development in kinematic accuracy and control algorithms is crucial. This will not only elevate the robot's operational precision but also expand its applicability across various industrial and exploration tasks.

Challenges and Insights

1. **Inverse Kinematics and Singularities**:
   - **Challenge**: The project encountered singularities in the inverse kinematics calculations, leading to unpredictable arm behavior and occasionally causing instability in the robot.
   - **Insight**: This highlighted the need for a deeper understanding of the kinematic structure to identify and avoid configurations that lead to singularities.
2. **Gripper Difficulties with a Screwdriver**:
   - **Challenge**: The vacuum gripper faced difficulties in securely gripping objects with elongated and slender profiles, such as a screwdriver. This issue was compounded by the fact that the gripper is not attached directly at the end effector but to a joint, affecting its precision and stability.
   - **Insight**: This emphasized the necessity for designing grippers that can adapt to various shapes and sizes of objects, especially those with non-standard geometries.
3. **Unanticipated Movement Due to Manipulator**:
   - **Challenge**: Movements of the manipulator arm inadvertently induced movement in the entire robot platform, impacting its overall stability.
   - **Insight**: This underlined the importance of considering the dynamic interplay between the manipulator and the mobile platform in the design and control of the robotic system.

Conclusion

The development of our 6-degree-of-freedom joint manipulator robot marks a significant stride in the field of robotics, especially for tasks in environments where human accessibility is limited. Employing Denavit-Hartenberg parameters for forward kinematics and the Inverse Jacobian matrix for inverse kinematics has been pivotal in achieving precise control over the robot's motion. However, it's important to note that the robot operates on an open-loop control system, which precludes autonomous navigation capabilities. This project, with its current

[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)

progress and identified areas for improvement, showcases the vast potential of robotics in transforming operations in challenging or hazardous environments. The lessons learned and the knowledge gained lay a strong foundation for future advancements in this dynamic field.

Google Drive Link:
https://drive.google.com/drive/folders/1a2_dajlm_lnQSF9W9A22jNttBCeRedm?usp=drive_link

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[1] Universal Robots - DH Parameters for calculations of kinematics and dynamics (universal-robots.com)