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Adaptive neural control for non-strict-feedback nonlinear systems with input delay☆

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ABSTRACT

In this research, the controller design problem is considered for non-strict-feedback systems with input delay. An appropriate auxiliary system is utilized to deal with the difficulties appeared in input delay. By the utilization of backstepping and adaptive neural control, a state-feedback stabilization controller is developed. The designed controller enables the variables in the closed-loop system to be semi-globally uniformly ultimately bounded in a small interval around the origin. The main significance of this research is that an intelligent control scheme is extended to a class of nonlinear systems with non-strict-feedback form and input delay, simultaneously. Finally, an example is given to show the effectiveness of the control method.

1. Introduction

Since the adaptive control architecture is developed for dealing with the controller design of uncertain nonlinear systems with the help of neural network of Gaussian radial basis functions [18,30,36,45,46] and fuzzy logic systems [15,37,38,40,42,43]. Approximation-based adaptive fuzzy/neural control has attracted considerable attention because of their approximation capability to unavailable nonlinearities. Through the combination of backstepping technique [10,13,44,47], adaptive fuzzy/neural control approaches are applied to control nonlinear systems without satisfying matching conditions. With the approximation ability of RBF neural networks, many adaptive backstepping neural control methods are presented for strict-feedback systems in [4–9,16,17,24,25,35,41]. Alternatively, some fuzzy adaptive backstepping control schemes are designed to different types of nonlinear systems with the utilization of fuzzy approximation in [2,19,21–23,31,33,39]. It should be mentioned that the above literatures are only suitable the control systems with strict-feedback form.

It is well known that non-strict-feedback nonlinear system represents a broader form than the system with strict feedback. In practical applications, there are a large number classes of nonlinear systems with non-strict-feedback form, such as the ball and beam system [28] and hyper-chaotic LC oscillation circuit system [32]. A typical feature of a non-strict-feedback system is that system functions are related to all system states, which makes it quite difficult and challenging to design a control scheme via backstepping. This means it is an interesting topic both in the theory and practice. To achieve an

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effective control scheme, some research scientists have done a lot of effort and received much developments. In order to overcome the restrictions, Chen et al. [3] first proposed a variable separation method based on the structural characteristic and the monotonously increasing property of the system bounding functions to overcome the difficulty from the non-strict-feedback structure. Then, in [34], by combining the approximation ability of radial basis function neural network and adaptive backstepping method, an adaptive neural network control method is proposed for non-strict-feedback stochastic nonlinear systems. More recently, in [20], a variable separation way is introduced to solve the controller design problem of a class of switched non-strict-feedback nonlinear systems. The input delay is not taken into account in the above literature, nevertheless it is very common in the practical application of engineering.

Because of the time taken for transportation of materials and transmission of signals, input delay is unavoidable in practical applications, which may result in instability of control systems [1]. Therefore, it is especially important to study the control system with input-delay. After the decades of the intensive research, there have been several meaningful methods to deal with input delay, such as the predictor-based approach [11,12], the Laplace transform method [14] and so on. Specifically, in [12], the predictor-based control approach is introduced for nonlinear systems with input delay. In [14], based on the Laplace transform to compensate for the input delay, a class of strict-feedback nonlinear systems is investigated, and an adaptive fuzzy backstepping tracking control technique is proposed. More recently, in [27], it introduces an adaptive backstepping neural network tracking control for MIMO nonlinear strict-feedback switching systems with input delay. In [26], based on a method of introducing a suitable auxiliary system, an adaptive neural control method is proposed for a class of nonlinear strict-feedback systems with input delays. Although there have been a large amount of interesting results on the input time-delay nonlinear systems, the proposed control schemes are merely suitable for the control systems with strict-feedback form. Theoretically, the aforementioned results in [14,26] cannot be directly extended to the nonlinear systems in non-strict-feedback form which is in a broader form than the strict-feedback ones. This means that the research on the nonlinear systems with non-strict-feedback form and input delay, simultaneously, is an valuable issue, which motivates us this study.

In this paper, adaptive neural control of non-strict feedback nonlinear systems with input delay is considered. The difficulty of designing a input-delay free controller is handled by introducing the auxiliary system method. In the process of designing the controller, by using the RBF neural network to estimate the uncertain nonlinear function, and combining with the classical adaptive control method, the designed controller is proposed. The proposed controller guarantees that all the signals within the closed-loop systems are semi-globally uniformly ultimately bounded. The main contributions of this article are listed as follows:

(1) The adaptive neural control, a main way to deal with nonlinear control problems, is extended to a class of nonlinear systems with input delay and non-strict feedback form.
(2) An auxiliary system is provided, which is convenient to overcome the design difficulty on the input-delay systems theoretically.
(3) According to the characteristics of the radial basis function of the neural network, the processing for the non-strict-feedback structure is simplified, which reduces the design difficulty of the controller.

The reminder of this research is arranged as follows. The basic knowledge and plant dynamics are given in Section 2. The controller design process is presented in Section 3. The feasibility of the proposed controller is given in Section 4, and the final conclusion is given in Section 5.

2. Basic knowledge and plant dynamics

Consider a class of non-strict-feedback nonlinear systems with input delay as follows

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x), i = 1, 2, \dots, n-1 \\ \dot{x}_n = u(t-\tau) + f_n(x) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, \dots, x_n]^T \in R^n$ and $y \in R$ represent the system state and output; $u(t-\tau)$ is the input signal with delayed τ being a known non-negative constant; $f_i(x)$ represents nonlinear smooth function.

In the controller design process, the following radial basis functions neural network is applied to model the continuous nonlinear function $f(Z)$: $R^n \to R$

$$f_{nn}(Z) = W^T \Phi(Z) \tag{2}$$

where the vector $Z \in \Omega_Z \subset R^q$ is called the input vector, the vector $W = [w_1, \dots, w_l]^T \in R^l$ is called the weight vector, $l > 1$ is node number, and the vector $\Phi(Z) = [s_1(Z), \dots, s_l(Z)]^T \in R^l$ is the radial basis function vector where $s_i(Z)$ is selected as the Gaussian-like function.

$$s_i(Z) = e^{-\frac{(Z-v_i)^T(Z-v_i)}{\eta^2}}, 1 \le i \le l \tag{3}$$

where $v_i = [v_{i1}, \dots, v_{iq}]^T$ are the center of the receptive domain and η denotes the width of the Gaussian function.

As mentioned in [30], if $f(Z)$ is continuous and the elements in $f(Z)$ belong to compact set Ω_Z, then for any given constant $\varepsilon > 0$, there is neural network $W^{*T}\Phi(Z)$ such that

$$f(Z) = W^{*T}\Phi(Z) + \delta(Z), \forall Z \in \Omega_Z \tag{4}$$

where W^* is the ideal weight vector and defined as

$$W^* = \arg\min_{W\in R^l} \sup_{Z\in\Omega_Z} |f(Z) - W^{*T}\Phi(Z)| \tag{5}$$

$\delta(Z)$ denotes the approximation error that satisfies the inequality $|\delta(Z)| < \varepsilon$.

Lemma 1 [29]**.** *It is assumed that the radial basis function of the neural network is represented by $\Phi(Z_n) = [s_1(Z_n), \ldots, s_l(Z_n)]^T$, and the input vector is represented by $Z_n = [z_1, \ldots, z_n]^T$. Then, the following inequality holds.*

$$\|\Phi(Z_n)\|^2 \le \|\Phi(Z_m)\|^2 \tag{6}$$

where $m \le n$.

In order to eliminate the influence of input delay on the controller design, we introduce the following auxiliary system [26]:

$$\begin{cases} \dot{\lambda}_1 = \lambda_2 - p_1\lambda_1 \\ \dot{\lambda}_i = \lambda_{i+1} - p_i\lambda_i, i = 2, \ldots, n-1 \\ \dot{\lambda}_n = -p_n\lambda_n + u(t-\tau) - u(t) \end{cases} \tag{7}$$

where $p_1 > \frac{1}{2}$ and $p_i > 1 (i = 2, \ldots, n)$ are design parameters with $\lambda_i(0) = 0$.

Remark 1. If the control system (1) is input time-delay free, i.e., $\tau = 0$, the variables λ_i in (7) remain zero when $\lambda_i(0) = 0$.

3. Controller design process

In this section, we will design an neural networks based controller via backstepping which is carried out based on the following coordinated transformation:

$$\begin{cases} z_1 = x_1 - \lambda_1 \\ z_i = x_i - \alpha_{i-1} - \lambda_i, i = 2, \ldots, n. \end{cases} \tag{8}$$

Now, it is time to start the controller design process.

Step 1. From (8), the time derivative of z_1 is

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{\lambda}_1 \\ &= x_2 + f_1 - \lambda_2 + p_1\lambda_1 \\ &= z_2 + \alpha_1 + \lambda_2 + f_1 - \lambda_2 + p_1\lambda_1 \\ &= z_2 + \alpha_1 + f_1 + p_1\lambda_1 \end{aligned} \tag{9}$$

Select a Lyapunov function V_1 as

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2r_1}\tilde{\theta}_1^2 \tag{10}$$

where $z_1 = x_1 - \lambda_1$, and $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ is the estimation error with $\hat{\theta}_1$ being the estimation of the unknown parameter θ_1 given later, and $r_1 > 0$ is a design constant. Then, the derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= z_1\dot{z}_1 - \frac{1}{r_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &= z_1(z_2 + \alpha_1 + f_1 + p_1\lambda_1) - \frac{1}{r_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &= z_1(z_2 + \alpha_1 + \bar{f}_1(Z_1) + p_1\lambda_1) \\ &\quad - \frac{1}{2}z_1^2 - \frac{1}{r_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \end{aligned} \tag{11}$$

where

$$\bar{f}_1(Z_1) = f_1 + \frac{1}{2}z_1. \tag{12}$$

It can be seen from (12) that $\bar{f}_1(Z_1)$ contains the unknown smooth function f_1. Then, neural network $W_1^{*T}\Phi_1(Z_1)$ is applied to model $\bar{f}_1(Z_1)$ such that for any constant $\varepsilon_1 > 0$

$$\bar{f}_1(Z_1) = W_1^{*T}\Phi_1(Z_1) + \delta_1(Z_1), |\delta_1(Z_1)| \le \varepsilon_1 \tag{13}$$

where $Z_1 = [x_1, \ldots, x_n, \lambda_1]^T$, $\delta_1(Z_1)$ is estimate error.

By using Young's inequality and Lemma 1, one has

$$\begin{aligned} z_1 \bar{f}_1(Z_1) &= z_1 \Big(W_1^{*T} \Phi_1(Z_1) + \delta_1(Z_1) \Big) \\ &\le |z_1| \Big(\|W_1^*\| \|\Phi_1(Z_1)\| + \varepsilon_1 \Big) \\ &\le |z_1| (\|W_1^*\| \|\Phi_1(X_1)\| + \varepsilon_1) \\ &\le \frac{1}{2a_1^2} z_1^2 \theta_1 \Phi_1^T(X_1) \Phi_1(X_1) + \frac{a_1^2}{2} + \frac{z_1^2}{2} + \frac{\varepsilon_1^2}{2} \end{aligned} \tag{14}$$

where $\theta_1 = \|W_1^*\|^2$, $X_1 = [x_1, \lambda_1]^T$ and $a_1 > 0$ is a constant. Furthermore, substituting (14) into (11) produces

$$\begin{aligned} \dot{V}_1 &\le z_1 (z_2 + \alpha_1 + p_1 \lambda_1 + \frac{1}{2a_1^2} z_1 \theta_1 \Phi_1^T(X_1) \Phi_1(X_1)) - \frac{1}{r_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} \\ &= z_1 (z_2 + \alpha_1 + p_1 \lambda_1 + \frac{1}{2a_1^2} z_1 \hat{\theta}_1 \Phi_1^T(X_1) \Phi_1(X_1)) \\ &\quad + \frac{1}{r_1} \tilde{\theta}_1 \Big(\frac{r_1}{2a_1^2} z_1^2 \Phi_1^T(X_1) \Phi_1(X_1) - \dot{\hat{\theta}}_1 \Big) + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} \end{aligned} \tag{15}$$

Next, the intermediate control signal α_1 and adaptive law $\dot{\hat{\theta}}_1$ are constructed as

$$\alpha_1 = -k_1 z_1 - p_1 \lambda_1 - \frac{1}{2a_1^2} z_1 \hat{\theta}_1 \Phi_1^T(X_1) \Phi_1(X_1) \tag{16}$$

$$\dot{\hat{\theta}}_1 = \frac{r_1}{2a_1^2} z_1^2 \Phi_1^T(X_1) \Phi_1(X_1) - \delta_1 \hat{\theta}_1, \hat{\theta}_1(0) \ge 0 \tag{17}$$

where $k_1 > 0$ and $\delta_1 > 0$ are design parameters.

By substituting (16) and (17) into (15), the following result holds.

$$\dot{V}_1 \le -k_1 z_1^2 + z_1 z_2 + \frac{1}{r_1} \delta_1 \tilde{\theta}_1 \hat{\theta}_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} \tag{18}$$

Step 2. Based on the coordinate transformation $z_2 = x_2 - \alpha_1 - \lambda_2$, the time derivative of z_2 is

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 - \dot{\lambda}_2 \\ &= x_3 + f_2 - \dot{\alpha}_1 - \lambda_3 + p_2 \lambda_2 \end{aligned} \tag{19}$$

where

$$\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1) + \frac{\partial \alpha_1}{\lambda_1} \dot{\lambda}_1 + \frac{\partial \alpha_1}{\hat{\theta}_1} \dot{\hat{\theta}}_1 \tag{20}$$

Take the Lyapunov function V_2 candidate as

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2r_2} \tilde{\theta}_2^2 \tag{21}$$

where $z_2 = x_2 - \alpha_1 - \lambda_2$, $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ is the estimation error and $r_2 > 0$ is a design constant. Thus, following a straightforward calculation gives

$$\begin{aligned} \dot{V}_2 &\le -k_1 z_1^2 + z_1 z_2 + \frac{1}{r_1} \delta_1 \tilde{\theta}_1 \hat{\theta}_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + z_2 (z_3 + \alpha_2 + f_2 - \dot{\alpha}_1 + p_2 \lambda_2) - \frac{1}{r_2} \tilde{\theta}_2 \dot{\hat{\theta}}_2 \\ &= -k_1 z_1^2 + z_1 z_2 + \frac{1}{r_1} \delta_1 \tilde{\theta}_1 \hat{\theta}_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + z_2 (z_3 + \alpha_2 + \bar{f}_2(Z_2) + p_2 \lambda_2) - \frac{1}{r_2} \tilde{\theta}_2 \dot{\hat{\theta}}_2 \\ &\quad - \frac{1}{2} z_2^2 \end{aligned} \tag{22}$$

where $\bar{f}_2(Z_2) = f_2 - \dot{\alpha}_1 + \frac{1}{2} z_2$. Since the function f_2 is uncertain, $\bar{f}_2(Z_2)$ cannot be used to design virtual control signal α_2 directly. Then, neural network $W_2^{*T} \Phi_2(Z_2)$ is used to approximate $\bar{f}_2(Z_2)$ such that, for any constant $\varepsilon_2 > 0$,

$$\bar{f}_2(Z_2) = W_2^{*T} \Phi_2(Z_2) + \delta_2(Z_2), |\delta_2(Z_2)| < \varepsilon_2 \tag{23}$$

with $Z_2 = [x_1, \ldots, x_n, \lambda_1, \lambda_2]^T$. By taking the same method as (14), one has

$$z_2 \bar{f}_2 = z_2 \Big(W_2^{*T} \Phi_2(Z_2) + \delta_2(Z_2) \Big)$$

$$
\begin{aligned}
&\leq |z_2|\Big(\|W_2^*\|\|\Phi_2(Z_2)\| + \varepsilon_2\Big) \\
&\leq |z_2|\Big(\|W_2^*\|\|\Phi_2(X_2)\| + \varepsilon_2\Big) \\
&\leq \frac{1}{2a_2^2} z_2^2 \theta_2 \Phi_2^T(X_2)\Phi_2(X_2) \\
&\quad + \frac{a_2^2}{2} + \frac{z_2^2}{2} + \frac{\varepsilon_2^2}{2}
\end{aligned} \tag{24}
$$

where $X_2 = [x_1, x_2, \lambda_1, \lambda_2]^T, \theta_2 = \|W_2^*\|^2$ and $a_2 > 0$ is a parameter. By combining (22)–(24), we can obtain

$$
\begin{aligned}
\dot{V}_2 \leq & -k_1 z_1^2 + z_1 z_2 + \frac{1}{r_2}\delta_1 \tilde{\theta}_1 \hat{\theta}_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + z_2\Big(z_3 + \alpha_2 + p_2\lambda_2 + \frac{1}{2a_2^2} z_2 \hat{\theta}_2 \Phi_2^T(X_2)\Phi_2(X_2)\Big) \\
& + \frac{1}{r_2}\tilde{\theta}_2\Big(\frac{r_2}{2a_2^2} z_2^2 \Phi_2^T(X_2)\Phi_2(X_2) - \dot{\hat{\theta}}_2\Big) + \frac{a_2^2}{2} + \frac{\varepsilon_2^2}{2}
\end{aligned} \tag{25}
$$

Currently, we are ready to design the virtual control input α_2 and adaptive law $\dot{\hat{\theta}}_2$ in the following form:

$$
\alpha_2 = -k_2 z_2 - z_1 - \frac{z_2}{2a_2^2}\hat{\theta}_2 \Phi_2^T(X_2)\Phi_2(X_2) - p_2\lambda_2 \tag{26}
$$

$$
\dot{\hat{\theta}}_2 = \frac{r_2}{2a_2^2} z_2^2 \Phi_2^T(X_2)\Phi_2(X_2) - \delta_2\hat{\theta}_2 \tag{27}
$$

where k_2, r_2 and δ_2 are positive design constants.

Furthermore, it follows from substituting (26) and (27) into (25) that

$$
\dot{V}_2 \leq -\sum_{j=1}^{2} k_j z_j^2 + z_2 z_3 + \sum_{j=1}^{2}\frac{1}{r_j}\delta_j\tilde{\theta}_j\hat{\theta}_j + \sum_{j=1}^{2}\left(\frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2}\right) \tag{28}
$$

Step $i(3 \leq i \leq n-1)$. Following a similar process to Step 2, consider a Lyapunov function as

$$
V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2r_i}\tilde{\theta}_i^2 \tag{29}
$$

where $z_i = x_i - \alpha_{i-1} - \lambda_i$, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ is the estimation error and $r_i > 0$ is a design constant.

Further, the dynamics of V_i is

$$
\begin{aligned}
\dot{V}_i \leq & -\sum_{j=1}^{i-1} k_j z_j^2 + z_{i-1}z_i + \sum_{j=1}^{i-1}\frac{1}{r_j}\delta_j\tilde{\theta}_j\hat{\theta}_j + \sum_{j=1}^{i-1}\left(\frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2}\right) + z_i(z_{i+1} + \alpha_i + f_i \\
& - \dot{\alpha}_{i-1} + p_i\lambda_i) - \frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\
= & -\sum_{j=1}^{i-1} k_j z_j^2 + z_{i-1}z_i + \sum_{j=1}^{i-1}\frac{1}{r_j}\delta_j\tilde{\theta}_j\hat{\theta}_j + \sum_{j=1}^{i-1}\left(\frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2}\right) + z_i(z_{i+1} + \alpha_i + \bar{f}_i(Z_i) + p_i\lambda_i) \\
& - \frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i - \frac{1}{2}z_i^2.
\end{aligned} \tag{30}
$$

where $\bar{f}_i(Z_i) = f_i - \dot{\alpha}_{i-1} + \frac{1}{2}z_i$. Then, the neural network $W_i^T\Phi_i(Z_i)$ is employed to approximate the unknown function $\bar{f}_i(Z_i)$ such that, for any positive $\varepsilon_i > 0$,

$$
\bar{f}_i(Z_i) = W_i^T\Phi_i(Z_i) + \delta_i(Z_i), |\delta_i(Z_i)| < \varepsilon_i \tag{31}
$$

with $Z_i = [x_1, \ldots, x_n, \hat{\theta}_1, \ldots, \hat{\theta}_{i-1}, \lambda_1, \ldots, \lambda_i]^T$. By taking the same method as (14), the following inequality is true.

$$
z_i\bar{f}_i \leq \frac{1}{2a_i^2} z_i^2 \theta_i \Phi_i^T(X_i)\Phi_i(X_i) + \frac{a_i^2}{2} + \frac{z_i^2}{2} + \frac{\varepsilon_i^2}{2} \tag{32}
$$

where $X_i = [x_1, \ldots, x_i, \hat{\theta}_1, \ldots, \hat{\theta}_{i-1}, \lambda_1, \ldots, \lambda_i]^T$, $\theta_i = \|W_i^*\|^2$ and $a_i > 0$ is a design parameter. By merging (30) and (32), one has

$$
\dot{V}_i \leq -\sum_{j=1}^{i-1} k_j z_j^2 + z_{i-1}z_i + \sum_{j=1}^{i-1}\frac{1}{r_i}\delta_j\tilde{\theta}_j\hat{\theta}_j + \sum_{j=1}^{i-1}\left(\frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2}\right) + z_i\Big[z_{i+1} + \alpha_i + p_i\lambda_i
$$

$$+\frac{1}{2a_i^2}z_i\hat{\theta}_i\Phi_i^T(X_i)\Phi_i(X_i)\Big]+\frac{a_i^2}{2}+\frac{\varepsilon_i^2}{2}+\frac{1}{r_i}\tilde{\theta}_i\Big[\frac{r_i}{2a_i^2}z_i^2\Phi_i^T(X_i)\Phi_i(X_i)-\dot{\hat{\theta}}_i\Big] \tag{33}$$

By observing the above formula (33), the virtual control input signal α_i and adaptive law $\dot{\hat{\theta}}_i$ can be selected are as follows.

$$\alpha_i=-k_iz_i-z_{i-1}-\frac{1}{2a_i^2}z_i\hat{\theta}_i\Phi_i^T(X_i)\Phi_i(X_i)-p_i\lambda_i \tag{34}$$

$$\dot{\hat{\theta}}_i=\frac{r_i}{2a_i^2}z_i^2\Phi_i^T(X_i)\Phi_i(X_i)-\delta_i\hat{\theta}_i \tag{35}$$

where k_i and δ_i are positive design constants. Furthermore, it follows from substituting (34) and (35) into (33) that

$$\dot{V}_i\le-\sum_{j=1}^{i}k_jz_j^2+z_{i+1}z_i+\sum_{j=1}^{i}\frac{1}{r_j}\delta_j\tilde{\theta}_j\hat{\theta}_j+\sum_{j=1}^{i}\left(\frac{a_j^2}{2}+\frac{\varepsilon_j^2}{2}\right) \tag{36}$$

Step *n*. In this step, the actual control law will be constructed. The function shown below is selected as the Lyapunov function.

$$V_n=V_{n-1}+\frac{1}{2}z_n^2+\frac{1}{2r_n}\tilde{\theta}_n^2 \tag{37}$$

where $r_n>0$ is a design constant and $\tilde{\theta}_n=\theta_n-\hat{\theta}_n$ is the estimation error.

Then, with the consideration of $\dot{V}_i$ in (36) with $i=n-1$ and the coordinate transformation $z_n=x_n-\alpha_{n-1}-\lambda_n$, we can obtain

$$\begin{aligned}\dot{V}_n\le&-\sum_{j=1}^{n-1}k_jz_j^2+\sum_{j=1}^{n-1}\frac{1}{r_j}\delta_j\tilde{\theta}_j\hat{\theta}_j+\sum_{j=1}^{n-1}\left(\frac{a_j^2}{2}+\frac{\varepsilon_j^2}{2}\right)+z_n(u(t)+\bar{f}_n(Z_n)+z_{n-1}+p_n\lambda_n)\\&-\frac{1}{2}z_n^2-\frac{1}{r_n}\tilde{\theta}_n\dot{\hat{\theta}}_n\end{aligned} \tag{38}$$

where $\bar{f}_n(Z_n)=f_n-\dot{\alpha}_{n-1}+\frac{1}{2}z_n$.

Again, a neural network $W_n^{*T}\Phi_n(Z_n)$ is employed to estimate the unknown function $\bar{f}_n(Z_n)$. Then, similar to (39), we have

$$z_n\bar{f}_n\le\frac{1}{2a_n^2}z_n^2\theta_n\Phi_n^T(X_n)\Phi_n(X_n)+\frac{a_n^2}{2}+\frac{z_n^2}{2}+\frac{\varepsilon_n^2}{2} \tag{39}$$

where $X_n=[x_1,\dots,x_n,\hat{\theta}_1,\dots,\hat{\theta}_{n-1},\lambda_1,\dots,\lambda_n]^T$, $\theta_n=\|W_n^*\|^2$ and $a_n>0$ is a design parameter.

Then, following the same deviations as those used at Step *i*, one can get

$$\begin{aligned}\dot{V}_n\le&\ z_n\left(\frac{1}{2a_n^2}z_n\hat{\theta}_n\Phi_n^T(X_n)\Phi_n(X_n)+u(t)+p_n\lambda_n\right)+\frac{1}{r_n}\tilde{\theta}_n\left[\frac{r_n}{2a_n^2}z_n^2\Phi_n^T(X_n)\Phi_n(X_n)-\dot{\hat{\theta}}_n\right]\\&-\sum_{j=1}^{n-1}k_jz_j^2+z_{n-1}z_n+\sum_{j=1}^{n-1}\frac{1}{r_i}\delta_j\tilde{\theta}_j\hat{\theta}_j+\sum_{j=1}^{n}\left(\frac{a_j^2}{2}+\frac{\varepsilon_j^2}{2}\right)\end{aligned} \tag{40}$$

with $a_n>0$ being a design parameter. Then, take the controller u and adaptive law $\dot{\hat{\theta}}_n$ as follows

$$u(t)=-k_nz_n-p_n\lambda_n-z_{n-1}-\frac{1}{2a_n^2}z_n\hat{\theta}_n\Phi_n^T(X_n)\Phi_n(X_n) \tag{41}$$

$$\dot{\hat{\theta}}_n=\frac{r_n}{2a_n^2}z_n^2\Phi_n^T(X_n)\Phi_n(X_n)-\delta_n\hat{\theta}_n \tag{42}$$

where $k_n>0$ and $\delta_n>0$ are design constants and $X_n=Z_n$. Next, substituting (41) and (42) into (40) gives

$$\dot{V}_n\le-\sum_{j=1}^{n}k_jz_j^2-\sum_{j=1}^{n}\frac{1}{r_j}\delta_j\tilde{\theta}_j\hat{\theta}_j+\sum_{j=1}^{n}\left(\frac{a_j^2}{2}+\frac{\varepsilon_j^2}{2}\right) \tag{43}$$

Through the above discussion, we can state our main result as follows:

Theorem 1. *Consider the closed-loop system consisting of the system (1), control law (41) and adaptive laws (35) with $i=1,\dots,n$. Under bounded initial conditions, all the signals of the closed-loop system remain bounded.*

Proof. By studying the definition of $\tilde{\theta}_j$, one has

$$\tilde{\theta}_j\hat{\theta}_j\le\frac{\theta_j^2}{2}-\frac{\tilde{\theta}_j^2}{2} \tag{44}$$

By substituting (44) into (43), we have

$$\begin{aligned}\dot{V}_n &\le -\sum_{j=1}^{n} k_j z_j^2 + \sum_{j=1}^{n} \frac{1}{r_j}\delta_j\theta_j^2 - \sum_{j=1}^{n} \frac{1}{r_j}\delta_j\tilde{\theta}_j^2 + \sum_{j=1}^{n}\left(\frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2}\right) \\ &= -\sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n} \frac{1}{r_j}\delta_j\tilde{\theta}_j^2 + \sum_{j=1}^{n}\left(\frac{\delta_j\theta_j^2}{r_j} + \frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2}\right) \\ &\le -c\sum_{j=1}^{n}\left(\frac{1}{2}z_j^2 + \frac{1}{2r_i}\tilde{\theta}_j^2\right) + d \\ &\le -cV_n + d \end{aligned} \tag{45}$$

with $c = 2m,\ m = \min\{k_1, \dots, k_n, \delta_1, \dots, \delta_n\}$ and $d = \sum_{j=1}^{n}(\frac{\delta_j\theta_j^2}{r_j} + \frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2})$.

By integrating (45) over [0, t], the following inequality can be obtained.

$$V_n(t) \le V_n(0)e^{-ct} + \frac{d}{c}(1 - e^{-ct}) \tag{46}$$

From the definition V_n in (37) and the inequality (46), it can be concluded that the signals $z_i = x_i - \alpha_{i-1} - \lambda_i$ and $\tilde{\theta}_i$ are bounded. To guarantee the boundedness of the system variables x_i, then we discuss the boundedness of λ_i in the auxiliary system (7). First select the following function as the Lyapunov function.

$$V_{\lambda 0} = \frac{1}{2}\sum_{j=1}^{n}\lambda_j^2 + \frac{1}{\mu}\int_{t-\tau}^{t}\int_{\theta}^{t}\|\dot{u}(s)\|^2 ds d\theta \tag{47}$$

After a simple calculation, one gets

$$\begin{aligned}\dot{V}_{\lambda 0} \le & \sum_{j=1}^{n-1}\lambda_j(\lambda_{j+1} - p_j\lambda_j) + \lambda_n(-p_n\lambda_n + u(t-\tau) \\ & - u(t)) + \frac{\tau}{\mu}\|\dot{u}(t)\|^2 - \frac{1}{\mu}\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds \end{aligned} \tag{48}$$

$$\begin{aligned} \le & -\sum_{j=1}^{n}\bar{p}_j\lambda_j^2 - \frac{1}{\mu}\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds \\ & + \frac{1}{2\omega}\|u(t-\tau) - u(t)\|^2 + \frac{\tau}{\mu}\|\dot{u}(t)\|^2 \end{aligned} \tag{49}$$

where $\bar{p}_1 = p_1 - \frac{1}{2}, \bar{p}_i = p_i - 1 (i = 1, 2, \dots, n-1)$ and $\bar{p}_n = p_n - \frac{1}{2} - \frac{\omega}{2}$; ω is a constant.

By using the Cauchy-Schwartz inequality, we can obtain

$$\frac{1}{2\omega}\|u(t-\tau) - u(t)\|^2 \le \frac{\tau}{2\omega}\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds \tag{50}$$

Then, substituting (50) into (49) results in

$$\dot{V}_{\lambda 0} \le -\sum_{j=1}^{n}\bar{p}_j\lambda_j^2 - \left(\frac{1}{\mu} - \frac{\tau}{2\omega}\right)\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds + \frac{\tau}{\mu}\|\dot{u}(t)\|^2 \tag{51}$$

Then we will deal with the term $\frac{\tau}{\mu}\|\dot{u}(t)\|^2$. Form (8), (16), (17), (26), (27), (34) and (41), the following results about $u(t)$ and $\dot{u}(t)$ can be easily obtained.

$$u(t) = \zeta_1(z_{n-1}, z_n, \hat{\theta}_n) + \zeta_4\lambda_n \tag{52}$$

$$\begin{aligned}\dot{u}(t) =& \zeta_2(z_{n-3}, z_{n-2}, z_{n-1}, z_n, \hat{\theta}_{n-2}, \hat{\theta}_{n-1}, \hat{\theta}_n) \\ & + \sum_{j=1}^{n}\zeta_{5j}(z, \hat{\theta})\lambda_j + \zeta_3(z, \hat{\theta})u(t-\tau)\end{aligned} \tag{53}$$

where $\zeta_1(.)$, $\zeta_2(.)$, $\zeta_3(.)$, $\zeta_4(.)$ and $\zeta_{5j}(.)(j = 1, \dots, n)$ are C^1 functions. Based on the boundedness of $z_i(i = n-3, n-2, n-1, n)$ and $\hat{\theta}_i(i = n-2, n-1, n)$, we can easily get the following results.

$$||\zeta_i|| \le \rho_i, i = 1, 2, 3, 4 \tag{54}$$

$$||\zeta_{ik}|| \leq \rho_{jk}, \tag{55}$$

where ρ_i and $\rho_{jk}(j = 5; k = 1, \ldots, n)$ are the positive constants.

$$\begin{aligned} ||u(t)||^2 &= (||\zeta_1(z_{n-1}, z_n, \hat{\theta}_n) + \zeta_4\lambda_n||)^2 \\ &\leq (||\zeta_1(z_{n-1}, z_n, \hat{\theta}_n)|| + ||\zeta_4||\lambda_n)^2 \\ &\leq (\rho_1 + \rho_4\lambda_n)^2 \\ &\leq 2\rho_1^2 + 2\rho_4^2\lambda_n^2 \\ &\leq \rho_1^{'} + \rho_4^{'}\lambda_n^2 \end{aligned} \tag{56}$$

where $\rho_1^{'} = 2\rho_1^2$ and $\rho_4^{'} = 2\rho_4^2$. Thus, a straightforward calculation shows that

$$||u(t-\tau)||^2 \leq \rho_1^{'} + \rho_4^{'}\lambda_n^2(t-\tau) \tag{57}$$

Then, we can get the following result.

$$\begin{aligned} \frac{\tau}{\mu}||\dot{u}(t)||^2 &\leq \frac{\tau}{\mu}||\zeta_2 + \zeta_5\lambda_n + \zeta_3 u(t-\tau)||^2 \\ &\leq \frac{\tau}{\mu}(\rho_2 + \rho_5\lambda_n + \rho_3 u(t-\tau))^2 \\ &\leq \frac{\tau}{\mu}3(\rho_2^2 + \rho_5^2\lambda_n^2 + \rho_3^2 u^2(t-\tau)) \\ &\leq \frac{\tau}{\mu}(3\rho_2^2 + 3\rho_3^2\rho_1^{'} + 3\rho_5^2\lambda_n^2 + 3\rho_3^2\rho_4^{'}\lambda_n^2(t-\tau)) \\ &= \frac{\tau}{\mu}(\rho_2^{'} + \rho_5^{'}\lambda_n^2 + \rho_3^{'}\lambda_n^2(t-\tau)) \end{aligned} \tag{58}$$

where $\rho_2^{'} = 3\rho_2^2 + 3\rho_3^2\rho_1^{'}$, $\rho_5^{'} = 3\rho_5^2$ and $\rho_3^{'} = 3\rho_3^2\rho_4^{'}$. And $\rho_i(i = 1, \ldots, 5)$ is a positive constant.

With the utilization of (58), we can rewrite (51) as

$$\dot{V}_{\lambda 0} \leq -\sum_{j=1}^{n}\tilde{p}_j\lambda_j^2 - (\frac{1}{\mu} - \frac{\tau}{2\omega})\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds + \frac{\tau}{\mu}\rho_2^{'} + \frac{\tau\rho_3^{'}}{\mu}\lambda_n^2(t-\tau) \tag{59}$$

where $\tilde{p}_j = \bar{p}_j, i = 1, \ldots, n-1$ and $\tilde{p}_n = \bar{p}_n - \frac{\tau}{\mu}\rho_5^{'}$.

Now, we select the following Lyapunov function for the whole auxiliary system (7):

$$V_\lambda = V_{\lambda 0} + \frac{\tau\rho_3^{'}}{\mu}\int_{t-\tau}^{t}\lambda_n^2(s)ds + \frac{1}{v}\int_{t}^{t-\tau}\int_{\theta}^{t}\lambda_n^2(s)dsd\theta \tag{60}$$

Deriving a derivative of V_λ can be as follows.

$$\dot{V}_\lambda \leq -\sum_{j=1}^{n}\hat{p}_j\lambda_j^2 - (\frac{1}{\mu} - \frac{\tau}{2\omega})\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds - \frac{1}{v}\int_{t-\tau}^{t}\lambda_n^2(s)ds + \frac{\tau}{\mu}\rho_2^{'} \tag{61}$$

where $\hat{p}_j = \tilde{p}_j(j = 1, 2, \ldots, n-1)$, and $\hat{p}_n = \tilde{p}_n + \frac{\tau}{\mu}\rho_3^{'} - \frac{\tau}{v}$.

The following inequalities can be satisfied by appropriately selecting the parameters p_i, μ, v and ω.

$$\begin{aligned} \hat{p}_j &> 0 \\ \frac{1}{\mu} - \frac{\tau}{2\omega} &> 0 \end{aligned} \tag{62}$$

□

Remark 2. The results in (62) are true by choosing the design parameters suitably, which are helpful to get the boundedness of λ_i in (66).

In addition, the following results are clearly established.

$$\begin{aligned} \int_{t-\tau}^{t}\int_{\theta}^{t}\|\dot{u}(s)\|^2 dsd\theta &\leq \tau \sup_{\theta\in[t-\tau,t]}\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds \\ &\tau\int_{t-\tau}^{t}\|\dot{u}(s)\|^2 ds \end{aligned} \tag{63}$$

$$\int_{t-\tau}^{t}\int_{\theta}^{t}\lambda_n^2(s)dsd\theta \leq \tau \sup_{\theta\in[t-\tau,t]}\int_{t-\tau}^{t}\lambda_n^2(s)ds$$

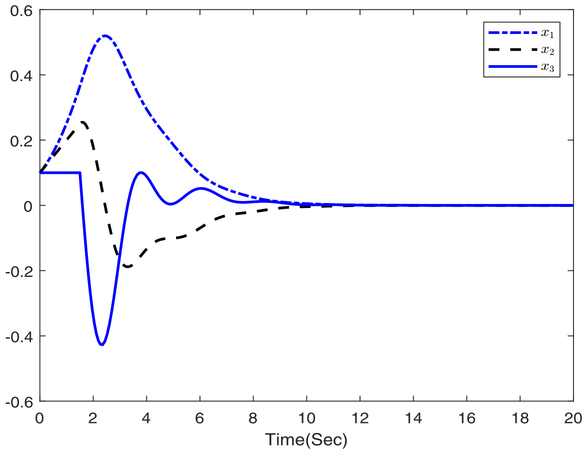

Fig. 1. The state variables x_1, x_2 and x_3.

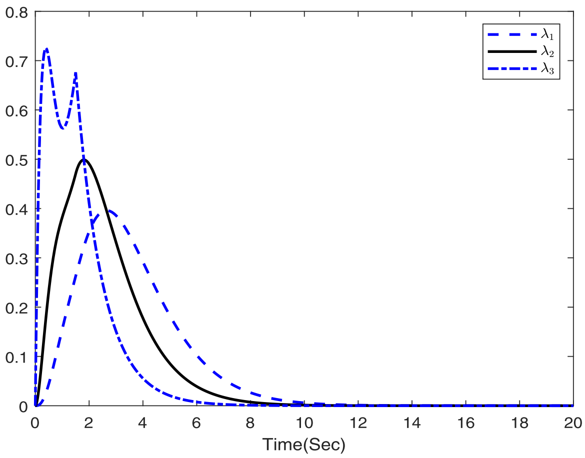

Fig. 2. The auxiliary system's states λ_1, λ_2 and λ_3.

$$= \tau \int_{t-\tau}^{t} \lambda_n^2(s)ds \tag{64}$$

By substituting (63) and (64), we can rewrite V_λ as

$$\begin{aligned}
\dot{V}_\lambda \leq & -\sum_{j=1}^{n} \hat{p}_j \lambda_j^2 - \left(\frac{1}{\mu} - \frac{\tau}{2\omega}\right) \int_{t-\tau}^{t} \|u'(s)\|^2 ds + \frac{\tau}{\mu}\rho_2' \\
& - \left(\frac{1}{v} - \frac{\tau\rho_3'}{\mu}\right) \int_{t-\tau}^{t} \lambda_n^2(s)ds - \frac{\tau\rho_3'}{\mu} \int_{t-\tau}^{t} \lambda_n^2(s)ds \\
\leq & -\sum_{j=1}^{n} \hat{p}_j \lambda_j^2 - \left(\frac{1}{\tau} - \frac{\mu}{2\omega}\right) \frac{1}{\mu} \int_{t-\tau}^{t} \int_{\theta}^{t} \|u'(s)\|^2 ds d\theta \\
& - \left(\frac{1}{\tau} - \frac{v\rho_3'}{\mu}\right) \frac{1}{v} \int_{t-\tau}^{t} \int_{\theta}^{t} \lambda_n^2(s) ds d\theta - \frac{\rho_3'}{\tau} \int_{t-\tau}^{t} \lambda_n^2(s)ds + \frac{\tau}{\mu}\rho_2' \\
\leq & -\varsigma V_\lambda + \rho
\end{aligned} \tag{65}$$

where $\varsigma = \min\{2\hat{p}_j, \frac{1}{\tau} - \frac{\mu}{2\omega}, 1, \frac{1}{\tau} - \frac{v\rho_3'}{\mu}, j = 1, \ldots, n.\}$ and $\rho = \frac{\tau}{\mu}\rho_2^{;}$.

By integrating (65) from 0 to t, we get

$$V_\lambda(t) \leq V_\lambda(0)e^{-\varsigma t} + \frac{\rho}{\varsigma}(1 - e^{-\varsigma t}) \tag{66}$$

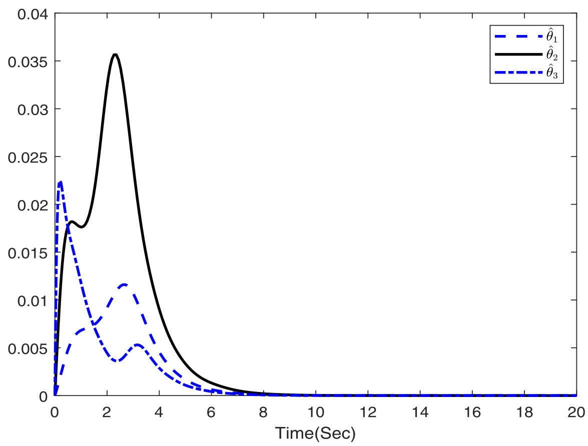

Fig. 3. The adaptive laws' trajectories $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\theta}_3$.

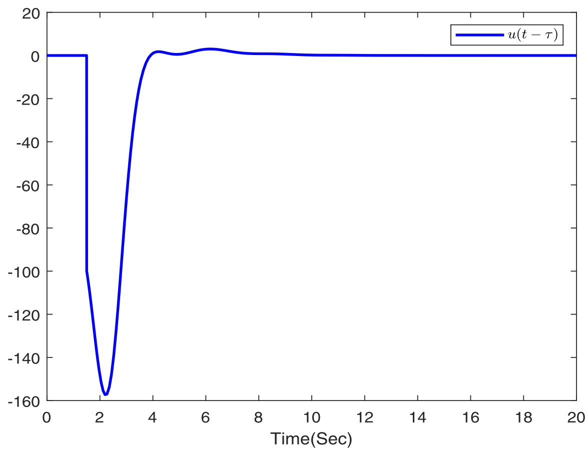

Fig. 4. The control signal $u(t-\tau)$ with $\tau = 1.5$.

which shows that λ_i is uniformly ultimately bounded. Since $z_1 = x_1 - \lambda_1, z_i = x_i - \alpha_{i-1} - \lambda_i$, it can be proven that x_i is bounded. Thus, it is concluded that all signals in the closed-loop system remain bounded.

4. Simulation

To verify the feasibility of the presented control scheme, the following nonlinear systems with input delay is considered.

$$\begin{cases} \dot{x}_1 = x_2 + 0.5x_2^2 x_3(1 + x_1^2) \\ \dot{x}_2 = x_3 + 0.2x_1 x_2 \sin(x_3^2) \\ \dot{x}_3 = u(t-\tau) + x_2^2 x_3^2 \sin(x_1^2) \\ y = x_1 \end{cases} \tag{67}$$

in which, the states are denoted by x_1, x_2 and x_3, the system output is denoted by y, the input is denoted by $u(t-\tau)$ with the time delay $\tau = 1.5s$. The task is to propose an adaptive neural network control method such that the signals in the closed-loop system are bounded.

According to Theorem 1, the virtual control input α_i and actual controller are designed as

$$\alpha_i = -k_i z_i - z_{i-1} - \frac{1}{2a_i^2} z_i \hat{\theta}_i \Phi_i^T(X_i)\Phi_i(X_i) - p_i\lambda_i, i = 1, 2 \tag{68}$$

$$u = -k_3 z_3 - p_3\lambda_3 - z_2 - \frac{1}{2a_3^2} z_3 \hat{\theta}_3 \Phi_3^T(X_3)\Phi_3(X_3) \tag{69}$$

And the adaptive law as shown below can be selected.

$$\dot{\hat{\theta}}_i = \frac{r_i}{2a_i^2} z_i^2 \Phi_i^T(X_i)\Phi_i(X_i) - \delta_i\hat{\theta}_i i = 1, 2, 3 \tag{70}$$

where $z_1 = x_1 - \lambda_1, z_2 = x_2 - \alpha_1 - \lambda_2, z_3 = x_3 - \alpha_2 - \lambda_3, Z_1 = [x_1, \lambda_1]^T$, $Z_2 = [x_1, x_2, \lambda_1, \lambda_2]^T$ and $Z_3 = [x_1, x_2, x_3, \lambda_1, \lambda_2, \lambda_3]^T$.

In the simulation, the design parameters are taken as the following: $k_1 = k_2 = 2, k_3 = 6, a_i = p_i = r_i = \delta_i = 1, i = 1, 2, 3,$ and $\tau = 1.5$. The simulation is run with the initial conditions $[x_1(0), x_2(0), x_3(0), \lambda_1(0), \lambda_2(0), \lambda_3(0)]^T = [0.1, 0.1, 0.1, 0, 0, 0]^T$ and $[\hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0)] = [0, 0, 0]$.

The simulation results are shown in Figs. 1–4. Fig. 1 shows the state trajectories x_1, x_2 and x_3. Fig. 2 shows the state trajectories of auxiliary system λ_1, λ_2 and λ_3. Fig. 3 displays the adaptive laws $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\theta}_3$ are bounded. Fig. 4 shows the control signal $u(t - \tau)$ with $\tau = 1.5s$. From the simulation results, it can be seen that the proposed controller guarantees the boundedness of all the signals in the closed-loop system.

From Figs. 1–4, we can see that, by using the designed controller, all the closed-loop system signals are bounded by adjusting the parameters appropriately. This means that the simulation results verify the validity of the proposed controller.

5. Conclusion

In this paper, a state control scheme for non-strict-feedback systems with input delay is proposed by using backstepping, where the input delay problem is solved by an appropriate auxiliary system. In the process of controller design, neural networks are utilized to approximate the unknown nonlinear functions and its structural characteristic is employed to simplify the design difficulty of non-strict-feedback form. The adaptive neural network based controller proposed in this paper can make the states in the closed loop system bounded. The simulation results show the feasibility of the presented control scheme.

It should be pointed out that our proposed adaptive neural control scheme does not consider the disturbance of external environment, which may limit performance of the system. Our future research may investigate this potential problem associated with the non-strict-feedback system with input delay.

Declaration of Competing Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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